GameOfGoals: Extended example

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1 Example

Consider a business that is seeking to diversify its current lines of business. Its goal is to develop a high-revenue, high-profit business unit that complements its existing units and provides an additional avenue for growth (in Fig. 1 in the paper, this is goal G_1). The Board is currently considering two options: (1) Establishing a new telecom service provider (in Fig. 1 in the paper, this is subgoal G_{11}) or (2) Establishing a new business process outsourcing (BPO) unit (in Fig. 1 in the paper, this is subgoal G_{12}).

In the remainder of this example, we will use following vocabulary (we use propositional logic, so the vocabulary in presented by listing the propositional letters and their denotation). In the following, RPA stands for Robotic Process Automation.

Proposition	Used to mean
t-ml	Telecom service market leadership
s-o	Spectrum license ownership
s-t-rs	Steady telecom revenue stream
t-pu	Telecom price undercutting
t-mp	Telecom market pricing
c-t-pu	Competitor telecom price undercutting
c-t-mp	Competitor telecom market pricing
bpo-o	BPO business ownership
bpo-ml	BPO business market leadership
s-bpo-rs	Steady BPO revenue stream
bpo-pu	BPO price undercutting
bpo-mp	BPO market pricing
c-rpa	Competitor establishing RPA business unit
c-bpo-pu	Competitor BPO price undercutting

Using the vocabulary above, we formalize the two subgoals as follows:

Goal	Formal assertion
G_{11}	s-o, t-ml, s-t-rs
G_{12}	bpo-o, bpo-ml, s-bpo-rs

We use the standard convention that a set of propositional literals refers to the conjunction of those literals. Thus, G_{11} refers to $s - o \land t - ml \land s - t - rs$.

We now formulate two separate Augmented Game Trees, one for each sub-goal. The two trees have non-ntersecting state vocabularies (i.e., they have no propositional letters in common). We describe below the states in the Augmented Game Tree corresponding to subgoal G_{11} (which appears in the figure below).

State	Formal description
S	$\neg s\text{-}o$, $\neg t\text{-}ml$, $\neg s\text{-}t\text{-}rs$, $\neg t\text{-}pu$, $\neg t\text{-}mp$, $\neg c\text{-}t\text{-}pu$, $\neg c\text{-}t\text{-}mp$
S_1	$s\text{-}o$, $t\text{-}ml$, $s\text{-}t\text{-}rs$, $t\text{-}pu$, $\neg t\text{-}mp$, $\neg c\text{-}t\text{-}pu$, $\neg c\text{-}t\text{-}mp$
S_2	$s\text{-}o$, $\neg t\text{-}ml$, $s\text{-}t\text{-}rs$, $\neg t\text{-}pu$, $t\text{-}mp$, $\neg c\text{-}t\text{-}pu$, $\neg c\text{-}t\text{-}mp$
S_{11}	$s\text{-}o$,$\neg t\text{-}ml$, $s\text{-}t\text{-}rs$, $t\text{-}pu$, $\neg t\text{-}mp$, $c\text{-}t\text{-}pu$, $\neg c\text{-}t\text{-}mp$
S_{12}	$s\text{-}o$, $t\text{-}ml$, $s\text{-}t\text{-}rs$, $t\text{-}pu$, $\neg t\text{-}mp$, $\neg c\text{-}t\text{-}pu$, $c\text{-}t\text{-}mp$

We simplify matters by ignoring condition-action rules. In other words, we assume all actions are always feasible.

We will now re-tell the story of the table above (a description of the states) and the figure below (the Augmented Game Tree involving these states), in informal terms. *In the initial state S, our business does not own a spectrum license (and hence has no telecom business), has no revenue stream from the telecom business, nor any market leadership position in the telecom sector. Neither our business nor its competitors have any pricing strategy (either market pricing or price undercutting). By acquiring a spectrum license (and thence a telecom business) and engaging in active price-cutting, our business obtains a steady telecom revenue stream and a market leadership position in the telecom sector in state S_1. An alternative available to our business is to acquire a telecom unit but price telecom services at market prices, leading to a steady revenue stream from telecom, but not a market leadership position (state S_2). In state S_1, the adversarial player (our business' competitors) have the option of also offering telecom services with price undercutting, leading again to loss of the market leadership position for our busineess, but still offering a steady revenue stream (state S_{11}). Another option for the adversarial player to maintain revenue and profit levels by pricing telecom services at market prices (state S_{12}).*

We will refer to the set the set of propositional letters referred to in a sentence in propositional logic as its *signature*. Thus, the signature of G_{11} is *s-o, t-ml, s-t-rs*. We will use the *projection* of a set of propositional literals X onto a set of propositional letters Y to denote the subset of X that only refers to letters contained in Y.

We will use the *Hamming distance* between a goal G and the projection of a state on the signature of G as the value of the *game tree evaluation function* when applied to that state. Thus, the evaluation function value of state S above, with respect to goals G_{11} is 3.

In terms of game tree search, we will take our business (whose strategic choices, in terms of which OR-refinement of a goal to pursue, we seek to inform) as a *minimizing player*. In other words, this player prefers states with lower values for the evaluation function. This makes intuitive sense since a lower Hamming distance implies a greater proximity to a goal-satisfying state.

The figure below shows the Augmented Game Tree for subgoal G_{11}. We cutoff search after looking ahead 2 moves (to simplify matters and to avoid a tedious and repetitive example, we only do 1-move lookahead on the branch involving state S_2). After MINIMAX search over this game tree, we obtain the following labels (upward propagated evaluation function values). In the case of state S_2, we assume that the label is 1.

State	Label after MINIMAX search
S_{11}	1
S_{12}	0
S_1	1
S_2	1
S	1

In the Augmented Game Tree in the figure below, the first move is made by the minimizing player (our business) while the next move is made by the maximizing player (the adversarial player - the competition).

The labels for S_{11} and S_{12} (these are the pseudo-leaf nodes at the cutoff depth) are obtained by applying the evaluation function. Since this is the move of the maximizing player, the label of S_1 is 1. We assume the label of S_2 to be 1. The move at this level is by the minimizing player and the label of S is 1.

We now formalize the states in the Augmented Game Tree for subgoal G_{12} shown below.

State	Formal description
S	$\neg bpo\text{-}o,\ \neg bpo\text{-}ml,\ \neg s\text{-}bpo\text{-}rs,\ \neg bpo\text{-}pu,\ \neg bpo\text{-}mp,\ \neg c\text{-}bpo\text{-}pu,\ \neg c\text{-}rpa$
S_3	$bpo\text{-}o,\ \neg bpo\text{-}ml,\ s\text{-}bpo\text{-}rs,\ \neg bpo\text{-}pu,\ bpo\text{-}mp,\ \neg c\text{-}bpo\text{-}pu,\ \neg c\text{-}rpa$
S_4	$bpo\text{-}o,\ bpo\text{-}ml,\ s\text{-}bpo\text{-}rs,\ bpo\text{-}pu,\ \neg bpo\text{-}mp,\ \neg c\text{-}bpo\text{-}pu,\ \neg c\text{-}rpa$
S_{41}	$bpo\text{-}o,\ \neg bpo\text{-}ml,\ \neg s\text{-}bpo\text{-}rs,\ bpo\text{-}pu,\ \neg bpo\text{-}mp,\ \neg c\text{-}bpo\text{-}pu,\ c\text{-}rpa$
S_{42}	$bpo\text{-}o,\ \neg bpo\text{-}ml,\ s\text{-}bpo\text{-}rs,\ bpo\text{-}pu,\ \neg bpo\text{-}mp,\ c\text{-}bpo\text{-}pu,\ \neg c\text{-}rpa$

The figure below shows the Augmented Game Tree for subgoal G_{12}. We cutoff search after looking ahead 2 moves (as before, to simplify matters and to avoid a tedious and repetitive example, we only do 1-move lookahead on the branch involving state S_3). After MINIMAX search over this game tree, we obtain the following labels (upward propagated evaluation function values). In the case of state S_3, we assume that the label is 3.

State	Label after MINIMAX search
S_{41}	2
S_{42}	1
S_4	2
S_3	3
S	2

In the Augmented Game Tree in the figure below, the first move is made by the minimizing player (our business) while the next move is made by the maximizing player (the adversarial player - the competition).
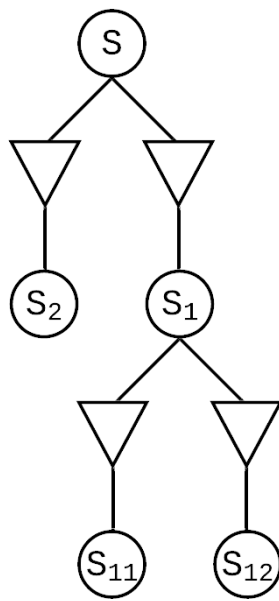
Fig. 1. Consequences of investing in telecom

The labels for S_{41} and S_{42} (these are the pseudo-leaf nodes at the cutoff depth) are obtained by applying the evaluation function. Since this is the move of the maximizing player, the label of S_4 is 1. We assume the label of S_3 to be 3. The move at this level is by the minimizing player and the label of S is 2.
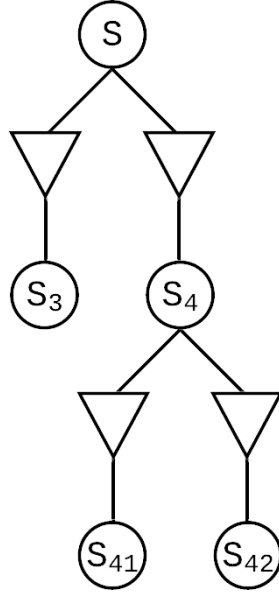

Fig. 2. Consequences of investing in BPO

Note that the Augmented Game Trees might not always be as cleanly separated as in this example. They might share propositions in common. This will only require us to extend the state description vocabularies (i.e., the set of propositional letters).

Ultimately, we obtain a label of 1 for state S by pursuing subgoal G_{11} and a label of 2 by pursuing subgoal G_{12}. Our business being a minmizing player will clearly seek to pursue subgoal G_{11}.