```
(*log utility case
  c==consumption when young
  d==debt
  tau==tau_2
  b==beta
  a==alpha
  sig==sigma
  w==wage
  rf==risk-free rate
*)
Clear[f]
```

In[98]:=
```
f[c_, d_, tau_] := (1 - b) Log[c] + b / 2 (Log[a (1 + sig) (w - d - c)^a (1 - tau) + d rf] +
     Log[a (1 - sig) (w - d - c)^a (1 - tau) + d rf])
```

In[99]:=
```
Assuming[0 < a < 1 && 0 < b < 1 && 0 < c < w && 0 < d < w && rf > 0 && 0 < sig < 1,
 Simplify[Solve[D[f[c, d, tau], c] == 0 && D[f[c, d, tau], d] == 0 &&
    1 - tau == ((w - d - c) / b w)^(1 - a), {c, rf, tau}]]]
```

Out[99]= $Aborted

In[100]:=
```
(*constant young utility case
  c==consumption when young
  d==debt
  tau==tau_2
  b==beta
  a==alpha
  sig==sigma
  w==wage
  rf==risk-free rate
*)
Clear[g]
```

In[101]:=
```
g[c_, d_, tau_] := c + b / 2 (Log[a (1 + sig) (w - d - c)^a (1 - tau) + d rf] +
     Log[a (1 - sig) (w - d - c)^a (1 - tau) + d rf])
```

In[102]:=
```
Assuming[0 < a < 1 && 0 < b < 1 && 0 < c < w && 0 < d < w && rf > 0 && 0 < i < 1,
 Simplify[Solve[D[g[c, d, tau], c] == 0 && D[g[c, d, tau], d] == 0 &&
    1 - tau == ((w - d - c) / b w)^(1 - a), {c, rf, tau}]]]
```

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

... Solve: Equations may not give solutions for all "solve" variables.

Out[102]=
$$\left\{\left\{rf \to \left(a\, b^{-1+a}\, w\, \left(w\, \left(-c-d+w\right)\right)^{-a} \left(2\, a\, \left(b-d\right)\, \left(b^2+2\, d\, sig^2\, \left(2\, d+\sqrt{b^2-4\, b\, d\, sig^2+4\, d^2\, sig^2}\right)-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.b\, \left(4\, d\, sig^2+\sqrt{b^2-4\, b\, d\, sig^2+4\, d^2\, sig^2}\right)\right)\, \left(-c-d+w\right)^a+\right.\right.\right.$$
$$\left.\left.\sqrt{2}\, sig^2\, \left(b^2+2\, d\, \left(2\, d\, sig^2+\sqrt{b^2-4\, b\, d\, sig^2+4\, d^2\, sig^2}\right)-\right.\right.\right.$$
$$\left.\left.\left.b\, \left(4\, d\, sig^2+\sqrt{b^2-4\, b\, d\, sig^2+4\, d^2\, sig^2}\right)\right)\, \left(c+d-w\right)\, \sqrt{\left(\frac{1}{sig^4}\right.\right.\right.$$
$$\left.\left.\left.a\, \left(a\, \left(b^2+2\, d^2\, sig^2-b\, \left(2\, d\, sig^2+\sqrt{b^2-4\, b\, d\, sig^2+4\, d^2\, sig^2}\right)\right)+4\, sig^2\, \left(b-2\, b\, sig^2+\right.\right.\right.\right.$$

$$2\,d\,sig^2-\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\ (c+d-w)\Big)\ (-c-d+w)^{2\,(-1+a)}\Big)\Big)\Big/$$
$$\Big(8\,(b-d)\,d\,sig^2\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big),\ tau\to 1-$$
$$\left(\frac{w\,(-c-d+w)}{b}\right)^{1-a}\Big\},\ \{rf\to$$
$$-\Bigg(\Bigg(a\,b^{-1+a}\,w\,(w\,(-c-d+w))^{-a}\Big(-2\,a\,(b-d)\,\Big(b^2+2\,d\,sig^2\,\Big(2\,d+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)-$$
$$b\,\Big(4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (-c-d+w)^a+$$
$$\sqrt{2}\ sig^2\,\Big(b^2+2\,d\,\Big(2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)-$$
$$b\,\Big(4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (c+d-w)$$
$$\sqrt{\Big(\frac{1}{sig^4}a\,\Big(a\,\Big(b^2+2\,d^2\,sig^2-b\,\Big(2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)+$$
$$4\,sig^2\,\Big(b-2\,b\,sig^2+2\,d\,sig^2-\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\ (c+d-w)\Big)$$
$$(-c-d+w)^{2\,(-1+a)}\Big)\Big)\Big/\Big(8\,(b-d)\,d\,sig^2\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big),$$
$$tau\to 1-\left(\frac{w\,(-c-d+w)}{b}\right)^{1-a}\Big\},\ \{rf\to-\Bigg(\Bigg(a\,b^{-1+a}\,w\,(w\,(-c-d+w))^{-a}$$
$$\Big(2\,a\,(b-d)\,\Big(b^2+2\,d\,sig^2\,\Big(2\,d-\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)+$$
$$b\,\Big(-4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (-c-d+w)^a+$$
$$\sqrt{2}\ sig^2\,\Big(b^2+b\,\Big(-4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)-$$
$$2\,d\,\Big(-2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (c+d-w)$$
$$\sqrt{\Big(\frac{1}{sig^4}a\,\Big(a\,\Big(b^2+2\,d^2\,sig^2+b\,\Big(-2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)+$$
$$4\,sig^2\,\Big(b-2\,b\,sig^2+2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\ (c+d-w)\Big)$$
$$(-c-d+w)^{2\,(-1+a)}\Big)\Big)\Big/\Big(8\,(b-d)\,d\,sig^2\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big),$$
$$tau\to 1-\left(\frac{w\,(-c-d+w)}{b}\right)^{1-a}\Big\},\ \{rf\to\Bigg(a\,b^{-1+a}\,w\,(w\,(-c-d+w))^{-a}$$
$$\Big(-2\,a\,(b-d)\,\Big(b^2+2\,d\,sig^2\,\Big(2\,d-\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)+$$
$$b\,\Big(-4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (-c-d+w)^a+$$
$$\sqrt{2}\ sig^2\,\Big(b^2+b\,\Big(-4\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)-$$
$$2\,d\,\Big(-2\,d\,sig^2+\sqrt{b^2-4\,b\,d\,sig^2+4\,d^2\,sig^2}\Big)\Big)\ (c+d-w)$$

$$\sqrt{\left(\frac{1}{\text{sig}^4}\text{a}\left(\text{a}\left(\text{b}^2+2\,\text{d}^2\,\text{sig}^2+\text{b}\left(-2\,\text{d}\,\text{sig}^2+\sqrt{\text{b}^2-4\,\text{b}\,\text{d}\,\text{sig}^2+4\,\text{d}^2\,\text{sig}^2}\right)\right)+\right.\right.$$
$$\left.\left.4\,\text{sig}^2\left(\text{b}-2\,\text{b}\,\text{sig}^2+2\,\text{d}\,\text{sig}^2+\sqrt{\text{b}^2-4\,\text{b}\,\text{d}\,\text{sig}^2+4\,\text{d}^2\,\text{sig}^2}\right)\right.\right.$$
$$\left.\left.\left(\text{c}+\text{d}-\text{w}\right)\right)\left(-\text{c}-\text{d}+\text{w}\right)^{2\,(-1+\text{a})}\right)\right)\Big/$$
$$\left(8\left(\text{b}-\text{d}\right)\,\text{d}\,\text{sig}^2\sqrt{\text{b}^2-4\,\text{b}\,\text{d}\,\text{sig}^2+4\,\text{d}^2\,\text{sig}^2}\right),\ \text{tau}\to 1-$$
$$\left.\left(\frac{\text{w}\left(-\text{c}-\text{d}+\text{w}\right)}{\text{b}}\right)^{1-\text{a}}\right\}\}$$