

Prove that for any natural number n ,

$$2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$$

Proof: by mathematical induction

Let $A(n)$ be the equality relation $2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$.

For $n = 1$, $A(1)$ is the equality $2 = 2^2 - 2$, which obviously holds.

To proof $\forall n[A(n) \Rightarrow A(n+1)]$, pick an arbitrary n and assume equality $A(n)$ holds:

$$2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$$

and deduce $A(n+1)$ also holds:

$$2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} = 2^{n+2} - 2$$

The left hand side of the equation:

$$\begin{aligned} 2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} &= (2 + 2^2 + 2^3 + \cdots + 2^n) + 2^{n+1} \\ &= 2^{n+1} - 2 + 2^{n+1} \text{ (by induction assumption)} \\ &= 2(2^{n+1}) - 2 \\ &= 2^{n+2} - 2 \end{aligned}$$

This is equal to the right hand side of the equation, so $A(n+1)$ holds. Therefore, the required is proved by principle of mathematical induction.