```
(*log utility case
                                                             c==consumption when young
                                                            d==debt
                                                            tau≕tau 2
                                                            b==beta
                                                            a==alpha
                                                            sig==sigma
                                                           w==wage
                                                            rf≕risk-free rate
                                             *)
                                            Clear[f]
         ln[98] = f[c_, d_, tau_] := (1-b) log[c] + b/2 (log[a (1+sig) (w-d-c)^a (1-tau) + drf] + b/2 (log[a (1+sig) (w-d-c)^a (w-d-c)^a (u-tau) + drf] + b/2 (log[a (1+sig) (w-d-c)^a (u-tau) + drf] + b/2 (log[a (1
                                                                                    Log[a (1 - sig) (w - d - c)^a (1 - tau) + drf])
         ln[99]: Assuming [0 < a < 1 \& \& 0 < b < 1 \& \& 0 < c < w \& \& 0 < d < w \& & rf > 0 \& \& 0 < sig < 1,
                                                    Simplify [Solve[D[f[c, d, tau], c] = 0 \& D[f[c, d, tau], d] = 0 \& \& D[f[c
                                                                            1 - tau = ((w - d - c) / bw)^{(1 - a)}, \{c, rf, tau\}]
      Out[99]= $Aborted
     In[100]:= (*constant young utility case
                                                            c==consumption when young
                                                           d==debt
                                                            tau≕tau 2
                                                            b==beta
                                                            a≕alpha
                                                            sig=sigma
                                                           w==wage
                                                            rf==risk-free rate
                                             *)
                                            Clear[g]
     Log[a (1 - sig) (w - d - c)^a (1 - tau) + drf])
     log[102] Assuming 0 < a < 1 & 0 < b < 1 & 0 < c < w & 0 < d < w & rf > 0 & 0 < i < 1
                                                    Simplify [Solve[D[g[c, d, tau], c] = 0 \&\& D[g[c, d, tau], d] = 0 \&\&
                                                                             1 - tau = ((w - d - c) / bw)^{(1 - a)}, \{c, rf, tau\}]
                                              .... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete
                                                                              solution information.
                                              Solve: Equations may not give solutions for all "solve" variables.
b \, \left( 4 \, d \, \text{sig}^2 \, + \, \sqrt{b^2 \, - \, 4 \, b \, d \, \text{sig}^2 \, + \, 4 \, d^2 \, \text{sig}^2} \, \, \right) \, \left( - \, c \, - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, \left( - \, d \, + \, w \right)^{\, a} \, + \, d^2 \, 
                                                                                                          \sqrt{2} \; sig^2 \; \left( b^2 + 2 \; d \; \left( 2 \; d \; sig^2 + \sqrt{b^2 - 4 \; b \; d \; sig^2 + 4 \; d^2 \; sig^2} \; \right) \; - \right. \\
                                                                                                                                 b \left( 4 d sig^2 + \sqrt{b^2 - 4 b d sig^2 + 4 d^2 sig^2} \right) \left( c + d - w \right) \sqrt{\left( \frac{1}{sig^4} \right)^2}
                                                                                                                                   a \left( a \left( b^2 + 2 \ d^2 \ sig^2 - b \left( 2 \ d \ sig^2 + \sqrt{b^2 - 4 \ b \ d \ sig^2 + 4 \ d^2 \ sig^2} \ \right) \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right) + 4 \ sig^2 \left( b - 2 \ b \ sig^2 + 4 \ d^2 \ sig^2 \ d^2 \right)
```

$$2 \, d \, sig^2 \, - \sqrt{b^2 - 4} \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right] \, \left(c + d - w \right) \, \left(-c - d + w \right)^{2 \, (-1 + a)} \right) \bigg] \bigg] \bigg/ \\ \left(8 \, \left(b - d \right) \, d \, sig^2 \, \sqrt{b^2 - 4} \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \, , \, tau \, \to 1 \, - \\ \left(\frac{w \, \left(- c - d + w \right)}{b} \right)^{-3 \, a} \, \bigg\} \, , \, \left\{ rf \, \to \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(w \, \left(- c - d + w \right) \right]^{-3 \, a} \, \left[- 2 \, a \, \left(b - d \right) \, \left(b^2 + 2 \, d \, sig^2 \, \left(2 \, d + \sqrt{b^2 - 4} \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right. \right) \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(w \, \left(- c - d + w \right) \right]^{-3 \, a} \, \left[- 2 \, a \, \left(b - d \right) \, \left(b^2 + 2 \, d \, sig^2 \, \left(2 \, d + \sqrt{b^2 - 4} \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right. \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(w \, \left(- c - d + w \right) \right]^{-3 \, a} \right. \right. \right. \right. \right. \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(w \, \left(- c \, d + w \right) \right]^{-3 \, a} \right. \right. \right. \right. \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(w \, \left(- c \, d + w \right) \right]^{-3 \, a} \right. \right. \right. \right. \right. \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(- c \, d + w \right) \right]^{-3 \, a} \right. \right. \right. \left. \left. \left(c \, d \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right] \left. \left. \left(c \, d \, d \, w \right) \right] \right. \\ \left. - \left(\left[a \, b^{-1 + a} \, w \, \left(- c \, d \, d \, w \right) \right] \right. \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left(\left[a \, b \, d \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right] \right) \left. \left(c \, d \, d \, w \right) \right] \right. \\ \left. \left. \left(\left[a \, b \, d \, d \, sig^2 \, \left(b^2 - 4 \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right. \right) \right. \right. \right. \right. \\ \left. \left. \left(\left[a \, b \, d \, d \, sig^2 \, \left(b^2 - 2 \, d \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right. \right) \right. \right. \\ \left. \left. \left(\left[a \, b \, d \, d \, sig^2 \, \left(b^2 - 2 \, d \, b \, d \, sig^2 + 4 \, d^2 \, sig^2 \, \right) \right. \right) \right. \right. \\ \left. \left. \left(\left[a \, b \, d \, d \, sig^2 \, + 4 \, d^2 \, sig^2 \, \right. \right) \right. \right. \right. \\ \left. \left. \left(\left[a \, b \, d \, d \, sig^2 \, + 4 \, d^2 \, sig^2 \, \right] \right) \right. \left. \left. \left(c \, d \, d \, w \right) \right. \right. \\ \left. \left. \left(\left[a \, d \, b \, d \, sig^2 \, + 4 \, d^2 \, sig^2 \, \right] \right. \right) \left. \left. \left(\left[a \, d \, d \, sig^2 \, + 4 \, d^2 \, sig^2 \, \right] \right. \right. \right. \right. \\ \left. \left. \left[\left[a \, d \, b \, d \, sig^2 \, + 4 \, d^2 \, sig^2 \, \right] \right. \right. \right. \right. \right. \right. \right. \\ \left. \left. \left[\left[a \, d \, b \, d \, sig^2 \, + 4 \, d^2 \, sig^2$$

$$\begin{split} \sqrt{\left(\frac{1}{\text{sig}^4} a \, \left(a \, \left(b^2 + 2 \, d^2 \, \text{sig}^2 + b \, \left(-2 \, d \, \text{sig}^2 + \sqrt{b^2 - 4 \, b \, d \, \text{sig}^2 + 4 \, d^2 \, \text{sig}^2} \, \right)\right) + } \\ & 4 \, \text{sig}^2 \, \left(b - 2 \, b \, \text{sig}^2 + 2 \, d \, \text{sig}^2 + \sqrt{b^2 - 4 \, b \, d \, \text{sig}^2 + 4 \, d^2 \, \text{sig}^2} \, \right) \\ & \left(c + d - w\right) \, \left(-c - d + w\right)^2 \, \frac{(-1 + a)}{2} \, \left(-\frac{1}{2}\right) \, \right) \, \Big| \, \\ & \left(8 \, \left(b - d\right) \, d \, \text{sig}^2 \, \sqrt{b^2 - 4 \, b \, d \, \text{sig}^2 + 4 \, d^2 \, \text{sig}^2} \, \right) \, , \, \, \text{tau} \rightarrow 1 \, - \\ & \left(\frac{w \, \left(-c - d + w\right)}{b}\right)^{1 - a} \, \right) \, \Big\} \, \end{split}$$