

# ICPSR 2015 “Advanced Maximum Likelihood”: Survival Analysis

Day One

August 3, 2015

# Preliminaries

- Instructor: [Christopher Zorn](#) ([zorn@psu.edu](mailto:zorn@psu.edu))
- Teaching Assistant: [Dominique Lewis](#) ([lewisd2@gmail.com](mailto:lewisd2@gmail.com))
- Class: 1:00-3:00 p.m. ET, in Room 296 [Weiser](#) (formerly [Dennison](#)) Hall
- Office Hours: 3:00 - ??? daily at [Espresso Royale](#) (State St.); also by appointment
- The syllabus is [here](#)
- Slides, readings, code, etc. are on the [github repo](#) (<https://github.com/PrisonRodeo/ICPSR-2015-git>)



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## R

- All examples, plots, etc.
- Current version is 3.2.1
- Packages you'll use (see the [survival analysis task view](#) for more):
  - `survival` (nearly everything you need)
  - `eha`
  - `timereg`

## Stata

- Current version is 14
- Mostly use the `-st-` series of commands (for “survival time”)

# Survival Analysis

- Models for *time-to-event data*.
- Roots in biostats/epidemiology, plus engineering, sociology, economics.
- Examples...
  - Political careers, confirmation durations, position-taking, bill cosponsorship, campaign contributions, policy innovation/adoption, etc.
  - Cabinet/government durations, length of civil wars, coalition durability, etc.
  - War duration, peace duration, alliance longevity, length of trade agreements, etc.
  - Strike durations, work careers (including promotions, firings, etc.), criminal careers, marriage and child-bearing behavior, etc.

# Characteristics of Time-To-Event Data

- Discrete events (i.e., not continuous),
- Take place over time,
- May not (or *never*) experience the event (i.e., possibility of censoring).

# Survival Data Basics: Terminology

$Y_i$  = the duration until the event occurs,

$Z_i$  = the duration until the observation is “censored”

$T_i$  =  $\min\{Y_i, Z_i\}$ ,

$C_i$  = 0 if observation  $i$  is censored, 1 if it is not.

# Survival Data Basics: The Density

$$f(t) = \Pr(T_i = t)$$

Issues:

- $T_i = t$  iff  $T_i > t - 1, t - 2$ , etc.
- $C_i = 0$  (censoring)



# Survival Data Basics: Survivor Function

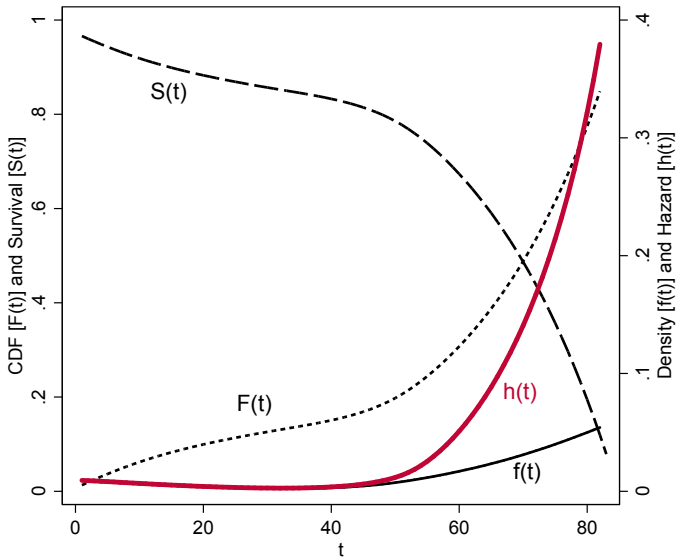
$$\Pr(T_i \leq t) \equiv F(t) = \int_0^t f(t) dt$$

$$\begin{aligned}\Pr(T_i \geq t) \equiv S(t) &= 1 - F(t) \\ &= 1 - \int_0^t f(t) dt\end{aligned}$$

# Survival Data Basics: The Hazard

$$\begin{aligned}\Pr(T_i = t | T_i \geq t) \equiv h(t) &= \frac{f(t)}{S(t)} \\ &= \frac{f(t)}{1 - \int_0^t f(t) dt}\end{aligned}$$

# Example: Human Mortality



# Some Useful Equivalencies

$$f(t) = \frac{-\partial S(t)}{\partial t}$$

Implies

$$\begin{aligned} h(t) &= \frac{\frac{-\partial S(t)}{\partial t}}{S(t)} \\ &= \frac{-\partial \ln S(t)}{\partial t} \end{aligned}$$

# More Useful Things: Integrated Hazard

Define

$$H(t) = \int_0^t h(t) dt.$$

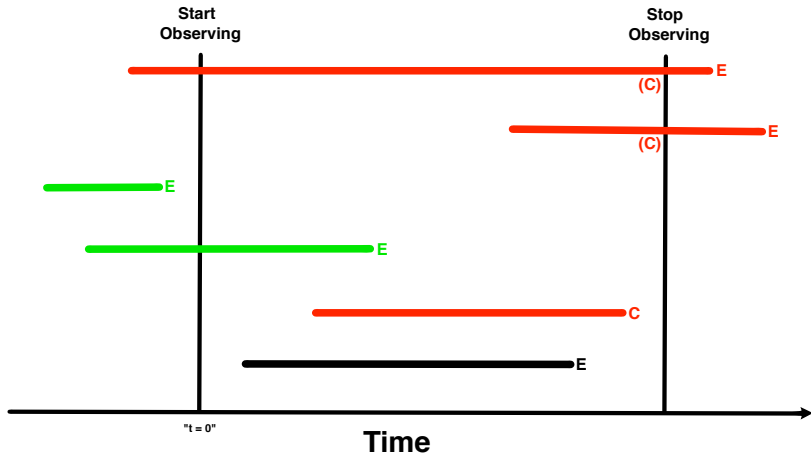
Implies

$$\begin{aligned} H(t) &= \int_0^t \frac{-\partial \ln S(t)}{\partial t} dt \\ &= -\ln[S(t)] \end{aligned}$$

and

$$S(t) = \exp[-H(t)]$$

# Censoring and Truncation



- Defined by the researcher
- Conditionally independent of both  $T_i$  and  $\mathbf{X}_i$
- Doesn't mean that the observation provides no information

# Estimating $S(t)$

Assume  $N$  observations, *absorbing* events, and no ties. Then define

$n_t$  = number of observations “at risk” for the event at  $t$ , and  
 $d_t$  = number of observations which experience the event at time  $t$ .

Then

$$\widehat{S(t_k)} = \prod_{t \leq t_k} \frac{n_t - d_t}{n_t}$$



# Variance of $\widehat{S}(t)$

$$\text{Var}[\widehat{S}(t_k)] = \left[ \widehat{S}(t_k) \right]^2 \sum_{t \leq t_k} \frac{d_t}{n_t(n_t - d_t)}$$

Note:

- $\text{Var}[\widehat{S}(t_k)]$  is increasing in  $S(t)$ ,
- is also increasing in  $d_t$ , but
- is decreasing in  $n_t$ .

# Estimating $H(t)$

“Nelson-Aalen”:

$$\widehat{H}(t_k) = \sum_{t \leq t_k} \frac{d_t}{n_t}$$

...which gives an alternative estimator for the survival function equal to:

$$\begin{aligned}\widehat{S}(t_k) &= \exp[-\widehat{H}(t_k)] \\ &= \exp\left[-\sum_{t \leq t_k} \frac{d_t}{n_t}\right]\end{aligned}$$

# Bivariate Hypothesis Testing

	Treatment	Placebo	Total
Event	$d_{1t}$	$d_{0t}$	$d_t$
No Event	$n_{1t} - d_{1t}$	$n_{0t} - d_{0t}$	$n_t - d_t$
Total	$n_{1t}$	$n_{0t}$	$n_t$

Log-Rank Test:

$$Q = \frac{\left[ \sum (d_{1t} - \frac{n_{1t}d_t}{n_t}) \right]^2}{\left[ \frac{n_{1t}n_{0t}d_t(n_t - d_t)}{n_t^2(n_t - 1)} \right]}$$
$$\sim \chi_1^2$$

# A Diversion: Survival Models and Counting Processes

Assume

- Event is *absorbing*,
- $Y_i$  is duration to the event
- $Z_i$  is duration to censoring
- Observe  $T_i = \min(Y_i, Z_i)$ , and
- $C_i$ :
  - $C_i = 0$  if  $T_i = Z_i$ ,
  - $C_i = 1$  if  $T_i = Y_i$ .
- $T_i \neq T_j \forall i \neq j$  (no “ties”)

# Three Key Variables

1. *Counting Process* Indicator:

$$N_i(t) = I(T_i \leq t, C_i = 1)$$

2. *Risk* Indicator:

$$R_i(t) = I(T_i > t)$$

3. *Intensity Process*:

$$\lambda_i(t) dt = R_i(t)h(t)$$

# Additional Things

With

$$\Lambda_i(t) = \int_0^t \lambda_i(t) dt$$

we can think of

$$N_i(t) = \Lambda_i(t) + M_i(t)$$

or

$$M_i(t) = N_i(t) - \Lambda_i(t).$$

# Martingales!

$$E(X_{t+s} | X_0, X_1, \dots, X_i, \dots, X_t) = X_t \quad \forall s > 0$$

# Data Structure and Organization: Non-Time-Varying

id	durat	censor	timein	timeout	X
1	4	0	30	34	0.12
2	2	1	12	14	0.19
3	5	1	5	10	0.09
...	...	...	...	...	...
N	10	1	21	31	0.22



# Time-Varying Data

id	durat	censor	timein	timeout	X	Z
1	1	0	30	31	0.12	331
1	2	0	31	32	0.12	412
1	3	0	32	33	0.12	405
1	4	0	33	34	0.12	416
2	1	0	12	13	0.19	226
2	2	1	13	14	0.19	296
3	1	0	5	6	0.09	253
3	2	0	6	7	0.09	311
3	3	0	7	8	0.09	327
3	4	0	8	9	0.09	344
3	5	1	9	10	0.09	301
...	...	...	...	...	...	...

# Analyzing Survival Data in R

survival object (non-time-varying):

```
library(survival)
NonTV<-read.csv(NonTVdata.csv)
NonTV.S<-Surv(NonTV$duration, NonTV$censor)
```

survival object (time-varying):

```
TV<-read.csv(TVdata.csv)
TV.S<-Surv(TV$starttime, TV$endtime, TV$censor)
```

# An Example

OECD Cabinet survival [Strom (1985); King et al. (1990)],

$N = 314$  cabinets in 15 countries

Outcome: Duration of cabinet, in months

Covariates (all non-time varying):

- *Fractionalization*
- *Polarization*
- *Formation Attempts*
- **Investiture**
- *Numerical Status*
- *Post-Election*
- *Caretaker*

Also: Indicator for whether the cabinet ended within 12 months of the end of the “constitutional inter-election period” (→ censored)

# KABL Data

```
> head(KABL)
```

	id	country	durat	ciep12	fract	polar	format	invest	numst2	eltime2	caret2
1	1		1	0.5	1	656	11	3	1	0	1
2	2		1	3.0	1	656	11	2	1	1	0
3	3		1	7.0	1	656	11	5	1	1	0
4	4		1	20.0	1	656	11	2	1	1	0
5	5		1	6.0	1	656	11	3	1	1	0
6	6		1	7.0	1	634	6	4	1	1	1

```
> KABL.S<-Surv(KABL$durat,KABL$ciep12)
```

```
> KABL.S[1:50,]
```

[1]	0.5	3.0	7.0	20.0	6.0	7.0	2.0	17.0	27.0	49.0+
[11]	4.0	29.0	49.0+	6.0	23.0	41.0+	10.0	12.0	2.0	33.0
[21]	1.0	16.0	2.0	9.0	3.0	5.0	5.0	6.0	45.0+	23.0
[31]	41.0	7.0	49.0+	46.0	9.0	51.0+	10.0	32.0	28.0	3.0
[41]	53.0+	17.0	59.0+	9.0	52.0+	3.0	23.0	33.0	1.0	30.0

# Example survfit Object

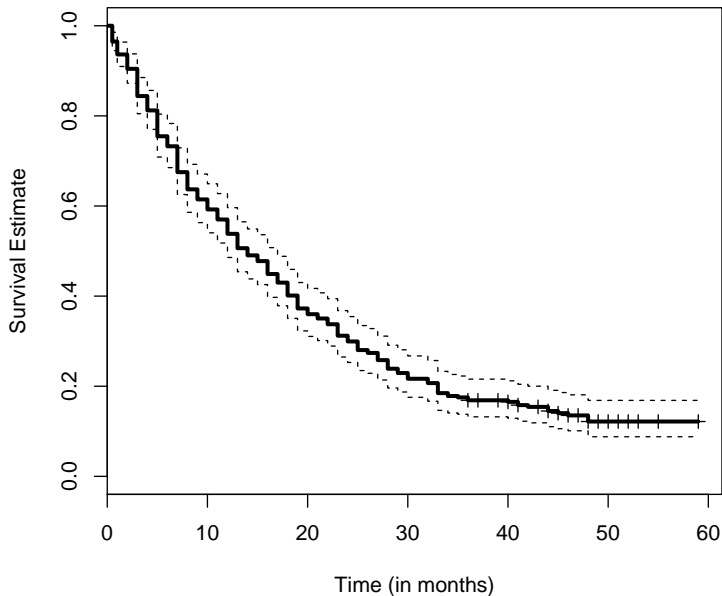
```
> KABL.fit<-survfit(KABL.S~1)
```

```
> str(KABL.fit)
```

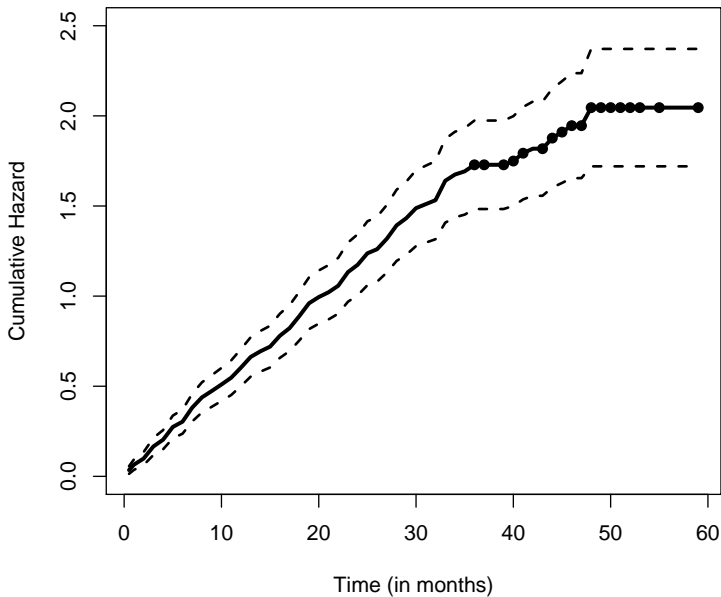
List of 13

```
$ n      : int 314
$ time   : num [1:54] 0.5 1 2 3 4 5 6 7 8 9 ...
$ n.risk  : num [1:54] 314 303 294 284 265 255 237 230 212 200 ...
$ n.event : num [1:54] 11 9 10 19 10 18 7 18 12 7 ...
$ n.censor : num [1:54] 0 0 0 0 0 0 0 0 0 0 ...
$ surv    : num [1:54] 0.965 0.936 0.904 0.844 0.812 ...
$ type    : chr "right"
$ std.err  : num [1:54] 0.0108 0.0147 0.0183 0.0243 0.0271 ...
$ upper    : num [1:54] 0.986 0.964 0.938 0.885 0.856 ...
$ lower    : num [1:54] 0.945 0.91 0.873 0.805 0.77 ...
$ conf.type: chr "log"
$ conf.int : num 0.95
$ call     : language survfit(formula = KABL.S ~ 1)
- attr(*, "class")= chr "survfit"
```

# Plotting $\widehat{S}(t)$



# Plotting $\widehat{H}(t)$



# Comparing $\widehat{S}(t)$ s

## Log-rank test:

```
> survdiff(KABL.S~invest,data=KABL,rho=0)
```

Call:

```
survdiff(formula = KABL.S ~ invest, data = KABL, rho = 0)
```

	N	Observed	Expected	(O-E) <sup>2</sup> /E	(O-E) <sup>2</sup> /V
invest=0	172	137	178.7	9.72	30.5
invest=1	142	134	92.3	18.81	30.5

Chisq= 30.5 on 1 degrees of freedom, p= 3.26e-08



# Comparing $\widehat{S}(t)$ s

