



Transportation Science

Publication details, including instructions for authors and subscription information:
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To cite this article:

Sara Reed, Ann Melissa Campbell, Barrett W. Thomas (2022) Impact of Autonomous Vehicle Assisted Last-Mile Delivery in Urban to Rural Settings. Transportation Science

Published online in Articles in Advance 01 Apr 2022

. <https://doi.org/10.1287/trsc.2022.1142>

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Impact of Autonomous Vehicle Assisted Last-Mile Delivery in Urban to Rural Settings

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Received: June 10, 2021

Revised: December 8, 2021

Accepted: March 8, 2022

Published Online in Articles in Advance:

April 1, 2022

<https://doi.org/10.1287/trsc.2022.1142>

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Abstract. We seek to determine in what geographies autonomous vehicle assisted delivery is most valuable for last-mile delivery. To build insights across urban-to-rural settings, we conduct a case study by generating test instances that reflect real-world geographies. We integrate real-world data for these instances, including driving and walking times, as well as obstacles, such as one-way streets, and their impact on last-mile delivery. We model the capacitated autonomous vehicle assisted delivery problem as an integer program on a general graph. To solve this model on realistically sized instances, we exploit the structure of the optimal solution to develop a number of preprocessing techniques to reduce the large number of variables present in the generic problem. We also introduce valid inequalities that raise the lower bound and reduce the size of the branch-and-bound tree. Autonomous vehicle assisted delivery reduces the completion time of the delivery tour and provides the most cost-effective business model in all customer geographies. In particular, a delivery person saves more time in urban environments than in rural environments. These savings are the result of both a reduction in the time to park but also in the amount of walking that the delivery person does. This increased productivity could reduce fleet size and ultimately the number of vehicles on the road. These conclusions support businesses with urban deliveries considering investment in this technology. However, higher savings in rural environments may be achieved by reducing the loading time.

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/trsc.2022.1142>.

Keywords: autonomous • routing • last-mile delivery

1. Introduction

Many companies are showing interest in combating the challenges of last-mile delivery with autonomous vehicles. For example, the U.S. Postal Service (2017) plans to build an autonomous rural delivery vehicle to use on rural routes by 2025. In rural areas, customers are further apart, resulting in longer routes for the delivery person (Joeress et al. 2016). The use of the autonomous vehicle would allow the delivery person to engage in productive activities such as sorting mail while the vehicle travels the relatively greater distances between customers in rural areas. However, autonomous vehicles potentially deliver significant savings in urban areas by avoiding the time-consuming task of finding parking and reducing the time that the delivery person spends walking back to parking locations after making multiple deliveries. Recent studies explore the challenge of parking for the delivery person in urban areas (Chiara et al. 2020) as well as analyze proposed changes to city infrastructure to increase parking efficiency (Abhishek and Fransoo 2021; Stephan, Weidinger, and Boysen 2021). One study shows that

autonomous vehicle assisted delivery could reduce the completion time of these urban routes by as much as 50% (Reed, Campbell, and Thomas 2022b).

In this paper, we seek to determine in what geographies autonomous vehicle assisted delivery is valuable for last-mile delivery and what parameters affect the potential savings. To answer these questions, we model the capacitated autonomous vehicle assisted delivery problem (CAVADP) on a general graph. The CAVADP is the problem of serving a set of customers using an autonomous vehicle assisted by a delivery person. The delivery person has a carrying capacity and must return to the vehicle to load more packages. The autonomous vehicle can transport the delivery person to customers as well as travel between customer locations when the delivery person is not on board the vehicle. Thus, the vehicle can pick up the delivery person after he or she delivers the last package that he or she was carrying.

Because of the interaction of the delivery person and the vehicle as well as the delivery person's limited carrying capacity, the CAVADP is challenging to solve.

Reed, Campbell, and Thomas (2022b) restrict the customer geography to a complete grid of customers and provide an analytical solution to the problem, eliminating the need to solve the mixed integer program (MIP). Although the restriction to a complete grid models urban environments well, it is less appropriate for studying the continuum of geographies from rural to urban. To gain insights across all customer geographies, we instead consider the problem on a general graph of customers. The results of Reed, Campbell, and Thomas (2022b) do not hold for general graphs, and we must instead solve MIPs to gain insights. Many of the advancements found in the vehicle routing literature do not apply because of the structural differences in the solutions. New advances are necessary and may benefit other problems that study the synchronization between routes, particularly in the case of drone routing problems. To solve this model on realistically sized instances, we exploit properties of optimal solutions to develop a number of preprocessing techniques that reduce the large number of variables present in the generic problem. We also introduce valid inequalities that raise the lower bound and reduce the size of the branch-and-bound tree.

To build insights across the urban-to-rural continuum, we conduct a case study on various counties in Illinois. We integrate real-world data for the driving times of the vehicle and the walking times of the delivery person. The driving time restricts the vehicle to the road network. In doing so, we capture the effect of real-world obstacles, such as one-way streets, that may cause walking between customers to be faster than driving.

We benchmark the CAVADP with multiple other business models to analyze effects on productivity and operational costs. The capacitated delivery problem with parking (CDPP), introduced in Reed, Campbell, and Thomas (2022b) and further explored in Reed, Campbell, and Thomas (2022a), models the traditional delivery model where the delivery person must park the vehicle prior to servicing customers on foot. The delivery person can serve multiple sets of customers from the same parking location, but must return to the parked vehicle to reload packages. We also consider a fully autonomous benchmark without a delivery person where the vehicle must visit every customer location for a given amount of time to allow the customer to retrieve the package from the vehicle. Finally, we consider a two-person model where there is a driver and delivery person on board a traditional vehicle. The optimal solution to the two-person model is equivalent to the CAVADP in terms of the time of the delivery tour but differs when we consider the cost to the business.

In addition to the methodological developments needed to solve the CAVADP, this paper offers the following managerial insights:

- Autonomous vehicle assisted delivery is the most cost-effective business model across all customer geographies. On average, increases in productivity from autonomous-assisted delivery are more significant in urban environments than in rural environments. Therefore, businesses with urban deliveries should carefully examine investment in this technology. However, higher savings relative to delivery practices with parking in rural environments may be achieved by reducing the loading time.

- Reductions in the time of the delivery tour by autonomous vehicle assisted delivery relative to delivery practices with parking come from both a reduction in the time needed to find parking but also in the time that the delivery person spends walking. If the investment in autonomous technology is not feasible, the productivity gains of autonomous-assisted delivery may be achieved by a delivery driver and delivery person on a traditional vehicle at a higher operational cost.

- Pairing an autonomous vehicle with a delivery person in last-mile delivery leads to increased productivity for the delivery person. Furthermore, this increased productivity could reduce fleet size and ultimately the number of vehicles on the road. Thus, businesses could reduce operational costs while improving sustainability.

The remainder of this paper is outlined as follows. Section 2 reviews the related literature. Section 3 details the assumptions, service times, and integer program for the CAVADP on a general graph. Section 4 addresses the necessary model improvements to reduce the size of the model. When we restrict the analysis to cases where driving between customers is faster than walking, Section 4.3 presents valid inequalities for the CAVADP. Section 5 presents the experimental design for the case study. In addition, we present the benchmark problems to evaluate the CAVADP. In Section 6, we analyze the effect of the model improvements on computational performance. Section 7 presents our experimental results and insights from the case study. Conclusions and future work are discussed in Section 8.

2. Literature Review

In this section, we discuss literature related to addressing the challenges of last-mile delivery in urban-to-rural settings. In this review, we focus on last-mile delivery models that use autonomous technology, including previous results on the CAVADP in urban settings. Then, we discuss studies that explicitly address challenges in last-mile delivery, that is, parking and walking. A broad review of last-mile delivery literature can be found in Boysen, Fedtke, and Schwerdfeger (2020).

2.1. Autonomous Delivery

Reed, Campbell, and Thomas (2022b) introduce the CAVADP and characterize the optimal solution on a

grid of customers representing an urban environment. Reed, Campbell, and Thomas (2022b) is the only work of which the authors are aware that addresses autonomous vehicle assisted delivery. In addition, the authors introduce the CDPP that models traditional package delivery services where the delivery person must park the vehicle prior to servicing customers. Computational experiments show that the CAVADP reduces the completion time relative to the CDPP on a grid by 30%–77%, with higher savings achieved when it takes longer to find parking, with smaller capacities for the delivery person, and with lower fixed times for loading packages. The conclusions of Reed, Campbell, and Thomas (2022b) rely on the customer geography being a complete grid (i.e., a customer located at every intersection). Outside of the street networks of urban environments, like New York City, a complete grid of customers is no longer a reasonable assumption for the density of customer locations. This paper provides model improvements that are necessary in order to solve the mixed integer program to optimality for realistically sized instances and real-world geographies.

The methodology of the CAVADP is also related to the literature on routing combinations of trucks and drones. The closest problem to the CAVADP in the drone literature is the multivisit drone routing problem (MVDRP) introduced by Poikonen and Golden (2020). (Other related literature is discussed in Reed, Campbell, and Thomas (2022b).) Similar to the CAVADP, in the MVDRP, the vehicle acts as a mobile depot and does not make deliveries. The drone departs from the vehicle to deliver one or more packages prior to returning to the vehicle. Similar to the delivery person's package capacity, the travel of the drone is restricted by a fixed energy capacity. Because of the need to partition customers and determine launching points, the potential number of interactions between the drone and the vehicle becomes very large. Therefore, Poikonen and Golden (2020) introduce a heuristic solution to create a visiting order sequence of the customers and partition the sequence into service sets for drone deliveries. Similarly, solving the CAVADP on a general graph requires finding a sequence between all customers, the rendezvous locations for the delivery person and autonomous vehicle, and which customers should be served on foot between visits with the vehicle. In this paper, we introduce new technology to make all of these decisions simultaneously and reduce the size of the problem. These improvements to the model allow us to solve to optimality. If we restrict the launch points in the MVDRP to customer locations, generate only service sets that are energy feasible, and exchange the cost of loading packages for the launch penalty, the mixed integer programming formulation for the CAVADP in Section 3.2 is an alternative formulation for the MVDRP.

Therefore, the preprocessing techniques introduced in this paper may also be able to control the size of the MVDRP as well.

2.2. Parking

A key feature of many last-mile delivery applications is that the delivery person must park the vehicle prior to serving customers on foot. Nguyen et al. (2019) use predefined sets of customers in London when optimizing package delivery with parking. The authors consider two partitions of customers: one based on an observational study and the other on geographical location. The routing decisions are the order in which to serve the customers and at which customer the delivery person will park. Martinez-Sykora et al. (2020) generalize Nguyen et al. (2019) by making the clustering of the customers a decision in the optimization problem. However, similar to Nguyen et al. (2019), Martinez-Sykora et al. (2020) require a parking spot in each customer service set. The CDPP generalizes Martinez-Sykora et al. (2020) by allowing multiple customer service sets from the same parking location. In addition, the search time for parking is included in the objective to minimize the completion time of the delivery tour. Reed, Campbell, and Thomas (2022a) explore how including the search time for parking impacts optimal routing decisions. The CDPP outperforms industry practice and other models in the literature. Thus, we use the CDPP as the benchmark to the CAVADP.

2.3. Walking

McLeod et al. (2020) take another approach to last-mile delivery in London and consider the use of on-foot porters and cycling couriers. The porters/couriers have a weight and volume carrying capacity. Their routes begin and end at collection and delivery points (CDPs). Vehicle drivers transfer packages from the depot to the CDPs. The authors use a randomized constructive procedure to create the customer clusters assigned to porters. Then, a tabu search heuristic improves the cluster assignments. The routing decisions are the routes of the van driver and the routes of the porters to service their customers. Again, the CAVADP makes both the routing and customer set selection decisions in the optimization problem. Combining autonomous vehicles with other innovations, such as on-foot porters, cycling couriers, or drones, may provide additional value in these technologies for last-mile delivery.

One of the benefits of autonomous-assisted delivery is the flexibility to drop off the delivery person at a customer location and pick up the delivery person at an alternative location. After being dropped off, the delivery person serves customers on foot. The ability to walk between customers is also shown to be

advantageous when paired with carpooling. Fikar and Hirsch (2015) show the potential benefit of allowing nurses to walk between jobs to reduce the number of vehicles as well as reliance on parking spaces. Coindreau, Gallay, and Zufferey (2019) consider the effects of combining walking and carpooling for on-site technician services. In urban areas, these strategies yield significant improvements over a model that assigns one vehicle to each worker without considering the option to walk between jobs. Fikar and Hirsch (2015) and Coindreau, Gallay, and Zufferey (2019) develop heuristics to solve these problems. This paper identifies when the option to drive dominates walking and may be beneficial to identify structure for other service operations, such as those considered in Fikar and Hirsch (2015) and Coindreau, Gallay, and Zufferey (2019).

3. Model

In this paper, we generalize the CAVADP introduced by Reed, Campbell, and Thomas (2022b) to **consider a general geography of customers**. Below we present a problem description similar to that in Reed, Campbell, and Thomas (2022b), but we introduce that the delivery person may need to wait for the autonomous vehicle, an important consideration when considering real-world data.

3.1. Problem Description and Assumptions

The CAVADP seeks to serve a set C of n customers using an autonomous vehicle assisted by a delivery person. Each customer requires one package delivery. The vehicle leaves and returns to the depot with the delivery person on board. We denote the depot by 0 and define $\bar{C} = C \cup \{0\}$. The vehicle also transports the delivery person in between servicing sets of customers on foot. We restrict the movement of the vehicle to the road network.

To deliver a set of packages, the delivery person is dropped off by the vehicle at a customer location, loads the packages to serve the specified set, and is picked up by the vehicle at a customer location that is potentially different than the drop-off location. The delivery person can carry up to q packages at one time. We let S be the set of all possible customer service sets σ of size less than or equal to q . Let $m = |S|$. For each $i \in C$, let $J_i = \{\sigma \in S \mid i \in \sigma\}$ be the customer service sets that include customer i . The fixed time for loading packages f is independent of the number of customers being served in a set. It is straightforward to generalize the results from Reed, Campbell, and Thomas (2022b) that the case of package-dependent loading times is equivalent to the case in which $f = 0$. The parameters of the CAVADP are summarized in Table 1.

Let $D(i, k)$ denote the time (in minutes) it takes to drive from customer i to customer k . Similarly, let $W(i, k)$ denote the time (in minutes) it takes to walk from customer i to customer k . In our computational study, these times will be based on real-world data. For the triple $(i, \sigma, k) \in C \times S \times C$, we refer to customer i as the loading point and customer k as the rendezvous point for service set σ . Let $(c_1^{i\sigma k}, c_2^{i\sigma k}, \dots, c_{|\sigma|}^{i\sigma k})$ be the permutation of customers in set σ that achieves the shortest path, with respect to walking times, when loading at i and returning to k . Note that customers i and k may or may not be in set σ . The time to walk the shortest walking path between customers in set σ when loading at customer i and returning to customer k is $w_{i\sigma k} = W(i, c_1^{i\sigma k}) + W(c_1^{i\sigma k}, c_2^{i\sigma k}) + \dots + W(c_{|\sigma|-1}^{i\sigma k}, c_{|\sigma|}^{i\sigma k}) + W(c_{|\sigma|}^{i\sigma k}, k)$. Table 2 summarizes the service times of the CAVADP.

Because of one-way streets and other obstacles, the delivery person may be able to arrive to a rendezvous point earlier than the vehicle, and may need to wait for the vehicle to arrive. The waiting time is $\max\{D(i, k) - w_{i\sigma k}, 0\}$ for servicing set σ when loading at customer i and returning to customer k . The objective of the CAVADP is to minimize the completion time of the delivery tour including this waiting time.

For the purposes of the analysis in this paper, we make the following assumptions in the model:

- Each customer has a single delivery of the same size.
- Every customer set of size less than or equal to capacity q is considered.
- The vehicle can pick up and drop off the delivery person at all customers.
- The driving time satisfies the triangle inequality.
- The walking time satisfies the triangle inequality.

3.2. Mixed Integer Program

Reed, Campbell, and Thomas (2022b) present an integer programming formulation for the CAVADP. Here, we present an **updated formulation**. Table 3 defines the binary decision variables x_{ik} and $y_{i\sigma k}$ as well as the integer valued variables v_{ik} . This formulation simplifies the model in Reed, Campbell, and Thomas (2022b) by eliminating the z variable representing autonomous movement of the vehicle. In this formulation, the $y_{i\sigma k}$ variables also define the autonomous

Table 1. Definition of Parameters in CAVADP

Notation	Description
n	Number of customers
C	Set of customers
\bar{C}	Set of customers and depot 0
m	Number of customer service sets
S	Set of customer service sets
f	Fixed time for loading packages from vehicle (minutes)
q	Capacity of delivery person (number of packages)

Table 2. Definition of Service Times in CAVADP

Notation	Description
$D(i, k)$	Time to drive from customer i to customer k (minutes)
$W(i, k)$	Time to walk from customer i to customer k (minutes)
w_{iok}	Time to walk the shortest walking path between customers in set σ when loading at i and returning to k (minutes)

movement of the vehicle. If $y_{iok} = 1$ for $i, k \in C$ and $\sigma \in S$, then the autonomous vehicle must travel between loading and rendezvous locations at customer i and customer k , respectively. In addition, we strengthen the formulation of Reed, Campbell, and Thomas (2022b) by replacing the Miller–Tucker–Zemlin subtour elimination constraints with single commodity flow subtour constraints. The objective function now includes potential waiting time for instances where walking between customers is faster than driving.

The CAVADP can then be modeled with the following integer program:

$$\min \sum_{i \in \bar{C}} \sum_{k \in C \setminus \{i\}} D(i, k) x_{ik} + \sum_{i \in C} \sum_{\sigma \in S} \sum_{k \in C} (w_{iok} + f + \max\{D(i, k) - w_{iok}, 0\}) y_{iok} \quad (1)$$

$$\text{s.t.} \sum_{i \in C} x_{i0} = 1, \quad (2)$$

$$\sum_{i \in C} x_{0i} = 1, \quad (3)$$

$$\sum_{i \in C} \sum_{\sigma \in J_l} \sum_{k \in C} y_{iok} = 1 \quad \forall l \in C, \quad (4)$$

$$\sum_{\sigma \in S} \sum_{k \in C} y_{iok} = \sum_{l \in \bar{C}} x_{li} \quad \forall i \in C, \quad (5)$$

$$\sum_{i \in C} \sum_{\sigma \in S} y_{iok} = \sum_{l \in \bar{C}} x_{kl} \quad \forall k \in C, \quad (6)$$

$$\sum_{k \in C} v_{0k} = n, \quad (7)$$

$$v_{0i} \leq n \cdot x_{0i} \quad \forall i \in C, \quad (8)$$

$$v_{ik} \leq n \left(x_{ik} + \sum_{\sigma \in S} y_{iok} \right) \quad \forall i, k \in C \text{ s.t. } i \neq k, \quad (9)$$

$$\sum_{k \in \bar{C} \setminus \{i\}} v_{ki} - \sum_{k \in C \setminus \{i\}} v_{ik} = \sum_{k \in C} x_{ik} + \sum_{\sigma \in S} \sum_{k \in C} (|\sigma| - 1) y_{iok} \quad \forall i \in C, \quad (10)$$

$$x_{ik} \in \{0, 1\} \quad \forall i, k \in \bar{C}, \quad (11)$$

$$y_{iok} \in \{0, 1\} \quad \forall i, k \in C, \sigma \in S, \quad (12)$$

$$v_{ik} \in \mathbb{Z}_+ \quad \forall i \in \bar{C}, k \in C \text{ s.t. } i \neq k. \quad (13)$$

The objective function (1) minimizes the completion time of the delivery tour for the delivery person. Constraints (2) and (3) ensure that the delivery person is with the vehicle to and from the depot, respectively. Constraints (4) ensure that all customers are served. Specifically, each customer l belongs in one customer service

Table 3. Definition of Decision Variables in CAVADP

Notation	Description
x_{ik}	$x_{ik} = 1$ if the vehicle drives from location i to location k with the delivery person on board for $i, k \in \bar{C}$
y_{iok}	$y_{iok} = 1$ if the delivery person loads at customer i , serves set σ , and meets the vehicle at customer k for $i, k \in C$ and $\sigma \in S$
v_{ik}	The flow of packages on board the vehicle from location i to location k for $i \in \bar{C}$ and $k \in C$ such that $i \neq k$

set $\sigma \in J_l$. When the delivery person loads at customer i (i.e., $y_{iok} = 1$ for some $\sigma \in S$ and $k \in C$), Constraints (5) verify that the vehicle will visit customer i . Constraints (6) ensure that when the delivery person returns to customer k after serving some set of customers σ (i.e., $y_{iok} = 1$ for some $\sigma \in S$ and $i \in C$), the vehicle will visit customer k to transport the delivery person to the next customer.

Constraints (7)–(10) and (13) are adapted from the single commodity flow subtour constraints (Gavish and Graves 1978). Constraint (7) indicates that there are n packages to be delivered at the start of the tour. Constraints (8) indicate the only way for the vehicle to visit a customer from the depot is with the delivery person on board (i.e., $x_{0i} = 1$ for some customer i). For all other vehicle travel between customers i and k , Constraints (9) capture that the vehicle can either travel with the delivery person on board (i.e., $x_{ik} = 1$) or autonomously (i.e., there exists a service set σ such that the delivery person loads at customer i and returns to customer k). Constraints (10) capture the change in flow at each customer. When the vehicle travels with the delivery person on board, the flow from the last customer to the next customer should change by one. When the vehicle travels autonomously (i.e., $y_{iok} = 1$), then the flow should change by the number of customers in the service set σ minus one. Note that the minus one avoids double counting of customers in the flow constraints.

Finally, the binary constraints on variables x_{ik} and y_{iok} are given in Constraints (11) and (12), respectively. The integer constraints on v_{ik} are given in Constraints (13).

4. Model Improvements

The size of the set of service sets S grows combinatorially, that is, $|S| = m = \sum_{i=1}^q \binom{n}{i}$. Therefore, the number of y_{iok} variables is $n^2 m$ and is large for realistically sized instances. In Sections 4.1 and 4.2, we discuss how to control the growth in the number of service sets in the model and y_{iok} variables, respectively. Then, Section 4.3 introduces valid inequalities that raise the lower bound and reduce the size of the branch-and-bound tree. Section 6 evaluates the impact of these changes to the model. In summary, these model improvements

are necessary in order to solve the mixed integer program in Section 3.2.

4.1. Service Set Reduction

In this section, we identify the structure of the optimal solution that will allow us to eliminate service sets from a problem instance. On the route of the delivery person, he or she is either on board the vehicle or servicing customers on foot. Let $G_d = (\bar{C}, E_d)$ be the driving graph, the graph over which the vehicle travels. With arcs $(i, k) \in E_d$ for all $i, k \in \bar{C}$, G_d is a complete graph.

Similarly, $G_w = (C, E_w)$ is the walking graph, the graph over which the delivery person can travel while walking. With arcs $(i, k) \in E_w$ for all $i, k \in C$, G_w is a complete graph. Observe that if it takes less (or equivalent) time to drive from customers i to k and load package(s) at k than to walk from i to k (i.e., $D(i, k) + f \leq W(i, k)$), the delivery person would never walk between i and k . This observation induces the restricted walking graph $\hat{G}_w = (C, \hat{E}_w)$. For $i, k \in C$, there exists edge $(i, k) \in \hat{E}_w$ if and only if $D(i, k) + f > W(i, k)$. This edge represents the potential to walk between customers i and k .

Figure 1 presents an example of the restricted walking graph for two different instances. These instances are classified based on their densities. A higher density corresponds to a restricted walking graph with nodes of larger degree. Figure 1(a) gives the restricted walking graph in a rural environment, that is, a less dense instance where some points are isolated. Figure 1(b) gives the restricted walking graph in an urban environment, that is, a more dense instance with only one connected component in the restricted walking graph.

We can use the restricted walking graph to develop structure for the optimal solution and eliminate service sets. To do so, we first need to consider the relationship between the loading/rendezvous locations and the service set. Consider set $\sigma \in S$ such that driving is faster than walking to/from all customers in σ and potential loading/rendezvous locations C . In this case, the autonomous vehicle is able to visit the customers in less time than the delivery person needs on foot and can arrive to the rendezvous point before the delivery person. Therefore, Lemma 1 states that the delivery person does not need to wait for the vehicle at the end of servicing set σ . This and all other proofs can be found in Online Appendix A.

Lemma 1. Consider $\sigma \in S$ such that $D(i, k) \leq W(i, k)$ and $D(k, i) \leq W(k, i)$ for all $i \in \sigma$ and $k \in C$. For all $a, b \in C$, it follows that

$$\max\{D(a, b) - w_{a\sigma b}, 0\} = 0. \quad (14)$$

If the delivery person does not have to wait for the vehicle, then the restricted walking graph provides insight into whether the service set can be eliminated. Claim 1 states that if there does not exist a path in the

restricted walking graph to visit all customers in σ , then σ will not be served in the optimal solution. Therefore, σ can be removed from potential service sets.

Claim 1. Consider $\sigma \in S$ such that $D(i, k) \leq W(i, k)$ and $D(k, i) \leq W(k, i)$ for all $i \in \sigma$ and $k \in C$. If there does not exist a permutation $(c_1, \dots, c_{|\sigma|})$ of the customers in σ that has a feasible path in the restricted walking graph, then set σ will not be served in the optimal solution (i.e., $y_{i\sigma k} = 0$ for all $i, k \in C$).

The result of Claim 1 may also identify service sets that must be used in the solution. Consider an isolated point i in the restricted walking graph. All other customers are “far enough” away that the delivery person will never walk to these customers from customer i . Thus, customer i will be served individually. Furthermore, the delivery person will be on board the vehicle to and from the location of customer i . Corollary 1 presents this result.

Corollary 1. Consider $i \in C$. If $D(i, k) + f \leq W(i, k)$ and $D(k, i) + f \leq W(k, i)$ for all $k \in C \setminus \{i\}$, then customer i will be served individually with loading and rendezvous locations at customer i (i.e., $y_{i\sigma i} = 1$, where $\sigma = \{i\}$). Furthermore, the following equations hold:

$$\sum_{k \in \bar{C}} x_{ki} = 1, \quad (15)$$

$$\sum_{k \in \bar{C}} x_{ik} = 1 \quad (16)$$

(i.e., the delivery person will drive to and from customer i).

4.2. Variable Reduction

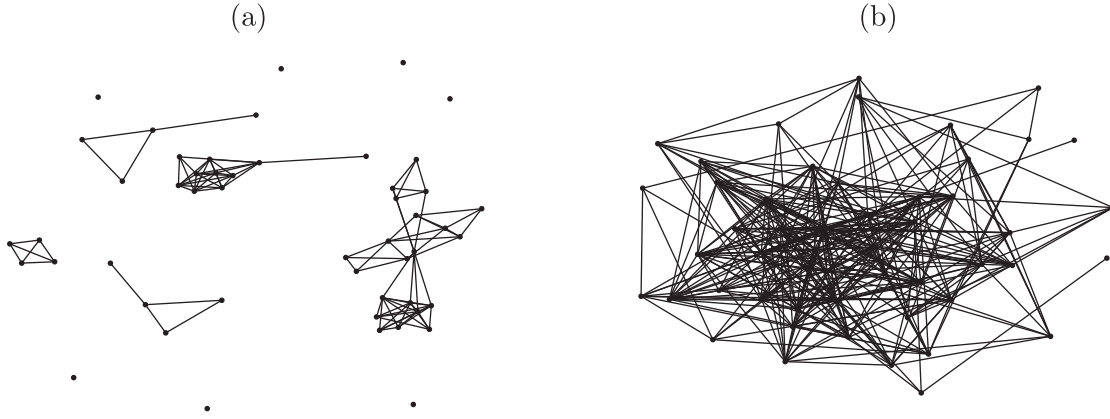
The elimination of service sets in Section 4.1 directly reduces the number of $y_{i\sigma k}$ variables. To further reduce the number of variables, we aim to understand the relationship between the loading and rendezvous points with the service set.

Claim 2 identifies when the loading and rendezvous points are restricted to members of the service set. In particular, if $D(i, k) \leq W(i, k)$ for all $i, k \in C$, then the delivery person would never walk outside of the service sets.

Claim 2. Consider $\sigma \in S$ such that $D(i, k) \leq W(i, k)$ and $D(k, i) \leq W(k, i)$ for all $i \in \sigma$ and $k \in C$. There exists an optimal solution such that if $y_{i\sigma k} = 1$ for some $i, k \in C$, then $i, k \in \sigma$.

Now, we relax the assumption that driving is faster than walking between all customers in the set. We aim to understand the trade-offs between walking and driving times. If the loading point is not within the service set, then the delivery person must walk from the loading point to the first customer served. If driving to the first customer is faster than walking, it

Figure 1. Restricted Walking Graphs of a (a) Less Dense, Rural Instance and (b) More Dense, Urban Instance



would seem that the delivery person would prefer to drive. However, it is possible that driving to the first customer in the service set will result in additional waiting time at the end of the set. Claim 3 eliminates the service tuple (i, σ, k) if the time saved by driving is greater than the additional waiting time.

Claim 3. Consider $(i, \sigma, k) \in C \times S \times C$ such that $i \notin \sigma$. Let $(c_1^{i\sigma k}, c_2^{i\sigma k}, \dots, c_{|\sigma|}^{i\sigma k})$ be the permutation of customers in set σ that achieves the shortest path, with respect to walking times, when loading at i and returning to k . If

$$W(i, c_1^{i\sigma k}) - D(i, c_1^{i\sigma k}) \geq \max\{D(c_1^{i\sigma k}, k) - w_{c_1^{i\sigma k} \sigma k}, 0\}, \quad (17)$$

then $y_{i\sigma k} = 0$, and the variable $y_{i\sigma k}$ is removed from the model.

Analogous to Claim 3, Claim 4 provides a particular case where the time saved by driving from the last customer to the rendezvous location is greater than the waiting time for a particular service set.

Claim 4. Consider $(i, \sigma, k) \in C \times S \times C$ such that $k \notin \sigma$. Let $(c_1^{i\sigma k}, c_2^{i\sigma k}, \dots, c_{|\sigma|}^{i\sigma k})$ be the permutation of customers in set σ that achieves the shortest path, with respect to walking times, when loading at i and returning to k . If

$$W(c_{|\sigma|}^{i\sigma k}, k) - D(c_{|\sigma|}^{i\sigma k}, k) \geq \max\{D(i, c_{|\sigma|}^{i\sigma k}) - w_{i\sigma c_{|\sigma|}^{i\sigma k}}, 0\}, \quad (18)$$

then $y_{i\sigma k} = 0$, and the variable $y_{i\sigma k}$ is removed from the model.

4.3. Valid Inequalities

In this section, we restrict our focus to cases where $D(i, k) \leq W(i, k)$ for all $i, k \in C$. In this case, we derive valid inequalities to strengthen the relationship between the x and y variables. Otherwise, the valid inequalities do not hold. Note that few test instances do not satisfy the inequality for all customer pairs.

We begin by identifying structure in the CAVADP solution. Claim 2 and Lemma 1 imply that each customer i will be visited only once. If the vehicle visits customer i , then customer i will be a loading or rendezvous point. Lemma 2 states that if the loading and rendezvous customer locations are the same, then that customer will be the only customer serviced.

Lemma 2. Consider $i \in C$. If $y_{i\sigma i} = 1$ for some $\sigma \in S$, then $|\sigma| = 1$. Furthermore, $\sigma = \{i\}$.

Similarly, Lemma 3 states that if the delivery person is on board the vehicle to customer i and leaves on the vehicle from customer i , then the service set is the singleton $\{i\}$.

Lemma 3. Consider $i \in C$. If $x_{ki} = 1$ and $x_{il} = 1$ for $k, l \in C \setminus \{i\}$ such that $k \neq l$, then i is served alone (i.e., $y_{i\sigma i} = 1$, where $\sigma = \{i\}$).

Then, Claim 5 concludes that if the delivery person is on board the vehicle from customers i to k such that $i \neq k$, then customers i and k will be served in different sets.

Claim 5. For $i, k \in C$ such that $i \neq k$, the following inequality holds:

$$\sum_{(a, \sigma, b) \in J} y_{a\sigma b} + x_{ik} \leq 1 \quad (19)$$

where $J = \{(a, \sigma, b) \mid a, b \in C, \sigma \in J_i \cap J_k\}$.

Next, consider $i, k \in C$ such that $i \neq k$. If the delivery person is on board from i to k (or k to i), then he or she will not serve a set of customers on foot between i and k . On the other hand, if the delivery person loads at i , serves some set σ , and returns to k , then the delivery person will not be on the vehicle between customers i and k . Claim 6 formalizes this observation. Note that

we can take $i < k$ to reduce the number of inequalities recognizing that duplicate constraints are added.

Claim 6. For all $i, k \in C$ such that $i \neq k$, the following inequality holds:

$$\sum_{\sigma \in S} (y_{k\sigma i} + y_{i\sigma k}) + x_{ik} + x_{ki} \leq 1. \quad (20)$$

5. Experimental Design

In this section, we discuss the generation of test instances and choice of parameter values that aim to reflect real-world geographies as well as real-world delivery conditions. In addition, we discuss benchmarks for the CAVADP. As discussed previously, the CDPP models traditional delivery practices where the delivery person must park the vehicle to service customers. The CDPP is challenging in its own right and needs more research. Importantly, we do not have preprocessing techniques for the CDPP that reduce the number of service sets or variables as discussed in Section 4 for the CAVADP. Therefore, using the CDPP as a benchmark restricts our analysis to $n = 50$ customers. The integer programming models for the CAVADP and CDPP are implemented in Python 3.7.0 using the Gurobi 9.0.0 solver with a 32 thread count.

5.1. Case Study: Illinois

In this section, we develop test instances to reflect real-world changes in customer geographies based on their population densities. The U.S. Department of Agriculture (USDA) classifies counties based on population size and adjacency to metro areas to be able to analyze trends related to population density (U.S. Department of Agriculture 2013). Therefore, all counties in a particular urban-to-rural continuum code, independent of the state of each county, should have similar population densities. Like the USDA, we refer to these classifications as the urban-to-rural continuum and use their nine codes to categorize counties. Because it has a county for each urban-to-rural code, we focus our case study on the state of Illinois. Table 4 lists the county in Illinois with the largest population per code.

Real-world address data are publicly available for Cook County (Cook County Government 2020) and Adams County (Adams County, Illinois 2019). However, address data is not available for the other seven counties. To complete a study across all counties, we use the approach introduced in this section to develop test instances for all counties. Online Appendix B compares the results from the generated test instances using this approach to results for the real-world address data sets for the two counties. The results using the real-world address data are comparable to

those for the generated test instances and demonstrate that the generated test instances are a good proxy.

Our approach identifies areas inside each county that would include service to $n = 50$ customers. Recent work finds that 15% of New York City households receive a package every day (Haag and Hu 2019). Given that we do not have data on the percentage of people likely to receive packages in each of the urban-to-rural-continuum codes, we assume that, for any code, 15% of the county population provided by the USDA receives a package on a given day (U.S. Department of Agriculture 2013). Assuming the customers are uniformly distributed over the county, we use the total square miles per county from the National Association of Counties (2010) to determine the population density. Assume county X has a population density of 100 per square mile. Then, if 15% of people in a square mile receive packages, we need a region just over three square miles in county X to represent the 50 customers that would receive packages on a given day.

For each county, we generate 10 random square service regions within the county borders defined by the Illinois Geospatial Data Clearinghouse (2003) to yield 10 service regions with 50 customers each. As we consider nine counties on the urban-to-rural continuum, our computational experiments include a total of 90 service regions (i.e., test instances). For each test instance, we use the VeRoViz package in Python to generate a uniform distribution of locations for the depot and customers within the service region (Peng and Murray 2019). These locations are placed on the nearest road and defined by their longitude and latitude coordinates. The square service region is divided into four quadrants. The test instances are constructed to satisfy the following conditions:

- All customer and depot locations are within the given square service region.
- There exists at least 10% of the customers in each quadrant of the square service region.

All test instances are posted at <https://doi.org/10.25820/data.006124>.

Table 4. Illinois Counties Along the Urban (1) to Rural (9) Continuum

Urban-to-rural code	County name
1	Cook County
2	Winnebago County
3	Champaign County
4	La Salle County
5	Adams County
6	Fulton County
7	Jefferson County
8	Johnson County
9	Cumberland County

5.2. Parameters

We set the parameters of the fixed time for loading packages f and capacity of the delivery person q based on discussion in the literature. Reed, Campbell, and Thomas (2022b) derive an estimate of $f = 2.8$ minutes. Therefore, we use $f = 2.8$ minutes as our base case here. Recall that the delivery person has a capacity q on the number of packages. A study in London shows drivers deliver on average three items per service (Allen et al. 2018). Therefore, we will use a capacity of three packages in our base case. We also consider smaller capacities for the delivery person of one and two packages to represent deliveries of larger items.

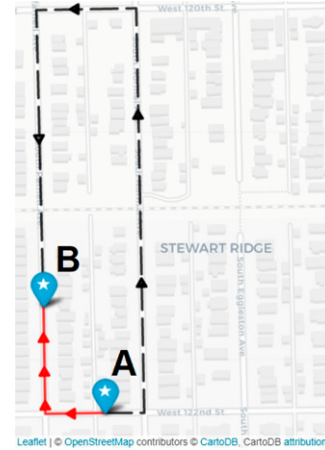
We utilize real-world data for the driving times of the vehicle and the walking times of the delivery person. The **VeRoViz package** uses OpenRouteService as a data provider for generating the walking and driving times between locations. OpenRouteService uses data from OpenStreetMap to restrict the driving route to the road network and define the pedestrian path of the delivery person. The incorporation of real-world data allows the model to capture features of different customer geographies, such as one-way streets in urban environments, and their impact. Figure 2 shows the solution for the CAVADP on the service set of customers A and B to analyze the effect of South Parnell Avenue in Chicago, Illinois, being a one-way street where vehicles travel north to south. The red solid path indicates the walking path of the delivery person and the black dashed path indicates the path of the autonomous vehicle. The shortest path for the vehicle requires a route around the block. To walk from customer A to customer B takes 2.0 minutes, whereas the autonomous vehicle takes 2.6 minutes to drive from customer A to customer B. Therefore, the delivery person must wait 0.6 minutes at customer B for the vehicle. This waiting time is captured in objective function (1).

5.3. Benchmarks

We consider multiple benchmark problems to analyze the effects of autonomous-assisted delivery on productivity and operational costs. Table 5 summarizes key characteristics of these benchmarks, but they will be described in detail in Sections 5.3.1–5.3.3.

5.3.1. CDPP. We use the CDPP as our benchmark to model traditional delivery practices. See Reed, Campbell, and Thomas (2022b) for a math programming formulation of the problem. The CDPP considers a set of n customers to be served by a delivery person. However, the delivery person must park the delivery vehicle prior to serving customers. The vehicle can park at any customer location with an expected time to find parking p (in minutes). The delivery person can serve multiple sets from the same parking location, but

Figure 2. (Color online) Example of Using Real-World Data to Capture Features of the Geography, Such as One-Way Streets



must return to the vehicle to reload packages. We integrate the same walking and driving times as used for the CAVADP. The objective of the CDPP is to minimize the completion time of the delivery tour. Jennings and Figliozzi (2020) use \$40 per hour for the cost of operation with a conventional van and a human driver. Therefore, we use \$40 for the operational cost per hour of the CDPP.

To represent changes in parking times across the urban-to-rural continuum, we use location-dependent parking times. INRIX Research reports that the average time to find parking in Chicago, Illinois, is nine minutes (Cookson and Pishue 2017). Therefore, we will use $p = 9$ for urban instances. We expect parking times to decrease along the urban-to-rural continuum and will represent this decrease as a one-minute decrease per continuum code (i.e., $p = 9$ minutes for Cook County and $p = 1$ minute for Cumberland County). Online Appendix C provides comparisons to the CDPP when we ignore parking time (i.e., $p = 0$ minutes across all counties.)

5.3.2. Fully Autonomous Model. The fully autonomous benchmark considers a model where the vehicle must visit every customer location for a given amount of time T . During this time, the customer must retrieve his or her package from the vehicle because there is no delivery person. Jennings and Figliozzi (2020) consider a three-minute delivery time penalty for the fully autonomous model. Therefore, we estimate T by adding three minutes to the loading time f and use $T = 5.8$ minutes as our base case. Because the objective is to minimize the completion time of the delivery tour and the vehicle must visit each customer, the fully autonomous benchmark reduces to solving the

Table 5. Summary of Benchmarks to the CAVADP

	CAVADP	CDPP	Fully autonomous model	Two-person model
Vehicle	Autonomous	Traditional	Autonomous	Traditional
Employees	1 delivery person	1 delivery person	None	1 driver, 1 delivery person

traveling salesman problem (TSP) through all customer locations with respect to driving times. We use the cost estimate by Jennings and Figliozzi (2020) of \$30 per hour of operation for an autonomous vehicle.

5.3.3. Two-Person Model. The optimal solution to the CAVADP can be achieved with a traditional vehicle using a driver and delivery person. The time of the delivery tour is the same, but the operational costs differ. Jennings and Figliozzi (2020) estimate the operational costs of an autonomous van to be \$30 per hour by removing the wage cost and adding a 15% increase for more expensive autonomous technology. This estimation implies a \$13.91 wage cost per hour for the delivery person. Adding the wage of a delivery person to the estimates of Jennings and Figliozzi (2020) for a traditional vehicle and a single driver (i.e., \$40 per hour), we estimate the operational costs of the two-person model to be \$53.91 per hour.

6. Computational Results

In this section, we analyze the effect of the preprocessing techniques and valid inequalities discussed in Section 4. We show that these techniques are successful in reducing the size of the problem and increasing the bound of the linear programming relaxation.

6.1. Preprocessing Techniques

This section shows the effects of the preprocessing techniques introduced in Sections 4.1 and 4.2. Table 6 provides details on the first test instance of each county when $n = 50$ customers, $q = 3$ packages, and $f = 2.8$ minutes. We denote the first test instance of Cook County as Cook_1 and follow a similar convention when referencing test instances. The second column of the table, average degree, indicates the average node degree in the restricted walking graph. In general, urban instances have higher average node degree indicating the delivery person is willing to walk to more customers. The third column of the table, customer pairs, indicates the number of customer pairs such that walking between the customers is faster than driving between these customers.

The maximum number of service sets with $n = 50$ customers and $q = 3$ packages is $m = 20,875$. Let S_1 be the set of customer service sets after eliminating sets due to Claim 1. On average, we can reduce the number of service sets by 98.2% for instances where $D(i,k) \leq W(i,k)$ for all customers $i,k \in C$. For instances

in which the inequality does not hold for all customer pairs, the reduction in service sets ranges from 15% to 28%.

Reducing the number of service sets also reduces the number of y_{iok} variables that need to be considered. Let $Y = \{y_{iok} \mid i,k \in C, \sigma \in S\}$ be the set of y_{iok} variables in the model. Without Claim 1, $|Y| = n^2m = 52,187,500$. Define $Y_1 = \{y_{iok} \mid i,k \in C, \sigma \in S_1\}$. The fifth column of Table 6, labeled $|Y_1|$, gives the number of y_{iok} variables resulting from the reduced number of service sets based on Claim 1. Let Y_2 be the set of y_{iok} variables in Y_1 considered after Claim 2. The sixth column of Table 6 gives $|Y_2|$. For instances where $D(i,k) \leq W(i,k)$ for all customers $i,k \in C$, we reduce Y_1 by at least 99.7%. For other instances, the reduction in variables range from 83% to 91%. Winnebago_1 achieves the lowest reduction of 83% as the instance with the largest number of customer pairs.

Understanding the time trade-offs between walking and driving allows us to further reduce the size of Y_1 . For cases where **driving is faster than walking** for all customer pairs, Claim 2 eliminates the possibility of walking outside of the set as it shows $i,k \in \sigma$ for all $y_{iok} = 1$. Therefore, we focus on the cases where there exist customers $i,k \in C$ such that $W(i,k) \leq D(i,k)$. Let $Y_{3,4}$ be the set of y_{iok} variables in Y_1 considered after Claims 3 and 4. The seventh column of Table 6 gives $|Y_{3,4}|$. For cases where there exist customer pairs such that $W(i,k) \leq D(i,k)$, the average reduction in variables is 88%. The eighth column gives the service sets that need to be considered in the model after implementing Claims 2, 3, and 4. Using all claims, the average reduction in variables is 98%.

Finally, the ninth column provides the time (seconds) to complete these preprocessing techniques as well as compute the w_{iok} walking time for each $y_{iok} \in |Y_2 \cap Y_{3,4}|$. For instances where $D(i,k) \leq W(i,k)$ for all $i,k \in C$, the preprocessing time is less than four seconds. Otherwise, the preprocessing time significantly increases.

6.2. Valid Inequalities

In this section, we analyze the value of the valid inequalities introduced in Section 4.3. Claims 5 and 6 strengthen the relationship between the x and y variables. Recall that the valid inequalities hold only for cases in which $D(i,k) \leq W(i,k)$ for all $i,k \in C$. We show that these valid inequalities reduce the optimality gap at the root node and run time to find the optimal

Table 6. Computational Results from Preprocessing Techniques Discussed in Section 4

Instance	Average degree	Customer pairs	$ S_1 $	$ Y_1 $	$ Y_2 $	$ Y_{3,4} $	$ Y_2 \cap Y_{3,4} $	Preprocessing time (sec)
Maximum	—	—	20,875	52,187,500	—	—	—	—
Cook_1	12.84	36	17,824	44,560,000	4,178,023	4,913,916	488,777	141
Winnebago_1	6.32	60	15,127	37,817,500	6,536,100	4,325,899	869,279	171
Champaign_1	7.40	0	766	1,915,000	5,569	1,915,000	5,569	4
LaSalle_1	4.72	0	351	877,500	2,169	877,500	2,169	2
Adams_1	3.08	0	201	502,500	1,024	502,500	1,024	1
Fulton_1	3.16	0	206	515,000	1,059	515,000	1,059	1
Jefferson_1	3.48	0	232	580,000	1,253	580,000	1,253	1
Johnson_1	5.44	0	449	1,122,500	2,961	1,122,500	2,961	2
Cumberland_1	3.96	36	14,976	37,440,000	4,147,156	4,385,635	514,661	132

solution. We use the first test instance from each urban-to-rural continuum code that satisfies that driving is faster than walking for all customers.

Table 7 shows the optimality gap at the root node and run time to find the optimal solution for $n = 50$ customers, $q = 3$ packages, and $f = 2.8$ minutes. The third and seventh columns in Table 7 give the optimality gap at the root node and run time, respectively, when using the valid inequalities in Claim 5. On average, Claim 5 provides a 44% reduction in the optimality gap at the root node and a 41% reduction in run time. The fourth and eighth columns in Table 7 show the impact of Claim 6 on the optimality gap at the root node and the run time, respectively. The results show that Claim 6 alone provides on average a 1% reduction in the optimality gap at the root node and a 29% reduction in run time. Finally, the fifth and ninth columns in Table 7 provide the optimality gap at the root node and run time, respectively, including both claims. Using both claims improves the optimality gap at the root node in all cases and run times in 9 out of 10 cases. On average, Claims 5 and 6 provide, on average, a 50% reduction in the optimality gap at the root node and a 49% reduction in the run time.

6.3. Computational Limitations

Section 6.1 shows the preprocessing techniques are more successful on instances where driving is faster than walking between all customer pairs. The valid inequalities in Section 4.3 are restricted to this case. For the test instances generated in Illinois, 78% of instances satisfy that driving is faster than walking between all customer pairs. In this subset of test instances, we are able to successfully solve the CAVADP to optimality for larger capacities for the delivery person. In particular, for $q = 4$ and 5 packages, we can solve 86% and 83% of instances to optimality within five minutes, respectively. When increasing the number of customers to $n = 100$, we can often solve to within a 5% optimality gap in less than 20 minutes. However, if there exist customer pairs such that

walking is faster than driving, additional model improvements need to be developed.

7. Experimental Comparison

In this section, we analyze the impact of autonomous-assisted delivery across the urban-to-rural continuum. We compare the solutions of the CAVADP to the benchmarks. We present the average results for the 10 test instances for each county. Detailed results are provided in Online Appendix D. In each county, the results are fairly consistent across the 10 instances. We begin by summarizing the cost-effectiveness of autonomous-assisted delivery in the base case ($q = 3$ packages, $f = 2.8$ minutes, and $T = 5.8$ minutes).

7.1. Summary

In this section, we analyze the cost-effectiveness of autonomous-assisted delivery in urban-to-rural settings. Because the CAVADP uses an autonomous vehicle with a delivery person, we assume the cost per hour of operation includes the operational costs of the autonomous vehicle and the wage cost of one delivery person (i.e., \$43.91 per hour). Figure 3 shows the average cost per delivery tour for the CAVADP and benchmark problems in the base case. On average, the CAVADP costs less than all benchmarks across urban-to-rural settings, demonstrating the cost-effectiveness of autonomous-assisted delivery in operations. Insight 1 summarizes this result.

Insight 1. Autonomous vehicle assisted delivery is the most cost-effective business model across all customer geographies.

Any productivity gains from the CAVADP can be achieved using the two-person model. However, the two-person model comes at the cost of additional wages by adding a second person on the vehicle. The CAVADP reduces the operational cost per hour by 19%. If the investment in autonomous technology is not feasible for the business, the two-person model may be a viable option to reduce costs, particularly in urban environments. Figure 3 shows that using the

Table 7. Optimality Gap (%) at the Root Node and Run Time (Seconds) to Find the Optimal Solution Using the Valid Inequalities from Section 4.3

Instance	Optimality gap at root node (%)				Run time (sec)			
	None	Claim 5	Claim 6	Both claims	None	Claim 5	Claim 6	Both claims
Cook_2	23.0	10.9	23.0	9.2	174	36	86	30
Winnebago_2	7.3	5.7	7.3	5.4	284	87	221	184
Champaign_1	13.7	7.8	13.7	7.3	213	102	49	97
LaSalle_1	9.8	4.5	9.8	4.3	9.8	4.5	9.8	4.3
Adams_1	11.9	4.7	11.9	3.7	26	5	17	4
Fulton_1	23.1	10.6	21.8	6.7	39	42	43	37
Jefferson_1	14.4	9.5	14.4	8.91	48	56	56	54
Johnson_1	8.5	3.8	8.5	3.5	35	6	10	4
Cumberland_2	26.6	20.4	26.6	19.3	27	33	28	13

two-person model reduces operational costs up to 39% relative to the CDPP in urban environments.

The operational costs per hour of the CAVADP are higher than those for the CDPP and fully autonomous model. However, Figure 3 shows that cost savings of autonomous-assisted delivery relative to the CDPP range from 18% to 50%, with higher cost savings in urban environments. Relative to the fully autonomous model, the CAVADP reduces the operational costs by, on average, 47%. Thus, the time savings from autonomous-assisted delivery are significantly greater than the increase in operational costs. Figure 4 summarizes savings in the tour completion time of the CAVADP relative to the CDPP and fully autonomous model in the base case. We see significant savings from autonomous-assisted delivery across all types of geographies. In comparison with the CDPP, we see that the level of savings is higher in urban environments, with 53% average savings, compared with rural environments, with 25% average savings. Similarly, the savings relative to the fully autonomous model range from a reduction of 61% in the completion time

of the delivery tour in rural environments to as high as 68% in urban environments. Sections 7.2 and 7.3 further explore the productivity gains of autonomous-assisted delivery relative to the CDPP and fully autonomous model, respectively. Insight 2 summarizes this result.

Insight 2. *Increases in productivity from the CAVADP with respect to both the CDPP and fully autonomous model are greater than the increased operational costs of using an autonomous vehicle. On average, a delivery person saves more time during the delivery tour in urban environments than in rural environments.*

7.2. CDPP Comparisons

In this section, we explore how changes in the structures of the CAVADP and CDPP solutions across the urban-to-rural continuum and parameter values explain different levels of savings. Figure 5 shows a significant decrease in the completion time of the delivery tour comes from the walking time. The additional walking time in the CDPP includes walking to service customers as well as the need to walk back to

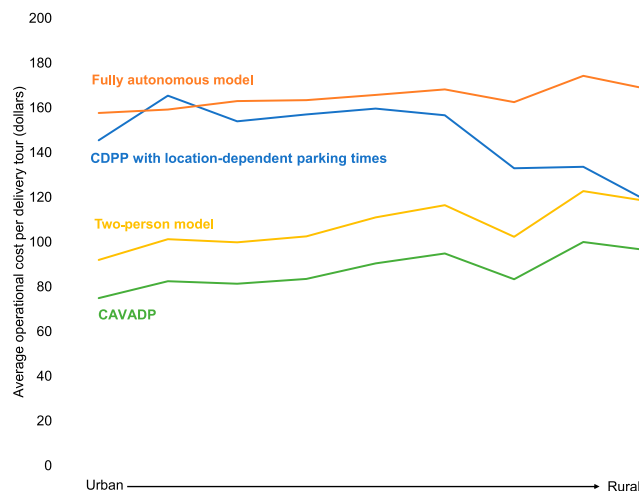
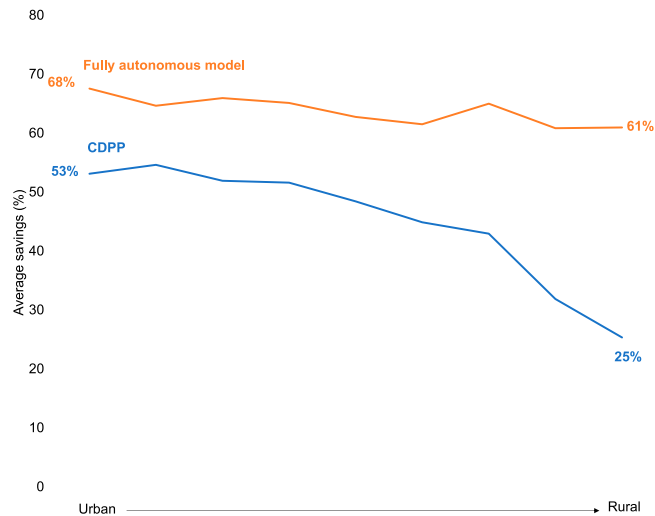
Figure 3. (Color online) The Average Cost per Delivery Tour for the CAVADP and Benchmark Problems in the Base Case

Figure 4. (Color online) The Average Amount of Time Savings from Autonomous-Assisted Delivery Compared with the CDPP and Fully Autonomous Model in the Base Case



the parked vehicle. In urban environments, the delivery person walks, on average, 81 fewer minutes in the CAVADP delivery tour compared with the CDPP delivery tour. The savings in walking time persists in rural environments, but the level of savings reduces to walking 6 fewer minutes. Insight 3 summarizes this observation.

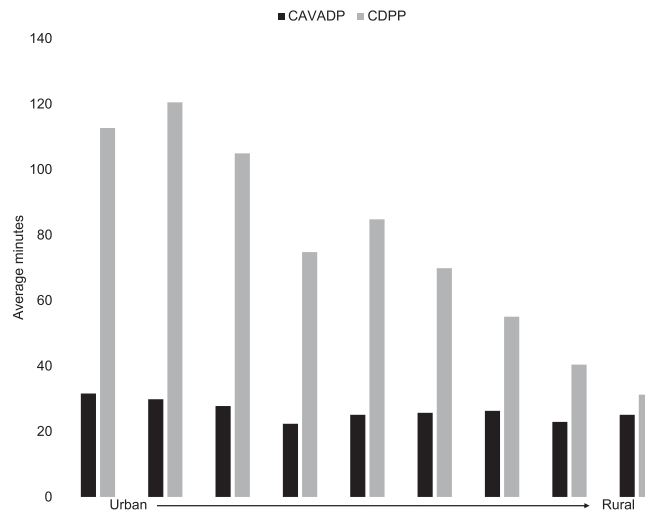
Insight 3. *Reductions in the time of the delivery tour by autonomous vehicle assisted delivery are greater than the time to find parking across all instances. In particular, the delivery person spends significantly less time walking in urban environments.*

Thus, it is clear that the delivery person spends her or his time differently in the CAVADP and CDPP models. Figure 6 decomposes the delivery tour for the CAVADP and CDPP into the percentages of time that the delivery person spends parking, on board the vehicle, walking, loading, and waiting for the vehicle. This decomposition also changes over the urban-to-rural continuum for both problems. For the CAVADP, Figure 6(a) shows the delivery person spends 31% of time walking in dense urban instances compared with 19% in less dense rural instances. Figure 6(b) shows a more significant change in walking time for the delivery person in the CDPP. In urban instances, the delivery person spends 51% of the delivery tour walking compared with 18% in rural instances. We also see that the percentage of time the delivery person spends on board the vehicle increases across the urban-to-rural continuum in both the CAVADP and CDPP. When customers are further apart in less dense rural environments, the delivery person spends more time on board the vehicle. Therefore, the solutions become more similar in rural environments due to more time spent

traveling between customers and less time needed for parking. This observation explains the reduced time savings resulting in converging costs for rural environments.

Figures 7 and 8 show the optimal solutions for the CAVADP and CDPP in an urban instance and rural instance, respectively. In each solution, the darker black lines indicate the delivery person is on board the vehicle and the lighter orange lines indicate the delivery person is walking. The parking spots in the CDPP are indicated with blue flags. Figure 7 shows an urban instance in Cook County. Figure 7(b) provides the optimal solution to the CDPP, where the delivery person parks four times. Parking in fewer locations forces the delivery person to walk more to service customers in these urban instances. Figure 8 shows a rural instance in Cumberland County. Compared with Cook County in Figure 7, the customer geography is less dense because of the size of region and the structure of the road network. We observe that the solutions of the CAVADP and CDPP become more similar in rural environments, with the delivery person spending a significant portion of time on board the vehicle in both delivery tours. Autonomous vehicle assisted delivery reduces the time of the delivery tour by 24% for the specific instance of Cumberland County in Figure 8. The time to find parking accounts for 55% of this savings. However, other delivery practices such as time spent on board the vehicle, walking, and loading packages account for 45% of the savings, highlighting the difference in the solution structure between the CAVADP and CDPP.

7.2.1. Impact of Capacity of the Delivery Person. To evaluate the sensitivity of the results to the capacity of

Figure 5. Mean Time Spent Walking (Minutes) in the Optimal Solutions of the CAVADP and CDPP for the Base Case

the delivery person, we consider the effect of different capacities. For this analysis, we restrict comparisons to at most $q = 3$ packages as further work is needed to solve the CDPP at larger capacities for the delivery person. Figure 9 presents the average savings from autonomous vehicle assisted delivery across capacities for the delivery person of one to three packages. Outside of urban environments, higher capacities result in higher savings. In urban environments, increasing from $q = 1$ to 2 packages also results in higher savings. However, when increasing to $q = 3$ packages, the delivery person may realize greater reductions in the CDPP relative to the CAVADP by parking in fewer locations when the time to find parking is high. Insight 4 summarizes these results.

Insight 4. Outside of urban environments, increasing the capacity of the delivery person generally leads to higher savings from autonomous vehicle assisted delivery with location-dependent parking times. At large capacities in urban environments with high parking times, the delivery person may realize greater reductions in traditional delivery practices by parking the vehicle in fewer locations without significantly increasing the walking time.

7.2.2. Impact of Loading Time in Rural Settings. The use of an autonomous vehicle may allow the delivery person to reduce loading time by locating and sorting packages while en route (U.S. Postal Service 2017). This is confirmed by Figure 6(a), which shows the

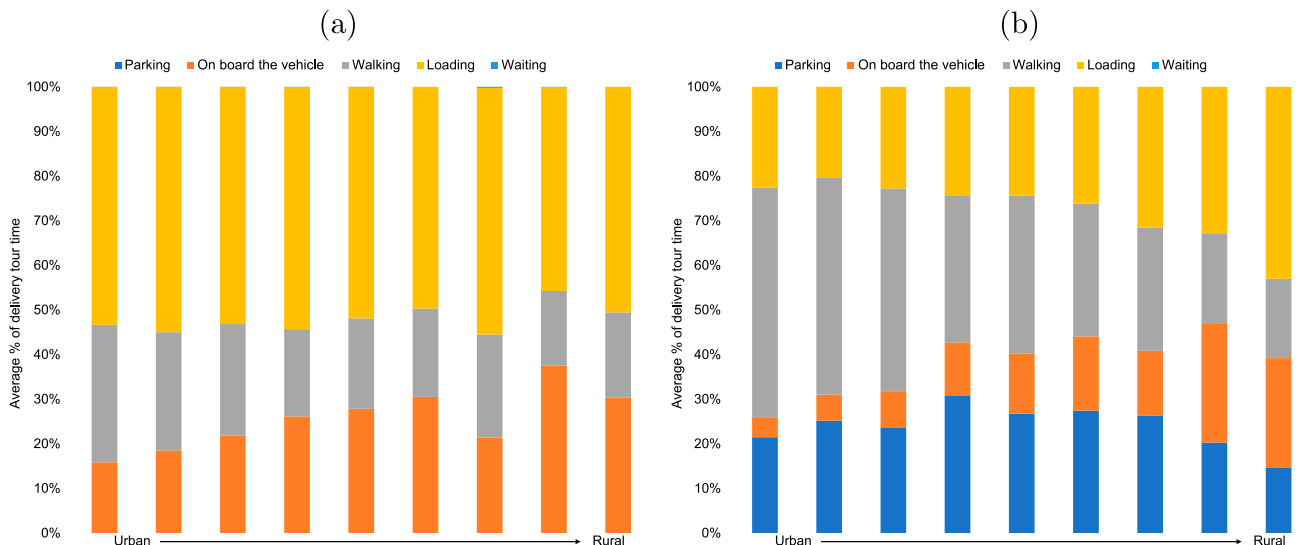
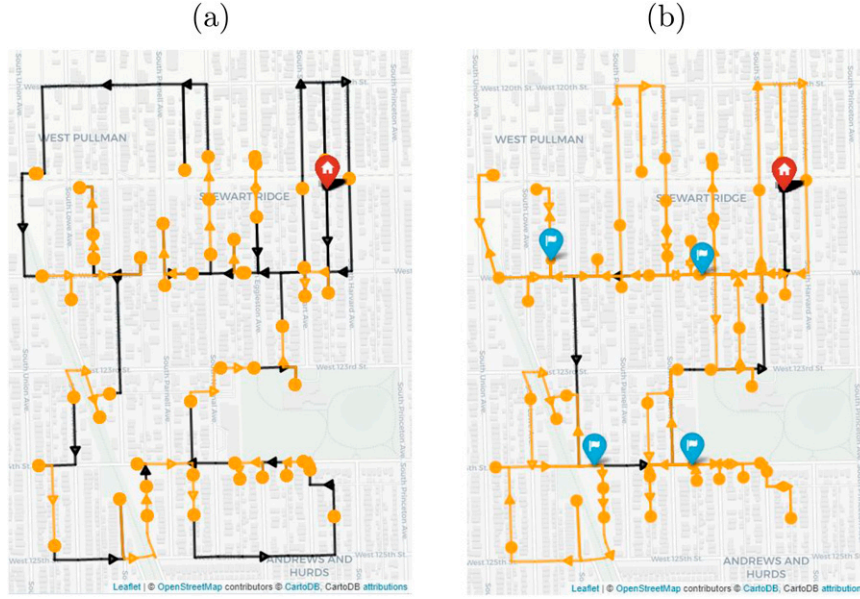
Figure 6. (Color online) Decomposition of the (a) CAVADP and (b) CDPP Solutions for the Base Case

Figure 7. (Color online) Optimal Solutions for the (a) CAVADP and (b) CDPP in an Urban Instance of Cook County for the Base Case



delivery person spends a greater portion of time on board the vehicle on the delivery tour in rural environments. This additional time on board the vehicle may allow for productivity gains that reduce the loading time. In urban areas, Figure 6(a) shows that the delivery person spends a greater portion of time walking than on board the vehicle and therefore may not be able to reduce the loading time. Thus, we focus on the impact of reducing the loading time in rural settings. We aim to determine whether a delivery person in rural environments can achieve level of savings similar to that observed in urban environments by reducing the loading time.

Figure 10 shows the average savings from autonomous vehicle assisted delivery at various loading times f for the last three urban-to-rural continuum codes with $q = 3$ packages and location-dependent parking times. We compare the optimal solution of the CDPP at $f = 2.8$ minutes to the optimal solution of the CAVADP at a reduced loading time. As a point of comparison, the average savings from autonomous vehicle assisted delivery is 53% in urban environments based on the first three urban-to-rural continuum codes. We indicate this 53% saving in urban environments with a dashed line in Figure 10. When we consider a 50% reduction in loading time ($f = 1.4$ minutes), average savings increase to 52% in rural environments. A further reduction to $f = 0.7$ minutes leads to average savings of 63%. Therefore, businesses

with rural deliveries must be able to significantly reduce the loading time to achieve savings similar to those in urban environments. Insight 5 summarizes this result.

Insight 5. To achieve the level of savings observed in urban environments, the loading time for the delivery person using an autonomous vehicle in rural environments must be significantly reduced.

Figure 8. (Color online) Optimal Solutions for the (a) CAVADP and (b) CDPP in a Rural Instance of Cumberland County for the Base Case

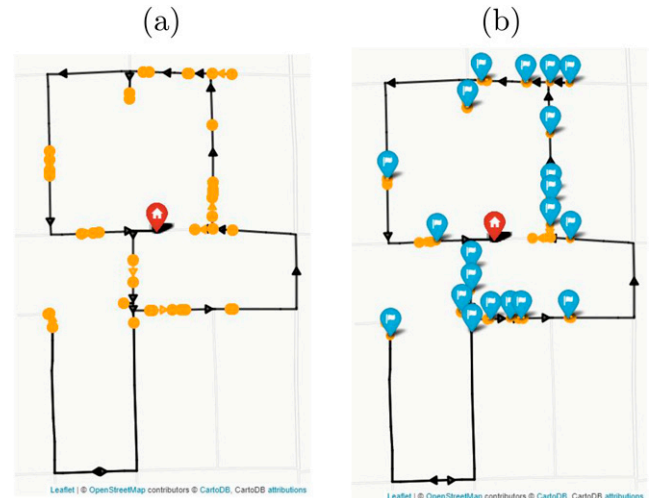
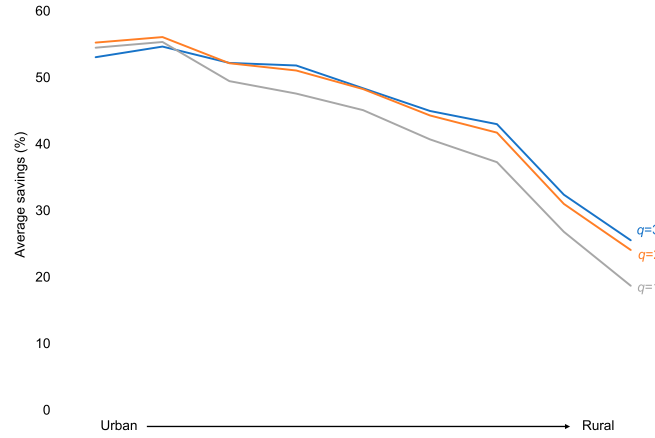


Figure 9. (Color online) The Average Savings from Autonomous Vehicle Assisted Delivery Across Capacities for the Delivery Person of One to Three Packages in the Base Case

7.3. Fully Autonomous Model Comparisons

In this section, we show that the time spent at each customer, T , in the fully autonomous model must be small (i.e., less than the three-minute delivery penalty for the fully autonomous model considered by Jennings and Figliozzi (2020)) to achieve performance similar to that of the CAVADP. The solution to the fully autonomous model reduces to finding the TSP with respect to driving times. Along this route, each customer location is visited for T minutes to give the customer time to retrieve the package from the vehicle. In the case where driving is always faster than walking, this TSP solution is also the solution to the CAVADP when $q = 1$ package and $f = T$ minutes. However, the time T allocated for the customer to retrieve his or her package is likely longer than the

time for the delivery person to unload the package f . When $q = 1$ and $T > f$, the CAVADP reduces the completion time of the delivery tour by $n(T - f)$ minutes. In cases where walking can be faster than driving and $T \geq f$, the CAVADP, unlike the fully autonomous model, has the flexibility of the delivery person walking to an alternative location if doing so is more advantageous than driving. Therefore, if $q = 1$ and $T \geq f$, the completion time of the fully autonomous model is at least the completion time of the CAVADP.

Because the loading time is independent of the number of packages, reducing the number of service sets reduces the time spent loading and, ultimately, the length of the delivery tour. Figure 11(a) shows the length of the delivery tour for the CAVADP in the base case ($q = 3$ packages and $f = 2.8$ minutes) and the

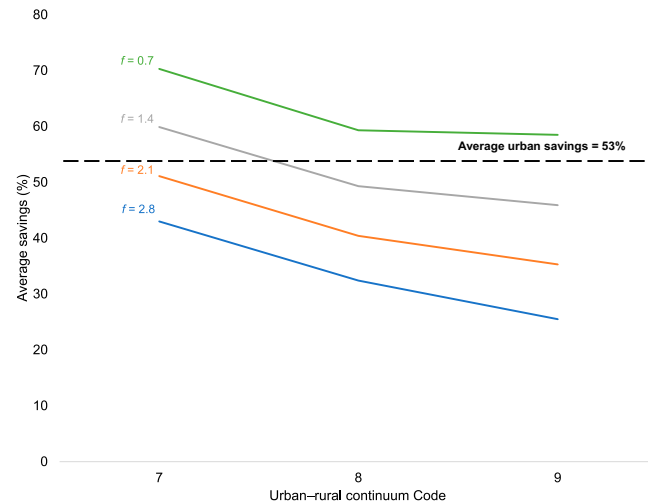
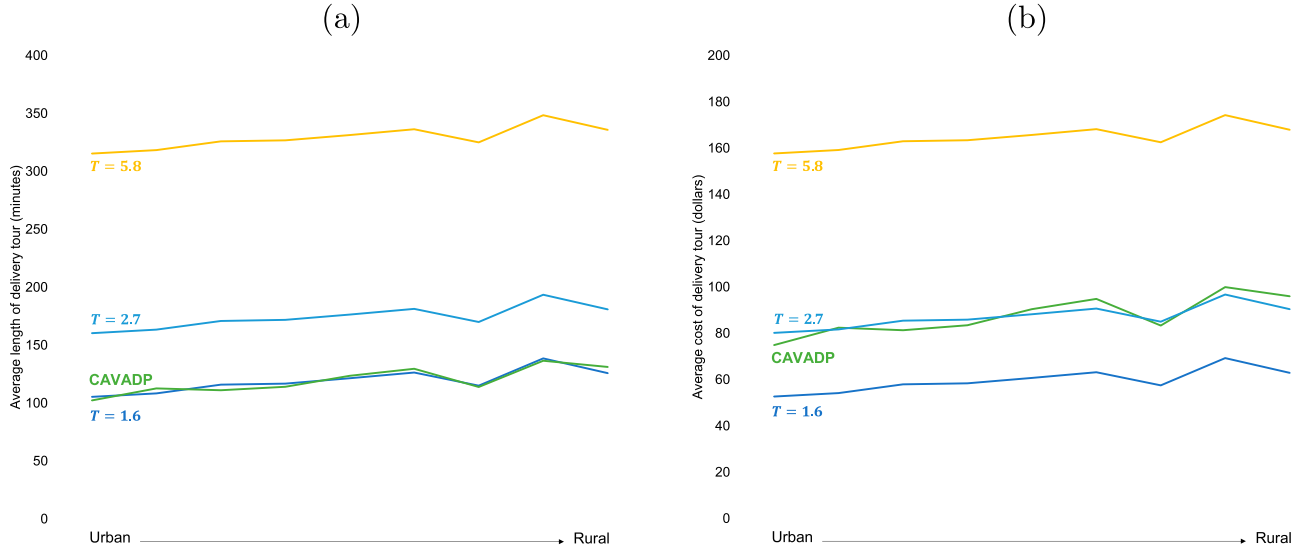
Figure 10. (Color online) The Average Savings from the Use of Autonomous Vehicles in Rural Environments Varying the Loading Time f in the CAVADP When $q = 3$ Packages

Figure 11. (Color online) Average (a) Completion Time and (b) Cost of Delivery Tour for the CAVADP in the Base Case and the Fully Autonomous Benchmark with Varying Values of T (Minutes)



fully autonomous benchmark for varying values of T . On average, for the capacity of $q = 3$, the CAVADP yields a shorter delivery tour completion time than the fully autonomous model when $T > 1.6$ minutes. Because of the increased wage cost of the CAVADP relative to the fully autonomous model, Figure 11(b) shows that T must be greater than 2.7 minutes on average for the CAVADP to be cost beneficial compared with the fully autonomous model. In our base case ($T = 5.8$ minutes), the CAVADP reduces the operational costs on average 47% compared with the fully autonomous benchmark. Insight 6 summarizes this result.

Insight 6. If $T > 1.6$ minutes, the delivery person's ability to deliver more than one package per service set results in the CAVADP having a lower completion time on average than the fully autonomous model. If $T > 2.7$ minutes, the increased productivity with the wage cost of the delivery person in the CAVADP are significant enough to realize an overall lower operational cost than the fully autonomous model.

Next, we analyze the effects of the capacity of the delivery person on time savings. Figure 12 shows the average savings from autonomous-assisted delivery relative to the fully autonomous model across varying capacities for the delivery person q when $f = 2.8$ and $T = 5.8$ minutes. To test $q = 4$ and 5 packages, we needed to limit the test instances to those that satisfy the assumption that driving is faster than walking for all customer pairs. Dashed lines in Figure 12 indicate the use of this modified set of test instances. The solution to the fully autonomous model is independent of q . However, the objective value of the CAVADP decreases as q increases. Therefore, we see savings relative to the fully

autonomous case increase as q increases. In particular, we observe decreasing marginal savings due to the decreasing marginal reductions of the objective value in the CAVADP. Insight 7 summarizes this observation.

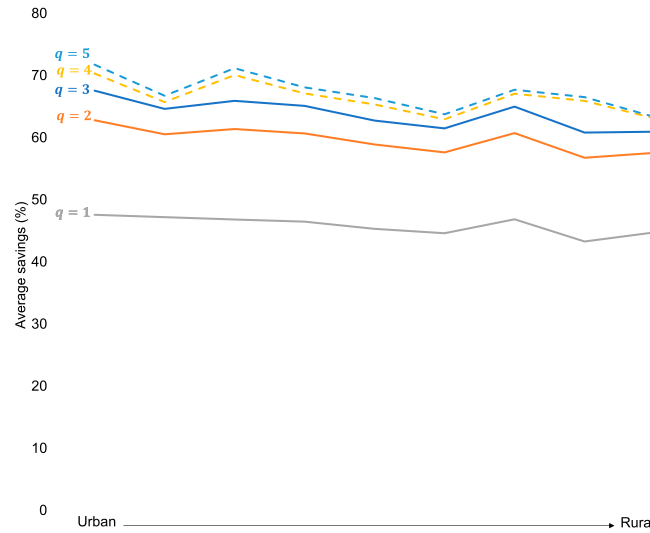
Insight 7. Increasing the capacity of the delivery person in the CAVADP increases time savings for the delivery person relative to the fully autonomous model. Marginal savings decrease at higher capacities.

8. Conclusions and Future Work

In this paper, we generalize the CAVADP model to analyze savings from pairing an autonomous vehicle with a delivery person in last-mile delivery on the urban-to-rural continuum. We conduct a case study in Illinois and evaluate savings by using the fully autonomous model, two-person model, and CDPP as benchmarks. Comparing the CAVADP to the fully autonomous model, we show that the time spent at each customer in the fully autonomous model must be small to be competitive with autonomous-assisted delivery. The two-person model achieves the productivity gains of the CAVADP but increases the overall wage cost. Autonomous-assisted delivery outperforms the CDPP and is the most cost-effective model across urban-to-rural settings.

While there is value in autonomous vehicle assisted delivery in all customer geographies, the biggest benefit may be realized in urban environments. Therefore, businesses with urban deliveries should carefully examine investment in this technology. With a 56% savings in completion time relative to the CDPP, one autonomous vehicle and delivery person can complete

Figure 12. (Color online) The Average Savings from Autonomous-Assisted Delivery Relative to the Fully Autonomous Model Across Varying Capacities for the Delivery Person q in the Base Case



the work of two drivers with traditional vehicles reducing operational costs for the business by 50% on average. This increased productivity could reduce fleet size and ultimately the number of vehicles on the road. Fewer delivery vehicles in urban environments could lead to additional benefits such as less traffic congestion in these areas. Thus, businesses could reduce operational costs while improving sustainability.

The U.S. Postal Service (2017) plans to utilize autonomous vehicles on rural routes, and we show that autonomous-assisted delivery may reduce route completion time significantly (26%) in these areas relative to delivery practices with parking. With the largest savings in urban environments, our conclusions demonstrate potentially more valuable environments for introducing autonomous-assisted delivery. In urban environments, a significant portion of savings relative to the CDPP is realized in the walking time of the delivery person, specifically, the need for the delivery person to walk back to the parked vehicle. However, with greater driving times between customers in rural environments, the potential to reduce loading times due to the delivery person preparing while the vehicle drives may help autonomous-assisted delivery become as attractive in rural as urban environments. Even if parking time is set to zero in all environments, the CAVADP still achieves significant savings. This occurs because a significant amount of time is spent loading packages in the CDPP compared with the CAVADP. We explore the impact of the capacity of the delivery person and demonstrate increasing capacity generally results in higher time savings relative to the CDPP and fully autonomous models. However, in urban environments, larger capacities for the

delivery person allow the delivery person to reduce the number of parking locations in the CDPP without significantly increasing the walking time. Therefore, we observe that a larger capacity for the delivery person may result in less savings in urban settings.

This paper provides necessary model improvements to continue the discussion of exploring autonomous vehicle assisted delivery in last-mile delivery. The ability to solve the CAVADP on a general graph of customers allows us to understand the significant savings that may be realized from autonomous-assisted delivery across the urban-to-rural continuum. Thus, we look to examine extensions to the CAVADP. For example, we can examine the impact of additional delivery people on board the vehicle, a fleet of autonomous vehicles, pickup and delivery services, and customer time windows. It is also important to see how these savings are impacted by stochastic travel times. Furthermore, we may achieve additional savings by combining autonomous-assisted delivery with other innovations in last-mile delivery, such as in Poikonen and Golden (2020); Agatz, Bouman, and Schmidt (2018); McLeod et al. (2020); and Boysen, Fedtke, and Schwerdfeger (2018). Solving these extensions to optimality may require new methodology for cases where walking is faster than driving between customer locations.

As companies look to autonomous technology, developments in the benchmark problems become crucial to understanding the benefit of implementing this new technology. For example, the vehicle may not need to visit each customer location in the fully autonomous model but instead a location close enough to each customer. To solve larger instances of the CDPP, additional work needs to be done in developing technology to

control the growth in the size of the CDPP. A column generation approach may be an efficient way to handle the large number of service variables in the model. We are also looking at adapting the heuristic and other ideas introduced for the variant of the CDPP studied in Reed, Campbell, and Thomas (2022a). Once we can better solve larger instances of the CDPP, we can introduce stochastic elements such as times to find parking and availability of parking locations. Any improvements to the CDPP may help leading delivery companies analyze their current operations.

Acknowledgment

The authors thank the associate editor and reviewers for their constructive feedback to improve the paper.

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