# IMAGE MULTI-SCALE EDGE DETECTION USING 3-D HIDDEN MARKOV MODEL BASED ON THE NON-DECIMATED WAVELET

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## ABSTRACT

Edge detection plays an important role in digital image processing. Based on the non-decimated wavelet which is shift-invariant, in this paper, we develop a new edge detecting technique using 3-D Hidden Markov Model. Our proposed model can not only capture the relationship of the wavelet coefficients inter-scale, but also consider the intra-scale dependence. A computationally efficient maximum likelihood (ML) estimation algorithm is employed to compute parameters and the hidden state of each coefficient is revealed by maximum a posteriori (MAP) estimation. Experimental results of natural images are provided to evaluate the algorithm. In addition, the proposed model has the potential to be an efficient multi-scale statistical modeling tool for other image or video processing tasks.

***Index Terms***— Edge detection, non-decimated wavelet, 3-D NWHMM, inter-scale, intra-scale

## 1. INTRODUCTION

Edges are one of the most important elements in image analysis and computer vision, because they play quite a significant role in many applications of image processing, in particular for machine vision. A lot of computer vision methods rely on edge detection as a pre-processing stage. However, no single edge detection algorithm can successfully discover edges for diverse images and no specific quantitative measure of the quality for edge detection is widely adopted at present.

Conventional edge detection mechanisms examine the image pixels for abrupt changes compared to their neighborhoods. This is often done by detecting the maximal value of grads, such as Roberts, Prewitt, Sobel, and Canny, all of which are classical edge detection methods. Alternatively, we can detect the zero-crossing points to find edge maps. Laplacian and Mar algorithms are based on the idea. Mallat established the edge detection technique in a multi-scale manner using the dyadic wavelet transform. Recently, many multi-scale transform tools based on statistics have been developed. As we know, transform coefficients have statistical properties because of their dependencies intra-scale and inter-scale. In order to fuse the multi-scale wavelet information, Hidden Markov Tree (HMT) model [1] based on the wavelet transform was developed, which pioneered a new research area on multi-scale statistical signal processing.

After that, many researchers applied and improved this HMT model, such as local contextual hidden markov model (LCHMM) [2], a new four-state HMT model called HMT-2 [3], contextual hidden markov model (CHMM) [4] and so on. These HMT-based models have been applied to various aspects in image analysis, including denoising, compression, fusion, segmentation, searching and so on. Edge detection algorithms based on the HMT model have also been developed [5]-[7]. In addition, new multi-scale tools, for example, Complex wavelet, Ridgelet, Contourlet, and Curvelet were developed to tackle different problems of wavelet.

In [8], we developed an edge detection method using Hidden Markov Chain (HMC) model based on the non-decimated wavelet which is shift-invariant and able to locate edges with good precision. But the HMC model only captured the inter-scale dependencies of the wavelet coefficients. Inspired by the non-decimated wavelet and three-dimensional Hidden Markov Model (3-D HMM) [9], we propose a new multi-scale statistical model 3-D NWHMM in this paper. The model can fuse both inter and intra-scale dependencies of the wavelet coefficients efficiently. Edge detection results for natural images in section 3.2 show the efficiency of the 3-D NWHMM. Moreover, the model has a potential application in other image and video processing tasks.

## 2. THREE-DIMENSIONAL HMM BASED ON THE NON-DECIMATED WAVELET (3-D NWHMM)

### 2.1. 3-D Hidden Markov Model (3-D HMM)

Over the past years, HMMs have been explored as efficient statistical modeling techniques in speech, image and video understanding, because observed signals can be modeled through a first order Markov process. In this section, we introduce the three-dimensional Hidden Markov Model (3-D HMM) in its general form. A graphical illustration for the model is in Fig 1. Assume a 3-D point *(i,j,k)*, where *i,j,k* are coordinates along the *X*,*Y* and *Z* axes respectively. A lexicographic order of the 3-D point is defined as below:

$$\begin{aligned}(i',j',k') < (i,j,k), if\ k' < k \text{ or } k' = k, j' < j \\ \text{or } k' = k, j' = j, i' < i\end{aligned} \quad . \tag{1}$$

We denote the observed signal value at the point *(i,j,k)* by $u_{i,j,k}$ and its hidden state by $s_{i,j,k}$. The model attempts to capture the statistical dependencies among 3-D points through the state transition probability:

$$P\{s_{i,j,k}=l|(s_{i',j',k'},u_{i',j',k'}):(i',j',k')<(i,j,k)\}=a_{p,m,n,l}, \quad (2)$$

where $(i',j',k')$ are all points preceding *(i,j,k)* in the lexicographic order. Because of the Markov property, there are three preceding points $p_1,p_2,p_3$ that affect the state of point *(i,j,k)* as shown in Fig 1. The hidden states of the three neighboring points are $s_{i-1,j,k}=p$, $s_{i,j-1,k}=m$, $s_{i,j,k-1}=n$ separately.

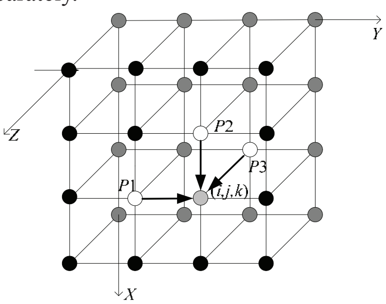


Fig 1. 3-D HMM graphical illustration. $p_1,p_2,p_3$ are three neighboring points that affect the state at *(i,j,k)*

**2.2. Construction of The 3-D NWHMM**

The discrete wavelet transform (DWT) has been developed for a long time and applied widely in signal processing. However, we know that the traditional DWT suffers a drawback: it is not a time-invariant transform. As a result, the non-decimated wavelet which is shift-invariant is designed. As to the transform, the signal is never sub-sampled and instead the filters are up-sampled at each level of decomposition. It is an inherently redundant scheme. These properties are useful for several applications such as image denoising, point detection and so on.

Suppose the non-decimated wavelet is performed at $T$ levels, then we can have the wavelet coefficients at $T$ different scales. As to each wavelet coefficient at the point *(i,j,k)* at a certain scale $t$, $t=1,2,\ldots T$, there are three neighboring points: the two points *(i-1,j,k)* *(i,j-1,k)* consider the intra-scale dependence and the point *(i,j,k-1)* capture the inter-scale dependence with the coefficient at *(i,j,k)*. Fig 2 shows the structure of the 3-D NWHMM.

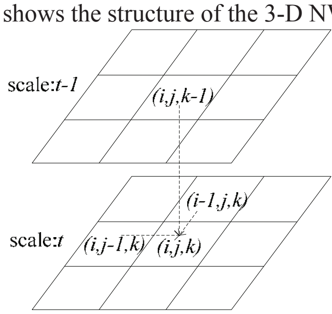


Fig 2. The structure of the 3-D NWHMM

**2.3. Probabilistic Model for Individual Wavelet Coefficients**

The wavelet coefficients can be treated as random realizations, thus, they can be represented by probabilistic models. Suppose they have two hidden states: the "small" one $s=1$ and the "big" one $s=2$. There exists a classical zero-mean, two-state Gaussian Mixture Model to represent each coefficient $x$. Therein, the hidden state $s=2$ corresponds to a high-variance Gaussian probability density function (PDF) and $s=1$ corresponds to a low-variance Gaussian PDF. Consequently, we model the coefficients which are jointly Gaussian in this way:

$$f(x)=\sum_{m=1}^{2} f(x|s=m)p(s=m). \quad (3)$$

However, the model is in conflict with the compression property, which dictates the sparse distribution of wavelet coefficients. That is to say, only a few coefficients contain most energy and others contribute little to signal's energy, as a result, PDF of the wavelet coefficients should be highly concentrated around zero and heavy-tailed at both sides, all of which can not be expressed using the previous Gaussian models. General Gaussian Distribution (GGD) can represent peak around zero and long heavy tail approximately [10], but parameter estimation is too complex. In paper [10], it proved that for "big" coefficients, the GGD fit is approximately a special case, Laplacian model. As a result, we use individual Gaussian model for "small" coefficients and Laplacian model for "big" coefficients to simplify the GGD fit. They form a new zero-mean, two-state mixture model which can express the wavelet coefficients precisely and easily. We denote each wavelet coefficient value at the point *(i,j,k)* by $u_{i,j,k}$ and its hidden state by $s_{i,j,k}$, then the probabilistic model for each coefficient will be:

PDF of "small" states:

$$f(u|s=1)=\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{u^2}{2\sigma^2}). \quad (4)$$

PDF of "big" states:

$$f(u|s=2)=\frac{1}{\sqrt{2}\xi}\exp(-\frac{\sqrt{2}|u|}{\xi}). \quad (5)$$

**2.4. Training Parameters and Searching for Hidden States**

Suppose the images used are of size $w\times w$ and the number of the non-decimated wavelet decomposition is $T$, then we have $C=\{(i,j,k):0\le i<w, 0\le j<w, 1<k\le T\}$. As to the 3-D NWHMM model, state transition probability $a_{p,m,n,l}$, $p,m,n,l=1,2$ and the parameters $\sigma$, $\xi$ of the PDF for the two states constitute the parameter set $\psi$ that need to be estimated. Generally, an iterative expectation maximization (EM) approach was employed, which had high computational complexity. In order to reduce the complexity, we apply the direct ML estimation proposed in paper [9] to estimate $\psi$.

The parameters are computed by assuming that the sates $s^*_{i,j,k}$ are the true underlying states. If the true states were known, the ML estimation of the parameters would be easy to obtain in (6)-(8). $I(\cdot)$ is the indicator function that equals to 1 when the argument is true and 0 otherwise.

variance of "small" states:

$$\sigma^2 = \frac{\sum_{(i,j,k)\in C} u_{i,j,k}^2 I(s_{i,j,k}^* = 1)}{\sum_{(i,j,k)\in C} I(s_{i,j,k}^* = 1)}, \quad (6)$$

shape parameter of "big" states:

$$\xi = \sqrt{2}\frac{\sum_{(i,j,k)\in C} \left|u_{i,j,k}\right| I(s_{i,j,k}^* = 2)}{\sum_{(i,j,k)\in C} I(s_{i,j,k}^* = 2)}, \quad (7)$$

transition probability:

$$a_{p,m,n,l} = \frac{\sum_{(i,j,k)\in C} I(s_{i,j,k}^* = l) I(s_{i-1,j,k}^* = p) I(s_{i,j-1,k}^* = m) I(s_{i,j,k-1}^* = n)}{\sum_{(i,j,k)\in C} I(s_{i-1,j,k}^* = p) I(s_{i,j-1,k}^* = m) I(s_{i,j,k-1}^* = n)}. \quad (8)$$

Various methods can be used to initialize the true underlying states. K-means clustering algorithm is applied to generate the initial states $s_{i,j,k}^*$. We classify the coefficients into two classes using clustering algorithm and then the iteration starts with the ML estimation.

After the parameters were estimated, we can obtain the optimal hidden states with the MAP probability. A locally optimal algorithm is adopted to search for the set of states. The points at the same position of different scales: $R_{i,j} = \{(i,j,k): 1 \le k \le T\}$ form a sequence of states $s_{i,j} = \{s_{i,j,1}, s_{i,j,2}, ..., s_{i,j,T}\}$ ,observed coefficients value $u_{i,j} = \{u_{i,j,1}, u_{i,j,2}, ..., u_{i,j,T}\}$. The procedure to iteratively update the states is in the lexicographic order as follow:

(1) Let the iteration number *r=1*.

(2) Let the point coordinate *i=0,j=0*.

(3) Given the observed coefficient in the current point and the hidden states in all the other sequences $s_{i'j'}^{r-1}, (i',j') < (i,j)$, $s_{i''j''}^{r-1}, (i,j) < (i'',j'')$, we can acquire the states $s_{i,j,k}^r$ with MAP approach as follow:

*a.* Let $\theta_l(0) = 0, l = 1,2$. Then for *k=1,2,…T*,

$$\theta_l(k) = \max_{m=1,2}(\theta_m(k-1) + \lambda_{m,l}(k)), \text{ where}$$

$$\lambda_{m,l}(k) = \log f_l(u_{i,j,k}) + \log a_{m, s_{i-1,j,k}, s_{i,j-1,k}, l} + \log a_{s_{i,j+1,k-1}, s_{i-1,j+1,k}, l, s_{i,j+1,k}} + \log a_{s_{i+1,j,k-1}, l, s_{i+1,j-1,k}, s_{i+1,j,k}}$$

$$\eta_l(k) = \arg\max_{m=1,2}(\theta_m(k-1) + \lambda_{m,l}(k)).$$

*b.* Let $s_{i,j,T}^r = \arg\max_{l=1,2} \theta_l(T)$. For *k=T-1,…1*,

$$s_{i,j,k}^r = \eta_{s_{i,j,k+1}^r}(k+1).$$

(4) Let $j+1 \to j$.

(5) If $j<w$, go to step (3). Otherwise

*a.* Let $j=0, i+1 \to i$;

*b.* If $i<w$, go to step (3), otherwise

*1.* Let $r+1 \to r$;

*2.* If stopping criterion is achieved, stop; else go to step (3).

From the searched optimal hidden states, we can obtain the corresponding edge and non-edge parts.

## 3. EXPERIMENTAL RESULTS AND ANALYSIS

### 3.1. The Algorithm Implementation

Our proposed method for multi-scale edge detection is based on the 3-D NWHMM. We use the three-scale Haar wavelet to transform images because of its efficiency in edge detection. The wavelet coefficients are in two high subbands: horizontal (LH), and vertical (HL). For the searched hidden states using the 3-D NWHMM in each subband, we adopt Bool operation "and" to combine their information and obtain the final edge detection results. The detailed algorithm flowchart is given in Fig 3.

The proposed 3-D NWHMM is an iteration based method. K-means algorithm is efficient for classification, so we do not need many iterations after obtaining the initial states using K-means. Through experiments we found that the model training will converge after two or three iterations. We set the iteration number for the 3-D NWHMM as two, because edge maps obtained after two iterations are nearly the same.

### 3.2. Comparisons With Canny and Two Multi-scale Techniques

Evaluation is very difficult due to the ad hoc nature of segmentation and highly dependent upon the intended use of the segmented images. Even now, the classical Canny edge detector is still a well-known technique and widely used. The Canny algorithm contains three steps: (1) the original image is smoothed by a Gauss filter, (2) the magnitude and direction of the gradient for each pixel are calculated, (3) post-processing including the non-maxima suppression and edge linking. The Gaussian pyramid edge detection algorithm is one of the traditional multi-scale techniques. It contains the following three steps: (1) the Gaussian pyramid representation of an original image is obtained by iteratively using the Gaussian filter through down-sampling, (2) Sobel edge detector is applied to the image at the coarsest resolution, (3) the algorithm detects edges at a finer resolution guided by the edge information at a coarser resolution. The multi-scale WD-VHMT technique [6] is based on the down-sampled wavelet which is not shift-invariant and uses HMT to model inter-scale dependence of the wavelet coefficients. However, the proposed algorithm (3-D NWHMM) adopts the non-decimated wavelet which is shift-invariant and 3-D HMM is built on the wavelet coefficients.

Fig 4 shows the edge detection results for the image "house" using Canny, Gaussian pyramid algorithm, WD-VHMT and 3-D NWHMM separately. It appears that edges detected from the three multi-scale methods are

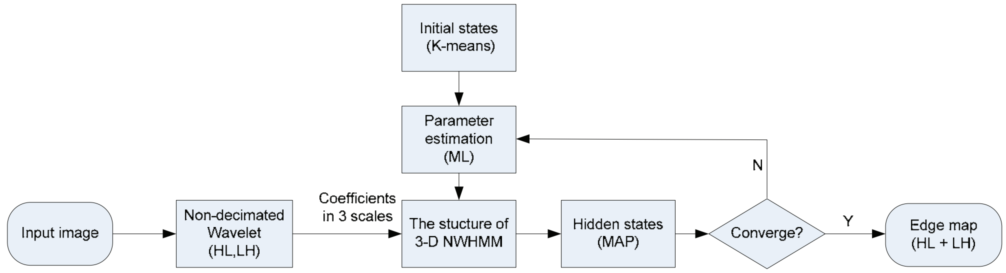


Fig 3. Flowchart of the proposed multi-scale edge detection algorithm

thicker than Canny, as the extra post-processing operation is applied for Canny algorithm. However, the three multi-scale methods can capture edges of main objects and remove redundant details. Main objects are more likely to be highlighted in the multi-scale methods.

Edge maps of the Gaussian pyramid edge detection algorithm are worse than those of WD-VHMT and 3-D NWHMM methods. The main reason is that it does not fuse multi-scale information efficiently, but WD-VHMT and 3-D NWHMM methods model the statistics of both edges and non-edges and take the causal dependencies of the wavelet coefficients into account using HMM.

Comparing the detected results of WD-VHMT and our proposed 3-D NWHMM methods, we can say that edge maps of our method have less isolated points that are useless, since intra-scale dependence of the wavelet coefficients is captured. In addition, some important edges are detected, for example, edges in the marked rectangular region in Fig 4.(*e*), but not available for the WD-VHMT. Our method is more accurate at some edge points, taking the detected edges in the marked circular region for example. We also test some other natural images and similar results can be obtained in most cases. These advantages of the proposed 3-D NWHMM are mainly because the adopted non-decimated wavelet is shift-invariant and the inter-scale and intra-scale dependencies between the wavelet coefficients can be captured efficiently.

## 4. CONCLUSIONS

The multi-resolution property of the wavelet transform has led to its efficiency in singularity detection as a multi-scale tool. In this paper, we propose a new edge detecting algorithm using 3-D NWHMM model based on the non-decimated shift invariant wavelet transform. HMMs can model the statistics and take the causal dependencies of the wavelet coefficients. The proposed 3-D NWHMM can fuse multi-scale information statistically, both inter-scale and intra-scale. Experimental results show its efficiency in edge detection. Also it has the potential to be applied for other image and video processing tasks.

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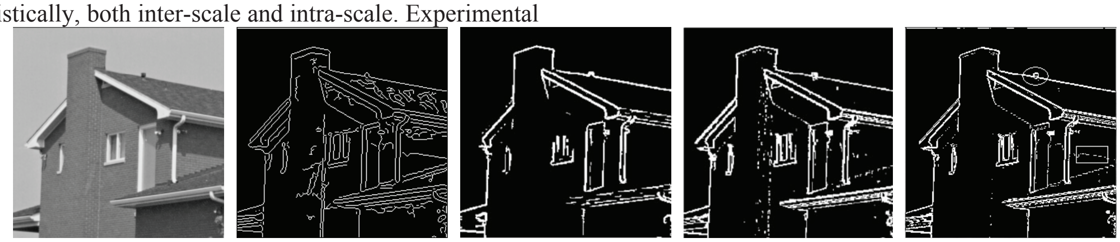

(*a*) (*b*) (*c*) (*d*) (*e*)

Fig 4. Edge detection results of Canny and three multi-scale algorithms. (*a*) the original image (*b*) Canny algorithm. (*c*) Gaussian pyramid algorithm. (*d*) WD-VHMT. (*e*) 3-D NWHMM