Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof:

Since $a_n \to L$ as $n \to \infty$, we have the following true statement by the definition of limit of a sequence:

$$(\forall \epsilon > 0)(\exists n \in \mathcal{N})(\forall m \geq n)\,[|a_m - L| < \epsilon] \tag{1}$$

Let ϵ be an arbitrary real number and $\epsilon > 0$. Let $\epsilon' = \frac{1}{M}\epsilon$.
Since $M > 0$, ϵ' is also a real number and $\epsilon' > 0$. Therefore, by statement (1), there exists a natural number n such that

$$\begin{aligned} &(\forall m \geq n)\,[|a_m - L| < \epsilon'] \\ \Rightarrow &(\forall m \geq n)\left[|a_m - L| < \frac{1}{M}\epsilon\right] \end{aligned} \tag{2}$$

Let m be an arbitrary natural number and $m \geq n$, we have

$$\begin{aligned} |a_m - L| &< \frac{1}{M}\epsilon \text{ (by 2)} \\ M\,|a_m - L| &< M\frac{1}{M}\epsilon \text{ (mutiply both side by } M,\ M > 0) \\ |Ma_m - ML| &< \epsilon \end{aligned} \tag{3}$$

Therefore, for an arbitrary positive real number ϵ, there exists a natural number n such that, for an arbitrary natural number m and $m \geq n$ (3) is true, meaning

$$(\forall \epsilon > 0)(\exists n \in \mathcal{N})(\forall m \geq n)\,[|Ma_m - ML| < \epsilon]$$

This is the definition of of sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML as $n \to \infty$, as required.