For office use only	Team Control Number	For office use only
T1 ________	**33650**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**A**	F4 ________

2015 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Abstract

Ebola, a dreadful threat to human beings, draws worldwide attention after its latest outbreak in west Africa. Aiming at eradicating Ebola, we decompose our mission into three tasks: predicting the spread of the disease, drawing up the production plan of the medicine and establishing the optimal international and domestic delivery system.

The Model of Spread of Ebola borrows the theory of SIR model, but we refine it with an infectious rate which adjusts itself with respect to the ratio of infected population. By comparing the Natural Model with Predicting Model, we conclude that the interference of medicine is urgent which brooks no delay. We can also obtain the demand curve of the medicine.

The Model of Production uses linear and dynamic optimization to build an optimal manufacturing model in order to satisfy the demand of the medicine at the lowest possible cost.

The Model of Delivery is enlightened by the ant colony algorithm to obtain the most cost-effective and delivery-effective route.But we make progress by weighing the edges with comprehensive factors: the city's severity level and the distance,etc.

Take Action to Eradicate Ebola

Team #33650

February 9, 2015

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Keywords: Ebola; SIR; Dynamic optimization; ACO algorithm

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1 Introduction

Ebola, previously known as Ebola hemorrhagic fever, is a rare and deadly disease caused by infection with one of the Ebola virus species. It has a high case fatality rates typically around 60-70%.

Ebola was first discovered in 1976 near the Ebola River.Since then, outbreaks have appeared sporadically in Africa. But none of them has been as large and persistent as the current epidemic began in Guinea in December 2013.

As of 4th February 2015, a total of 22495 cases including 8981 deaths have been reported from the affected countries. Below is the Ebola epidemic's distribution map:

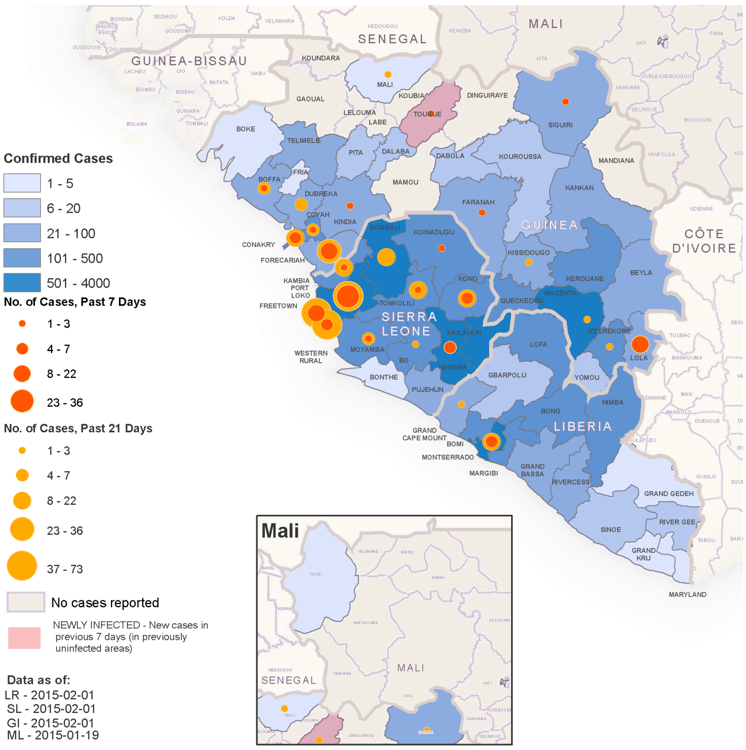

Figure 1: Geographical distribution of new and total confirmed cases

The unprecedented size and scale of this ongoing outbreak has the potential to destabilise already fragile local economies and healthcare systems, and fears of international spread of a Category A Priority Pathogen have made this a massive focus for international public health.

1.1 Restatement of the Problem

We need to build a mathematical model to optimize the eradication of Ebola. We know that some factories in America, Canada and China had successfully developed a new medication that is capable of stopping Ebola and curing patients whose disease is not advanced. We decompose the problem into five sub-problems:

- **Build a model to simulate the process of the spread of the disease** and determining the quantity of the medicine needed in future 60 months.
- **Build an sensible distribution plan.** Including how to distribute the quantity of drugs among all infected countries and the shortest route of drug.
- **Determine the production functions for different factories.**In particular, how many drugs should they produce each months to minimize its cost. distribution.
- **Simulate the process of domestic drug delivery.** Find the best route taking the cities' severity level and the geographical location into consideration.
- **Prepare a 1-2 page non-technical letter** for the world medical association to use in their announcement.

2 Assumption and Justifications

According to the data released by WHO on 24th December 2014, 27 036 out of 27 085 reported cases of Ebola come from Guinea, Liberia and Sierra Leone. So we decided to base our study on these three countries' situations to predict the future of infected population and determine the optimal medicine production and delivery. Our paper will be focusing on the epidemic spreading, medicine production and delivery for the **following 60 months.**

We will make the following simplifying assumptions:

1. **There are no further infection between countries.**

2. **Compared with the three countries mentioned, countries with relatively low infected population would not be taken into consideration.**

3. **We define the spread of Ebola is under control, if the infected people among the whole population is under .01.**

4. **People cured from the disease will be permanently immuned from Ebola.**

5. **Drug resistance can be ignored due to the short disease period.**

6. **During the time inspected from March 22 to November 23 in 2014, no medicine is used in the infected areas.**

7. **The epidemic areas begin to use medicine from February 2015**

8. **The mandatory drug usage for each patient is 30 units for a month on average**

9. **The cost of medicine transportation is independent with the amount of medicine transported, be it by air or by road. It is only a function of distance.**

10. **There is no inventory cost in the regions where the medicine is in need.**

3 Model Overview

In our paper, we choose Sierra Leone, Guinea and Liberia, where the virus spread especially rampant, to be our observational units. With three main models, we decompose the problem of Eradicating Ebola into three simplified tasks: the spread of the disease, the production of the medicine and the delivery of the medicine.

In the model of the Spread of Ebola, we first adapt the data release by WHO recording the infected population in nine inspected months in absence of medicine to build the Natural Spread Model. Then we obtain the fitting parameters λ, σ and K_I, which denote for contact rate, death rate caused by the Ebola and the upper limit of infective population that a country can tolerate. Using the parameters obtained along with a new recover parameter μ, which means the medicine will be used, we can visualize the transfer of population among susceptible, infected and dead or recovered groups for the following 50 months. Comparing the data with and without medicine, we can conclude the necessity of medicine. Furthermore, it provides medicine manufacturers with useful information about how to determine the quantity of medicine produced in each month.

In the model of Production of Medicine, we assume only three manufacturers in the world are producing the medicine. With the method of linear optimization, we can figure out the minimum amount of products to supply in order to satisfy the demand in the infected areas. With the method of dynamic optimization, we can construct a scheme for each manufacturer about how to produce for each month so that they can not only satisfy the demand of the medicine, but also minimize their cost, which consists of production cost and inventory cost.

In the model of Delivery of Medicine, we are inspired by the method of ant colony algorithm to set a goal to find a shortest route. But we make some progress by weighing the edge with comprehensive factor, which is the city's severity level and the distance.

The advantage of our model is that it is logic, concise and comprehensive. It can be very easy for readers to understand, because the three parts that we decompose our model into are connected logically. The disadvantage is the insufficient of data and information, which could probably leads to some inaccurate result.

4 The Model of Spread of Ebola

We base our model on the situation of Sierra Leone. Our data is from WHO official website, recording the cases infected with Ebola virus from March 22 to Novermber 23 in 2014. According to our data, we obtain our fitting parameters λ_0, σ and K_I. Then we use the parameters and data of initial infective population i_0 to build differential equations model, which is noted as **Natural Model**. By analyzing the results and figures of the model, we know the spread of Ebola when there is no medicine. Compared with **Natural Model**, we refine our model considering the effect of medicine, and name it **Predict Model**. Then we can study the transfer of population among susceptible, infective and removed groups and the behavior of infective population curve. This information will be vital to the determination of the production of the medicine.

4.1 Assumptions

1. **Since the time inspected is short, death rate and birth rate have no influence on the effective contact λ_0 and λ.**
2. **During the disease spread period, we divide people into susceptible people, infective people and removed people(in which people are cured or dead), and the total number N stays constant.**

4.2 Notations

All the variables and constants used in this paper are listed in **Table 1** and **Table 2**.

Table 1: Symbol Table-**Constants**

Symbol	Units	Definition
N	unitless	The total number of people during the inspected time.
K_I	unitless	The maximum proportion of infected population that a country can tolerate.
μ	unitless	The proportion of cured people because of the medicine every year.
σ	unitless	The proportion of people who is dead of the disease.
λ_0	unitless	Infective rate in natural condition.We define it as**natural effective contact**.
i_0	unitless	The proportion of patient in the initial time.

Table 2: Symbol Table-**Variables**

Symbol	**Units**	**Definition**
$s(t)$	unitless	The proportion of susceptible in total number.
$i(t)$	unitless	The proportion of infective in total number.
$r(t)$	unitless	The proportion of removed people in total number, consisting of people recovered from and dead of the disease.
λ	unitless	Infective rate under the interference of government. We define it as **effective contact**.

4.3 Natural Model

4.3.1 Model Establishment

We choose our data not only because of the rampant spread of Ebola virus and the instant update of information during the period from March 22 to November 23 in 2014, but also because there is no interference of medicine during that period. In this sense, we can simulate the natural spread of Ebola virus with abundant and accurate data.

Basically, governments and organizations would not be at the mercy of a virus, instead, they would take measures after the outbreak of a disease. For example, isolate suspected patients from public.contact. This leads to a non-constant effective contact, which becomes smaller as the epidemic gets severe. In our case, the effective contact

$$\lambda = (1 - \frac{i(t)}{K_I})\lambda_0 \tag{1}$$

An infective person can have average effective contact to a susceptible person at rate of λ, infecting the susceptible person .This is to say, the $Ni(t)$ infective people account for $\lambda Ns(t)i(t)$ infected people everyday. In the natural condition, we obtain

$$N\frac{dr}{dt} = \sigma Ni \tag{2}$$

$$N\frac{di}{dt} = \lambda Nsi - \sigma Ni \tag{3}$$

according to our assumption, we know

$$s(t) + i(t) + r(t) = 1 \tag{4}$$

Since the susceptible people could convert to infective people,

$$\frac{ds}{dt} = -\lambda si \tag{5}$$

if we let the initial ratio of infective people be i_0, the initial ratio of susceptible people be s_0, we build our model as below:

$$\frac{di}{dt} = \lambda si - \sigma i, i(0) = i_0 \tag{6}$$

$$\frac{ds}{dt} = -\lambda si, s(0) = s_0 \tag{7}$$

4.3.2 Parameter Evaluation

We obtain the fitting parameter from the actually data of reported cases of illness and death toll from March 2014 to November 2014 in Sierra Leone, Guinea and Liberia.

Date	Sierra Leone		
	cases of illness	Death toll	Infection rates
2014/3/22	0	0	0
2014/4/22	0	0	0
2014/5/22	0	0	0
2014/6/22	140.4	39.5	2.18011E-05
2014/7/22	448.3	210.7	6.97634E-05
2014/8/22	910	392	0.000014171
2014/9/22	1813	593	0.000028232
2014/10/22	3706	1539	0.00057711
2014/11/22	6210.5	1280.7	0.00096428

Figure 2: Sierra Leone Data

Date	Guinea		
	cases of illness	Death toll	Infection rates
2014/3/22	49	29	0.000004375
2014/4/22	205.5	132.5	1.83482E-05
2014/5/22	255.5	172.3	2.28125E-05
2014/6/22	392.2	4.2	3.50255E-05
2014/7/22	424	317.7	3.78571E-05
2014/8/22	910	392	5.57653E-05
2014/9/22	1048	641	9.35714E-05
2014/10/22	1540	904	0.000140952
2014/11/22	2119.5	1252.3	0.000189241

Figure 3: Guinea Data

Date	Liberia		
	cases of illness	Death toll	Infection rates
2014/3/22	0	0	0
2014/4/22	30.1	12.4	0.47540E-05
2014/5/22	12.9	9.6	0.04423E-05
2014/6/22	49	32.1	0.77649E-05
2014/7/22	211.6	121.9	3.35317E-05
2014/8/22	1082	624	0.000171462
2014/9/22	3022	1578	0.000478889
2014/10/22	4665	2705	0.000737667
2014/11/22	7112.3	2998.1	0.001124606

Figure 4: Liberia Data

The results of parameter evaluation are shown in **Table 3**

Table 3: Parameters-**Natural Model**

Parameters	**Sierra Leone**	**Guinea**	**Liberia**
λ_0	.92	.61	.93
σ	.15	.15	.13
K_I	.5	.5	.5

4.3.3 Model Solution and Analysis

Furthermore, the following figures show the relationship between $i(t)$ and $s(t)$

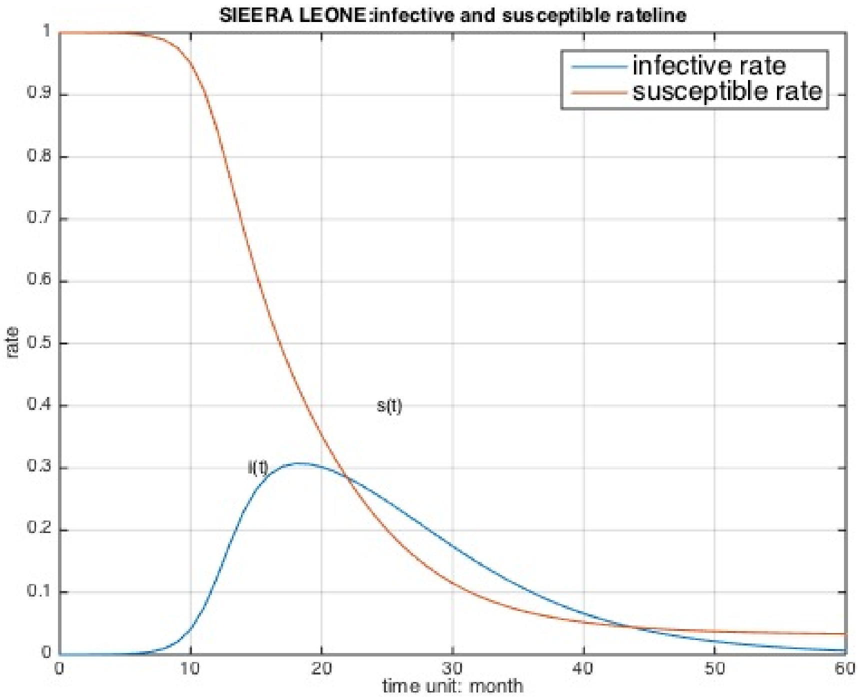

Figure 5:

We obtain the similar graphs of Guinea and Liberia in the same way[1]:

[1]For the complete data table, please check the appendix

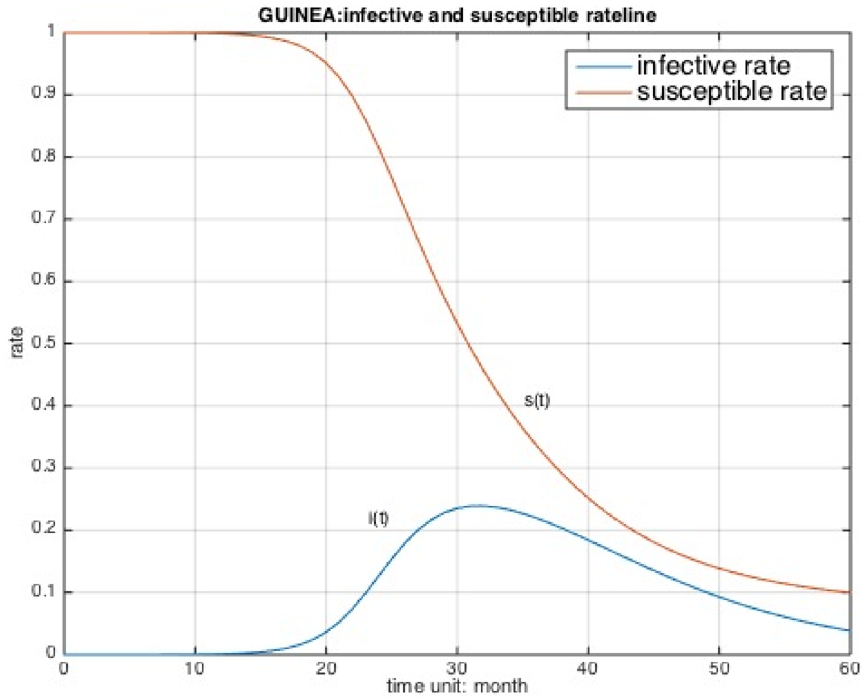

Figure 6:

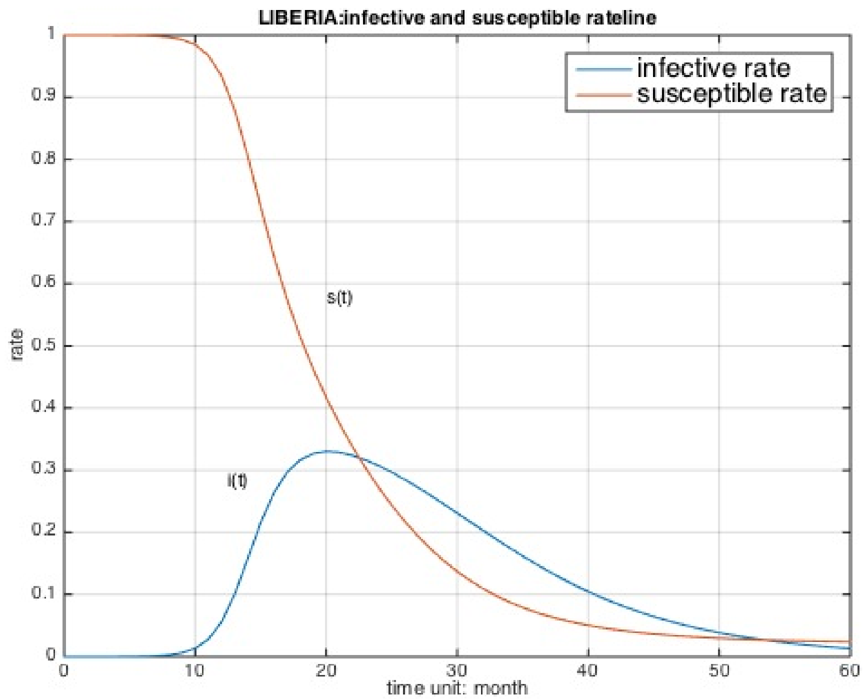

Figure 7:

The three graphs together give us an intuitive view that, with no use of medicine, the infective rate will hit the peak at about .3 around the 30th month, and the susceptible rate would keep decrease until it goes to the lower limit.

4.4 Predicting Model

4.4.1 Model Establishment

According to our assumption, the epidemic areas begin to use medicine from February 2015. This means that infected people can be cured due to the medicine, which is to say

$$r(t) = \sigma + \mu \tag{8}$$

in which μ can be interpreted as the effect of the medicine Having modified the Natural Model, and plugged in the fitting parameters we build the Predicting Model in the very same manner and obtain

$$\frac{di}{dt} = \lambda si - \sigma i - \mu i, i(0) = i_0 \tag{9}$$

$$\frac{ds}{dt} = -\lambda si, s(0) = s_0 \tag{10}$$

4.4.2 Parameter Evaluation, Model Solution

We assume that:

$$\mu = 0.4 \tag{11}$$

and we still use the parameters λ_0, σ and K_I in **Table 3** in our Predicting Model.

The chart below shows how we fitted $i(t)$ and $s(t)$ with respective to t [2]:

[2]For the compose type of the article, we do not show the whole data here, but please check them in the appendix if necessary.

	Sierra Leone			
	Natural		Prediction	
Time	i(t)	s(t)	i(t)	s(t)
0	2.18E-05	0.999978	2.18E-05	0.999978
1	4.71E-05	0.999948	4.71E-05	0.999948
2	0.000102	0.999883	0.000102	0.999883
3	0.000219	0.999743	0.000219	0.999743
4	0.000473	0.999439	0.000473	0.999439
5	0.001019	0.998787	0.001019	0.998787
6	0.002191	0.997385	0.002191	0.997385
7	0.004685	0.994398	0.004685	0.994398
8	0.009914	0.988121	0.009914	0.988121
9	0.020522	0.975321	0.020404	0.975327
10	0.040709	0.950692	0.035197	0.952395
11	0.074922	0.908004	0.057085	0.916897
12	0.123507	0.844683	0.085619	0.867088
13	0.17858	0.766949	0.117284	0.804994
14	0.228355	0.686537	0.146399	0.736162
15	0.265576	0.612103	0.16865	0.666473
16	0.289469	0.546451	0.18222	0.600065
17	0.302437	0.488947	0.187609	0.539034
18	0.307285	0.438297	0.186498	0.483883
19	0.306479	0.393035	0.180485	0.43473
20	0.301661	0.352173	0.17096	0.391558
21	0.293905	0.315219	0.159264	0.35378
22	0.283999	0.281773	0.146434	0.320709

Figure 8: The relationship between $i(t)$ and $s(t)$ in Sierra Leone

Figure 9 shows the relationship between $i(t)$ and $s(t)$ in Sierra Leone:

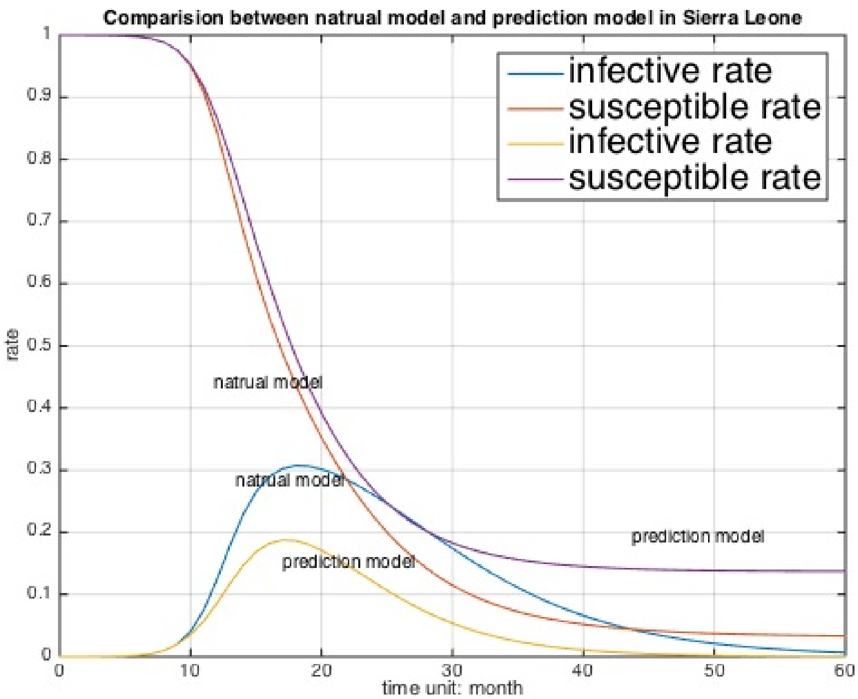

Figure 9: Comparison between natural model and prediction model in Sierra Leone

As we can see, when there is no medicine, which is shown in blue and brown curve, the infective rate would hit the peak at .3 in the 18th month, and the susceptible rate would keep going downward and become stable at.05 from 40th month. In contrast, with the use of medicine, which is shown in yellow and purple curve, it is notable that the height and duration of the peak in infective rate decreases. And the stable susceptible rate increases as well. Along with the similar results in Guinea and Liberia, the graph indicates that the medicine could be of great help in the epidemic areas. In the same manner, we can obtain the relationship between $s(t)$ and $i(t)$ in Guinea and Liberia,respectively:

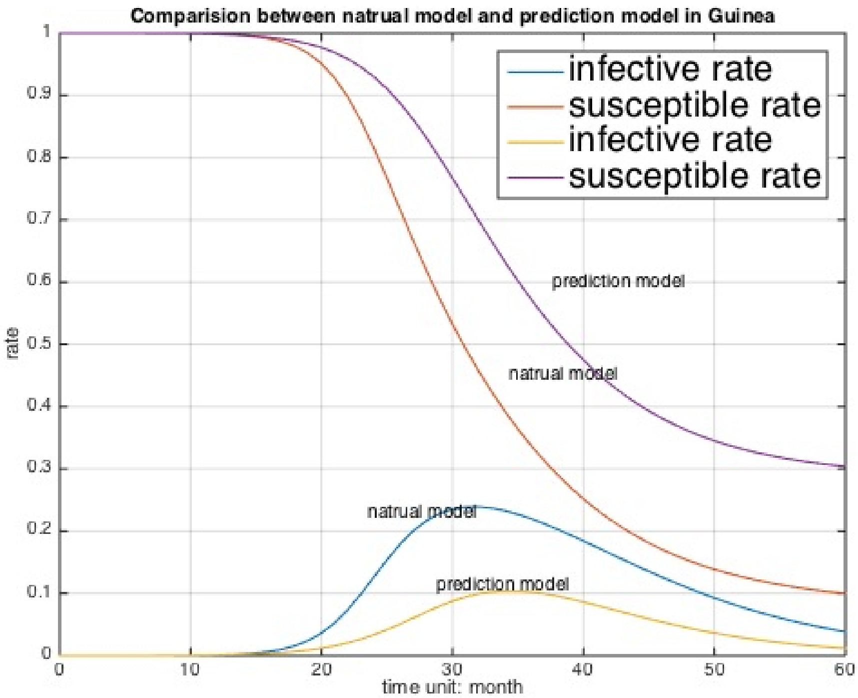

Figure 10: Comparison between natural model and prediction model in Guinea

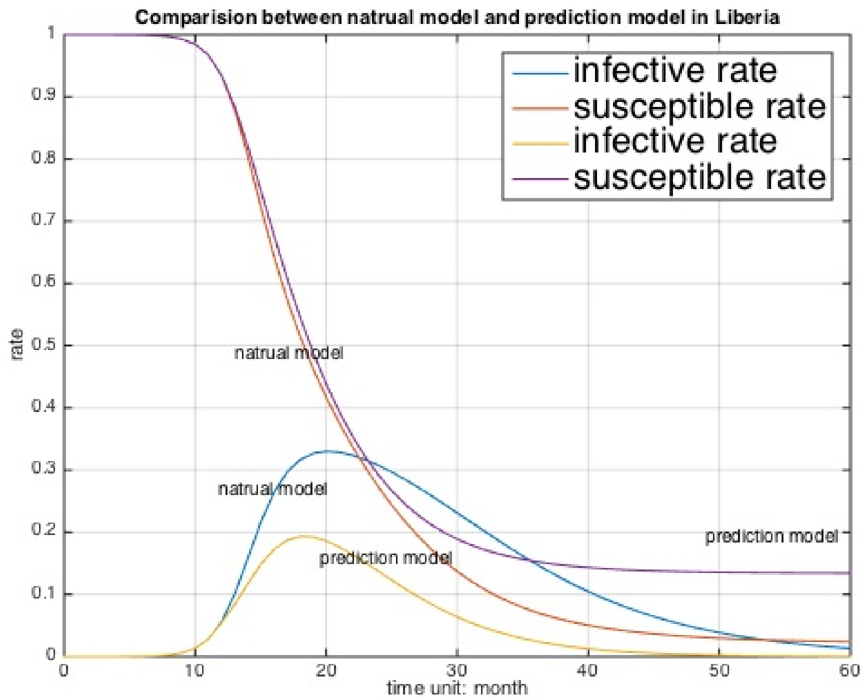

Figure 11: Comparison between natural model and prediction model in Liberia

5 The Model of Production

5.1 Distribution Optimization

We build this linear optimization model to determine:

The minimum demand for each factory to produce the medicine, in order to ensure that every country can get the amount of the medicine they need in a unit time. Furthermore, we will use the results of this model to decide the quantity of medicine produced in each factory in a time unit.

5.1.1 Notations

Table 4: Symbol Table-**Distribution Model**

Symbol	Value	Definition
$\boldsymbol{i}$	$1, 2, 3$	denotes for factory in the US, China and Canada, respectively.
$\boldsymbol{j}$	$1, 2, 3$	denotes for epidemic country Sierra Leone, Guinea and Liberia, respectively
$\boldsymbol{d_{ij}}$		denotes for the quantity of medicine transported from factory i to country j.
$\boldsymbol{D_i}$		donotes the total demand for factory i.
$\boldsymbol{\alpha}$	1.5	means the actual demand is α times the theoretical demand in case of emergency and delivery loss.
$\boldsymbol{\gamma}$	30	means the dose for a patient for one course of treatment is 30.
$\boldsymbol{i_j(t)}$		the infected rate in Sierra Leone, Guinea and Liberia, respectively.

5.1.2 Assumptions

1. Only 3 factories in the world are producing medicine, they locate in the US, China and Canada, respectively.

2. Medicine produced by each factory has identical effect on Ebola virus.

3. The demand for medicine is larger than the amount we calculated from $i(t)$ in the prediction model, in case of the loss during the delivery and emergency.

4. The course of treatment is $\gamma = 30$ doses for each patients.

5. The unit of time is one month, which means the demand we get from this model would be in terms of each month.

6. The transport of the medicine is from three designated airports in the US, China

and Canada to the capital airport of each country.

5.1.3 Model Establishment

$$\begin{aligned}
\mathbf{min} \quad & d_{11} + d_{12} + d_{13} \\
\mathbf{min} \quad & d_{21} + d_{22} + d_{23} \\
\mathbf{min} \quad & d_{31} + d_{32} + d_{33} \\
\mathbf{s.t} \quad & d_{11} + d_{21} + d_{31} \geq \alpha\gamma N i_1(t) \\
& d_{12} + d_{22} + d_{32} \geq \alpha\gamma N i_2(t) \\
& d_{13} + d_{23} + d_{33} \geq \alpha\gamma N i_3(t)
\end{aligned}$$

Since d_{ij} is non-negative, we can simplify the model by replacing the objective function with

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}$$

and we get

$$\begin{aligned}
\mathbf{min} \quad & D = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \\
\mathbf{s.t} \quad & d_{11} + d_{21} + d_{31} \geq \alpha\gamma N i_1(t) \\
& d_{12} + d_{22} + d_{32} \geq \alpha\gamma N i_2(t) \\
& d_{13} + d_{23} + d_{33} \geq \alpha\gamma N i_3(t)
\end{aligned}$$

If we plug in the data we get from Predicting Model of each $i_j(t)$, we can obtain d_{ij}, which means we can thus determine the quantity each factory should delivery to each country. The result is shown in **Table 12** [3]:

[3]For the compose type of the article, we do not show the whole data here, but please check them in the appendix if necessary.

5.1.4 Model Solution and Analysis

The demand of medicine for each factory												
time	d11	d12	d13	d21	d22	d23	d31	d32	d33	D1	D2	D3
0	2029.253	790.3808	313.541	2029.253	790.3808	313.541	2029.253	790.3808	313.541	3133.175	3133.175	3133.175
1	4382.352	1252.01	697.7923	4382.352	1252.01	697.7923	4382.352	1252.01	697.7923	6332.155	6332.155	6332.155
2	9455.641	1991.186	1557.34	9455.641	1991.186	1557.34	9455.641	1991.186	1557.34	13004.17	13004.17	13004.17
3	20371.13	3132.889	3392.149	20371.13	3132.889	3392.149	20371.13	3132.889	3392.149	26896.16	26896.16	26896.16
4	44025.58	4929.465	7596.967	44025.58	4929.465	7596.967	44025.58	4929.465	7596.967	56552.01	56552.01	56552.01
5	94815.35	7817.747	16922.83	94815.35	7817.747	16922.83	94815.35	7817.747	16922.83	119555.9	119555.9	119555.9
6	203929.4	12419.05	37526.65	203929.4	12419.05	37526.65	203929.4	12419.05	37526.65	253875.1	253875.1	253875.1
7	436111.1	19692.64	83418.59	436111.1	19692.64	83418.59	436111.1	19692.64	83418.59	539222.3	539222.3	539222.3
8	922824.2	31090.49	184492.4	922824.2	31090.49	184492.4	922824.2	31090.49	184492.4	1138407	1138407	1138407
9	1899179	49182.99	405498.4	1899179	49182.99	405498.4	1899179	49182.99	405498.4	2353861	2353861	2353861
10	3276120	77969.46	878432.9	3276120	77969.46	878432.9	3276120	77969.46	878432.9	4232522	4232522	4232522

Figure 12: The demand of medicine for each country

From **Table 12** we notice that the demand for three factories is the same, which is to say:

$$D_1 = D_2 = D_3 \tag{12}$$

This is because three factories face the same demand curve from three countries, so the optimal choice is to divided the demand into three equal parts. We plot the demand curve for each factory in **Figure13**.

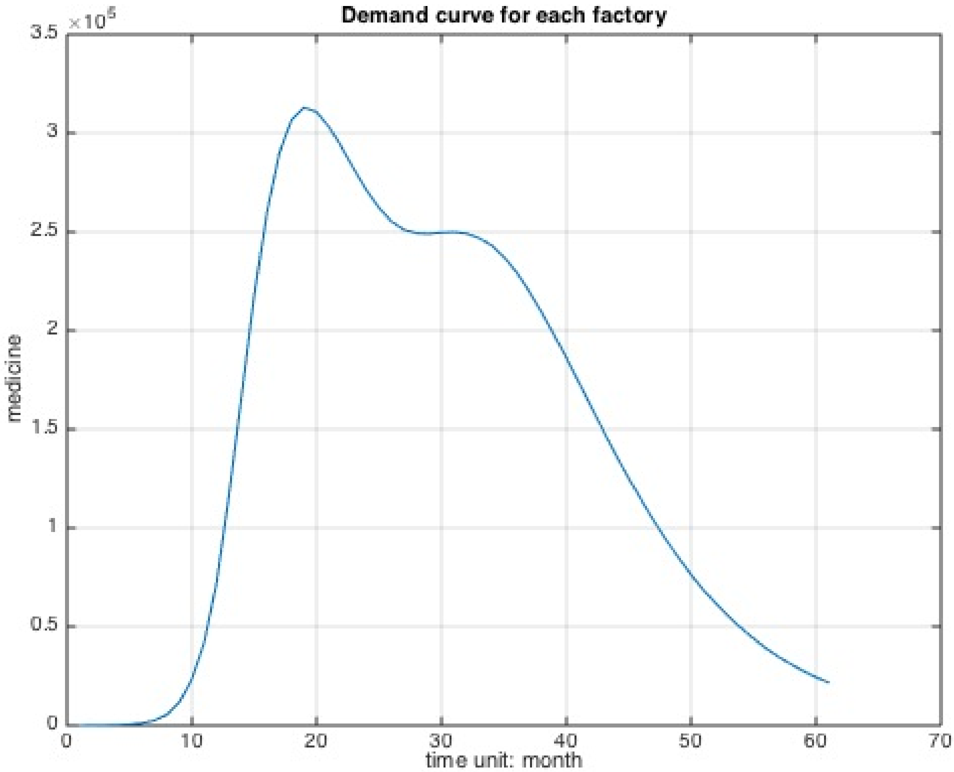

Figure 13: The demand of medicine for each country

5.2 Production Optimization

According to the results from distribution optimization, each factory faces the same demand curve. So we only need to analyze the production of one factory, say the Mapp Biopharmaceutical Inc[2] from the US, to minimize its cost.

5.2.1 Assumptions

1. **There are three factories[2][5][6] selling the identical medicine.**

2. **Due to the limitation of technology, the maximum production for each month is $\hat{X}_k$.**

3. **The quantity produced would increase every month by β.**

4. **The production would start from Feb 2015 and last for 60 month.**

5.2.2 Notations

Table 5: Symbol Table-**Production Optimization**

Symbol	Value	Definition
$\boldsymbol{x_k}$		denotes for quantity produced in kth month.
$\boldsymbol{i_k}$		denotes for inventory quantity at the beginning of kth month.
$\boldsymbol{c_0}$		donotes for the fixed cost of production of the factory.
$\boldsymbol{m}$		denotes for the marginal cost of production.
$\boldsymbol{c(x_k)}$	$c_0 + mx_k$	denotes for the production cost of kth month.
$\boldsymbol{I}$		means the cost for preserving unit inventory.
$\boldsymbol{h(i_k)}$	Ii_k	means the cost for preserving inventory in kth month.
$\boldsymbol{\hat{X}_k}$	5000	denotes for the maximum production in kth month.
$\boldsymbol{\beta}$	.08	denotes for the growth rate of production in each month.
$\boldsymbol{f_k}$	$c(x_k) + h(i_k)$	means the total cost in kth month.
$\boldsymbol{F_k}$	$F_{k+1} + f_k$	means the cost for preserving unit inventory.

5.2.3 Model Establishment

We base our model on dynamic optimization. It is obvious that

$$i_{k+1} = i_k + x_k - dk \tag{13}$$

$$f_k(x_k) = c(x_k) + h(i_k) \tag{14}$$

$$= c_0 + mx_k + Ii_k \tag{15}$$

$$= c_0 + mx_k + I(i_k + x_k - d_k) \tag{16}$$

From (13) to (16), we can easily imagine that, when demand is satisfied, the less the production is, the less it would have to cost.

However the factory has to produce more than what is demanded sometimes, since chances are, it can foresee that demand would be larger than their maximum production in the following months. So in the months when the demand is higher than the maximum production or after which the demand would be higher than maximum production, inventory would occur.

Inspired by greedy algorithm [7], we optimize production of each month respectively, from the last to the first. When we add those local optimized solution together, we would obtain the overall optimized result, thus determining the production of each month.

$$F_k = \text{min } c(x_k) + h(i_k) + F_{k+1}, i_{k+1} = i_k + x_k - dk \tag{17}$$

$$x_k \leq \hat{X}_k(1+\beta)^{k-1} \tag{18}$$

We run the codes in Matlab, with the assigned parameters $m = 10, h = 1$[4], and get the result as below[5]:

[4]The information about the production of medicine is so limited that we can only make a ball-park estimate of the proportional relationship between marginal cost and inventory cost.

[5]Here we only show the graph in the paper, please check the complete data in Appendix C

Time	0	1	2	3	4	5	6	7	8	9
Production	3394	3426	3492	3631	3927	4557	83853	91399	99624	108590
Demand	31.33175	63.32155	130.0417	268.9616	565.5201	1195.559	2538.751	5392.223	11384.07	23538.61
Time	10	11	12	13	14	15	16	17	18	19
Production	118363	129015	140626	153282	167077	182113	198503	216368	235841	257066
Demand	42325.22	72824.81	117392.7	166850.9	216992.2	259594.5	289799	306868.3	312964.2	310793
Time	20	21	22	23	24	25	26	27	28	29
Production	280201	321982	309697	298133	288244	280847	276313	274267	274013	274612
Demand	303160.9	292710.8	281543.2	271030.4	262039.8	255315.5	251193.6	249334	249103.1	249648.6

Figure 14: Partial Data for the Production and Demand

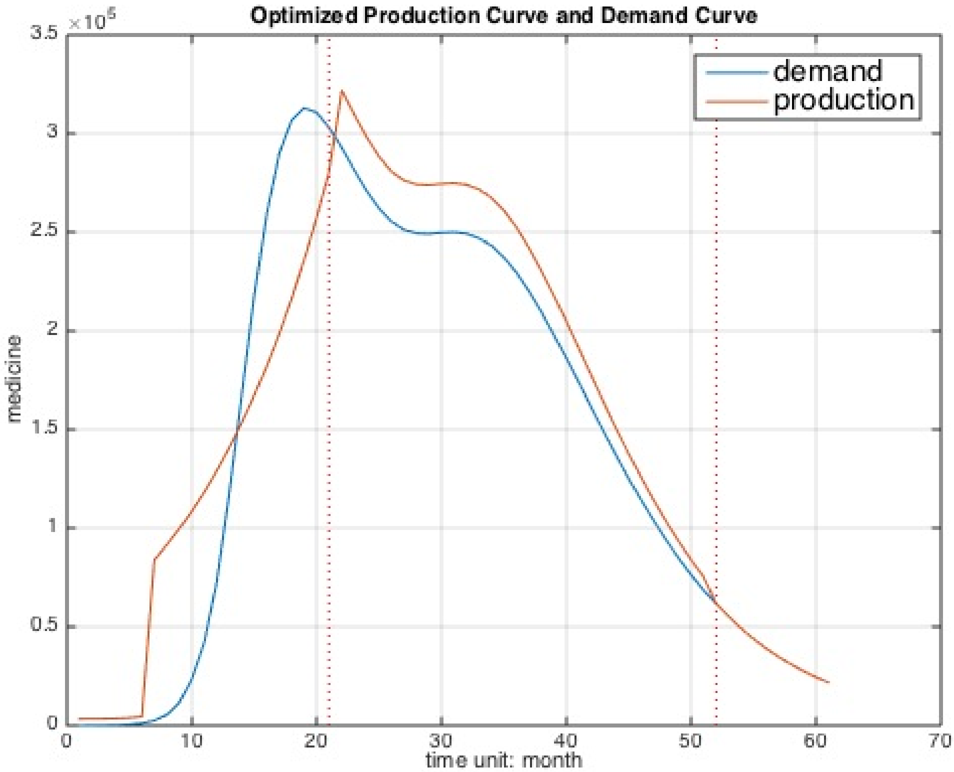

Figure 15: Optimized Production Curve and Demand Curve

As it is shown in the graph, during the first five months the amount produced is equal to the amount demanded. From 6th month to 14th month , the production is higher than what is demanded, accumulating the inventory for the peak of the demand. From month

14 to month 22 , the maximum production is less than the demand, from 22th month to 52th month the production would be larger than then demand. After the month 52, the production is equal to the demand.[6]

6 The Model of Delivery

Drugs are not common commodity, they usually requires a high standard of storage atmosphere, such as a certain range of temperature and humidity, avoid sunshine contact and be strictly sealed. So it is crucial that drugs arrived at where they needed timely and with as little damage as possible.

We simplify the cost of delivery as an linear function of transported distance, which is to say, the transport cost can be written as:

$$Z_i = Z_0 + k * L_i \tag{19}$$

where Z_0 is the fixed cost of each transportation and L_i is the distance of transportation. Therefore, the question can be simplified as how to determining the shortest route of delivery.

In our model, we use **the Ant colony optimization algorithms** for both International and Domestic Delivery System. Here's a brief summary of our algorithm: [3]

An ant is a simple computational agent in the ant colony optimization algorithm. It iteratively constructs a solution for the problem at hand. The intermediate solutions are referred to as solution states. At each iteration of the algorithm, each ant moves from a state x to state y, corresponding to a more complete intermediate solution. Thus, each ant k computes a set $A_k(x)$ of feasible expansions to its current state in each iteration, and moves to one of these in probability. For ant k , the probability p^k_{xy} of moving from state x to state y depends on the combination of two values, viz., the attractiveness η_{xy} of the move, as computed by some heuristic indicating the a priori desirability of that

[6]please see the detailed information in excel Appendix C

move and the trail level τ_{xy} of the move, indicating how proficient it has been in the past to make that particular move.

The trail level represents a posteriori indication of the desirability of that move. Trails are updated usually when all ants have completed their solution, increasing or decreasing the level of trails corresponding to moves that were part of "good" or "bad" solutions, respectively.

In general, the kth ant moves from state x to state y with probability

$$p_{xy}^{k} = \frac{(\tau_{xy}^{\alpha})(\eta_{xy}^{\beta})}{\sum_{y \in \mathrm{allowed}_y} (\tau_{xy}^{\alpha})(\eta_{xy}^{\beta})} \tag{20}$$

where

τ_{xy} is the amount of pheromone deposited for transition from state x to y , $\alpha \geq 0$ is a parameter to control the influence of τ_{xy}, η_{xy} is the desirability of state transition xy (a priori knowledge, typically $1/d_{xy}$,where d is the distance) and $\beta \geq 1$ is a parameter to control the influence of η_{xy}. τ_{xy} and η_{xy} represent the attractiveness and trail level for the other possible state transitions.

When all the ants have completed a solution, the trails are updated by

$$\tau_{xy} \leftarrow (1-\rho)\tau_{xy} + \sum_{k} \Delta\tau_{xy}^{k} \tag{21}$$

where τ_{xy} is the amount of pheromone deposited for a state transition xy, ρ is the pheromone evaporation coefficient and $\Delta\tau_{xy}^{k}$ is the amount of pheromone deposited by kth ant, typically given for a Travelling salesman problem problem (with moves corresponding to arcs of the graph) by

$$\Delta\tau_{xy}^{k} = \begin{cases} Q/L_k & \text{if ant } k \text{ uses curve } xy \text{ in its tour} \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

where L_k is the cost of the kth ant's tour (typically length) and Q is a constant.

The general algorithm is relatively simple and based on a set of ants, each making one of the possible round-trips along the cities. At each stage, the ant chooses to move from one city to another according to some rules:

1. It must visit each city exactly once;
2. A distant city has less chance of being chosen (the visibility);
3. The more intense the pheromone trail laid out on an edge between two cities, the greater the probability that that edge will be chosen;
4. Having completed its journey, the ant deposits more pheromones on all edges it traversed, if the journey is short;
5. After each iteration, trails of pheromones evaporate.

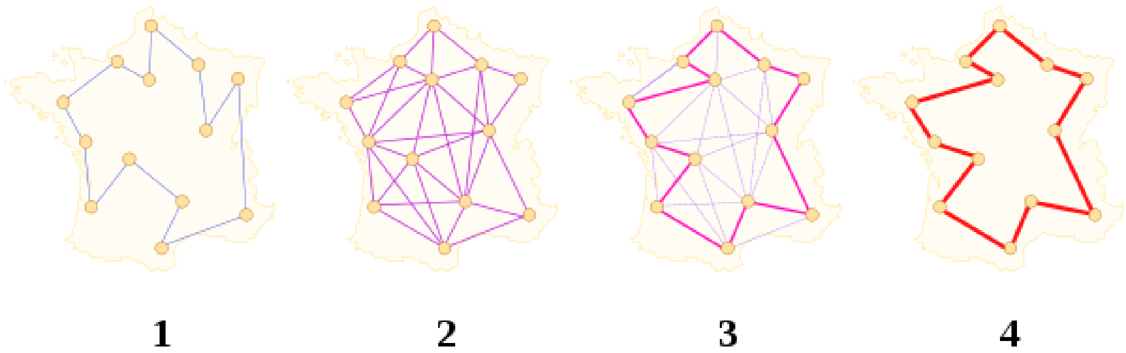

Figure 16: ACO algorithm

6.1 International Delivery Optimization

We assume that the drug is delivered at the beginning of each month from the international airport of the US, China and Canada to the international airport of Guinea, Liberia and Sierra Leone. Each flight should stop three times to delivery the drugs to Guinea, Liberia and Sierra Leone. We here using the geographical coordinates to determine the optimal route of airplane:[9]

		X	Y
Beijing		40.04	116.35
New York		40.38	-73.46
Vancouver		49.11	-123.11
Sierra Leone		8.36	-13.11
Liberia		6.14	-10.22
Guinea		9.34	-13.36

Figure 17: Geographical coordinates of international airports

Using the ACO algorithm, where L_{ij},the distance between $Region_i$ and $Region_j$ is defined as the Euler distance of the two cities:

$$L_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{23}$$

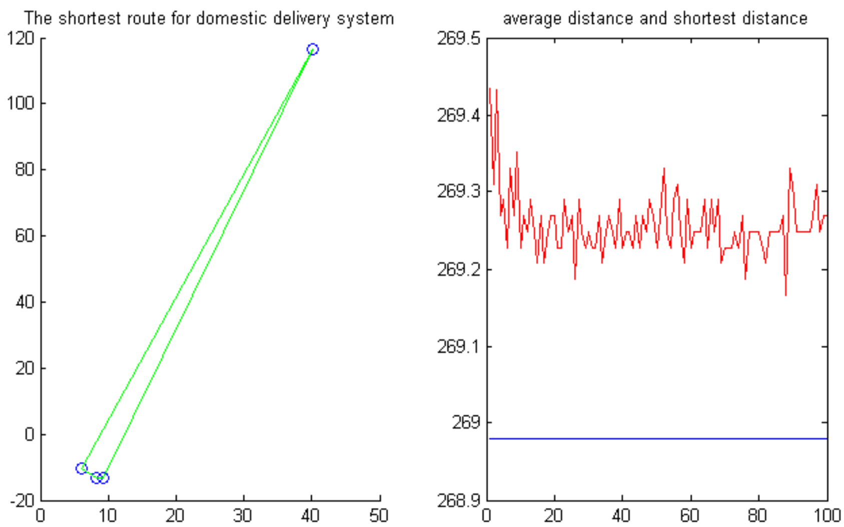

Figure 18: The optimal route departed from Beijing to Liberia, Sierra Leone and Guinea

Using Matlab and the data of coordinates, we can easily determine the optimal route for three flights, which are

$China : Beijing \longleftrightarrow Liberia \longleftrightarrow Sierra\ Leone \longleftrightarrow Guinea \longleftrightarrow Beijing$

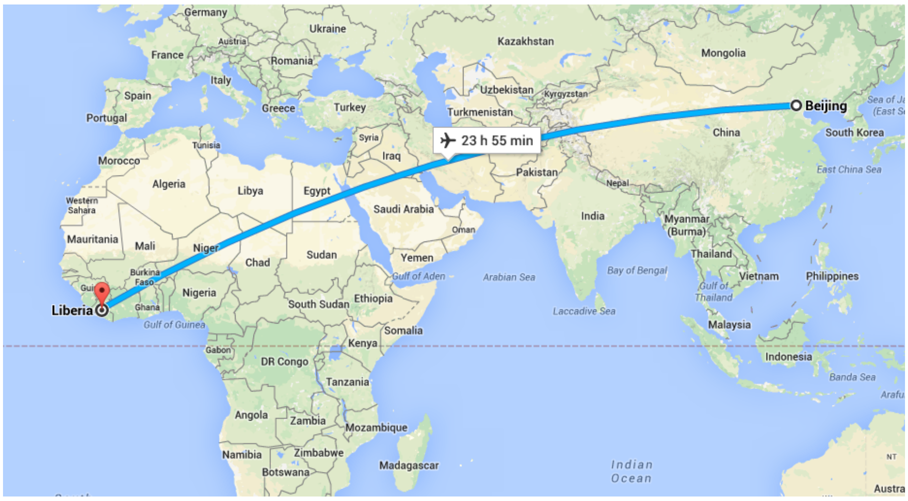

Figure 19: Beijing to Liberia Route

$US: \quad New\ York \longleftrightarrow Guinea \longleftrightarrow Liberia \longleftrightarrow Sierra\ Leone \longleftrightarrow New\ York$

Figure 20: New York to Guinea Route

$Canada: Vancouver \longleftrightarrow Sierra\ Leone \longleftrightarrow Liberia \longleftrightarrow Guinea \longleftrightarrow Vancouver$

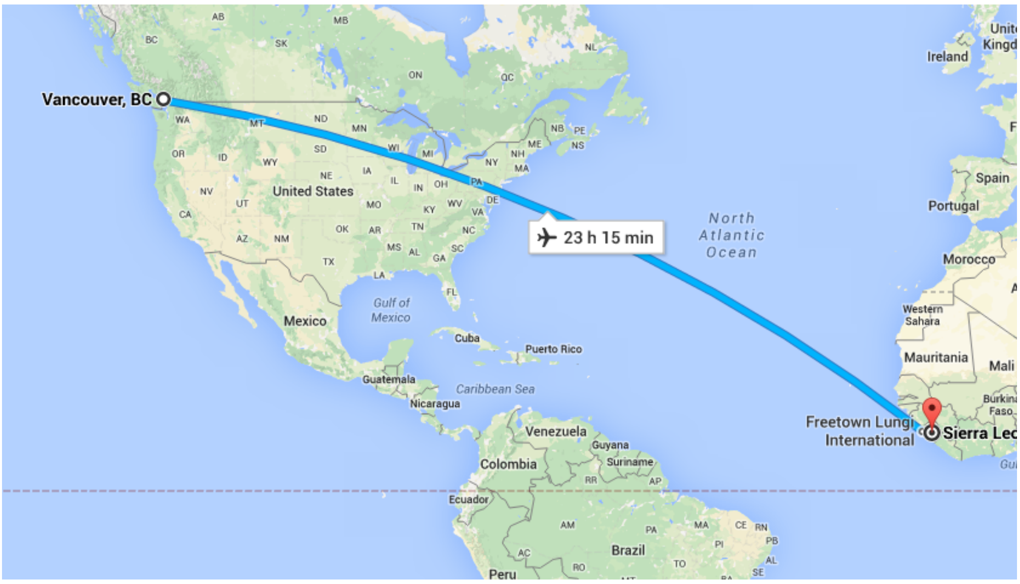

Figure 21: Vancouver to Sierra Leone Route

6.2 Domestic Delivery Optimization

We apply the same method for Domestic Delivery System, after the medicine arrived at the capital airport, it must be transported to local hospitals as soon as possible at the lowest cost. We assume that the time and cost of transportation are linear functions of the distance of transportation. The question can be therefore simplified as to find the shortest route of domestic delivery. We use Sierra Leone as an example to illustrate how it works:

The geographic coordinates and shortest route of Sierra Leone					
		W	X	Y	Shortest Route
1	KAMBIA	4	7.839946	-12.8271	3
2	PORTLOKO	5	8.766904	-12.7879	1
3	FREETOWN	5	8.483516	-13.2346	10
4	BOMBALI	5	8.886213	-12.0554	8
5	KOINADUGU	4	9.575572	-11.5628	9
6	KONO	4	8.474062	-10.7949	11
7	TONKOLILI	4	8.736843	-11.93	12
8	MOYAMBA	4	8.160825	-12.4335	13
9	BO	4	7.955323	-11.7425	6
10	BONTHE	1	7.526584	-12.5051	5
11	PUJEHUN	3	7.350485	-11.7184	7
12	KENEMA	5	7.863337	-11.198	4
13	KAILAHUN	5	8.094744	-10.7046	2

Figure 22: The optimal domestic delivey route of Sierra Leone[9]

In **Figure 22**, blue background emphasis represents the captical of the nation where international airport is located. X, Y represents the coordinates of local hospitals. W represents the severity level of each region, which takes value of 1-5 according to the number of confirmed cases of each region.[1]

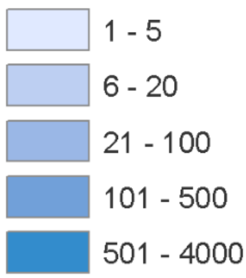

Figure 23: The severity level table: confirmed cases of each region

Using the ACO algorithm, where L_{ij},the distance between $Region_i$ and $Region_j$ is defined as not only the distance of the two cities, but weighted by the average severity level of the two cities:

$$L_{ij} = \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{\frac{W_i + W_j}{2}} \tag{24}$$

And the shortest route and the shortest distance are shown in **Table 24**

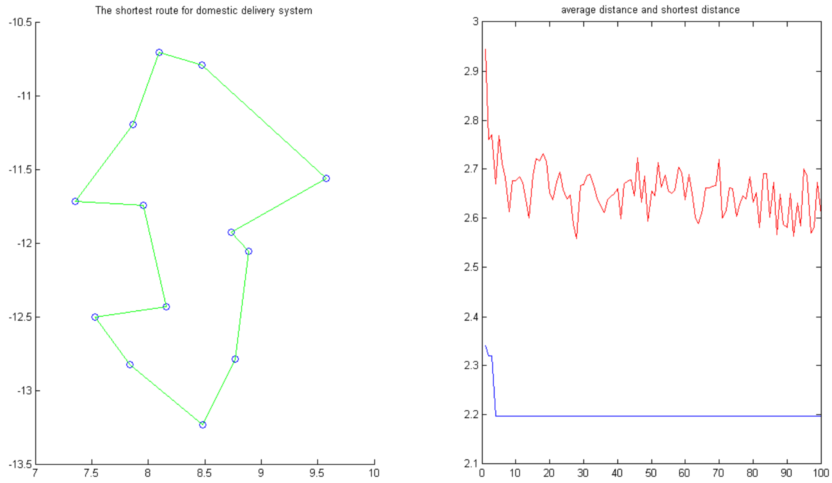

Figure 24: The shortest route and the shortest distance of Sierra Leone

By combining the severity map and the shortest route together, we get:

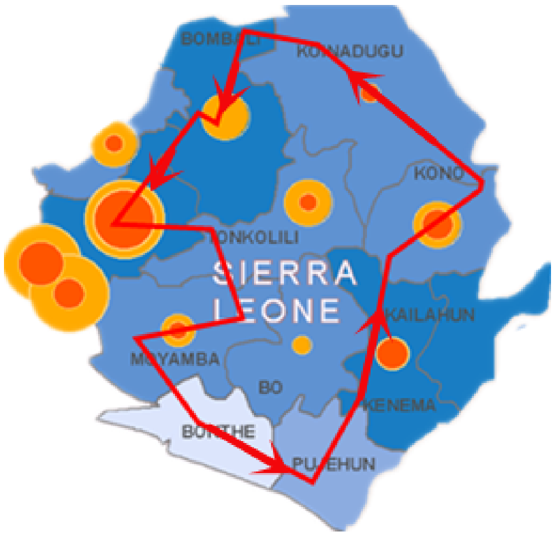

Figure 25: Sierra Leone Delivery Route Map [1]

Similarly, we can obtain the optimal route for Liberia and Guinea, the rest of the results are shown in appendix.

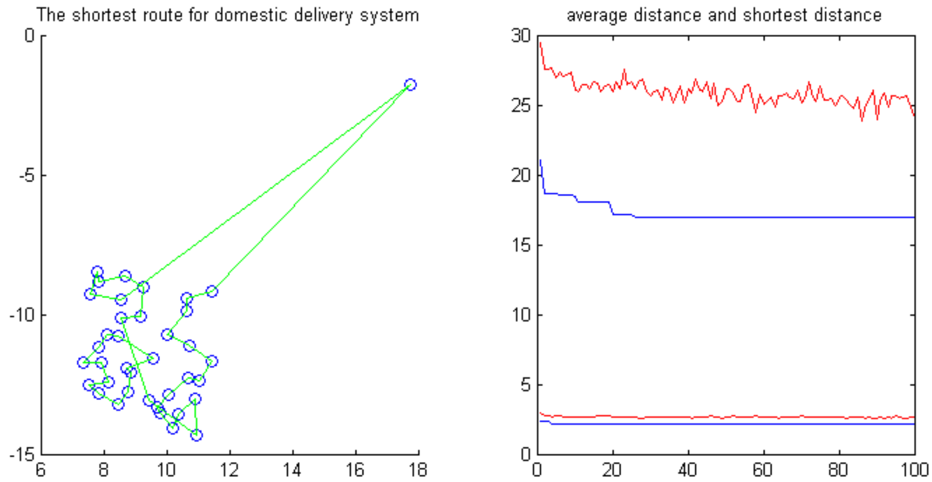

Figure 26: The shortest route and the shortest distance of Guinea

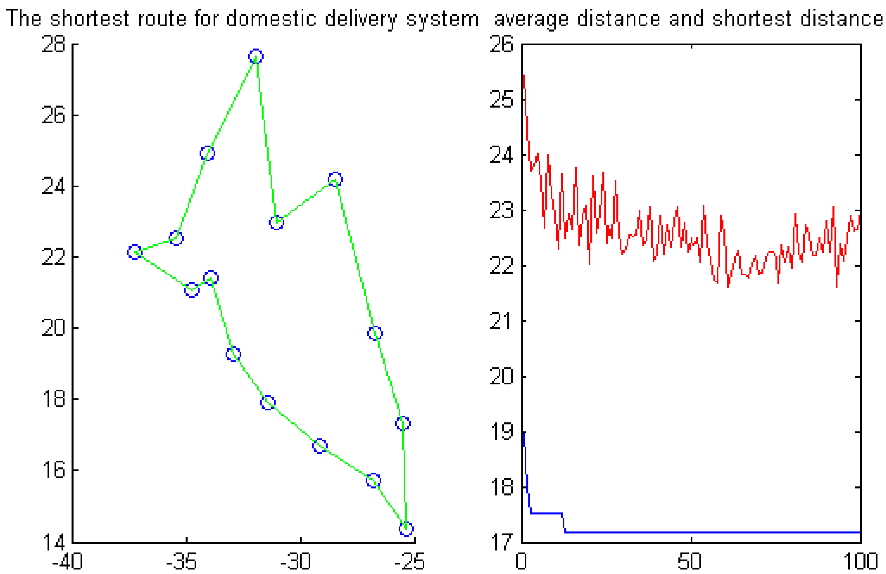

Figure 27: The shortest route and the shortest distance of Liberia

7 Conclusions

- By executing the model for the Spread of Ebola, we obtain the fitted parameters of infected rate, death rate and the upper limit of infected population that a country can tolerate shown in Table 3. Plugging in the fitted parameters, we can obtain the variation trends of infected rate and susceptible rate in Figure2, Figure3 and Figure4. And the complete data is shown in Appendix data 1.From these result, we draw the conclusions that the effect of the medicine is significant for the eradicating of Ebola, in that both the peak of infected population and duration of the peak would decrease with the use of medicine. The comparison of the situations between with and without medicine are shown in Figure9, Figure10 and Figure11. We then use these conclusion as the basis for quantity produced for manufacturers.

- The results in the Distribution Optimization Model shows how each manufacturer would distribute the product it has produced to three infected countries, as shown in Figure12, Figure13 and Appendix data 2. We come to the conclusion that total amount produced and the distribution to each country for each manufacturer are identical. And the demand curve of the medicine that the manufacturers are facing has the same trend as the infected rate curve, as we expect.

- The results of the Production Optimization Model shows how the manufacturers would balance their production and inventory each month in order to satisfy the demand for the medicine as well as reduce the production cost as much as possible. The results are shown in Figure15 and Appendix data 2. We can draw conclusions that the manufacturers can reduce their cost if they decide to produce the inventory as late as possible and as less as possible.

- The results of the Delivery System provide us with the blue print about the route along which the medicine will be delivered in order to deliver the medicine to the cities which are in most desperate need and cost least on traffic as well. After we run the codes in Matlab, we obtained the optimized international delivery route as below:
  the US: New York-> Guinea-> Liberia-> Sierra Leone-> New York
  China: Beijing-> Liberia-> Sierra Leone-> Guinea-> Beijing
  Canada: Vancouver-> Sierra Leone-> Liberia-> Guinea-> Vancouver

The domestic optimized route are displayed in Fig25, Fig27 and Fig26.

8 Letter for the world medical association

Dear World Medical Association staff,

Based on our mathematical model, we are honorable to provide this announcement as below:

West Africa is the critical region to control the spread of Ebola. With substantial proportion of population getting involved in this battle against Ebola, Sierra Leone, Guinea and Liberia have become the center of this medical assistance. We can makea a fairly conservative prediction based on mathematical computing. With no interference from medicine, a peak of 30% of the whole population in the infected area would occur at around September in 2015. In contrast, if we introduce the new medicine, which proved to be effective with the cure rate of .4, the peak of the infected population would be 18% of the whole population at around September in 2015, and the duration of the spike in the infected population would have a notable decrease. We can predict that we the disease will be under control in 30 months. In conclusion, it is necessary and urgent to introduce new medicine to eradicate the disease.

I would introduce the production system and delivery system respectively to show how we will endeavor to reduce the loss from Ebola at our best effort. Considering the backward economy of the area in question, medicine manufactures may have no incentive to research on and produce the medicine in need. However, considering the urgent situation of Ebola, joint efforts from governments, the UNs and NGOs must be made to raise fund for research and development of this medicine. Up until now, there are three countries in the world producing the medicine. They are company Mapp Biopharmaceutical Inc from the US, Sihuan Pharmacy from China and TekmiraPharmaceuticals from Cananda. And their supply of the medicine would satisfy the demand.

In the beginning of each month, the medicine produced would first be transported from each factory to the three countries' international airports. They then would be delivered to each capital city of each province. Since each city is infected at different levels, and the cost of traffic is different from city to city, the delivery route would base on the optimization of the comprehensive integral of infected degree and traffic cost. Which is to say, our delivery system is not only the most economic one, but also the most effective one in terms of saving lives.

In the meantime, we would definitely take action to prevent people from getting infected. We would try our best to improve the medical and sanitary condition of the infected area, to advocate the knowledge of preventing the disease. And we would keep testing and assessing our predict model, to keep track of the data, and to provide the most accurate and updated information to the public.

Best Regard,

XXX

9 Sensitivity Analysis

Some inputs of our model may be hard to obtain or there might be some uncertainty in our inputs. Both these kinds of deviation might influence the result of our model. To test the robustness of our model, we implement a sensitivity analysis. We test our model for the spread of Ebola in the cure rate μ ,and the model of manufacturing in the ratio of marginal production cost and marginal inventory cost. The analysis proves that our model does not demonstrate a chaotic behavior, showing a good sensitivity.

9.1 Sensitivity to μ

We alternate μ, the cure rate of the medicine, from 0.24 to 0.48, to test the sensitivity of our model with respect to μ.

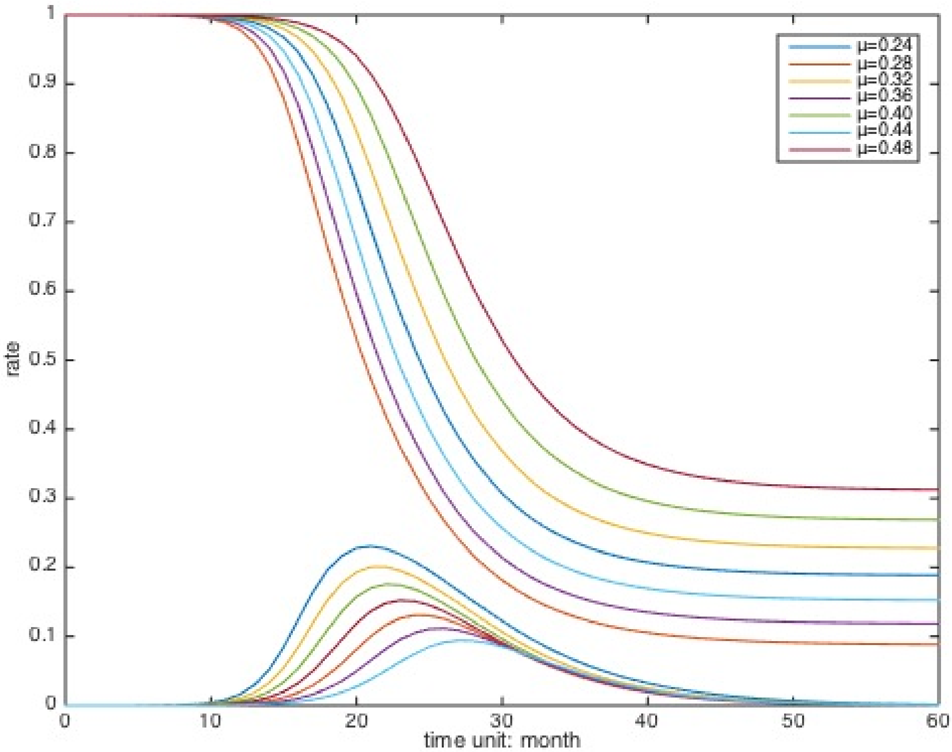

Figure 28: The prediction model with different μ

From **Figure28** ,we notice that, as μ increases, the highest infection rate decreases. Also, the peak of $i(t)$ arrives later. But the big trend and shape of the figure remain unchanged as we alternate μ, showing that our model is robust.

9.2 Sensitivity to production/inventory cost ratio

We define the ratio between marginal production cost of each factory m and marginal inventory cost I to be the production/inventory cost ratio:

$$\theta = \frac{m}{I} \tag{25}$$

This ratio represent the relationship between production plan and inventory plan. If keeping inventory is relatively expensive, the factory would have more incentive to produce as much as they needed in order to lower the cost. If production is relatively expensive, the factory would have more incentive to produce more before the peak of demand arrives to prepare themselves for the need in the future.We plot the curve of production/inventory at 8:1, 10:1 and 12:1 respectively.

Demand	Production		
	8:1	10:1	12:1
31.33	1395	3394	2473
63.32	2081	3426	2505
130.04	3968	3492	59405
268.96	5553	3631	61985
565.52	9940	3927	62281
1195.56	15490	4557	62911
2538.75	43853	83853	64255
5392.22	51399	91399	91399
11384.07	69624	99624	99624
23538.61	78590	108590	108590
42325.22	88363	118363	118363
72824.81	99015	129015	129015
117392.66	100634	140626	140626
166850.89	113228	153282	153282
216992.17	137073	167077	167077
259594.49	152133	182113	182113
289798.95	178503	198503	198503
306868.31	196334	216368	216368

Figure 29: The production model with different θ

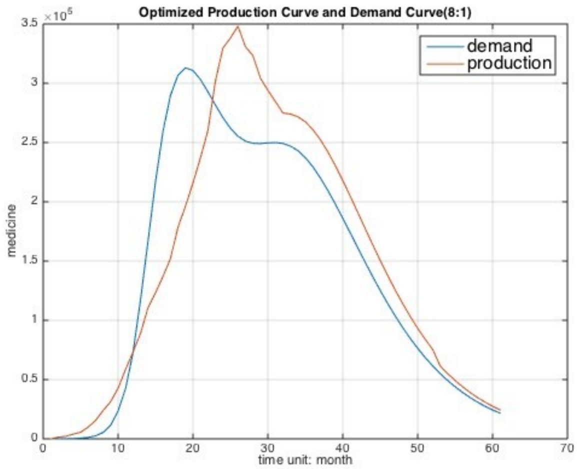

Figure 30: The production model with $\theta = 8:1$

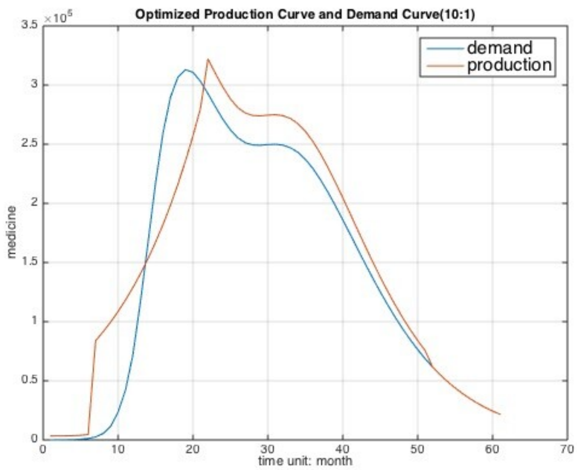

Figure 31: The production model with $\theta = 10:1$

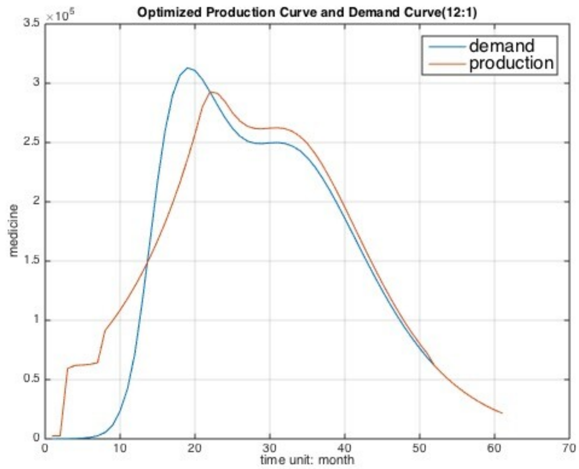

Figure 32: The production model with $\theta = 12 : 1$

From the table we notice that, as θ increases, the factory tends to produce more at the beginning of production and produce less when the demand reaches its peak. In realistic, the ratio is different from factory to factory, if we consider the ratio to be different for each factory, we need to rearrange the demand for each factory to optimize its manufacturing.

10 Model Assessment

10.1 Advantages

- The parameters in our Natural Model, are fitted from the data, which is more accurate than assigning some random data as most infectious disease models do.

- The effective contact λ in the Natural Model has a negative relationship with infected rate, which is more realistic because governments and organizations would take measure when the spread of the disease is rampant. The changing λ is more practical than a fixed one.

- In the Medicine Delivery Model, we weigh the edges of ant colony algorithm with comprehensive combination of city's severity level and distance. This could lead to not only a very cost-saving but also a life-saving route.

- We organize our tasks in a very logical way, which makes the whole paper understandable.

10.2 Weakness and Future Improvements

- Since time given is very limited, our model only analyzes some of the most concerned problems with relatively simplified parameters. If possible, we would like to make such improvements to refine our model as below:

- If the manufacturers also produce an effective vaccine, then the contact rateλ_0 would decrease with respect to time. Based on our model, we can speculate that, it would contract the peak of the infective rate further, and would control the spread of Ebola in shorter period of time.

- When we put weighs on the ants colony algorithm, we only take the severity level and distance between cities into consideration. In fact, we can refine our model by considering more factors like the city's population density, level of medical and sanitary conditions and climate etc.

- In our model, we only consider the scenario where medicine is imported from other counties. If with sufficient time, we could collect more information about the requirements of establishing a temporary factory in infected areas, and optimize the producing and delivery conditions. By comparing the conclusions with that of the importing case, we may end up with a different scheme.

- We assume that the unit of medicine production and delivery is counted in month.

But in reality, we can discuss the production cycle and delivery cycle to optimize the results.

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[8] $http : //maps.who.int/MapGallery/$

[9] http://www.google.cn/maps

[10] World Health organization.*Ebola Response Roadmap* 28 August 2014.

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[12] Qiyuan Jiang, Jinxing Xie, Jun Ye.*Mathematical Models* 4th Edition

[13] Zhonggeng Han, Xiaoli Guo, Jianping Du, Jingxu Liu, Yuan Tian*Analysis of American University students Mathematical Contest in Modeling* 2nd Edition

Appendices

Appendix A Matlab Code

Here are simulation programmes we used in our model as follow.

Natural Model:

```
ts=0:120;
x0=[0.000004375,0.999995625];
[t,x]=ode45('Guiill',ts,x0);
[t,x]
plot(t,x(:,1),t,x(:,2))
xlabel('i')
ylabel('s')
h=legend('infective rate','susceptible rate');
set(h,'Fontsize',15)
gtext('i(t)')
gtext('s(t)')
title('infective and susceptible rateline'),grid,pause,
plot(x(:,2),x(:,1)),grid,
```

Predicting Model:

```
%ts=0:100;
%x0=[0.000021801,0.999978199];
%[t,x]=ode45('ill',ts,x0);
%[t,x]
%[t,y]=ode45('ill2',ts+12,y0);
%plot(t,x(:,1),t,x(:,2),t,y(:,1),t,y(:,2))
%xlabel('i')
%ylabel('s')
%h=legend('infective rate','susceptible rateąŕ,ąŕinfective rate','susceptible rate');
%set(h,'Fontsize',20)
%gtext('i(t)')
%gtext('s(t)')
%title('infective and susceptible rateline'),grid,
%%plot(x(:,2),x(:,1)),grid,
ts=0:60;
```

```
x0=[0.000004375,0.999995625];
[t,x]=ode45('Gill',ts,x0);
[t,y]=ode45('Gill2',ts,x0);
%plot(t,x(:,1),t,x(:,2))
%tr=11:90;
%y0=[0.1235,0.8765];
%[t,y]=ode45('ill2',ts,y0);
plot(ts,x(:,1),ts,x(:,2),ts,y(:,1),ts,y(:,2))
%plot(ts,y(:,1),ts,y(:,2))
xlabel('time unit: month')
ylabel('rate')
h=legend('infective rate','susceptible rate','infective rate','susceptible rate');
set(h,'Fontsize',20)
gtext('natrual model')
gtext('prediction model')
gtext('natrual model')
gtext('prediction model')
title('Comparision between natrual model and prediction model in Guinea'),grid,
%plot(x(:,2),x(:,1)),grid,
```

Distribution Optimization:

```
for i=1:61

c=[1;1;1;1;1;1;1;1;1];
a=[1,0,0,1,0,0,1,0,0;0,1,0,0,1,0,0,1,0;0,0,1,0,0,1,0,0,1];
b=B(i,:)';
[x,y]=linprog(c,-a,-b,[],[],zeros(9,1))
X(i,:)=x'
Y(i,:)=y
end
```

Production Optimization:

```
function v = Obj(k,x,u,d,h,f,c)
if u==0,
t=0;
else
t=1;
```

```
end
v=C(k)*t+H(k)*u+F(k)*(x+u-d(k));
function [opt,fval]=prdcStoreProb(x0,x00,d,m,b,H,F,C,Decision Obj,Trans)
x=state(d,m,x0,x00);
[opt,fval]=dynprog(x0,x00,d,m,b,H,F,C,Decision,Obj,Trans)
function y = Trans(k,x,u,d)
y=x+u-d(k);
function x=state(d,m,x0,x00)
t=1;
n=length(d);
while t<=n;
g(t)=sum(d(t:n));
t=t+1;
end
tmax=max(g(2:n));
x=nan*ones(tmax+1,n);
x(1,1)=x0;
x(1,n)=x00;
tt=2;
while tt<=n-1;
t(tt)=min(m(tt),g(tt));
x(1:t(tt)+1,tt)=(0:t(tt))';
tt=tt+1;
end
function y = Trans(k,x,u,d)
y=x+u-d(k);
```

Ant Colony Optimization, the ACO model:

```
%function [R_best,L_best,L_ave,Shortest_Route,Shortest_Length]=ACATSP(C,NC_max,m,Alpha,Beta,Rho
m=40;
Alpha=1;
Beta=5;
Rho=0.2;
Q=10;
n=size(C,1);
D=zeros(n,n);
NC_max=100;
```

```
R_best=zeros(NC_max,n);
L_best=zeros(NC_max,1);
for i=1:n
for j=1:n
if i~=j
D(i,j)=(((C(i,1)-C(j,1))^2+(C(i,2)-C(j,2))^2)^0.5);
else
D(i,j)=eps;
D(j,i)=D(i,j);
end
end
Eta=1./D;
Tau=ones(n,n);
Tabu=zeros(m,n);
NC=1;
R_best=zeros(NC_max,n);
L_best=inf.*ones(NC_max,1);
L_ave=zeros(NC_max,1);
while NC<=NC_max
Randpos=[];
for i=1:(ceil(m/n))
Randpos=[Randpos,randperm(n)];
end
Tabu(:,1)=(Randpos(1,1:m))';
for j=2:n
for i=1:m
visited=Tabu(i,1:(j-1));
J=zeros(1,(n-j+1));
P=J;
Jc=1;
for k=1:n
if length(find(visited==k))==0
J(Jc)=k;
Jc=Jc+1;
end
end
for k=1:length(J)
P(k)=(Tau(visited(end),J(k))^Alpha)*(Eta(visited(end),J(k))^Beta);
```

```
end
P=P/(sum(P));
Pcum=cumsum(P);
Select=find(Pcum>=rand);
to_visit=J(Select(1));
Tabu(i,j)=to_visit;
end
end
if NC>=2
Tabu(1,:)=R_best(NC-1,:);
end
L=zeros(m,1);
for i=1:m
R=Tabu(i,:);
for j=1:(n-1)
L(i)=L(i)+D(R(j),R(j+1));
end
L(i)=L(i)+D(R(1),R(n));
end
L_best(NC)=min(L);
pos=find(L==L_best(NC));
R_best(NC,:)=Tabu(pos(1),:);
L_ave(NC)=mean(L);
NC=NC+1

Delta_Tau=zeros(n,n);
for i=1:m
for j=1:(n-1)
Delta_Tau(Tabu(i,j),Tabu(i,j+1))=Delta_Tau(Tabu(i,j),Tabu(i,j+1))+Q/L(i);

end
Delta_Tau(Tabu(i,n),Tabu(i,1))=Delta_Tau(Tabu(i,n),Tabu(i,1))+Q/L(i);
end
Tau=(1-Rho).*Tau+Delta_Tau;

Tabu=zeros(m,n);
end
```

```
Pos=find(L_best==min(L_best));
Shortest_Route=R_best(Pos(1),:)
Shortest_Length=L_best(Pos(1))
subplot(1,2,1)
DrawRoute(C,Shortest_Route)
subplot(1,2,2)
plot(L_best)
hold on
plot(L_ave,'r')
title('The shortest route')
```

Natural Model and Prediction Model Result Data Table for 60 months

	Sierra Leone			
	Natural		Prediction	
Time	i(t)	s(t)	i(t)	s(t)
0	2.18E-05	0.99998	2.18E-05	0.99998
1	4.71E-05	0.99995	4.71E-05	0.99995
2	0.0001	0.99988	0.0001	0.99988
3	0.00022	0.99974	0.00022	0.99974
4	0.00047	0.99944	0.00047	0.99944
5	0.00102	0.99879	0.00102	0.99879
6	0.00219	0.99738	0.00219	0.99738
7	0.00469	0.9944	0.00469	0.9944
8	0.00991	0.98812	0.00991	0.98812
9	0.02052	0.97532	0.0204	0.97533
10	0.04071	0.95069	0.0352	0.95239
11	0.07492	0.908	0.05708	0.9169
12	0.12351	0.84468	0.08562	0.86709
13	0.17858	0.76695	0.11728	0.80499
14	0.22835	0.68654	0.1464	0.73616
15	0.26558	0.6121	0.16865	0.66647
16	0.28947	0.54645	0.18222	0.60006
17	0.30244	0.48895	0.18761	0.53903
18	0.30728	0.4383	0.1865	0.48388
19	0.30648	0.39303	0.18049	0.43473
20	0.30166	0.35217	0.17096	0.39156
21	0.29391	0.31522	0.15926	0.35378
22	0.284	0.28177	0.14643	0.32071
23	0.27249	0.25153	0.13313	0.29196
24	0.25977	0.2243	0.11966	0.26746
25	0.24618	0.19995	0.10644	0.2468
26	0.232	0.17829	0.09393	0.22929
27	0.21748	0.1591	0.08232	0.21447
28	0.20284	0.1422	0.07172	0.20198
29	0.18827	0.12741	0.06214	0.19149
30	0.17395	0.11457	0.0536	0.18269
31	0.16005	0.10345	0.04605	0.17531
32	0.1467	0.09381	0.03942	0.16914
33	0.13399	0.08548	0.03365	0.16397
34	0.12197	0.07829	0.02866	0.15964
35	0.11071	0.07209	0.02436	0.15599
36	0.10021	0.06678	0.02066	0.15293
37	0.09049	0.06221	0.01749	0.15038
38	0.08154	0.05826	0.0148	0.14825
39	0.07333	0.05486	0.01251	0.14647
40	0.06584	0.05193	0.01057	0.14498
41	0.05903	0.04938	0.00892	0.14371
42	0.05285	0.04718	0.00752	0.14264
43	0.04726	0.04526	0.00634	0.14175
44	0.04222	0.0436	0.00534	0.14101
45	0.03768	0.04215	0.0045	0.14038
46	0.0336	0.04089	0.00379	0.13985
47	0.02994	0.03979	0.00319	0.13941
48	0.02667	0.03883	0.00268	0.13903
49	0.02373	0.03798	0.00226	0.13872
50	0.02111	0.03724	0.0019	0.13846
51	0.01877	0.03659	0.0016	0.13824
52	0.01669	0.03602	0.00134	0.13805
53	0.01483	0.03552	0.00113	0.1379
54	0.01317	0.03508	0.00095	0.13776
55	0.0117	0.03469	0.0008	0.13765
56	0.01039	0.03435	0.00067	0.13756
57	0.00922	0.03405	0.00056	0.13748
58	0.00818	0.03378	0.00047	0.13742
59	0.00726	0.03355	0.0004	0.13736
60	0.00644	0.03334	0.00033	0.13732

Liberia				
	Natural		Prediction	
Time	i(t)	s(t)	i(t)	s(t)
0	4.8E-06	1	4.8E-06	1
1	1.1E-05	0.99999	1.1E-05	0.99999
2	2.4E-05	0.99997	2.4E-05	0.99997
3	5.1E-05	0.99994	5.1E-05	0.99994
4	0.00012	0.99987	0.00012	0.99987
5	0.00026	0.9997	0.00026	0.9997
6	0.00057	0.99934	0.00057	0.99934
7	0.00126	0.99853	0.00126	0.99853
8	0.0028	0.99675	0.0028	0.99675
9	0.00615	0.99284	0.00615	0.99284
10	0.01332	0.98446	0.01332	0.98446
11	0.02798	0.96722	0.02798	0.96722
12	0.05548	0.93447	0.05421	0.9346
13	0.09983	0.88021	0.08343	0.8846
14	0.15741	0.80599	0.11692	0.82118
15	0.21582	0.72326	0.14816	0.75006
16	0.26351	0.64427	0.17219	0.67775
17	0.2966	0.57465	0.18695	0.60887
18	0.31665	0.51458	0.19308	0.54555
19	0.32673	0.46259	0.19209	0.48863
20	0.33003	0.41659	0.18589	0.43808
21	0.32856	0.37518	0.17618	0.39342
22	0.32371	0.33757	0.16424	0.35421
23	0.31637	0.30329	0.15085	0.32028
24	0.30718	0.27194	0.1368	0.29122
25	0.29654	0.24331	0.12278	0.26634
26	0.28478	0.21728	0.10919	0.24511
27	0.27213	0.19374	0.09633	0.22713
28	0.25881	0.17255	0.08435	0.21203
29	0.245	0.1536	0.07341	0.19935
30	0.23088	0.13676	0.06354	0.1887
31	0.21664	0.12191	0.05474	0.17978
32	0.20246	0.10888	0.04697	0.1723
33	0.18848	0.09747	0.04017	0.16605
34	0.17482	0.08751	0.03424	0.16081
35	0.1616	0.07886	0.02911	0.15643
36	0.1489	0.07136	0.0247	0.15279
37	0.13679	0.0649	0.02092	0.14977
38	0.12534	0.05932	0.0177	0.14723
39	0.11458	0.0545	0.01495	0.14509
40	0.10451	0.05034	0.01262	0.14329
41	0.09514	0.04673	0.01064	0.14179
42	0.08646	0.04361	0.00896	0.14054
43	0.07845	0.0409	0.00755	0.13949
44	0.07109	0.03856	0.00636	0.1386
45	0.06433	0.03651	0.00534	0.13786
46	0.05815	0.03474	0.00449	0.13724
47	0.05252	0.03319	0.00378	0.13672
48	0.04739	0.03183	0.00318	0.13628
49	0.04272	0.03064	0.00267	0.13592
50	0.03849	0.02959	0.00224	0.13561
51	0.03466	0.02868	0.00188	0.13535
52	0.03119	0.02787	0.00158	0.13513
53	0.02806	0.02716	0.00133	0.13495
54	0.02523	0.02653	0.00111	0.1348
55	0.02267	0.02597	0.00094	0.13467
56	0.02037	0.02548	0.00079	0.13456
57	0.01829	0.02505	0.00066	0.13448
58	0.01643	0.02466	0.00055	0.1344
59	0.01475	0.02432	0.00046	0.13434
60	0.01323	0.02401	0.00039	0.13428

Guinea				
	Natural		Prediction	
Time	i(t)	s(t)	i(t)	s(t)
0	4.4E-06	1	4.4E-06	1
1	6.9E-06	0.99999	6.9E-06	0.99999
2	1.1E-05	0.99999	1.1E-05	0.99999
3	1.7E-05	0.99998	1.7E-05	0.99998
4	2.7E-05	0.99997	2.7E-05	0.99997
5	4.3E-05	0.99994	4.3E-05	0.99994
6	6.9E-05	0.99991	6.9E-05	0.99991
7	0.00011	0.99986	0.00011	0.99986
8	0.00017	0.99977	0.00017	0.99977
9	0.00027	0.99964	0.00027	0.99964
10	0.00043	0.99943	0.00043	0.99943
11	0.00068	0.9991	0.00068	0.9991
12	0.00108	0.99857	0.00108	0.99857
13	0.00171	0.99774	0.00147	0.9978
14	0.00269	0.99643	0.002	0.99675
15	0.00423	0.99438	0.00271	0.99534
16	0.00663	0.99118	0.00367	0.99343
17	0.01033	0.98623	0.00495	0.99087
18	0.01594	0.97867	0.00666	0.98743
19	0.02427	0.96737	0.00892	0.98285
20	0.03621	0.95095	0.01186	0.97681
21	0.0526	0.92795	0.01565	0.96892
22	0.07372	0.89741	0.02044	0.95874
23	0.09895	0.85929	0.02636	0.94583
24	0.1265	0.81484	0.03345	0.9298
25	0.15406	0.76621	0.04166	0.91035
26	0.1792	0.71602	0.05078	0.88739
27	0.20027	0.66641	0.06043	0.86106
28	0.21663	0.61873	0.07014	0.83176
29	0.22816	0.57381	0.07937	0.80008
30	0.23525	0.53191	0.08754	0.76682
31	0.23861	0.493	0.09425	0.73275
32	0.23889	0.4569	0.09929	0.69862
33	0.23672	0.42336	0.10256	0.66506
34	0.23256	0.39227	0.10404	0.63258
35	0.22677	0.36358	0.10386	0.60156
36	0.21967	0.3372	0.10221	0.57227
37	0.21162	0.31291	0.09937	0.54485
38	0.20286	0.29058	0.09559	0.51937
39	0.1936	0.27009	0.09109	0.49585
40	0.18399	0.25136	0.08608	0.47426
41	0.17416	0.23432	0.08075	0.45456
42	0.16424	0.21886	0.07524	0.43668
43	0.15437	0.20485	0.0697	0.42049
44	0.14464	0.19216	0.06423	0.40588
45	0.13511	0.1807	0.05891	0.39273
46	0.12587	0.17037	0.0538	0.38092
47	0.11695	0.16108	0.04896	0.37035
48	0.1084	0.15273	0.04439	0.36092
49	0.10026	0.14523	0.04014	0.35251
50	0.09253	0.1385	0.03619	0.34501
51	0.08524	0.13246	0.03255	0.33835
52	0.07839	0.12705	0.02922	0.33242
53	0.07197	0.1222	0.02617	0.32717
54	0.06598	0.11784	0.02341	0.3225
55	0.06041	0.11394	0.0209	0.31836
56	0.05523	0.11045	0.01864	0.3147
57	0.05045	0.10731	0.0166	0.31146
58	0.04603	0.1045	0.01477	0.30859
59	0.04196	0.10197	0.01313	0.30605
60	0.03822	0.09971	0.01166	0.30381

The demand of medicine for each factory

time	d11	d12	d13	d21	d22	d23	d31	d32	d33	D1	D2	D3
0	2029.25	790.381	313.541	2029.25	790.381	313.541	2029.25	790.381	313.541	3133.17	3133.17	3133.17
1	4382.35	1252.01	697.792	4382.35	1252.01	697.792	4382.35	1252.01	697.792	6332.15	6332.15	6332.15
2	9455.64	1991.19	1557.34	9455.64	1991.19	1557.34	9455.64	1991.19	1557.34	13004.2	13004.2	13004.2
3	20371.1	3132.89	3392.15	20371.1	3132.89	3392.15	20371.1	3132.89	3392.15	26896.2	26896.2	26896.2
4	44025.6	4929.46	7596.97	44025.6	4929.46	7596.97	44025.6	4929.46	7596.97	56552	56552	56552
5	94815.3	7817.75	16922.8	94815.3	7817.75	16922.8	94815.3	7817.75	16922.8	119556	119556	119556
6	203929	12419.1	37526.6	203929	12419.1	37526.6	203929	12419.1	37526.6	253875	253875	253875
7	436111	19692.6	83418.6	436111	19692.6	83418.6	436111	19692.6	83418.6	539222	539222	539222
8	922824	31090.5	184492	922824	31090.5	184492	922824	31090.5	184492	1138407	1138407	1138407
9	1899179	49183	405498	1899179	49183	405498	1899179	49183	405498	2353861	2353861	2353861
10	3276120	77969.5	878433	3276120	77969.5	878433	3276120	77969.5	878433	4232522	4232522	4232522
11	5313479	123374	1845628	5313479	123374	1845628	5313479	123374	1845628	7282481	7282481	7282481
12	7969514	194269	3575483	7969514	194269	3575483	7969514	194269	3575483	1.2E+07	1.2E+07	1.2E+07
13	1.1E+07	265700	5502503	1.1E+07	265700	5502503	1.1E+07	265700	5502503	1.7E+07	1.7E+07	1.7E+07
14	1.4E+07	360914	7711386	1.4E+07	360914	7711386	1.4E+07	360914	7711386	2.2E+07	2.2E+07	2.2E+07
15	1.6E+07	489525	9771894	1.6E+07	489525	9771894	1.6E+07	489525	9771894	2.6E+07	2.6E+07	2.6E+07
16	1.7E+07	662606	1.1E+07	1.7E+07	662606	1.1E+07	1.7E+07	662606	1.1E+07	2.9E+07	2.9E+07	2.9E+07
17	1.7E+07	894478	1.2E+07	1.7E+07	894478	1.2E+07	1.7E+07	894478	1.2E+07	3.1E+07	3.1E+07	3.1E+07
18	1.7E+07	1203087	1.3E+07	1.7E+07	1203087	1.3E+07	1.7E+07	1203087	1.3E+07	3.1E+07	3.1E+07	3.1E+07
19	1.7E+07	1610789	1.3E+07	1.7E+07	1610789	1.3E+07	1.7E+07	1610789	1.3E+07	3.1E+07	3.1E+07	3.1E+07
20	1.6E+07	2143033	1.2E+07	1.6E+07	2143033	1.2E+07	1.6E+07	2143033	1.2E+07	3E+07	3E+07	3E+07
21	1.5E+07	2827126	1.2E+07	1.5E+07	2827126	1.2E+07	1.5E+07	2827126	1.2E+07	2.9E+07	2.9E+07	2.9E+07
22	1.4E+07	3692091	1.1E+07	1.4E+07	3692091	1.1E+07	1.4E+07	3692091	1.1E+07	2.8E+07	2.8E+07	2.8E+07
23	1.2E+07	4761733	9949188	1.2E+07	4761733	9949188	1.2E+07	4761733	9949188	2.7E+07	2.7E+07	2.7E+07
24	1.1E+07	6043115	9022634	1.1E+07	6043115	9022634	1.1E+07	6043115	9022634	2.6E+07	2.6E+07	2.6E+07
25	9907528	7526633	8097393	9907528	7526633	8097393	9907528	7526633	8097393	2.6E+07	2.6E+07	2.6E+07
26	8743385	9174395	7201583	8743385	9174395	7201583	8743385	9174395	7201583	2.5E+07	2.5E+07	2.5E+07
27	7662756	1.1E+07	6353328	7662756	1.1E+07	6353328	7662756	1.1E+07	6353328	2.5E+07	2.5E+07	2.5E+07
28	6675426	1.3E+07	5563397	6675426	1.3E+07	5563397	6675426	1.3E+07	5563397	2.5E+07	2.5E+07	2.5E+07
29	5784338	1.4E+07	4841658	5784338	1.4E+07	4841658	5784338	1.4E+07	4841658	2.5E+07	2.5E+07	2.5E+07
30	4989266	1.6E+07	4190648	4989266	1.6E+07	4190648	4989266	1.6E+07	4190648	2.5E+07	2.5E+07	2.5E+07
31	4286178	1.7E+07	3610252	4286178	1.7E+07	3610252	4286178	1.7E+07	3610252	2.5E+07	2.5E+07	2.5E+07
32	3669472	1.8E+07	3098022	3669472	1.8E+07	3098022	3669472	1.8E+07	3098022	2.5E+07	2.5E+07	2.5E+07
33	3132610	1.9E+07	2649247	3132610	1.9E+07	2649247	3132610	1.9E+07	2649247	2.4E+07	2.4E+07	2.4E+07
34	2668118	1.9E+07	2258355	2668118	1.9E+07	2258355	2668118	1.9E+07	2258355	2.4E+07	2.4E+07	2.4E+07
35	2267314	1.9E+07	1920055	2267314	1.9E+07	1920055	2267314	1.9E+07	1920055	2.3E+07	2.3E+07	2.3E+07
36	1922703	1.8E+07	1629041	1922703	1.8E+07	1629041	1922703	1.8E+07	1629041	2.2E+07	2.2E+07	2.2E+07
37	1628108	1.8E+07	1379980	1628108	1.8E+07	1379980	1628108	1.8E+07	1379980	2.1E+07	2.1E+07	2.1E+07
38	1377441	1.7E+07	1167517	1377441	1.7E+07	1167517	1377441	1.7E+07	1167517	2E+07	2E+07	2E+07
39	1164697	1.6E+07	986328	1164697	1.6E+07	986328	1164697	1.6E+07	986328	1.9E+07	1.9E+07	1.9E+07
40	983963	1.6E+07	832125	983963	1.6E+07	832125	983963	1.6E+07	832125	1.7E+07	1.7E+07	1.7E+07
41	830140	1.5E+07	701494	830140	1.5E+07	701494	830140	1.5E+07	701494	1.6E+07	1.6E+07	1.6E+07
42	699839	1.4E+07	591176	699839	1.4E+07	591176	699839	1.4E+07	591176	1.5E+07	1.5E+07	1.5E+07
43	589832	1.3E+07	498042	589832	1.3E+07	498042	589832	1.3E+07	498042	1.4E+07	1.4E+07	1.4E+07
44	497079	1.2E+07	419167	497079	1.2E+07	419167	497079	1.2E+07	419167	1.3E+07	1.3E+07	1.3E+07
45	418720	1.1E+07	352493	418720	1.1E+07	352493	418720	1.1E+07	352493	1.1E+07	1.1E+07	1.1E+07
46	352418	9719931	296354	352418	9719931	296354	352418	9719931	296354	1E+07	1E+07	1E+07
47	296530	8844405	249172	296530	8844405	249172	296530	8844405	249172	9390107	9390107	9390107
48	249531	8020331	209472	249531	8020331	209472	249531	8020331	209472	8479335	8479335	8479335
49	210003	7251177	175968	210003	7251177	175968	210003	7251177	175968	7637148	7637148	7637148
50	176652	6537935	147773	176652	6537935	147773	176652	6537935	147773	6862360	6862360	6862360
51	148524	5880570	124110	148524	5880570	124110	148524	5880570	124110	6153204	6153204	6153204
52	124879	5278051	104259	124879	5278051	104259	124879	5278051	104259	5507190	5507190	5507190
53	105028	4728352	87558.2	105028	4728352	87558.2	105028	4728352	87558.2	4920938	4920938	4920938
54	88334.9	4228608	73492.5	88334.9	4228608	73492.5	88334.9	4228608	73492.5	4390435	4390435	4390435
55	74252.3	3775831	61686.5	74252.3	3775831	61686.5	74252.3	3775831	61686.5	3911770	3911770	3911770
56	62404.3	3366951	51792.7	62404.3	3366951	51792.7	62404.3	3366951	51792.7	3481148	3481148	3481148
57	52459.8	2998820	43491.6	52459.8	2998820	43491.6	52459.8	2998820	43491.6	3094771	3094771	3094771
58	44112.8	2668226	36505.6	44112.8	2668226	36505.6	44112.8	2668226	36505.6	2748844	2748844	2748844
59	37085.3	2371883	30639.7	37085.3	2371883	30639.7	37085.3	2371883	30639.7	2439608	2439608	2439608
60	31170.8	2106598	25716.3	31170.8	2106598	25716.3	31170.8	2106598	25716.3	2163485	2163485	2163485

Production	**Demand**
3394	31.3317
3426	63.3215
3492	130.042
3631	268.962
3927	565.52
4557	1195.56
83853	2538.75
91399	5392.22
99624	11384.1
108590	23538.6
118363	42325.2
129015	72824.8
140626	117393
153282	166851
167077	216992
182113	259594
198503	289799
216368	306868
235841	312964
257066	310793
280201	303161
321982	292711
309697	281543
298133	271030
288244	262040
280847	255316
276313	251194
274267	249334
274013	249103
274612	249649
274932	249940
274159	249236
271766	247060
267411	243101
260953	237229
252451	229501
242188	220171
230561	209602
217949	198136
204680	186073
191046	173679
177312	161193
163720	148837
150474	136794
137711	125192
125545	114132
114055	103687
103291	93901.1
93272	84793.3
84008	76371.5
75486	68623.6
61532	61532
55072	55071.9
49209	49209.4
43904	43904.4
39118	39117.7
34812	34811.5
30944	30947.7
27488	27488.4
24396	24396.1
21635	21634.9

The geographic coordinates and shortest route of Guinea						
		W	X	Y		Shortest Route
1	YOMOU	2	7.56635	-9.2536		26
2	DABOLA	2	10.7507	-11.117		10
3	KOUROUSSA	2	10.6566	-9.8999		8
4	SIGUIRI	3	11.4272	-9.1552		12
5	FARANAH	3	10.0334	-10.734		9
6	KISSIDOUGO	3	9.18183	-10.088		1
7	KANKAN	3	10.6483	-9.4059		13
8	BEYLA	3	8.6814	-8.6353		11
9	LOLA	3	7.79803	-8.4651		6
10	KEROUANE	4	9.26679	-9.0169		5
11	GUECKEDOU	4	8.54317	-10.123		2
12	NZEREKORE	4	7.838	-8.801		18
13	MACENTA	5	8.55008	-9.4652		15
14	DALABA	2	10.6896	-12.257		14
15	PITA	2	11.0598	-12.395		20
16	MALI	1	17.7624	-1.7888		17
17	BOKE	1	10.9381	-14.302		21
18	TOUGUE	1	11.4356	-11.666		19
19	FRIA	1	10.368	-13.584		22
20	TELIMELE	3	10.9002	-13.033		25
21	BOFFA	3	10.1996	-14.051		24
22	DUBREKA	3	9.79285	-13.525		23
23	KINDIA	3	10.0692	-12.851		3
24	FORECARIAH	3	9.43212	-13.06		7
25	COYAH	4	9.71582	-13.344		4
26	CONAKRY	4	9.34	13.36		16

The geographic coordinates and shortest route of Liberia						
		W	X	Y		Shortest Route
1	GRAND GEDEH	1	-26.699	19.876		15
2	GRAND KUR	1	-26.789	15.75		14
3	MARYLAND	1	-25.329	14.385		9
4	SINOE	2	-29.186	16.684		7
5	GBARPOLU	2	-34.097	24.902		4
6	RIVER GEE	2	-25.482	17.328		2
7	RIVERCESS	3	-31.429	17.931		3
8	LOFA	3	-37.314	22.133		6
9	GRANDBASSA	3	-32.952	19.285		1
10	LOFA	4	-31.979	27.617		12
11	BONG	4	-31.078	22.956		11
12	NIMBA	4	-28.465	24.172		10
13	BOMI	4	-35.484	22.542		5
14	MARGIBI	4	-33.944	21.417		13
15	MONTSERRADO	5	-34.809	21.095		8

Figure 33: The geographic coordinates and shortest route of Liberia