$A_n, n = 1, 2, \ldots$ is a family of intervals. When $A_n = [0, \frac{1}{n}]$, $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number.

Prove $A_{n+1} \subset A_n$ for all n.

Let n be an arbitrary natural number, then $A_n = [0, \frac{1}{n}]$ and $A_{n+1} = [0, \frac{1}{n+1}]$. Let a be an arbitrary element in $[0, \frac{1}{n+1}]$, then $0 \leq a \leq \frac{1}{n+1}$. Since $n+1 > n > 0$, we have $\frac{1}{n} > \frac{1}{1+n}$. Therefore $0 \leq a \leq \frac{1}{n+1} < \frac{1}{n} \leq \frac{1}{n}$, which means a is in $[0, \frac{1}{n}]$. So for any element in $[0, \frac{1}{n+1}]$ is also in $[0, \frac{1}{n}]$, meaning $A_{n+1} \subseteq A_n$.
Let b be a real number such that $\frac{1}{n+1} < b \leq \frac{1}{n}$. b is not in A_{n+1} but in A_n, so $A_{n+1} \subset A_n$. This proves $\forall n[A_{n+1} \subset A_n]$ as required.

Prove $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number.

Let $\{a_n\}_{n=1}^{\infty}$ be an infinite real series that $a_n = \frac{1}{n}$.
Let ϵ be an arbitrary real number and $\epsilon > 0$. Let n be a natural number so that $n > \frac{1}{\epsilon}$, and m an arbitrary natural number so that $m \geq n$. Then we have

$$\begin{aligned}
&|\frac{1}{m} - 0| \\
=&\frac{1}{m} \\
\leq&\frac{1}{n} \text{ (because } m \geq n > 0) \\
<&\epsilon \text{ (because } n > \frac{1}{\epsilon} > 0)
\end{aligned}$$

Therefore, we have

$$(\forall \epsilon > 0)(\exists n \in \mathcal{N})(\forall m >= n)[|\frac{1}{m} - 0| < \epsilon]$$

This is the definition of $\lim\limits_{n \to \infty} a_n = 0$.

So $a_n = \frac{1}{n} \to 0$ as $n \to \infty$, and $A_n = [0, \frac{1}{n}] \to [0, 0]$ as $n \to \infty$. $[0, 0]$ is the set $\{x \in \mathcal{R} | 0 \leq x \leq 0\}$, which obviously contains a single element 0. Therefore, $A_n \to \{0\}$ as $n \to \infty$.

$\bigcap_{n=1}^{\infty} A_n$ is defined as the set $\{x | (\forall n)(x \in A_n)\}$, which is the set of elements that are in all of $A_1, A_2, A_3, \ldots$. Because A_n tends to $\{0\}$ as $n \to \infty$ and there is one single real number 0 in $\{0\}$, then 0 will be the only element in $\bigcap_{n=1}^{\infty} A_n$ if 0 is in all of A_n.

To prove 0 is in all of $A_1, A_2, A_3, \ldots$, let n be an arbitrary natural number. $A_n = [0, \frac{1}{n}]$ and $0 \leq \frac{1}{n}$, we have $0 \leq 0 \leq \frac{1}{n}$. Therefore, $0 \in [0, \frac{1}{n}]$ for arbitrary natural number n. This proves $(\forall n \in \mathcal{N})[0 \in A_n]$.

Therefore $\bigcap_{n=1}^{\infty} A_n = \emptyset$ as required.