Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Proof:

By Division Theorem, given an arbitrary natural number n, there exist unique integers q and r such that $n = 4q + r$ and $0 \leq r < 4$. For $r \in \mathcal{Z}$, r can only take one of four different values 0, 1, 2 and 3. Therefore n can be expressed in one of the four forms:

$$4n + 0$$

$$4n + 1$$

$$4n + 2$$

$$4n + 3$$

For $4n+0$ and $4n+2$, they can both be expressed as $2k, k \in \mathcal{Z}$ ($4n = 2k, k = 2n$ and $4n + 2 = 2k, k = (2n + 1)$). Therefore n is even when in one of these two forms.
For $4n + 1$ and $4n + 3$, they can both be expressed as $2k + 1, k \in \mathcal{Z}$ ($4n + 1 = 2k + 1, k = 2n$ and $4n + 3 = 2k + 1, k = (2n + 1)$). Therefore n is odd when in one of these two forms.
So, for any natural number n, if it's odd, it can be expressed as one of and only one of the forms $4n + 1$ and $4n + 3$ as required.