For any integer n, the number $n^2 + n + 1$ is odd.

The statement is true.

Proof:
Given an arbitrary integer n, it's either even or odd.
if n is even, it can be expressed as $n = 2k, k \in \mathcal{Z}$. Substitute into $n^2 + n + 1$ gives $4k^2 + 2k + 1 = 2(2k^2 + k) + 1$. Let $k' = 2k^2 + k$, the formula can be expressed as $2k' + 1, k \in \mathcal{Z}$ which is odd as required.

if n is odd, it can be expressed as $n = 2k + 1, k \in \mathcal{Z}$. Substitute into $n^2 + n + 1$ gives $4k^2 + 6k + 3 = 2(2k^2 + 3k + 1) + 1$. Let $k' = 2k^2 + 3k + 1$, the formula can be expressed as $2k' + 1, k \in \mathcal{Z}$ which is odd as required.

$n^2 + n + 1$ is odd when n is odd and when n is even, so for any integer n, it is odd as required.