# Appendices

## Appendix A: Extended Literature Review

In this appendix, we extend the discussion on the literature related to the CAVADP and CDPP 2. We note that the closest drone problem to ours is the multi-visit drone routing problem (MVDRP) [Poikonen and Golden, 2020b]. Table EC.1 highlights key features of the problem often indicating that the CAVADP is a special case of the MVDRP. As discussed, a key difference in the structures of the solution is the difference in traveling speeds.

| **Feature** | **MVDRP** | **CAVADP** | **Relating the MVDRP and CAVADP** |
|---|---|---|---|
| Speeds | The drone travels faster than the vehicle. | The vehicle travels faster than the delivery person. | The MVDRP must account for hovering time if the drone reaches a destination prior to the vehicle. Appendix D.2 discusses that if the walking speed is faster than the driving speed (similar to the drone operation), waiting time must be accounted for in the objective function and the optimal solution does not hold. |
| Packages | Heterogeneous packages | Homogeneous packages | The CAVADP is a special case of MVDRP in this area. |
| Launch/Loading Points | Separate launch points $V$ and customer locations $C$ | Customer locations are the loading points $C = V$ | The analysis of the CAVADP in this paper is a special case of MVDRP in this area. However, the CAVADP could be generalized to separate the loading points and the customer locations. |
| Synchronization | The drone is allowed to hover prior to the vehicle's arrival. | Assuming driving is faster than walking, the vehicle can always meet the delivery person. Therefore, there is no hovering time. | The CAVADP is a special case of the MVDRP where the delivery person is not allowed to "hover". Appendix D.2 shows that if waiting time is accounted for in the objective function, the optimal solution may not hold. |
| Depot | A drone can be launched from the depot. | The delivery person cannot leave the depot to deliver packages. | The CAVADP is a special case of the MVDRP by limiting the delivery activity at the depot. |
| Capacity Constraint | There is a fixed energy capacity. The drone loses energy based on the weight of packages. The only service sets that are considered are energy feasible. | There is a package capacity. All service sets of size less than or equal to $q$ are considered. | In the CAVADP, we would need to verify if the service sets were energy feasible. It may be the case that a service set greater than size $q$ is energy feasible, but we would not consider it due to the carrying capacity of the delivery person. Similarly, there may be some service sets that are not energy feasible that the delivery person could service. |

| | | | |
|---|---|---|---|
| Loading Time/Launch Penalty | There is a *Launch-Penalty* when the truck stops to launch one or more drones. | The CAVADP has a fixed cost of loading packages $f$ prior to servicing a set of customers. | The cost of loading packages $f$ is a key parameter in determining the optimal solution to the CAVADP. This factor may be important in the MVDRP, but the formulation does not make it clear how this time is incorporated. The alternative formulation that we provide allows us to prove theoretical results about the optimal solution based on this time. |

Table EC.1: Comparisons of the key features of the MVDRP and CAVADP.

## Appendix B: Proofs

### B.1. Notation

We introduce the definitions and notation of Itai et al. [1982] for the proofs of this paper. Take $R(g,h)=(V,E)$ to denote a solid rectangular grid with the bottom left corner at $(1,1)$, bottom right corner at $(g,1)$, top left corner at $(1,h)$, and top right corner at $(g,h)$. The set of vertices $V$ create a lattice of integer-valued points and $E$ represents the set of edges between these points. Let $(s_x, s_y)$ represent the coordinates of vertex $s \in V$ and $st \in E$ denote the edge between vertices $s,t \in V$. The grid is a bipartite graph as we can color it using Equation (17). Figure EC.1 presents an example of $R(4,3)$.

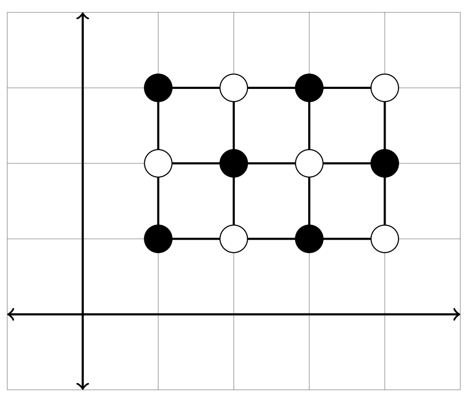


**Figure EC.1** **$R(4,3)$ with the coloring of Equation** (17).

A Hamiltonian path between two vertices is a path that visits each vertex exactly once. $(R,s,t)$ denotes the problem of determining the existence of a Hamiltonian path between vertices $s$ and $t$ in $R(g,h)$. $(R,s,t)$ is *acceptable* if it is color compatible and not forbidden. $(R,s,t)$ is said to be *color compatible* if

- $|V|$ is even and $s,t$ are different colors
- $|V|$ is odd and $s,t$ are the same color (being the majority color).

The following graphs are classified as *forbidden*:

- 1 rectangle (i.e. $R(1,a)$ or $R(b,1)$ for some $a,b \in \mathbb{N}$) and $s,t$ are not corners. Figure EC.2 gives an example.

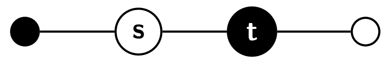



**Figure EC.2 An example of a 1 rectangle forbidden graph.**

- 2 rectangle (i.e. $R(2,a)$ or $R(b,2)$ for some $a,b \in \mathbb{N}$) and $st$ is a nonboundary edge. Figure EC.3 gives an example.

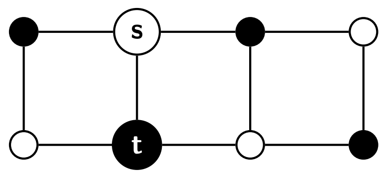
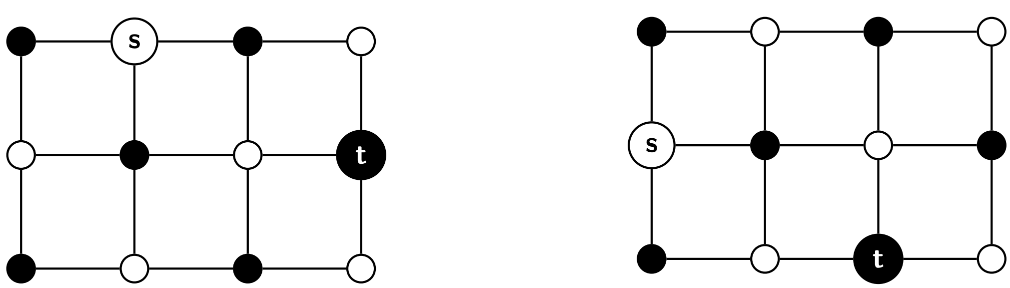



**Figure EC.3 An example of a 2 rectangle forbidden graph.**

- $(R,s,t)$ is isomorphic to $(R',s',t')$ satisfying:
  — $R' = R'(a,b)$ with $b=3$ and $a$ even.
  — $s'$ is colored differently from $t'$ and the left corners of $R'$
  — One of the following hold:
1. $s'_x < t'_x - 1$: Figure EC.4 part (a) gives an example.
2. $s'_y = 2$ and $s'_x < t'_x$: Figure EC.4 part (b) gives an example.




(a) $s_x < t_x - 1$ (b) $s_y = 2$ and $s_x < t_x$

**Figure EC.4 Examples of 3 rectangle forbidden graphs. Part (a) gives an example where $s_x < t_x - 1$. Part (b) gives an example where $s_y = 2$ and $s_x < t_x$.**

### B.2. Proof of Theorem 1

THEOREM 1. *The closest customer $c_1 = (i_1, j_1)$ to the depot will be located on the boundary of the grid and is unique.*

*The set of second closest customers to the depot $\mathcal{C}_2$ can include the following forms:*

$$\mathcal{C}_2 \subset \{(i_1, j_1+1), (i_1, j_1-1), (i_1+1, j_1), (i_1-1, j_1)\}.$$

*For all $c_2 \in \mathcal{C}_2$, $D(0,c_2) = MinDistance + 1$. If $c_1$ is located on the corner of the grid, $|\mathcal{C}_2| = 2$. Otherwise, $|\mathcal{C}_2| = 3$. In addition, there exists $c_2 \in \mathcal{C}_2$ such that $c_1c_2$ is a boundary edge.*

*The set of third closest customers to the depot $\mathcal{C}_3$ can include the following forms:*

$$\mathcal{C}_3 \subset \{(i_1-1,j_1+1),(i_1-1,j_1-1),(i_1+1,j_1-1),(i_1-1,j_1+1),$$
$$(i_1-2,j_1),(i_1+2,j_1),(i_1,j_1-2),(i_1,j_1+2)\}.$$

*For all $c_3 \in \mathcal{C}_3$, $D(0,c_3) = MinDistance + 2$. If $c_1$ is located on the corner of the grid, $1 \le |\mathcal{C}_3| \le 3$. Otherwise, $2 \le |\mathcal{C}_3| \le 5$.*

*Proof.* The proofs are given in Lemmas 1, 2, and 3. □

LEMMA 1. *Consider a solid rectangular grid of size $g \times h$ where $g,h > 1$. The closest customer $c_1$ to the depot will be located on the boundary. Given the coordinates of the depot $(a,b)$, the closest customer $c_1$ is defined by:*

| $(a,b)$ | $c_1$ |
|---|---|
| $a \le 1, b \le 1$ | $(1,1)$ |
| $a \le 1, 1 \le b \le h$ | $(1,b)$ |
| $a \le 1, b \ge h$ | $(1,h)$ |
| $1 \le a \le g, b \ge h$ | $(a,h)$ |
| $a \ge g, b \ge h$ | $(g,h)$ |
| $a \ge g, 1 \le b \le h$ | $(g,b)$ |
| $a \ge g, b \le 1$ | $(g,1)$ |
| $1 \le a \le g, b \le 1$ | $(a,1)$ |

*Therefore, $c_1$ is unique.*

*Proof.* Consider $R(g,h)$. By way of contradiction, assume that the closest customer $c_1 = (i,j)$ to the depot is located on the interior of the grid. Find the shortest path from $(a,b)$ to $(i,j)$ with respect to Manhattan distance. This path must pass through one of the following lines: $y = 1$, $y = h$, $x = 1$, or $x = g$. Without loss of generality, assume it passes through $x = 1$ at the point $(1,s)$. In terms of Manhattan distance, we have

$$D((a,b),(i,j)) = D((a,b),(1,s)) + D((1,s),(i,j))$$

as $(1,s)$ lies on the shortest path between $(a,b)$ and $(i,j)$. Because $D((1,s),(i,j)) > 0$ as the points are distinct, we have $D((a,b),(i,j)) > D((a,b),(1,s))$ contradicting that $c_1$ is the closest customer to the depot. A similar argument can be made if the path passes through $y = h$, $y = 1$ or $x = g$. Thus, the closest customer $c_1$ to the depot will be located on the boundary.

Let $C$ be the set of coordinates representing customers. Consider the first case: $a \le 1, b \le 1$. We want to find $c_1 = (x_1,y_1)$ such that

$$\begin{aligned} D((a,b),(x_1,y_1)) &= \min_{(x,y)\in V} D((a,b),(x,y)) \\ &= \min_{(x,y)\in V} |a-x| + |b-y|. \end{aligned}$$

For $(x,y) \in \{(i,j) : i \in \{1,...,g\}, j \in \{1,...,h\}\}$, $a \le 1$, and $b \le 1$, we know $\min_{x\in\{1,...,g\}} |a-x|$ will occur when $x = 1$ and $\min_{y\in\{1,...,h\}} |b-y|$ will occur when $y = 1$. Therefore, $c_1 = (1,1)$.

A similar argument can be used for the remaining seven cases listed. Thus, we conclude $c_1$ is unique. □

LEMMA 2. *Under the assumptions of Lemma 1, take $c_1 = (i_1, j_1)$. The set of second closest customers to the depot $\mathcal{C}_2$ can include the following forms:*

$$\mathcal{C}_2 \subset \{(i_1, j_1+1), (i_1, j_1-1), (i_1+1, j_1), (i_1-1, j_1)\}.$$

*For all $c_2 \in \mathcal{C}_2$, $D(c_1, c_2) = 1$ and $D(0, c_2) = MinDistance + 1$. If $c_1$ is located on the corner of the grid, $|\mathcal{C}_2| = 2$. Otherwise, $|\mathcal{C}_2| = 3$. In addition, there exists $c_2 \in \mathcal{C}_2$ such that $c_1c_2$ is a boundary edge.*

*Proof.* Consider $R(g,h) = (V,E)$ and $c_1 = (i_1, j_1)$. The smallest distance between two vertices on the solid rectangular grid using Manhattan distance is assumed to be 1. Therefore, $D(c_1, c_2) = 1$ for all $c_2 \in \mathcal{C}_2$. It follows that candidates for $\mathcal{C}_2$ include $(i_1, j_1 \pm 1)$ and $(i_1 \pm 1, j_1)$. Since $c_1$ is on the boundary by Lemma 1, it follows that $c_1$ is on the shortest path from $c_2$ to the depot and

$$D(0, c_2) = D(0, c_1) + D(c_1, c_2) = MinDistance + 1.$$

To determine $|C_2|$, we consider two cases:

1. $c_1$ is located on the corner of the solid rectangular grid: A similar argument can be made for each corner, so we assume $c_1 = (1, h)$ (i.e. upper left corner). Figure EC.5 part (a) gives an example. Since $(1, h+1), (0, h) \notin V$, it follows that $\mathcal{C}_2 = \{(1, h-1), (2, h)\} \subset V$ since $g, h > 1$. Thus, we conclude $|\mathcal{C}_2| = 2$. For each $c_2 \in \mathcal{C}_2$, $c_1c_2$ is a boundary edge.
2. $c_1$ is not located on the corner of the solid rectangular grid: A similar argument can be made for all locations that are not corners, so we assume $c_1 = (i_1, h)$ for some $i_1 \in \{2, ..., g-1\}$. Figure EC.5 part (b) gives an example. Since $(i_1, h+1) \notin V$, it follows that $\mathcal{C}_2 = \{(i_1, h-1), (i_1+1, h), (i_1-1, h)\} \subset V$. Thus, we conclude $|\mathcal{C}_2| = 3$. For each $c_2 \in \{(i_1+1, h), (i_1-1, h)\} \subset \mathcal{C}_2$, $c_1c_2$ is on the boundary.

In each case, we have shown there exists $c_2 \in \mathcal{C}_2$ such that $c_1c_2$ is on the boundary. □

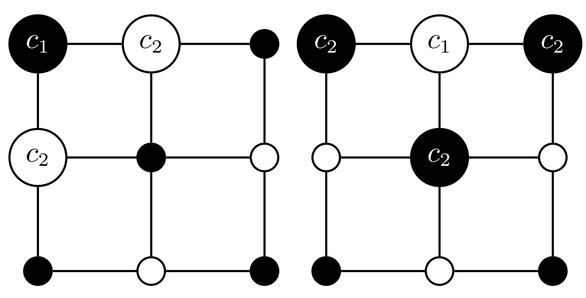



**Figure EC.5** **Examples of sets of second closest customers when $c_1$ is (a) located on the corner and (b) otherwise.**

LEMMA 3. *Under the assumptions of Lemma 1, take $c_1 = (i_1, j_1)$. The set of third closest customers to the depot $\mathcal{C}_3$ can include the following forms:*

$$\mathcal{C}_3 \subset \{(i_1-1, j_1+1), (i_1-1, j_1-1), (i_1+1, j_1-1), (i_1-1, j_1+1),$$
$$(i_1-2, j_1), (i_1+2, j_1), (i_1, j_1-2), (i_1, j_1+2)\}.$$

*For all $c_3 \in \mathcal{C}_3$, $D(c_1, c_3) = 2$ and $D(0, c_3) = MinDistance + 2$. If $c_1$ is located on the corner of the grid, $1 \le |\mathcal{C}_3| \le 3$. Otherwise, $2 \le |\mathcal{C}_3| \le 5$.*

*Proof* Consider $R(g,h) = (V,E)$ and $c_1 = (i_1, j_1)$. With adjacent edges assumed to be one unit away, the third closest customer will be all customers 2 units away from $c_1$. Therefore, $D(c_1, c_3) = 2$ for all $c_3 \in \mathcal{C}_3$. It follows that candidates for $\mathcal{C}_3$ include: $\{(i_1-1, j_1+1), (i_1-1, j_1-1), (i_1+1, j_1-1), (i_1-1, j_1+1), (i_1-2, j_1), (i_1+2, j_1), (i_1, j_1-2), (i_1, j_1+2)\}$. Since $c_1$ is on the boundary by Lemma 1, it follows that $c_1$ is on the shortest path from $c_3$ to the depot and

$$D(0, c_3) = D(0, c_1) + D(c_1, c_3) = MinDistance + 2.$$

To determine $|\mathcal{C}_3|$, we consider two cases:

1. $c_1$ is located on the corner of the solid rectangular grid: A similar argument can be made for each corner, so we assume $c_1 = (1, h)$ (i.e. upper left corner). Figure EC.6 part (a) gives an example. The candidates for $\mathcal{C}_3$ become:

$$\{(0, h+1), (0, h-1), (2, h-1), (0, h+1), (-1, h), (3, h), (1, h-2), (1, h+2)\}.$$

Since the grid is contained in the rectangle formed by $(1,1)$ and $(g,h)$, the feasible candidates become:

$$\{(2, h-1), (3, h), (1, h-2)\}.$$

Therefore, $|\mathcal{C}_3| \leq 3$. To show $|\mathcal{C}_3| \geq 1$, note $(2, h-1) \in \mathcal{C}_3$ as $g, h > 1$.

2. $c_1$ is not located on the corner of the solid rectangular grid: A similar argument can be made for all locations that are not corners, so we assume $c_1 = (i_1, h)$ for some $i_1 \in \{2, ..., h-1\}$. Figure EC.6 part (b) gives an example. The candidates for $\mathcal{C}_3$ become:

$$\{(i_1-1, h+1), (i_1-1, h-1), (i_1+1, h-1), (i_1-1, h+1), (i_1-2, h), (i_1+2, j_1), (i_1, h-2), (i_1, h+2)\}.$$

Since the grid is contained in the rectangle formed by $(1,1)$ and $(g,h)$, the feasible candidates become:

$$\{(i_1-1, h-1), (i_1+1, h-1), (i_1-2, h), (i_1+2, j_1), (i_1, h-2)\}.$$

Therefore, $|\mathcal{C}_3| \leq 5$. To show $|\mathcal{C}_3| \geq 2$, note $(i_1-1, h-1), (i_1+1, h-1) \in \mathcal{C}_3$ as $1 < i_1 < g$ and $g, h > 1$.

□

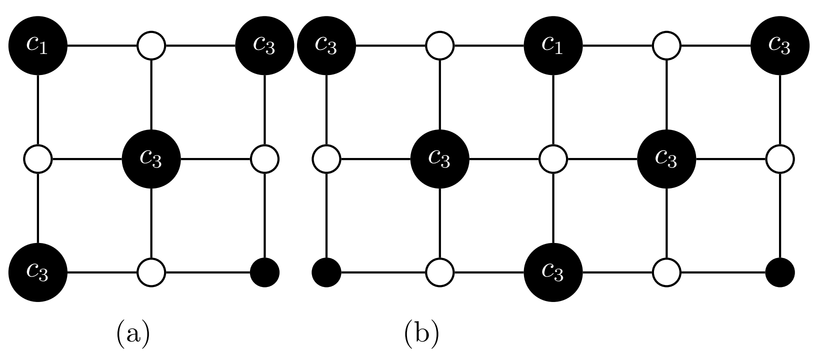



**Figure EC.6 Examples of sets of third closest customers when $c_1$ is (a) located on the corner and (b) otherwise.**

### B.3. Proof of Theorem 2

THEOREM 2. *Assume $n$ is even. For $c_2 \in \mathcal{C}_2$, there exists a Hamiltonian path from $s = c_1$ to $t = c_2$ through the solid rectangular grid.*

*If the grid is of size $2 \times g$, $g \times 2$, $3 \times g$, or $g \times 3$ for some $g \in \mathbb{N}$, choose $c_2$ such that $c_1c_2$ is a boundary edge. Otherwise, choose any $c_2 \in \mathcal{C}_2$.*

*Proof.* Consider $R(g,h) = (V,E)$. By Theorem 3.2 in Itai et al. [1982], to show the existence of a Hamiltonian path from $c_1$ to $c_2$, we must show $(R, c_1, c_2)$ is acceptable (i.e. color compatible and not forbidden). By Lemma 2, we know $D(c_1, c_2) = 1$. Therefore, in the coloring described in Equation (17), $c_1$ and $c_2$ are colored differently and color compatible, because $n$ is even. We need to verify that $(R, c_1, c_2)$ is not forbidden. We will show that given a forbidden case, we can choose another member of $\mathcal{C}_2$ to eliminate this case. So we consider the three forbidden cases:

1. 1 rectangle and $c_1, c_2$ are not corners. We assume $g, h > 1$ so this case does not apply.

2. 2 rectangle and $c_1c_2$ is a nonboundary edge. By Lemma 2, there exists $c_2 \in \mathcal{C}_2$ such that $c_1c_2$ is a boundary edge to eliminate this case.

3. $(R, c_1, c_2)$ is isomorphic to $(R', s', t')$ satisfying:

- $R' = R'(i,j)$ with $i = 3$ and $j$ even.
- $s'$ is colored differently from $t'$ and the left corners of $R'$
- One of the following hold:
  - (a) $s'_x < t'_x - 1$
  - (b) $s'_y = 2$ and $s'_x < t'_x$

Without loss of generality, we assume with a potential rotation that $R = R(3,h)$ for $h$ even and consider $s' = c_1$ and $t' = c_2$ as $c_1$ and $c_2$ are colored differently. We argue that neither case (a) nor (b) will hold.

Case (a): An equivalent statement to $c_{1_x} < c_{2_x} - 1$ is $c_{2_x} - c_{1_x} > 1$. By Lemma 2, $c_{1_x} - c_{2_x} = \{0,1\}$ since $D(c_1, c_2) = 1$. Therefore, statement (a) will never hold.

Case (b): By Lemma 1, $c_1$ is on the boundary. Assume $c_{1_y} = 2$. By Lemma 2, the only time when $c_{1_x} < c_{2_x}$ is when $c_2 = (c_{1_x} + 1, 2)$. Since $c_1$ is on the boundary, the only feasible case is when $c_{1_x} = 1$. Therefore, we have $c_1 = (1,2)$ and $c_2 = (2,2)$. Figure EC.7 gives an example. In this case, $c_1c_2$ is not a boundary edge. By Lemma 2, we can choose $c_2 \in \mathcal{C}_2$ such that $c_1c_2$ is a boundary edge (e.g. $c_2^1 = (1,3)$ and $c_2^2 = (1,1)$ in Figure EC.7). Therefore, we eliminate this case.

In summary, we can eliminate all forbidden cases. Thus, $(R, c_1, c_2)$ is acceptable for some $c_2 \in \mathcal{C}_2$ and there exists a Hamiltonian path from $c_1$ to $c_2$ in the solid rectangular grid. $\square$

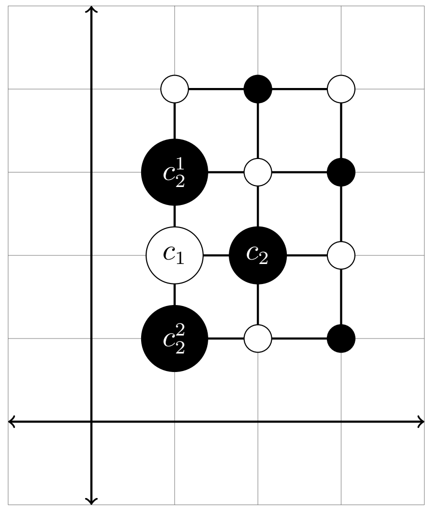



**Figure EC.7** **Example of 3 rectangle forbidden case with $c_{1_y} = 2$ and $c_{1_x} < c_{2_x}$.**

## B.4. Proof of Theorem 3

THEOREM 3. *Assume $n$ is odd and the solid rectangular grid of customers is colored using Equation* (17)*.*

- *If $c_1$ is colored by the majority color, let $s := c_1$ and $t \in \mathcal{C}_3$.*
- *If $c_1$ is not colored by the majority color, let $s, t \in \mathcal{C}_2$ such that $s \neq t$.*

*Then there exists a Hamiltonian path from $s$ to $t$ through the solid rectangular grid.*

*Proof.* Consider $R(g,h)$. By Theorem 3.2 in Itai et al. [1982], to show the existence of a Hamiltonian path from $s$ to $t$, we must show $(R,s,t)$ is acceptable (i.e. color compatible and not forbidden). To show it is color compatible, we consider two cases:

- $s = c_1$ is the majority color. Therefore, $t \in \mathcal{C}_3$ is the majority color as $D(t, c_1) = 2$ by Lemma 3. Since $n$ is odd and $s, t$ are the majority color, $(R,s,t)$ is color compatible.
- $c_1$ is not the majority color. Therefore, $s, t \in \mathcal{C}_2$ are the majority color since $D(c_1, c_2) = 1$ for all $c_2 \in \mathcal{C}_2$ by Lemma 2. Since $n$ is odd and $s, t \in \mathcal{C}_2$ are the majority color, $(R,s,t)$ is color compatible.

We need to verify that $(R,s,t)$ is not forbidden. So we consider the three forbidden cases. We assume $g, h > 1$ which eliminates the first forbidden case. The other two forbidden cases are not applicable as both result in an even number of vertices. In conclusion, $(R,s,t)$ is acceptable, and there exists a Hamiltonian path from $s$ to $t$ as defined. □

## B.5. Proof of Theorem 4.

THEOREM 4 *Assume a Hamiltonian path through the solid rectangular grid. We minimize terms (a), (b), and (c) of Equation 16 by traversing the Hamiltonian path and serving*

- *$n$ sets if $f \leq \frac{1}{w} - \frac{1}{d}$*

*and serving*

- *$\lceil \frac{n}{q} \rceil$ sets if $f \geq \frac{1}{w} - \frac{1}{d}$.*

*Proof.* Assume we serve the $n$ customers in $g$ sets along the Hamiltonian path of length $n-1$ to minimize the number of units traveled on the interior of the grid. It follows $\lceil \frac{n}{q} \rceil \leq g \leq n$. When serving $g$ sets of customers, we will drive $g-1$ edges between the sets and walk $n-1-(g-1)=n-g$ edges within the path. Given this grouping, the contribution of the objective function to terms (a), (b), and (c) of Equation (16) is given by:

$$\frac{1}{d} \cdot (g-1) + g \cdot f + \frac{1}{w} \cdot (n-g). \tag{EC.1}$$

Now, we show that it is sufficient to consider two specific number of sets in finding the optimal partition of the Hamiltonian path. The number of sets served and their respective objective values for terms (a), (b), and (c) of Equation (16) are given as follows:

1. Serve $n$ sets of customers (i.e. serve all customers individually). We drive $n-1$ edges and walk 0 edges. The contribution to the objective value is:

$$\frac{1}{d} \cdot (n-1) + n \cdot f \tag{EC.2}$$

2. Serve $\lceil \frac{n}{q} \rceil$ sets of customers. We drive $\lceil \frac{n}{q} \rceil - 1$ units and walk $n-1-\left(\lceil \frac{n}{q} \rceil - 1\right) = n - \lceil \frac{n}{q} \rceil$ units. The contribution to the objective value is:

$$\frac{1}{d} \cdot \left(\lceil \frac{n}{q} \rceil - 1\right) + MinSets \cdot f + \frac{1}{w} \cdot \left(n - \lceil \frac{n}{q} \rceil\right) \tag{EC.3}$$

Case 1 (serving $n$ sets) has a lower objective value than serving $g$ sets for $g < n$ when Equation (EC.2) is less than Equation (EC.1) which leads to the following calculations:

$$\frac{1}{d} \cdot (n-1) + n \cdot f \leq \frac{1}{d} \cdot (g-1) + g \cdot f + \frac{1}{w} \cdot (n-g) \tag{EC.4}$$

$$f \cdot (n-g) \leq \frac{1}{d} \cdot (g-n) + \frac{1}{w} \cdot (n-g) \tag{EC.5}$$

$$f \leq \frac{1}{w} - \frac{1}{d}. \tag{EC.6}$$

Rearranging the terms of Inequality (EC.4) gives us Inequality (EC.5). Note that since $g < n$, we know $n-g > 0$. Dividing both sides of Inequality (EC.5) by $n-g$ results in the desired Inequality (EC.6). In conclusion, serving them individually results in an optimal solution when

$$f \leq \frac{1}{w} - \frac{1}{d}. \tag{EC.7}$$

Similarly, Case 2 (serving $\lceil \frac{n}{q} \rceil$ sets) has a lower objective value than serving $g$ sets for $g > \lceil \frac{n}{q} \rceil$ when Equation (EC.3) is less than Equation (EC.1) which leads to the following calculations:

$$\frac{1}{d} \cdot \left(\lceil \frac{n}{q} \rceil - 1\right) + \lceil \frac{n}{q} \rceil \cdot f + \frac{1}{w} \cdot \left(n - \lceil \frac{n}{q} \rceil\right) \leq \frac{1}{d} \cdot (g-1) + g \cdot f + \frac{1}{w} \cdot (n-g) \tag{EC.8}$$

$$f \cdot \left(\lceil \frac{n}{q} \rceil - g\right) \leq \frac{1}{d} \cdot \left(g - \lceil \frac{n}{q} \rceil\right) + \frac{1}{w} \cdot \left(-g + \lceil \frac{n}{q} \rceil\right) \tag{EC.9}$$

$$f \cdot \left(\lceil \frac{n}{q} \rceil - g\right) \leq -\frac{1}{d} \cdot \left(\lceil \frac{n}{q} \rceil - g\right) + \frac{1}{w} \cdot \left(\lceil \frac{n}{q} \rceil - g\right) \tag{EC.10}$$

$$f \geq \frac{1}{w} - \frac{1}{d} \tag{EC.11}$$

Rearranging the terms of Inequality (EC.8) gives us Inequality (EC.9). The redistribution of the negative sign gives us Inequality (EC.10). Note that since $g > \lceil \frac{n}{q} \rceil$, we know $\lceil \frac{n}{q} \rceil - g < 0$. Dividing both sides of

Inequality (EC.10) by $\lceil \frac{n}{q} \rceil - g$ reverses the inequality sign and results in the desired Inequality (EC.11). In conclusion, serving $\lceil \frac{n}{q} \rceil$ number of sets results in an optimal solution when

$$f \geq \frac{1}{w} - \frac{1}{d}. \tag{EC.12}$$

By Inequalities (EC.7) and (EC.12), we conclude that if $f \leq \frac{1}{w} - \frac{1}{d}$, an optimal solution will be to serve $n$ sets and alternatively, if $f \geq \frac{1}{w} - \frac{1}{d}$, an optimal solution will be to serve $\lceil \frac{n}{q} \rceil$ sets. $\square$

### B.6. Proof of Theorem 5

We need the following result to show the solution constructed by Algorithm 1 is an optimal solution to the CAVADP. The starting $s$ and ending $t$ locations of the Hamiltonian paths in Table 4 minimize term (d) of Equation (16) by minimizing the number of units traveled on the exterior of the grid. In the case where $n$ is even, we minimize the total number of units to be traveled on the exterior and interior of the grid at $MinDistance + n - 1 + MinDistance + 1 = 2 \cdot MinDistance + n$ units. When $n$ is odd, Lemma 4 shows that the minimum number of units that can be traveled is $2 \cdot MinDistance + n + 1$.

Lemma 4. *Assume $n$ is odd. The minimum distance that must be traveled in the tour of customers from the depot is $2 \cdot MinDistance + n + 1$.*

*Proof.* Consider $R(g,h)$ where $n = gh$ is odd. We decompose the number of units traveled into the exterior and interior of the grid. If the number of units traveled on the exterior of the grid is minimized, we enter the grid at $c_1$ and exit the grid at $c_2$ for some $c_2 \in \mathcal{C}_2$. Therefore, there are $2 \cdot MinDistance + 1$ units traveled on the exterior of the grid. By Lemma 2, $D(c_1, c_2) = 1$. Therefore, by Equation (17), $c_1$ and $c_2$ are different colors which makes $(R, c_1, c_2)$ not color compatible as $n$ is odd. Thus, $(R, c_1, c_2)$ is not acceptable, and therefore, there does not exist a Hamiltonian path starting at $c_1$ and ending at $c_2$ by Theorem 3.2 in Itai et al. [1982]. Therefore, the units traveled on the interior of the grid is greater than $n-1$. Thus, the total distance traveled must be greater than $2 \cdot MinDistance + n$. Since the smallest distance between adjacent vertices is assumed to be one, it follows the total distance traveled must be greater than or equal to $2 \cdot MinDistance + n + 1$. To conclude, this bound can be realized realized by the Hamiltonian path in Theorem 3. $\square$

The additional unit to be traveled in the case when $n$ is odd can be chosen to occur on the exterior of the grid by the vehicle. Because we assume that the speed of driving is faster than walking, it is optimal to have the vehicle drive this unit on the exterior of the grid as opposed to the delivery person walking the unit.

Theorem 5. *Construct a solution using Algorithm 1 and call the resulting solution $S$. Table 5 presents the value of solution $S$ for each case, and these are the optimal solution values for the CAVADP.*

*Proof.* Constraints (4) require all $n$ customers must be visited once in the CAVADP. The shortest path through these customers on a grid is of length $n-1$ which is achieved by the existence of a Hamiltonian path as described in Table 4. Defining the tour as starting at the depot, following the Hamiltonian path from $s$ to $t$, and ending at the depot ensures there are no subtours. Theorem 4 gives the partitioning of the path that minimizes terms (a), (b), and (c) of Equation (16). Thus, the customers cannot be served in a faster way. By defining the sets and reloading points in this way, the Hamiltonian tour satisfies all constraints. Finally, starting and ending the tour as defined in Table 4 minimizes term (d) of Equation 16 by minimizing the distance between the depot and grid (Theorem 1). When $n$ is odd, Lemma 4 shows an additional unit must be traveled. This unit can be chosen to be driven on the exterior as $d < w$ to ensure the existence of the Hamiltonian path and maintain optimality for the case when $n$ is odd. In summary, the solution simulataneously minimizes each term of Equation (16) and is an optimal solution to the CAVADP. □

## Appendix C: CAVADP Solution on a Line

Theorem 12 describes the shortest path through the customers on a line where the depot is located on the exterior of the line.

THEOREM 12. *Consider a line of $n$ customers embedded into the plane by placing the $i$-th customer at $(i,0)$ with the depot located on the exterior at $(a,b)$. The shortest path starting/ending at the depot and visiting all customers is satisfied by visiting the customer at $(1,0)$ first and the customer at $(n,0)$ last.*

*Proof.* We propose the path of the first customer visited being $(1,0)$ and the last customer visited being $(n,0)$. Using Manhattan distance, the length of this path is

$$D((a,b),(1,0)) + n - 1 + D((n,0),(a,b)) = 2|b| + |a-1| + |n-b| + n - 1. \tag{EC.13}$$

Consider a path that starts at $(i,0)$ and ends at $(j,0)$ such that $i \neq 0$ or $j \neq n-1$. As we can change the direction of the path, we can assume $i < j$. The shortest distance to visit every customer would be to serve $(i,0)$, serve the customers to the left of $(i,0)$ ($2i$ units), serve the customers between $(i,0)$ and $(j,0)$ ($j-i$ units), and finally, serve the customers to the right of $(j,0)$ ($2(n-1-j)$ units). Using Manhattan distance, the total distance traveled would be

$$\begin{aligned} &D((a,b),(i,0)) + 2i + j - i + 2(n-1-j) + D((j,0),(a,b)) \\ &\qquad = |a-i| + 2|b| + |j-a| + i - j + 2n - 2. \end{aligned} \tag{EC.14}$$

As the location of the depot is fixed, we minimize Equation (EC.14) over $i,j \in \{1,...,n\}$ which is equivalent to the following minimization

$$\min_{i,j\in\{1,\ldots,n\}} |a-i| + |a-j| + i - j. \tag{EC.15}$$

We now consider three cases: $a < 1, 1 \le a \le n$ and $a > n$.

1. $a < 1$. Then we know $a < i$ and $a < j$. Equation (EC.15) can be written

$$\min_{i,j\in\{1,\ldots,n\}} |a-i| + |a-j| + i - j = \min_{i,j\in\{1,\ldots,n\}} i - a + j - a + i - j \tag{EC.16}$$

$$= \min_{i,j\in\{1,\ldots,n\}} 2i - 2a \tag{EC.17}$$

which is minimized when $i = 1$. If $i = 1$, the shortest path through the customers would be for $j = n$ which is the proposed solution.

2. $1 \leq a \leq n$. Equation (EC.15) is minimized at 0 when $j = a$ and $i = a$. In this case, the path length given by Equation (EC.14) is

$$2|b| + 2n - 2 = 2|b| + 2(n-1). \tag{EC.18}$$

Noting that $1 \leq a \leq n$, Equation (EC.13) giving the proposed solution becomes

$$2|b| + a - 1 + n - a + n - 1 = 2|b| + 2(n-1) \tag{EC.19}$$

showing that the proposed solution realizes the minimum distance traveled for this case.

3. $a > n$. Then we know $a > i$ and $a > j$. Equation (EC.15) can be written

$$\min_{i,j \in \{1,\ldots,n\}} |a-i| + |a-j| + i - j = \min_{i,j \in \{1,\ldots,n\}} a - i + a - j + i - j \tag{EC.20}$$

$$= \min_{i,j \in \{1,\ldots,n\}} 2a - 2j \tag{EC.21}$$

which is minimized when $j = n$. If $j = n$, the shortest path through the customers would be for $i = 1$ which is the proposed solution.

In summary, the path that starts at $(1,0)$ and ends at $(n,0)$ satisfies being the shortest path starting/ending at the depot and visiting all customers. □

Since the solution constructed in Theorem 12 utilizes a Hamiltonian path through the customers, Theorem 4 still holds. Therefore, we are able to characterize the solution to the CAVADP on the line. In summary, the vehicle drives from the depot to the first customer at $(1,0)$. The delivery person traverses the line from $(1,0)$ to $(n,0)$, serving either $n$ sets (when $f \leq \frac{1}{w} - \frac{1}{d}$) or $\lceil \frac{n}{q} \rceil$ sets (when $f \geq \frac{1}{w} - \frac{1}{d}$). The vehicle then drives from the customer at $(n,0)$ to the depot. Table EC.2 provides the objective value for the CAVADP on a line.

| $d, w, f$ | Sets Served | Objective Value |
|---|---|---|
| $f \geq \frac{1}{w} - \frac{1}{d}$ | $\lceil \frac{n}{q} \rceil$ sets | $\frac{1}{d}(2\lvert b\rvert + \lvert a\rvert + \lvert n-1-a\rvert + \lceil \frac{n}{q} \rceil - 1) + \frac{1}{w}(n - \lceil \frac{n}{q} \rceil) + \lceil \frac{n}{q} \rceil \cdot f$ |
| $f \leq \frac{1}{w} - \frac{1}{d}$ | $n$ sets | $\frac{1}{d}(2\lvert b\rvert + \lvert a\rvert + \lvert n-1-a\rvert + n - 1) + nf$ |

**Table EC.2 Optimal Solution for CAVADP on a line with the depot located at $(a, b)$.**

## Appendix D: CAVADP Extensions on Grid

In this appendix, we consider extensions to the CAVADP by having the loading time dependent on the number of packages and including pickup services on the delivery tour. In both cases, we characterize the optimal solution on the grid. In these extensions, we maintain the assumptions of Section 3.1, and continue to assume the solid rectangular grid in Section 3.4.

Then, we relax the assumptions regarding single package delivery, driving speed being faster than walking speed, and constant driving speeds. We explore the implications of these assumptions on the solution of the CAVADP on a complete grid.

### D.1. CAVADP with Loading Times Dependent on the Number of Packages

We will characterize the optimal solution when the loading time depends on the number of packages being delivered. Let $h$ be the loading time for one package (in minutes). Then, the objective function given in Equation (1) can be rewritten as follows:

$$\min \quad \sum_{i=0}^{n}\sum_{\substack{k=0\\k\neq i}}^{n} x_{ik}d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}(w_{ijk} + |\sigma_j|h) \tag{EC.22}$$

$$= \min \quad \sum_{i=0}^{n}\sum_{\substack{k=0\\k\neq i}}^{n} x_{ik}d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}w_{ijk} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}(|\sigma_j|h) \tag{EC.23}$$

$$= \min \quad \sum_{i=0}^{n}\sum_{\substack{k=0\\k\neq i}}^{n} x_{ik}d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}w_{ijk} + nh. \tag{EC.24}$$

Equation (EC.23) expands the second term of Equation (EC.22). Recall that Constraints (4) ensure each customer is served in exactly one set. Thus, Equation (EC.24) evaluates the third term to be $nh$ (i.e. the time it takes to load all $n$ packages).

Since $nh$ is constant, we can assume $h = 0$ and the solution to Equation (EC.24) is equivalent to the solution of the following objective function:

$$\min \quad \sum_{i=0}^{n}\sum_{\substack{k=0\\k\neq i}}^{n} x_{ik}d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}w_{ijk} \tag{EC.25}$$

This problem becomes equivalent to the CAVADP with $f = 0$ minutes. Under the assumption that $d > w$, it follows $f = 0 \leq \frac{1}{w} - \frac{1}{d}$. We can construct the optimal solution using Algorithm 1. The optimal solution of the CAVADP with package dependent loading time is for the delivery person to follow the constructed Hamiltonian path and serve $n$ sets (i.e. serve every customer individually). Theorem 13 formalizes this result.

THEOREM 13. *Construct a solution using Algorithm 1 where $f = 0$ and call the resulting solution $S$. Table EC.3 presents the value of solution $S$ for each case, and these are the optimal solution values for the CAVADP with package dependent loading time.*

| $n$ | **Optimal Objective Value** |
|---|---|
| Even | $\frac{1}{d}(2 \cdot MinDistance + n) + nh$ |
| Odd | $\frac{1}{d}(2 \cdot MinDistance + n + 1) + nh$ |

**Table EC.3 Optimal Solution for CAVADP on solid rectangular grid with package dependent loading time.**

### D.2. CAVADP with Pickups and Deliveries

The CAVADP with pickups includes delivery and pickup services to a set of customers using an autonomous vehicle assisted by a delivery person. Any packages picked up during the delivery tour must return to the depot. Similar to the CAVADP, servicing each set of customers requires the delivery person to be dropped off by the vehicle at a determined location and load the packages to be delivered. Then, at each customer, the delivery person delivers as well as picks up packages. The delivery person is then picked up by the vehicle where he/she loads the picked up packages. For the purposes of this analysis, we make the following assumptions:

- The fixed time for loading picked up packages $g$ is independent of the number of pickups.
- Every customer being considered has at most a single pickup.

We will show that Algorithm 1 constructs the optimal solution of the CAVADP with pickups and deliveries on the complete grid. First, consider service set $\sigma_j$. Assuming a single delivery and at most a single pickup per customer, service set $\sigma_j$ has $|\sigma_j|$ deliveries and at most $|\sigma_j|$ pickups. At each customer, the delivery person can deliver the customer's package and then pickup the package (if there is one) that is to be taken to the depot. If there is at most a single pickup at each customer location, the capacity of the delivery person $q$ is never exceeded during service because he/she will not be carrying more packages after visiting the customer than before. Define $f' = f + g$ to include the time to load packages to be delivered and load the packages to be picked up. Then, the objective function given in Equation (1) can be rewritten as follows:

$$\sum_{i=0}^{n}\sum_{\substack{k=0\\k\neq i}}^{n} x_{ik}d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}(w_{ijk} + f'). \tag{EC.26}$$

Theorem 5 shows that Algorithm 1 constructs the optimal solution to the CAVADP with pickups where $f'$ represents total loading time. Theorem 14 formalizes this result.

THEOREM 14. *Construct a solution using Algorithm 1 with the fixed time for loading packages as $f' = f + g$. Call the resulting solution $S$. Table 5 presents the value of solution $S$ for each case with $f$ replaced by $f'$, and these are the optimal solution values for the CAVADP with pickups.*

The assumption that every customer being considered has a single pickup is necessary to ensure the capacity of the delivery person is not exceeded while servicing customers on foot. If there are more pickups than deliveries at a customer, the delivery person's capacity along the service route would need to be considered in the construction of potential service sets. Future work includes adding this additional structure to the model presented in Section 3.3.

### D.3. Multiple Packages per Customer

The theoretical results of Section 3.4 for the CAVADP on the grid rely on the assumption that each customer has a single delivery. We relax this assumption to allow customer $i$ to have $d_i \geq 1$ deliveries for all $i \in \{1, ..., n\}$. We characterize the optimal solution to the CAVADP with multiple packages on a solid rectangular grid when $f \leq \frac{1}{w} - \frac{1}{d}$. When $f \geq \frac{1}{w} - \frac{1}{d}$, we provide a counter example to the delivery person following the Hamiltonian path.

### D.3.1. Low Loading Time

Assume $f \leq \frac{1}{w} - \frac{1}{d}$. If $d_i = 1$ for all $i \in \{1,...,n\}$, Theorem 4 concludes $n$ sets of size 1 are served in the optimal solution making each customer a reloading point. Lemma 5 generalizes this result when $d_i \geq 1$ for all $i \in \{1,...,n\}$.

LEMMA 5. *Assume $f \leq \frac{1}{w} - \frac{1}{d}$. The only service sets of size greater than one consist of customers with multiple packages (i.e. all customers are reloading points).*

*Proof.* Fix the route of the delivery person. Consider $y_{ajb} = 1$ for some $a, b \in \{1,...,n\}$. Denote $v := |\sigma_j|$ and let $(a = j_1, j_2, ..., j_v = b)$ be the optimal way to serve $\sigma_j$ when loading at $a$ and returning to $b$. We will show $D(j_l, j_{l+1}) = 0$ for all $l \in \{1,...,v-1\}$ (i.e. all packages come from the same customer location). By way of contradiction, assume there exists $l \in \{1,...,v-1\}$ such that $D(j_l, j_{l+1}) > 0$. Each distinct customer on the grid is at least one block apart, so $D(j_l, j_{l+1}) \geq 1$. By assumption, $f \leq \frac{1}{w} - \frac{1}{d} \leq (\frac{1}{w} - \frac{1}{d})D(j_l, j_{l+1})$ and can be written as

$$\frac{1}{d}D(j_l, j_{l+1}) + f \leq \frac{1}{w}D(j_l, j_{l+1}). \tag{EC.27}$$

Since $y_{ajb} = 1$, the contribution to the objective function in Equation (1) is:

$$\begin{aligned}
w_{ajb} + f &= \frac{1}{w}\Big(D(a, j_1) + D(j_1, j_2) + \cdots + D(j_{l-1}, j_l) + D(j_l, j_{l+1}) + D(j_{l+1}, j_{l+2}) + \cdots \\
&\qquad + D(j_{v-1}, j_v) + D(j_v, b)\Big) + f \qquad \text{(EC.28)}\\
&\geq \frac{1}{w}\Big(D(a, j_1) + D(j_1, j_2) + \cdots + D(j_{l-1}, j_l)\Big) + \frac{1}{d}D(j_l, j_{l+1}) + f \\
&\quad + \frac{1}{w}\Big(D(j_{l+1}, j_{l+2}) + \cdots + D(j_{v-1}, j_v) + D(j_v, b)\Big) + f \qquad \text{(EC.29)}\\
&= w_{aj'j_l} + f + d_{j_l j_{l+1}} + w_{j_{l+1}j''b} + f \qquad \text{(EC.30)}
\end{aligned}$$

where $\sigma_{j'} = \{j_1, ..., j_l\}$ and $\sigma_{j''} = \{j_{l+1}, ..., j_v\}$. Equation (EC.28) expands the service times by the definitions in Section 3.2. Inequality (EC.29) uses Inequality (EC.27). Using the definitions of service times from 3.2, we state Equation (EC.30) and conclude that the solution that services set $\sigma_j$ is bounded below by a solution that services sets $\sigma_{j'}$ and $\sigma_{j''}$ with the delivery person being on board the vehicle between customers $j_l$ and $j_{l+1}$. Thus, we can take $D(j_l, j_{l+1}) = 0$ for all $l \in \{1,...,v-1\}$. □

Now, we construct a solution of the CAVADP with multiple packages per customer on a grid as follows. Algorithm 2 summarizes this procedure. First, use Table 4 to determine the first customer $s$ and last customer $t$ that the vehicle will visit. Then, the delivery person follows the Hamiltonian path from customer $s$ to customer $t$. Since $f \leq \frac{1}{w} - \frac{1}{d}$, Lemma 5 concludes each customer is a reloading point. Therefore, the vehicle must visit each customer. Then, at each customer $i$, the delivery person services $\lceil \frac{d_i}{q} \rceil$ service sets.

Theorem 15 shows that the solution constructed in Algorithm 2 is an optimal solution to the CAVADP with multiple packages per customer on a grid when $f \leq \frac{1}{w} - \frac{1}{d}$.

THEOREM 15. *Assume $f \leq \frac{1}{w} - \frac{1}{d}$. Construct a solution using Algorithm 2 and call the resulting solution $S$. Table EC.4 presents the value of solution $S$ for each case, and these are the optimal solution values for the CAVADP with multiple packages per customer on a grid.*

**Algorithm 2** CAVADP on Solid Rectangular Grid with Multiple Packages when $f \leq \frac{1}{w} - \frac{1}{d}$

**Input:**
  Size of solid rectangular rectangular grid $g \times h$ with $n$ customers
  Location of the depot
  Demand of each customer $d_i$ $\forall i \in \{1,...,n\}$
  Driving speed of vehicle $d$
  Walking speed of delivery person $w$
  Fixed time for loading packages $f$
**Output:** Optimal Solution $S$
Find the closest customers to the depot.
Use Table 4 to determine the points of entrance $s$ and exit $t$ in the grid.
Find the Hamiltonian path from $s$ to $t$ through the grid.
Determine the number of services sets at each customer: $\lceil \frac{d_i}{q} \rceil$ service sets at customer $i$.

*Proof.* First, we show that at each unique customer location, the minimum number of sets will be served. Consider location $i$ with $d_i$ packages. Let $g_i$ be the number of service sets the delivery person will serve while loading and returning to location $i$. We will show $g_i = \lceil \frac{d_i}{q} \rceil$. We have shown that the only service sets of size greater than one consist of customers at the same location. Therefore, we know $\lceil \frac{d_i}{q} \rceil \leq g_i \leq d_i$. If $q = 1$, then $\lceil \frac{d_i}{q} \rceil = d_i = g_i$ as desired. When $q > 1$, by way of contradiction, assume $g_i = \lceil \frac{d_i}{q} \rceil + k$ for some $k \in \mathbb{Z}_+$ such that $\lceil \frac{d_i}{q} \rceil + k \leq d_i$. Let $\{(a^1, j^1, b^1), (a^2, j^2, b^2), ..., (a^{\lceil \frac{d_i}{q} \rceil + k}, j^{\lceil \frac{d_i}{q} \rceil + k}, b^{\lceil \frac{d_i}{q} \rceil + k})\}$ denote the rendevous points and service sets in the solution. Note that there exists a solution such that $g_i = \lceil \frac{d_i}{q} \rceil$. Denote this solution by $\{(\hat{a}^1, \hat{j}^1, \hat{b}^1), (\hat{a}^2, \hat{j}^2, \hat{b}^2), ..., (\hat{a}^{\lceil \frac{d_i}{q} \rceil}, \hat{j}^{\lceil \frac{d_i}{q} \rceil}, \hat{b}^{\lceil \frac{d_i}{q} \rceil})\}$. It follows:

$$\sum_{l=1}^{\lceil \frac{d_i}{q} \rceil + k} (f + w_{a^l j^l b^l}) y_{a^l j^l b^l} = \Big(\lceil \frac{d_i}{q} \rceil + k\Big) f \tag{EC.31}$$

$$\geq \Big(\lceil \frac{d_i}{q} \rceil\Big) f \tag{EC.32}$$

$$= \sum_{l=1}^{\lceil \frac{d_i}{q} \rceil} (f + w_{\hat{a}^l \hat{j}^l \hat{b}^l}) y_{\hat{a}^l \hat{j}^l \hat{b}^l} \tag{EC.33}$$

Equation (EC.31) evaluates the service times by the definitions in Section 3.2. Note that $w_{i j^l i} = 0$ for all $l$ by Lemma 5. Inequality (EC.32) follows since $k, f \geq 0$. Using the definitions of service times in 3.2, we state Equation (EC.33) and conclude that the solution that services $\lceil \frac{d_i}{q} \rceil + k$ sets for any $k \in \mathbb{Z}_+$ is bounded below by a solution that services $\lceil \frac{d_i}{q} \rceil$ sets as desired.

Now, consider the objective function in Equation (1). We have shown the delivery person will service $\lceil \frac{d_i}{q} \rceil$ sets at each location $i$ and will not walk between any customer locations during service, so Equation (1) can be written as follows:

$$\min \sum_{i=0}^{n} \sum_{\substack{k=0 \\ k \neq i}}^{n} x_{ik} d_{ik} + \sum_{i=1}^{n} \lceil \frac{d_i}{q} \rceil f. \tag{EC.34}$$

The second term of Equation (EC.34) is constant. Therefore, the optimal solution reduces to the delivery person being on board the vehicle traveling the shortest path through the customers. This path is achieved by following the Hamiltonian path in Table 4. □

| $n$ | Sets Served | Optimal Objective Value |
|---|---|---|
| Even | $\sum_{i=1}^{n} \lceil \frac{d_i}{q} \rceil$ sets | $\frac{1}{d}(2 \cdot MinDistance + n) + f \cdot \sum_{i=1}^{n} \lceil \frac{d_i}{q} \rceil$ |
| Odd | $\sum_{i=1}^{n} \lceil \frac{d_i}{q} \rceil$ sets | $\frac{1}{d}(2 \cdot MinDistance + n + 1) + f \cdot \sum_{i=1}^{n} \lceil \frac{d_i}{q} \rceil$ |

**Table EC.4 Optimal Solution for CAVADP with multiple packages on solid rectangular grid for $f \leq \frac{1}{w} - \frac{1}{d}$.**

#### D.3.2. High Loading Time

Assume $f \geq \frac{1}{w} - \frac{1}{d}$. At high loading times and $d_i = 1$ for all $i \in \{1, ..., n\}$, Theorem 4 concludes that the delivery person serves $\lceil \frac{n}{q} \rceil$ sets to minimize loading time. When $d_i \geq 1$ for all $i \in \{1, ..., n\}$, the varying demand per customer results in the potential for the delivery person to not follow the Hamiltonian path in order to realize fewer loadings. When $q = 3$, Figure EC.8 shows that the delivery person visits customer A twice on foot as opposed to servicing both packages on the first visit to customer A. The delivery person travels nine blocks within the grid as opposed to the Hamiltonian path of $n - 1 = 8$ blocks. The additional unit traveled within the grid allows for the delivery person to service $\lceil \frac{\sum_{i=1}^{n} d_i}{q} \rceil = 5$ service sets. At high loading times, the delivery person may prioritize minimizing the number of services sets over the travel in the grid.

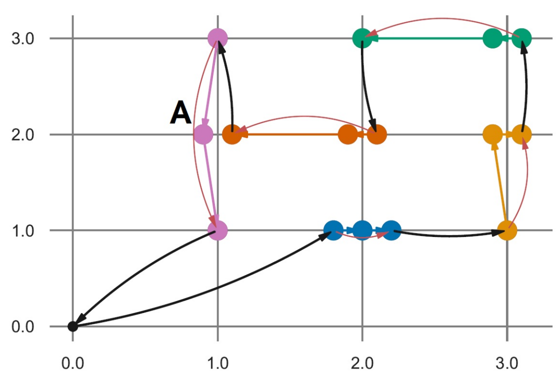
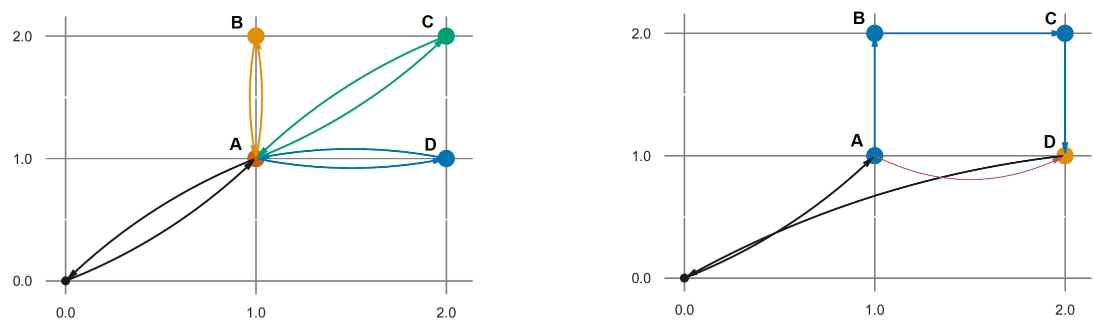



**Figure EC.8 CAVADP solution with multiple packages per customer when $f \geq \frac{1}{w} - \frac{1}{d}$ and $q = 3$. The delivery person does not follow the Hamiltonian path within the grid.**

### D.4. Walking Speed is Faster than Driving Speed

When walking speed is faster than driving speed $w > d$, the vehicle can no longer meet the delivery person at rendezvous points by following the same path without the delivery person now having to wait. The time that the delivery person needs to wait for the vehicle is not accounted for in the CAVADP objective function given in Equation (1). When waiting time is accounted for in the objective, travel time may not be minimized by following the Hamiltonian path.

Figure EC.9 gives examples where the Hamiltonian path is not followed by either the delivery person or the vehicle. When $q = 1$, case (a) gives an example where the walking speed is at least twice the driving speed. Walking to each customer from customer A takes less time than waiting for the vehicle at each customer along the Hamiltonian path. At higher capacities, such as $q = 3$, case (b) shows that the vehicle may be able to take a shorter path by driving directly from loading point A to loading point D as opposed to following the Hamiltonian path. At higher capacities, the vehicle does not necessarily visit all customers. Therefore, minimizing vehicle travel between loading points will reduce waiting time for the delivery person. Even though utilizing the Hamiltonian path minimizes distance traveled, the completion time of the delivery tour may not be minimized when accounting for waiting time for the delivery person.



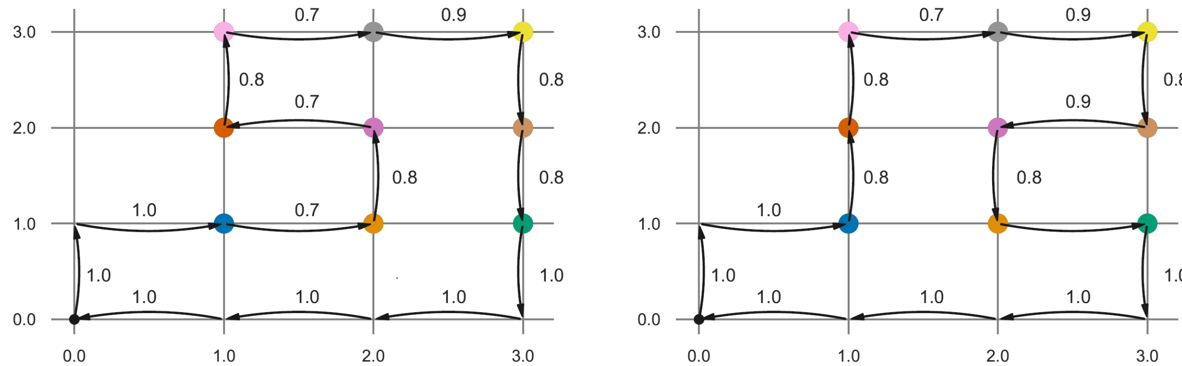

(a) $q = 1$ (b) $q = 3$

**Figure EC.9 CAVADP solutions when $w > d$ that reduce waiting time by (a) the delivery person walking to every customer or (b) the vehicle not following the Hamiltonian path.**

### D.5. Heterogeneous Driving Speeds

The theoretical results of Section 3.4 for the CAVADP on the grid rely on the assumption that the driving speed $d$ is constant. In this case, minimizing the time spent driving is equivalent to minimizing distance traveled. When we allow driving speed to vary on the blocks of the grid, the travel time of individual Hamiltonian paths may differ. Consider $q = 1$ packages. Figure EC.10 gives two solutions to the CAVADP that each utilize a Hamiltonian path and minimize distance traveled. In these figures, the edge labels are the individual block's driving speed (blocks/min). The travel time of the Hamiltonian path in part (a) is 16.4 minutes, but the travel time in part (b) is 15.8 minutes. When the driving speeds vary, the existence of the Hamiltonian path is no longer sufficient for constructing the optimal solution.

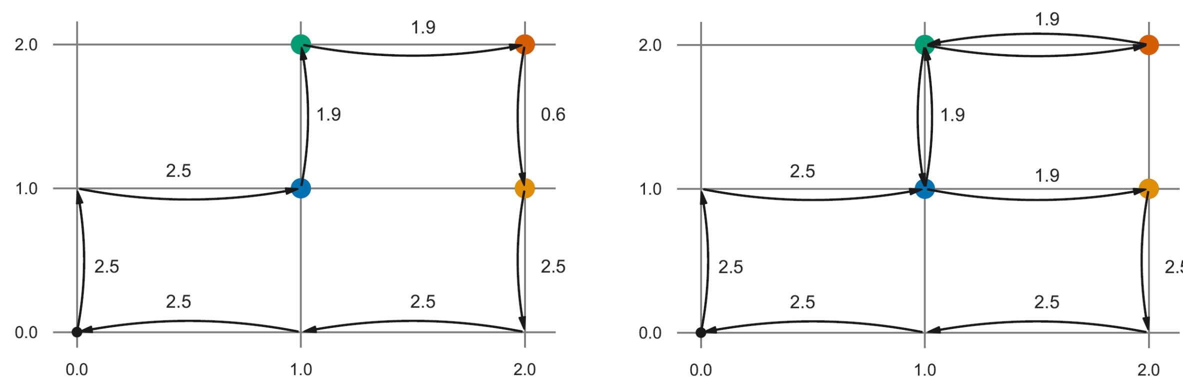
In some cases, the optimal solution may not even follow the Hamiltonian path. The path that minimizes travel time between two customers may not be the shortest path. Figure EC.11 gives an example where traveling more units results in a reduced completion time of the delivery tour. In case (a) of Figure EC.11, the vehicle follows the Hamiltonian path entering and exiting the grid from the closest and second closest customers, respectively. The completion time of the delivery tour is 4.7 minutes. However, in case (b), by




(a) 16.4 minutes (b) 15.8 minutes

**Figure EC.10 CAVADP solutions following an Hamiltonian path but resulting in different travel times due to varying driving speeds on blocks.**

traveling two additional blocks in total, the completion time of the delivery tour is reduced to 4.6 minutes. Therefore, for any two customers, all paths between these customers must be considered to minimize travel time.




(a) 4.7 minutes (b) 4.6 minutes

**Figure EC.11 CAVADP solutions when driving speeds vary on the blocks of the grid shows that traveling more units (case (b)) can reduce the travel time when compared to minimizing distance traveled (case (a)).**

## Appendix E: Capacitated Delivery Problem with Parking (CDPP)

In this appendix, we define the service times and provide the integer programming formulation for the CDPP.

### E.1. Service Times

The parameters and service times for the CDPP are summarized in Tables 1 and EC.5 respectively. We define $\hat{d}_{ik}$ to be the time to drive from customer $i$ to customer $k$ and park at $k$. Given the driving speed $d$, it follows $\hat{d}_{ik} = \frac{1}{d} \cdot D(i,k) + p$ for $i \in \{0,...,n\}$ and $k \in \{1,...,n\}$ such that $i \neq k$. In the case where $k = 0$ (i.e. the return to the depot), the vehicle does not need to park and $\hat{d}_{i0} = \frac{1}{d} \cdot D(i,0)$ for all $i \in \{1,...,n\}$. We define $\hat{w}_{ij}$ to be the time to walk from customer $i$ to the first customer to be served in set $\sigma_j$, walk among customers in $\sigma_j$, and walk back to customer $i$ where the vehicle is parked. Note that customer $i$ may or may not be in set $\sigma_j$. The time $\hat{w}_{ij}$ is equivalent to $w_{ijk}$ from Section 3.2 where $k = i$.

| Notation | Description |
|---|---|
| $\hat{d}_{ik}$ | Time to drive from customer $i$ to customer $k$ and park at $k$ for $i, k \in \{0, 1, ..., n\}$ such that $i \neq k$ and $k \neq 0$ (minutes) |
| $\hat{d}_{i0}$ | Time to drive from customer $i$ to depot for $i \in \{1, ..., n\}$ (minutes) |
| $\hat{w}_{ij}$ | Time to walk and serve set $\sigma_j$ while parked at customer $i$ (minutes) |

**Table EC.5 Definition of service times in CDPP.**

### E.2. Model

We define the binary decision variables $\hat{x}_{ik}$ and $\hat{y}_{ij}$ as well as the integer valued variable $\hat{u}_i$ in Table EC.6.

| Notation | Description |
|---|---|
| $\hat{x}_{ik}$ | $\hat{x}_{ik} = 1$ if the vehicle drives from customer $i$ to customer $k$ and parks at customer $k$ for $i, k \in \{0, 1, ..., n\}$ such that $i \neq k$ |
| $\hat{y}_{ij}$ | $\hat{y}_{ij} = 1$ if the delivery person is parked at customer $i$ and serves set $\sigma_j$ for $i \in \{1, ..., n\}$ and $j \in \{1, ..., m\}$ |
| $\hat{u}_i$ | Gives the position of the vehicle visiting customer $i$ in the tour given that the customer is visited for $i \in \{0, 1, ..., n\}$ |

**Table EC.6 Set of decision variables in CDPP.**

The CDPP can then be modeled with the following integer programming formulation.

$$\min \sum_{i=0}^{n} \sum_{\substack{k=0 \\ k \neq i}}^{n} \hat{x}_{ik} \hat{d}_{ik} + \sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij} (\hat{w}_{ij} + f) \tag{EC.35}$$

$$\text{s.t.} \sum_{i=1}^{n} \hat{x}_{0i} = 1 \tag{EC.36}$$

$$\sum_{i=1}^{n} \hat{x}_{i0} = 1 \tag{EC.37}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} I_{kj} \hat{y}_{ij} \geq 1 \qquad \text{for each } k \in \{1, ..., n\} \tag{EC.38}$$

$$\sum_{\substack{i=0 \\ i \neq k}}^{n} \hat{x}_{ik} = \sum_{\substack{l=0 \\ l \neq k}}^{n} \hat{x}_{kl} \qquad \text{for each } k \in \{1, ..., n\} \tag{EC.39}$$

$$\hat{y}_{ij} \leq \sum_{\substack{k=0 \\ k \neq i}}^{n} \hat{x}_{ki} \qquad \text{for all } i \in \{1, ..., n\}, j \in \{1, ..., m\} \tag{EC.40}$$

$$\hat{u}_0 = 0 \tag{EC.41}$$

$$\hat{u}_k \leq \sum_{i=1}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \hat{x}_{il} + 1 \qquad \text{for all } k \in \{1, ..., n\} \tag{EC.42}$$

$$\hat{u}_i - \hat{u}_k + 1 \leq n(1 - \hat{x}_{ik}) \qquad \text{for all } i \in \{0,1,...,n\}, k \in \{1,...,n\} \text{ such that } i \neq k \tag{EC.43}$$

$$\hat{x}_{ik} \in \{0,1\} \qquad \text{for all } i,k \in \{0,1,...,n\} \text{ such that } i \neq k \tag{EC.44}$$

$$\hat{y}_{ij} \in \{0,1\} \qquad \text{for all } i \in \{1,...,n\}, j \in \{1,...,m\} \tag{EC.45}$$

$$\hat{u}_i \in \mathbb{Z}_{\geq 0} \qquad \text{for all } i \in \{0,1,...,n\}. \tag{EC.46}$$

The objective function (EC.35) minimizes the completion time of the delivery tour. Constraints (EC.36) and (EC.37) ensure that the vehicle drives from the depot to a customer and returns to the depot by driving from a customer. Constraints (EC.38) ensure that all customers are served. When the vehicle visits a customer, Constraints (EC.39) verify that the vehicle will also leave that customer. Given that the delivery person serves set $\sigma_j$ from customer $i$ (i.e. $\hat{y}_{ij} = 1$), Constraints (EC.40) verify that the vehicle will visit customer $i$. The subtour elimination constraints are adapted from MTZ constraints and given in Constraints (EC.41), (EC.42), (EC.43), and (EC.46). Finally, the binary constraints on variables $\hat{x}_{ik}$ and $\hat{y}_{ij}$ are given in Constraints (EC.44) and (EC.45), respectively.

## Appendix F: Experimental Results

Tables EC.7 and EC.8 show the optimal objective values over a variety of settings for $f = 2.8$ and $n$ at most 36 and above 36, respectively. We report the optimal objective values of the CAVADP in the third column and the CDPP with parking times of 0, 6, 9, and 15 minutes in the fourth, sixth, tenth, and fourteenth columns, respectively. Similarly, Tables EC.9 and EC.10 show the optimal objective values for $f = 0.5$ with $n$ at most 36 and above 36, respectively. We report the optimal objective values of the CAVADP in the third column and the CDPP with parking times of 6, 9, and 15 minutes in the fourth, eighth, and twelfth columns, respectively. Each grid size is considered with capacities of 1, 2, 3, and 4 packages. Recall the CDPP is solved with Gurobi as stated in Section 4. An asterisk (*) indicates the IP was solved to an optimality gap of 1%. Two asterisks (**) indicate an optimality gap of 2.5%. Three asterisks (***) indicate an optimality gap of 5.1%.

Due to the rapid increase in variables, some instances could not be solved within these gaps and a heuristic is implemented as discussed in Section 4. The solutions to these modified IPs are indicated by italicized results. The case where the modified IP can only be solved within the defined gaps is indicated by the appropriate asterisks with the italicized values. Note the optimality gap is with respect to the modified IP. This solution gives an upper bound to the CDPP solution.

For each parking instance solved to optimality, we subtract the amount of time used to find parking from the CDPP objective value to report the CDPP without parking. For $f = 2.8$ minutes, Tables EC.7 and EC.8 show the objective value without parking in the eighth, twelfth, and sixteenth columns, for parking times of 6, 9, and 15 minutes, respectively. Similarly, for $f = 0.5$ minutes, Tables EC.9 and EC.10 show the objective value without parking in the sixth, tenth, and fourteenth columns, for parking times of 6, 9, and 15 minutes, respectively. This value encompasses the time spent driving, walking, and serving customers for the delivery person. When the same value is given for two different parking times, this indicates that the underlying solutions have the same structure, and the higher objective values in the CDPP are due solely to high parking times.

For parking times of $p = 0$, 6, 9, and 15 when $f = 2.8$, Tables EC.7 and EC.8 report the percentage of savings realized by the CAVADP with parking in the fifth, seventh, eleventh, and fifteenth columns, respectively. The percentage of savings realized by the CAVADP without parking is reported in the ninth, thirteenth, and seventeenth columns, for parking times $p = 6$, 9, and 15, respectively. When $f = 0.5$, Tables EC.9 and EC.10 report the percentage of savings realized by the CAVADP for parking times $p = 6$, 9, and 15, with parking in the fifth, ninth, and thirteenth columns, respectively, and without parking in the seventh, eleventh, and fifteenth columns, respectively.

| | | | $p=0$ | | $p=6$ | | without | | $p=9$ | | without | | $p=15$ | | without | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $q$ | CAVADP | CDPP | (%) | CDPP | (%) | parking | (%) | CDPP | (%) | parking | (%) | CDPP | (%) | parking | (%) |
| 4 | 1 | 21.2 | 21.2 | 0 | 38.2 | 45 | 32.2 | 34 | 41.2 | 49 | 32.2 | 34 | 47.2 | 55 | 32.2 | 34 |
| 4 | 2 | 17.1 | 21.1 | 19 | 28.6 | 40 | 22.6 | 24 | 31.6 | 46 | 22.6 | 24 | 37.6 | 55 | 22.6 | 24 |
| 4 | 3 | 17.1 | 18.6 | 8 | 24.6 | 30 | 18.6 | 8 | 27.6 | 38 | 18.6 | 8 | 33.6 | 49 | 18.6 | 8 |
| 4 | 4 | 15.05 | 15.8 | 5 | 21.8 | 31 | 15.8 | 5 | 24.8 | 39 | 15.8 | 5 | 30.8 | 51 | 15.8 | 5 |
| 9 | 1 | 42.7 | 42.7 | 0 | 79.7 | 46 | 61.7 | 31 | 88.7 | 52 | 61.7 | 31 | 98.2 | 57 | 83.2 | 49 |
| 9 | 2 | 34.5 | 41.3 | 16 | 62.0 | 44 | 56.0 | 38 | 65.0 | 47 | 56.0 | 38 | 71.0 | 51 | 56.0 | 38 |
| 9 | 3 | 30.4 | 40.0 | 24 | 52.4 | 42 | 46.4 | 34 | 55.4 | 45 | 46.4 | 34 | 61.4 | 50 | 46.4 | 34 |
| 9 | 4 | 30.4 | 37.2 | 18 | 45.9 | 34 | 39.9 | 24 | 48.9 | 38 | 39.9 | 24 | 54.9 | 45 | 39.9 | 24 |
| 16 | 1 | 69.8 | 69.8 | 0 | 136.8 | 49 | 112.8 | 38 | 148.8 | 53 | 112.8 | 38 | 172.8 | 60 | 112.8 | 38 |
| 16 | 2 | 53.4 | 69.5 | 23 | 107.4 | 50 | 89.4 | 40 | 115.4 | 54 | 97.4 | 45 | 127.4 | 58 | 112.4 | 52 |
| 16 | 3 | 49.3 | 67.0 | 26 | 88.8 | 44 | 82.8 | 40 | 91.8 | 46 | 82.8 | 40 | 97.8 | 50 | 82.8 | 40 |
| 16 | 4 | 45.2 | 58.2 | 22 | 75.2 | 40 | 69.2 | 35 | 78.2 | 42 | 69.2 | 35 | 84.2 | 46 | 69.2 | 35 |
| 25 | 1 | 107.5 | 107.5 | 0 | 208.5 | 48 | 160.5 | 33 | 230.5 | 53 | 176.5 | 39 | 263.5 | 59 | 188.5 | 43 |
| 25 | 2 | 82.9 | 105.7 | 22 | 164.4 | 50 | 140.4 | 41 | 176.4 | 53 | 140.4 | 41 | 196.2 | 58 | 151.2 | 45 |
| 25 | 3 | 74.7 | 102.0 | 27 | 136.0 | 45 | 112.0 | 33 | 145.7 | 49 | 118.7 | 37 | 159.2 | 53 | 144.2 | 48 |
| 25 | 4 | 70.6 | 90.8 | 22 | 121.1 | 42 | 109.1 | 35 | 127.1 | 44 | 109.1 | 35 | 137.6 | 49 | 122.6 | 42 |
| 36 | 1 | 150.8 | 150.8 | 0 | 296.3 | 49 | 224.3 | 33 | 327.3 | 54 | 237.3 | 36 | 374.3 | 60 | 284.3 | 47 |
| 36 | 2 | 113.9 | 150 | 24 | 231.4 | 51 | 195.4 | 42 | 245.0 | 54 | 209.0 | 46 | 269.0 | 58 | 209.0 | 46 |
| 36 | 3 | 101.6 | 145 | 29.9 | 190.6 | 47 | 154.6 | 34 | 204.1 | 50 | 168.1 | 40 | 226.4 | 55 | 181.4 | 44 |
| 36 | 4 | 95.45 | 124.7** | 23 | 165.2 | 42 | 141.2 | 32 | 176.2 | 46 | 149.2 | 36 | 192.7 | 50 | 162.7 | 41 |

**Table EC.7** **Optimal objective values of the CAVADP and the CDPP with varying parking times when $f = 2.8$ and $n$ is at most 36 customers. For each parking time $p$, the optimal objective value is given as well as when the time spent parking is removed for $p > 0$.**

| $n$ | $q$ | CAVADP | $p=0$ CDPP | (%) | $p=6$ CDPP | (%) | without parking | (%) | $p=9$ CDPP | (%) | without parking | (%) | $p=15$ CDPP | (%) | without parking | (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 49 | 1 | 204.7 | 204.7 | 0 | 401.2 | 49 | 317.2 | 35 | 440.7 | 54 | 332.7 | 38 | 505.7 | 60 | 370.7 | 45 |
| 49 | 2 | 155.5 | 202.3* | 23 | 312.0 | 50 | 264 | 41 | 332.5 | 53 | 278.5 | 44 | 367.0 | 58 | 292.0 | 47 |
| 49 | 3 | 139.1 | 194.9* | 29 | 255.6 | 46 | 213.6 | 35 | 272.6 | 49 | 236.6 | 41 | 296.6 | 53 | 236.6 | 41 |
| 49 | 4 | 130.9 | 169.6** | 23 | 224.4** | 42 | - | - | 234.4 | 44 | 198.4 | 34 | 289.8 | 55 | 229.8 | 43 |
| 64 | 1 | 264.2 | 264.2 | 0 | 520.2 | 49 | 400.2 | 34 | 570.2 | 54 | 426.2 | 38 | 651.7 | 59 | 471.7 | 44 |
| 64 | 2 | 198.6 | 262.7* | 24 | 403.1* | 51 | - | - | 429.9 | 54 | 357.9 | 45 | 474.9 | 58 | 369.9 | 46 |
| 64 | 3 | 178.1 | 251.7* | 29 | 334.6 | 47 | 280.6 | 37 | 359.9 | 50 | 296.6 | 40 | 381.2* | 53 | - | - |
| 64 | 4 | 165.8 | 212.8*** | 22 | 283.3 | 41 | 241.3 | 31 | 301.8 | 45 | 247.8 | 33 | 326.8 | 49 | 266.8 | 38 |
| 81 | 1 | 334.3 | 334.3 | 0 | 658.3* | 49 | - | - | 720.3* | 54 | - | - | 823.8 | 59 | 583.8 | 43 |
| 81 | 2 | 252.3 | 331.1* | 24 | 509.3** | 50 | - | - | 539.1 | 53 | 449.1 | 44 | 595.2 | 58 | 475.2 | 47 |
| 81 | 3 | 223.6 | 316.3* | 29 | 422.2** | 47 | - | - | 451.4 | 50 | 370.4 | 40 | 496.9 | 55 | 236.6 | 43 |
| 81 | 4 | 211.3 | 317.5*** | 33 | 393.0 | 46 | 339.0 | 38 | 418.0 | 49 | 346.0 | 39 | 463.0 | 54 | 358.0 | 41 |
| 100 | 1 | 410.0 | 410.0 | 0 | 815.5* | 50 | - | - | 886.5* | 54 | - | - | 1012.0 | 59 | 727.0 | 44 |
| 100 | 2 | 307.5 | 407.6* | 25 | 627.0** | 51 | - | - | 669.3** | 54 | - | - | 735.1 | 58 | 570.1 | 46 |
| 100 | 3 | 274.7 | 395.3*** | 31 | 519.8*** | 47 | - | - | 554.2*** | 50 | - | - | 604.2*** | 55 | - | - |
| 100 | 4 | 256.25 | 330.0*** | 22 | 435.5* | 41 | - | - | 461.5 | 44 | 380.5 | 33 | 508.0 | 50 | 403.0 | 36 |

**Table EC.8** **Optimal objective values of the CAVADP and the CDPP with varying parking times when $f = 2.8$ and $n$ is more than 36 customers. For each parking time $p$, the optimal objective value is given as well as when the time spent parking is removed for $p > 0$.**

| $n$ | $q$ | CAVADP | $p=6$ CDPP | (%) | without parking | (%) | $p=9$ CDPP | (%) | without parking | (%) | $p=15$ CDPP | (%) | without parking | (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 12.0 | 29.0 | 59 | 23.0 | 48 | 32.0 | 63 | 23.0 | 48 | 38.0 | 68 | 23.0 | 48 |
| 4 | 2 | 12.0 | 24.0 | 50 | 18.0 | 33 | 27.0 | 56 | 18.0 | 33 | 33.0 | 64 | 18.0 | 33 |
| 4 | 3 | 12.0 | 20.0 | 40 | 14.0 | 14 | 23.0 | 48 | 14.0 | 14 | 29.0 | 59 | 14.0 | 14 |
| 4 | 4 | 12.0 | 19.5 | 38 | 13.5 | 11 | 22.5 | 47 | 13.5 | 11 | 28.5 | 58 | 13.5 | 11 |
| 9 | 1 | 22.0 | 59.0 | 63 | 41.0 | 46 | 68.0 | 68 | 41.0 | 46 | 77.5 | 72 | 62.5 | 65 |
| 9 | 2 | 22.0 | 50.5 | 56 | 44.5 | 51 | 53.5 | 59 | 44.5 | 51 | 59.5 | 63 | 44.5 | 51 |
| 9 | 3 | 22.0 | 45.5 | 52 | 39.5 | 44 | 48.5 | 55 | 39.5 | 44 | 54.5 | 60 | 39.5 | 44 |
| 9 | 4 | 22.0 | 39.0 | 44 | 33.0 | 33 | 42.0 | 48 | 33.0 | 33 | 48.0 | 54 | 33.0 | 33 |
| 16 | 1 | 33.0 | 100.0 | 67 | 76.0 | 57 | 112.0 | 71 | 76.0 | 57 | 136.0 | 76 | 91.0 | 64 |
| 16 | 2 | 33.0 | 89.0 | 63 | 71.0 | 54 | 97.0 | 66 | 79.0 | 58 | 109.0 | 70 | 79.0 | 58 |
| 16 | 3 | 33.0 | 75.0 | 56 | 51.0 | 35 | 78.0 | 58 | 69.0 | 52 | 84.0 | 61 | 69.0 | 52 |
| 16 | 4 | 33.0 | 66.0 | 50 | 60.0 | 45 | 69.0 | 52 | 60.0 | 45 | 75.0 | 56 | 60.0 | 45 |
| 25 | 1 | 50.0 | 151.0 | 67 | 103.0 | 51 | 173.0 | 71 | 119.0 | 58 | 206.0 | 76 | 131.0 | 62 |
| 25 | 2 | 50.0 | 134.5 | 63 | 110.5 | 55 | 146.0 | 66 | 119.0 | 58 | 164.0 | 70 | 119.0 | 58 |
| 25 | 3 | 50.0 | 113.0 | 56 | 89.0 | 44 | 125.0 | 60 | 89.0 | 44 | 138.5 | 64 | 123.5 | 60 |
| 25 | 4 | 50.0 | 105.0 | 52 | 93.0 | 46 | 111.0 | 55 | 93.0 | 46 | 121.5 | 59 | 106.5 | 53 |
| 36 | 1 | 68.0 | 213.5 | 68 | 141.5 | 52 | 244.5 | 72 | 154.5 | 56 | 291.5 | 77 | 201.5 | 66 |
| 36 | 2 | 68.0 | 187.0 | 64 | 163.0 | 58 | 199.0 | 66 | 163.0 | 58 | 223.0 | 70 | 163.0 | 58 |
| 36 | 3 | 68.0 | 160.0 | 58 | 124.0 | 45 | 175.0 | 61 | 139.0 | 51 | 196.5 | 65 | 151.5 | 55 |
| 36 | 4 | 68.0 | 144.5 | 53 | 120.5 | 44 | 155.5 | 56 | 128.5 | 47 | 172.0 | 60 | 142.0 | 52 |

**Table EC.9** **Optimal objective values of the CAVADP and the CDPP with varying parking times when $f = 0.5$. For each parking time $p$, the optimal objective value is given as well as when the time spent parking is removed.**

| | | | $p=6$ | | without | | $p=9$ | | without | | $p=15$ | | without | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $q$ | CAVADP | CDPP | (%) | parking | (%) | CDPP | (%) | parking | (%) | CDPP | (%) | parking | (%) |
| 49 | 1 | 92.0 | 288.5 | 68 | 204.5 | 55 | 328.0 | 72 | 211.0 | 56 | 393.0 | 77 | 258.0 | 64 |
| 49 | 2 | 92.0 | 253.0 | 64 | 211.0 | 56 | 273.0 | 66 | 210.0 | 56 | 308.5 | 70 | 233.5 | 61 |
| 49 | 3 | 92.0 | 211.0 | 56 | 163.0 | 44 | 230.5 | 60 | 194.5 | 53 | 254.5 | 64 | 194.5 | 53 |
| 49 | 4 | 92.0 | 214.0 | 57 | 178.0 | 48 | 229.0 | 60 | 193.0 | 52 | 231.0* | 60 | - | - |
| 64 | 1 | 117.0 | 373.0 | 69 | 253.0 | 54 | 423.0 | 72 | 279.0 | 58 | 504.5 | 77 | 324.5 | 64 |
| 64 | 2 | 117.0 | 325.0 | 64 | 277.0 | 58 | 349.0 | 66 | 277.0 | 58 | 396.0 | 70 | 291.0 | 60 |
| 64 | 3 | 117.0 | 281.5 | 58 | 203.5 | 43 | 306.5 | 62 | 252.5 | 54 | 342.5 | 66 | 252.5 | 54 |
| 64 | 4 | 117.0 | 246.5 | 53 | 204.5 | 43 | 265.0 | 56 | 211.0 | 45 | 290.0 | 60 | 230.0 | 49 |
| 81 | 1 | 148.0 | 471.5* | 69 | - | - | 534.0 | 72 | 372.0 | 60 | 637.5 | 77 | 412.5 | 64 |
| 81 | 2 | 148.0 | 412.0** | 64 | - | - | 439.0 | 66 | 358.0 | 59 | 493.0 | 70 | 358.0 | 59 |
| 81 | 3 | 148.0 | 412.0*** | 64 | - | - | 386.0*** | 62 | - | - | 431.5 | 66 | 341.5 | 57 |
| 81 | 4 | 148.0 | 335.5 | 56 | 281.5 | 47 | 360.5 | 59 | 288.5 | 49 | 405.5 | 64 | 300.5 | 51 |
| 100 | 1 | 180.0 | 579.0** | 69 | - | - | 655* | 73 | - | - | 782.0 | 77 | 557.0 | 68 |
| 100 | 2 | 180.0 | 507.0*** | 64 | - | - | 547.5** | 67 | - | - | 615.5** | 71 | - | - |
| 100 | 3 | 180.0 | 432*** | 58 | - | - | 474.0*** | 62 | - | - | 527.5* | 66 | - | - |
| 100 | 4 | 180.0 | 378.0 | 52 | 324.0 | 44 | 404.0 | 55 | 323.0 | 44 | 450.5 | 60 | 345.5 | 48 |

**Table EC.10** **Optimal objective values of the CAVADP and the CDPP with varying parking times when $f = 0.5$. For each parking time $p$, the optimal objective value is given as well as when the time spent parking is removed.**

## Appendix G: Lower Loading Times

In this appendix, we further examine the situation where $f < \frac{1}{w} - \frac{1}{d}$ and let $f = 0.5$ minutes. By Theorem 4, the optimal solution to the CAVADP will be to serve $n$ sets of size 1 for any capacity $q$. Since the CDPP is an upper bound on the CAVADP and the optimal objective value increases as $p$ increases, we will see greater percent savings for higher levels of parking time. Figure EC.12 shows Insight 5 still holds when $q = 3$ packages and $f = 0.5$ minutes. There is similar behavior in savings for each value of $q$ when $f = 0.5$ minutes.

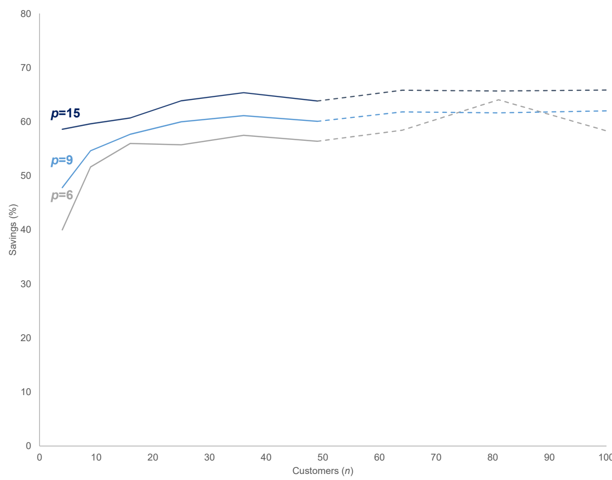



**Figure EC.12 The percentage of savings across all nonzero parking times for $q = 3$ packages and $f = 0.5$ minutes.**

Figure EC.13 confirms that higher savings occur at lower capacities when $f = 0.5$ for $p = 9$ minutes. Similar behavior is observed for all parking times when $f = 0.5$ minutes. With low values of $f$, the optimal solution of the CAVADP is independent of capacity as the delivery person always serves singleton sets. Any reduction in savings from increasing capacity is due to changes in the CDPP. The only time we see higher savings at higher capacities in Figure EC.13 is when heuristic solutions are reported. Insight 9 provides a summary of the change in savings with respect to capacity.

**Insight 9** *For every one package increase in capacity, there is on average a 5.3% decrease in savings when $f = 0.5$ minutes.*

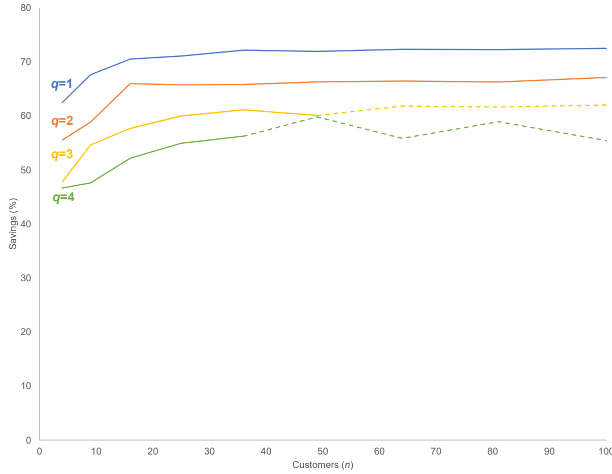



**Figure EC.13** **The percentage of savings across all capacities for $p = 9$ minutes and $f = 0.5$ minutes.**