$$Y_t = (1+\varepsilon_t)\cdot\left[\overset{K_t}{\cancel{\beta(1-\alpha)Y_{t-1}}}\right]^{\alpha}$$

$$K_{t+1} = \beta(1-\alpha)Y_t = (1+\varepsilon_t)\beta(1-\alpha)K_t^{\alpha}$$

$$= (1+\varepsilon_t)(1+\varepsilon_{t-1})^{\alpha}\beta^{1+\alpha}(1-\alpha)^{1+\alpha}K_{t-1}^{\alpha^2}$$

$$= \cdots$$

$$= (1+\varepsilon_t)(1+\varepsilon_{t-1})^{\alpha}(1+\varepsilon_{t-2})^{\alpha^2}\cdots(1+\varepsilon_1)^{\alpha^{t-1}}\left[\beta(1-\alpha)\right]^{1+\alpha+\cdots+\alpha^{t-1}}K_1^{\alpha^t}$$

$\cancel{1+\alpha+\cdots+\alpha}$

$$K_{t+1} = K_1^{\alpha^t}\prod_{i=1}^{t}\left[(1+\varepsilon_i)\beta(1-\alpha)\right]^{\alpha^{t-i}}$$

$$\frac{\alpha^t-1}{\alpha-1}$$

$$\boxed{K_t = K_1^{\alpha^{t-1}}\prod_{i=1}^{t-1}\left[(1+\varepsilon_i)\beta(1-\alpha)\right]^{\alpha^{t-1-i}}}$$

$$Y_t = (1+\varepsilon_t)\cdot K_t^{\alpha}$$

$$W_t = (1+\varepsilon_t)\cdot(1-\alpha)K_t^{\alpha}.$$ 开心吗v.

$$c_{y,t} = (1-\beta)(1-\alpha)(1+\varepsilon_t)K_t^{\alpha}$$

$$V_t = (1-\beta)\ln(c_{y,t}) + \beta\ln(c_{o,t+1})$$

$$= (1-\beta)\ln\left((1-\beta)(1-\alpha)\overset{Y_t}{\cancel{(1+\varepsilon_t)K_t^{\alpha}}}\right)$$

$$+ \beta\ln\left(\beta(1-\alpha)R^f_t\overset{Y_t}{\cancel{(1+\varepsilon_t)K_t^{\alpha}}}\right)$$

$$= \underbrace{\ln(1-\alpha)+(1-\beta)\ln(1-\beta)+\beta\ln\beta}_{C} + \beta\ln R^f_t$$

$$+ \ln Y_t$$

$$= C + \beta\ln\left[\frac{\alpha[1-\delta^2]}{(\beta(1-\alpha)Y_t)^{1-\alpha}}\right] + \ln Y_t$$

$$= C + \beta\ln\left[\alpha[1-\delta^2]\right] - (1-\alpha)\beta\ln\left[\beta(1-\alpha)Y_t\right) + \ln Y_t$$

$$= \overbrace{c + \beta \ln[\alpha[1-\delta^2]] - (1-\alpha)\beta \ln[\beta(1-\alpha)]}^{d}$$

$$- (1-\alpha)\beta \ln Y_t + \ln Y_t$$

$$= d + [1-(1-\alpha)\beta] \ln(1+\varepsilon_t) + \ln(K_t^{\alpha}) - (1-\alpha)\beta \ln(K_t^{\alpha})$$

$$= d + [1-(1-\alpha)\beta] \ln(1+\varepsilon_t) + \alpha \ln K_t - (1-\alpha)\alpha\beta \ln K_t$$

$$= d + [1-(1-\alpha)\beta] \ln(1+\varepsilon_t) + \alpha(1-\beta+\alpha\beta) \ln K_t$$

$$\cancel{1-(1-\alpha)\beta = 1-\beta+\alpha\beta}$$

$$= d + (1-\beta+\alpha\beta) \ln(1+\varepsilon_t) + \alpha(1-\beta+\alpha\beta) \ln K_t$$

$$\ln K_t = \alpha^{t-1} \ln K_1 + \sum_{i=1}^{t-1} \alpha^{t-1-i} \ln[(1+\varepsilon_i)\beta(1-\alpha)]$$

$$= \alpha^{t-1} \ln K_1 + \sum_{i=1}^{t-1} \alpha^{t-1-i} [\ln(\beta) + \ln(1-\alpha)]$$

$$+ \sum_{i=1}^{t-1} \alpha^{t-1-i} \ln(1+\varepsilon_i)$$

$$V = \ln(1-\alpha) + (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta\ln[\alpha(1-\delta^2)] - (1-\alpha)\beta\ln[\beta(1-\alpha)]$$
$$+ \alpha(1-\beta+\alpha\beta)\left[\alpha^{t-1}\ln K_1 + \sum_{i=1}^{t-1} \alpha^{t-1-i} \ln(\beta(1-\alpha))\right]$$
$$+ (1-\beta+\alpha\beta) \sum_{i=1}^{t} \alpha^{t-i} \ln(1+\varepsilon_i)$$

$$= \ln(1-\alpha) + (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta\ln[\alpha(1-\delta^2)] - (1-\alpha)\beta\ln[\beta(1-\alpha)]$$
$$+ \alpha(1-\beta+\alpha\beta) \cdot \alpha^{t-1} \ln K_1 + (1-\beta+\alpha\beta)\ln[\beta(1-\alpha)] \cdot \sum_{i=1}^{t-1} \alpha^{t-i}$$
$$+ (1-\beta+\alpha\beta) \sum_{i=1}^{t} \alpha^{t-i} \ln(1+\varepsilon_i)$$

$$= \ln(1-\alpha) + (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta\ln[\alpha(1-\sigma^2)] - (1-\alpha)\beta\ln[\beta(1-\alpha)]$$
$$+ (1-\beta+\alpha\beta)\alpha^t \ln k_1 + (1-\beta+\alpha\beta)\frac{\alpha(1-\alpha^{t-1})}{1-\alpha}\ln[\beta(1-\alpha)]$$
$$+ (1-\beta+\alpha\beta)\sum_{i=1}^{t}\alpha^{t-i}\ln(1+\varepsilon_i)$$

$$= (1-\beta)\ln\left(\frac{1-\beta}{\beta}\right) + \beta\ln[\alpha(1-\sigma^2)] + (1-(1-\alpha)\beta)\ln[\beta(1-\alpha)]$$
$$+ (1-\beta+\alpha\beta)\alpha^t \ln k_1 + (1-\beta+\alpha\beta)\frac{\alpha(1-\alpha^{t-1})}{1-\alpha}\ln[\beta(1-\alpha)]$$
$$+ (1-\beta+\alpha\beta)\sum_{i=1}^{t}\alpha^{t-i}\ln(1+\varepsilon_i)$$

$$= (1-\beta)\ln\left(\frac{1-\beta}{\beta}\right) + \beta\ln[\alpha(1-\sigma^2)]$$
$$+ (1-\beta+\alpha\beta)\alpha^t \ln k_1 + (1-\beta+\alpha\beta)\ln[\beta(1-\alpha)]\left(\frac{\alpha(1-\alpha^{t-1})}{1-\alpha} + 1\right)$$
$$+ (1-\beta+\alpha\beta)\sum_{i=1}^{t}\alpha^{t-i}\ln(1+\varepsilon_i)$$

$$= (1-\beta)\ln\left(\frac{1-\beta}{\beta}\right) + \beta\ln[\alpha(1-\sigma^2)]$$
$$+ (1-\beta+\alpha\beta)\alpha^t \ln k_1 + (1-\beta+\alpha\beta)\ln[\beta(1-\alpha)]\underbrace{\left(\frac{1-\alpha^t}{1-\alpha}\right)}_{=\sum_{i=1}^{t}\alpha^{t-i}}$$
$$+ (1-\beta+\alpha\beta)\sum_{i=1}^{t}\alpha^{t-i}\ln(1+\varepsilon_i)$$

$$= (1-\beta)\ln\left(\frac{1-\beta}{\beta}\right) + \beta\ln[\alpha(1-\sigma^2)]$$
$$+ (1-\beta+\alpha\beta)\alpha^t \ln k_1 + (1-\beta+\alpha\beta)\sum_{i=1}^{t}\alpha^{t-i}\ln[(1+\varepsilon_i)(1-\alpha)\beta]$$