Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7

Proof:

Given an arbitrary prime number n and $n \neq 3$, we test whether $n+2$ and $n+4$ can both be prime numbers.
Examine the even and odd cases for n.
If n is even, it can only be 2. It is obvious that $n+2=4$ and $n+4=6$ are not primes. Therefore, when n is even, there is no prime triples.
If n is odd, then n is not a multiple of 3, since the only prime divisible by 3 is 3 and $n \neq 3$. Therefore by Division Theorem, n can be expressed as one of $3k+1$ and $3k+2$, $k \in \mathcal{Z}$. Examine these two possibilities:
if $n = 3k+1$, then $n+2 = 3k+3 = 3(k+1)$, which is divisible by 3 and not equal to 3. Therefore $n+2$ is not prime in this case;
if $n = 3k+2$, then $n+4 = 3k+6 = 3(k+2)$, which is divisible by 3 and not equal to 3. Therefore $n+4$ is not prime in this case.
Therefore, for all possible cases of prime n and $n \neq 3$, at least one of $n+2$ and $n+4$ is not prime. This makes 3, 5, 7 the only prime triple, as required.