$A_n, n = 1, 2, \ldots$ is a family of intervals. When $A_n = (0, \frac{1}{n})$, $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \emptyset$.

Prove $A_{n+1} \subset A_n$ for all n.

Let n be an arbitrary natural number, then $A_n = (0, \frac{1}{n})$ and $A_{n+1} = (0, \frac{1}{n+1})$. Let a be an arbitrary element in $(0, \frac{1}{n+1})$, then $0 < a < \frac{1}{n+1}$. Since $n+1 > n > 0$, $\frac{1}{n} > \frac{1}{1+n}$. Therefore $0 < a < \frac{1}{n+1} < \frac{1}{n}$, which means a is in $(0, \frac{1}{n})$. So for any element in $(0, \frac{1}{n+1})$ is also in $(0, \frac{1}{n})$, meaning $A_{n+1} \subseteq A_n$.
Let b be a real number such that $\frac{1}{n+1} \leq b < \frac{1}{n}$. b is not in A_{n+1} but in A_n, so $A_{n+1} \subset A_n$. This proves $\forall n[A_{n+1} \subset A_n]$ as required.

Prove $\bigcap_{n=1}^{\infty} A_n = \emptyset$.

Let $\{a_n\}_{n=1}^{\infty}$ be an infinite real series that $a_n = \frac{1}{n}$.
Let ϵ be an arbitrary real number and $\epsilon > 0$. Let n be a natural number so that $n > \frac{1}{\epsilon}$, and m an arbitrary natural number so that $m \geq n$. Then we have

$$\begin{aligned} &|\frac{1}{m} - 0| \\ =&\frac{1}{m} \\ \leq&\frac{1}{n} \text{ (because } m \geq n > 0) \\ <&\epsilon \text{ (because } n > \frac{1}{\epsilon} > 0) \end{aligned}$$

Therefore, we have

$$(\forall \epsilon > 0)(\exists n \in \mathcal{N})(\forall m >= n)[|\frac{1}{m} - 0| < \epsilon]$$

This is the definition of $\lim_{n\to\infty} a_n = 0$.

So $a_n = \frac{1}{n} \to 0$ as $n \to \infty$, and $A_n = (0, \frac{1}{n}) \to (0, 0)$ as $n \to \infty$. $(0, 0)$ is the set $\{x \in \mathcal{R} | 0 < x < 0\}$, which is obviously empty. Therefore, $A_n \to \emptyset$ as $n \to \infty$.
$\bigcap_{n=1}^{\infty} A_n$ is defined as the set $\{x \mid (\forall n)(x \in A_n)\}$, which is the set of elements that are in all of $A_1, A_2, A_3, \ldots$. Because A_n tends to $\emptyset$ as $n \to \infty$ and there is no element in $\emptyset$, then there is no element $\bigcap_{n=1}^{\infty} A_n$. Therefore $\bigcap_{n=1}^{\infty} A_n = \emptyset$ as required.