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The Value of Autonomous Vehicles for Last-Mile Deliveries in Urban Environments

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Abstract. We demonstrate that autonomous-assisted delivery can yield significant improvements relative to today's system in which a delivery person must park the vehicle before delivering packages. We model an autonomous vehicle that can drop off the delivery person at selected points in the city where the delivery person makes deliveries to the final addresses on foot. Then, the vehicle picks up the delivery person and travels to the next reloading point. In this way, the delivery person would never need to look for parking or walk back to a parking place. Based on the number of customers, driving speed of the vehicle, walking speed of the delivery person, and the time for loading packages, we characterize the optimal solution to the autonomous case on a solid rectangular grid of customers, showing the optimal solution can be found in polynomial time. To benchmark the completion time of the autonomous case, we introduce a traditional model for package delivery services that includes the time to search for parking. If the time to find parking is ignored, we show the introduction of an autonomous vehicle reduces the completion time of delivery to all customers by 0%–33%. When nonzero times to find parking are considered, the delivery person saves 30%–77% with higher values achieved for longer parking times, smaller capacities, and lower fixed time for loading packages.

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1. Introduction

In a 2017 survey, 40% of shoppers said that they make at least seven purchases online in a three-month period (UPS 2017). This trend is likely to continue. A 2018 article on New York City predicts volumes to grow by 68% by 2045 (New York City Economic Development Corporation 2018). While increasing convenience for consumers, this boom in deliveries, and the associated increase in delivery trucks, worsens vehicle congestion problems in large cities, making driving slow and parking difficult. A recent study of last-mile delivery practices observes how the drivers' time was spent when delivering packages to customers. The authors observe that the drivers parked an average of 37 times per day, and the vehicles were parked for 62% of the time that deliveries and collections were being made (Allen et al. 2018b). The drivers walked an average of 7.94 km per day with a increasing portion of this due to the distances drivers are now needing to walk from their parking places to their delivery locations and back. Another study found that drivers are increasingly incurring fines for double parking in an effort to park closer to customers (Figliozzi and Tipagornwong 2017).

The time a driver spends looking for parking and walking from a parking spot is time that the driver is paid but not making deliveries. In large cities in the United States, a driver spends an average of nine minutes every time he or she needs to find a parking spot (Cookson and Pishue 2017). Increased demand for deliveries exacerbates this problem, and companies will eventually need to hire even more drivers. Hiring more drivers leads to more delivery vehicles and increased congestion in urban areas. Thus, delivery companies are actively looking for alternatives (Forger 2019). Amazon has developed Prime Air, which has the ability to deliver up to five pounds in 30 minutes or less using a small aerial drone (Amazon 2018). However, in many urban environments, the proposed drone operations would be challenging because of wind conditions and poor lines of communication at potential landing spots (e.g., rooftops and ground levels; Dayarian et al. 2020). Businesses are looking to autonomous vehicles to satisfy their delivery needs. For example, some are experimenting with compartmentalized sidewalk autonomous robots for the delivery of small items such as groceries. However, because of limitations such as the robot's capacity and its inability to ring a doorbell or climb a staircase, deliveries will still require a delivery person for the foreseeable future (Mims 2019).

Another option, and the one we consider here, is to use an autonomous vehicle with a delivery person on board. The vehicle drops off the delivery person at selected points in the city, the delivery person makes the deliveries to the final addresses on foot, and then the vehicle picks up the delivery person and travels to the next reloading point. In this way, the delivery person never needs to look for parking or walk back to a parking place. Such a use of autonomous vehicles is well motivated by recent media articles such as the report by the U.S. Postal Service Office of Inspector General (U.S. Postal Service 2017). In this paper, our goal is to demonstrate that autonomous-assisted delivery can yield significant improvements relative to today's system, in which a delivery person must park the vehicle before delivering packages. Specifically, we demonstrate how the completion time of the delivery to all customers decreases with the use of autonomous vehicles. Our study offers insights regarding how such a delivery model could change the number of drivers and therefore vehicles required to serve an urban area.

To this end, this paper introduces the capacitated autonomous vehicle assisted delivery problem (CAVADP). The CAVADP is the problem of serving a set of customers using an autonomous vehicle assisted by a delivery person. The delivery person can carry a limited number of packages and must return to the vehicle to replenish packages once a given set has been delivered. The vehicle can travel between customers with or without the delivery person on board. The autonomous nature of the vehicle allows for the delivery person to load at a given location, serve a set of customers, and be picked up by the vehicle at an alternate location. In between loading points, the vehicle can circle the area or find parking in a less congested area. Under the assumption that driving is faster than walking, the vehicle can meet the delivery person at the determined reloading points. A solution to this problem specifies rendezvous locations for the vehicle and delivery person as well as the sets of customers that should be grouped together for delivery. These decisions provide a sequence of stops by the autonomous vehicle, and the solution is measured by the time required to complete all deliveries. We gain insight by restricting our analysis to a grid network and comparing the autonomous case to deliveries that require vehicle parking.

To benchmark the CAVADP, we introduce the capacitated delivery problem with parking (CDPP). Like the CAVADP, the CDPP serves a set of customers, but in doing so, the vehicle must be driven by the delivery person while in use. For the delivery person to serve a set of customers, the vehicle must be parked. After parking and delivering on foot, the delivery person returns to the same parked location to either reload packages and deliver to another set of customers or drive to a new location to find a new parking spot. Moving to a new parking spot comes at the cost of again finding parking. A solution to this problem specifies a sequence of parking locations for the vehicle. For each parking spot, the solution also indicates which customers should be grouped together for delivery from the given parking location.

The contributions of this work can be summarized as follows:

- We introduce the CAVADP, which captures a realistic way in which autonomous vehicles can be used to assist with package delivery in the near future.
- We introduce the CDPP, which models the traditional model for package delivery services and includes the time to search for parking.
- Based on the number of customers, driving speed of the vehicle, walking speed of the delivery person, and fixed time for loading packages, we characterize the optimal solution to the CAVADP on a solid rectangular grid of customers, showing the optimal solution can be found in polynomial time.
- Computational experiments demonstrate that the CAVADP reduces the completion time of the CDPP by 0%–33% if parking time is ignored. When nonzero times to find parking are considered, the delivery person saves 30%–77%, with higher values achieved for longer parking times, smaller capacities, and lower fixed time for loading packages.

In Section 2, we review the literature related to the CAVADP and CDPP. Section 3 details the assumptions, service times, and integer program (IP) for the CAVADP. We characterize the structure of the optimal solution for the CAVADP on a solid rectangular grid in Section 3.4. In Section 4, we introduce the CDPP as the benchmark to the CAVADP. The parameters used in experiments are discussed in Section 5. Section 6 presents our experimental results and insights regarding the relationship between the CAVADP and CDPP. In Section 7, we explore the financial considerations as well as positive and negative externalities for a company investing in this technology. Future work is discussed in Section 8.

2. Literature Review

In this section, we discuss literature related to the CAVADP and CDPP, particularly the literature related to vehicle and drone routing. The majority of the literature on vehicle and drone routing is algorithmic. There are limited results that characterize optimal solutions or provide tight bounds.

2.1. Vehicle Routing Problems

Vehicle routing problems find optimal routes for a vehicle to visit a set of customers. The general framework of the CAVADP and CDPP is finding a sequence between all customers, determining the reloading/parking stops in this sequence for the vehicle, and deciding which customers should be served on foot in between stops. Making all of these decisions simultaneously differentiates the CAVADP and CDPP from other vehicle routing problems. With these decisions being optimized in the CAVADP and CDPP, the size of the model increases rapidly.

The closest routing problem to the CDPP is explored by Nguyen et al. (2019). They use predefined sets of customers when optimizing package delivery in London with the consideration of parking. Their decisions are the order in which the sets are served and at which customer in the set the delivery person will park. The approach can be viewed as a cluster-first, route-second approach (Bodin and Golden 1981). They test their model on two different partitions of customers with the objective of minimizing a weighted sum of the driving and walking time while respecting time windows. One partition is based on an observational study and the other on geographical location of the customers. The results show increased savings with the partition based on geographical location. The CDPP is a generalization of this problem, as the partition of customers is not predetermined, and the solution decides which sets should be served and from which parking spots. Also, the CDPP allows the flexibility for the delivery person to serve more than one set of customers from the same parking spot.

The cluster-first, route-second approach is further explored in the clustered traveling salesman problem (CTSP). Clusters are predefined with the requirement of visiting the vertices of each cluster consecutively (Chisman 1975). Bao et al. (2017) provide a 1.9-approximation algorithm for when the starting and ending vertices of each cluster are free to be selected. Pop et al. (2018) extend the CTSP into the clustered vehicle routing problem by routing capacitated vehicles among predefined clusters. A two-level approach consisting of routing between the clusters and then within the clusters is implemented. In the CAVADP and CDPP, the sets served are not predefined, but a decision variable in the optimization problem.

Next, we consider routing problems that do not require the vehicle to visit every customer. In the CAVADP, the vehicle only visits the customers where the delivery person is dropped off or picked up. In the CDPP, the vehicle only visits the customers where the vehicle parks, and the delivery person loads packages. The customers visited by the vehicle form a tour starting and ending at the depot. In both problems, the delivery person serves the remaining customers on foot. Typically, the delivery person serves customers near the place at which he or she loaded packages from the vehicle. In this manner, the CAVADP has similarities to the close enough traveling salesman problem (CETSP). In the CETSP, the tour must travel within a fixed radius of each customer (Gulczynski et al. 2006). Applications of this problem include meter reading (Cerrone et al. 2017) and other wireless sensor networks (Behdani and Smith 2014).

Other variants that do not require a visit to every customer by the vehicle include the general routing problem and the orienteering problem. A general routing problem, such as the rural postman problem, requires a subset of arcs or nodes in the network to be visited in the solution (Orloff 1974), but the orienteering problem chooses the subset of nodes to visit to maximize a collected reward (Vansteenwegen et al. 2011). The tour formed in these required/selected nodes is similar to the tour of the vehicle in our problems. In contrast, in the CAVADP and CDPP, there must also be a route through the customers not visited by the vehicle. In the CDPP and CAVADP, the delivery person services all customers, and his or her route must coordinate with the route of the vehicle.

2.2. Drones

The existence of autonomous vehicles in the literature is most prevalent with the discussion of drones. The drone routing literature of interest here focuses on the pairing of vehicles and drones, introduced as the flying sidekick problem by Murray and Chu (2015). The decisions in the drone literature often include which customers are served on the vehicle route, who is served by the drone(s), and the synchronization between the routes of the vehicle and drone(s). In the CAVADP and CDPP, the delivery person has the role similar to the drone(s). Unlike many of the current problems proposed in the drone literature, however, the autonomous vehicle in the CAVADP does not deliver packages, and the delivery person must visit all customers. The capacity constraint for the drone is on energy, whereas the delivery person is constrained by the number of packages. In addition, the drone travels in Euclidean space faster than the vehicle. In the CAVADP, we assume that the vehicle is faster than the delivery person, and this feature of the problem allows us to characterize the optimal solution on a grid. This feature (meaning driving faster than walking) methodologically differentiates the CAVADP from the drone literature. Online Appendix D, Section D.4, demonstrates that the results may not hold if this assumption is relaxed. The flexibility of synchronization present in the drone literature is less applicable to the CDPP, as the delivery person must return back to the parking spot before serving the next set of customers. Otto et al. (2018) provide a survey of optimization approaches for aerial drones.

The closest drone problem to ours is the multivisit drone routing problem (MVDRP) introduced by

Poikonen and Golden (2020b). Online Appendix A provides a detailed comparison of the MVDRP and CAVADP. Similar to the CAVADP, in the problem by Poikonen and Golden (2020b), the vehicle does not make deliveries and acts as a mobile depot from which the drone leaves to deliver one or more packages before returning to the vehicle. The capacity constraint is on the fixed energy capacity of the drone, which is assumed to decrease based on the total weight of the packages and the direction of travel. The drone must recharge when it returns to the vehicle. Both the MVDRP and CAVADP allow for the set of customers to differ from the set of launch points for the drone (or delivery person). Poikonen and Golden (2020b) propose a heuristic to solve the problem when the launch points for the vehicle and drone are restricted to a discrete set of points. If the energy capacity is exchanged for a capacity on the number of packages, the CAVADP is similar to the MVDRP. When we restrict the customer geography to a solid rectangular grid using Manhattan distance, we are able to provide the optimal solution to the CAVADP. Poikonen and Golden (2020b) also consider multiple drones on the vehicle. We reserve the consideration of multiple delivery people on a single vehicle for future work.

Poikonen and Golden (2020a) extend the rendezvous locations of the vehicle and drone from a discrete set of points to the Euclidean plane in the mothership and drone routing problem (MDRP). The MDRP considers the mothership as a ship or airplane allowed to travel in Euclidean space, and the ship again does not visit customers (like the CAVADP). The drone visits every customer by departing from the mothership, visiting a single customer, and returning to the mothership. There is a limit on the number of time units that the drone can be separated from the mothership. Poikonen and Golden (2020a) implement a route-first, cluster-second methodology in their exact branch-and-bound algorithm to define a visiting order sequence to the customers and then consider all groupings of the sequence to determine optimal launching and landing points for the drone. They also extend the MDRP to the mothership and infinite capacity drone routing problem, which allows for the drone to visit multiple customers without capacity constraints on the drone. When testing capacities of 1 and 20 items, the authors use the exact branch-and-bound algorithm to solve up to 20 and 10 nodes, respectively.

Carlsson and Song (2017) demonstrate an improvement in efficiency by utilizing a drone with vehicle delivery. The drone's ability to travel in Euclidean space allows the authors to use a continuous approximation paradigm for their theoretical analysis. They account for the vehicle's restriction to the road network by performing numerical simulations with the vehicle traveling in Euclidean space at a reduced speed. In this paper, we maintain the restrictions of both the vehicle and the delivery person to the road network when developing an analytical solution to the CAVADP. Analogous to Carlsson and Song (2017), we seek to demonstrate savings from the coordination between a delivery person and autonomous vehicle with a computational approach to the parking problem in urban delivery.

Other problems in the drone literature allow the vehicle and drone to make deliveries, unlike in the CAVADP. The first example in the literature that considers the synchronization between a single vehicle and drone is by Murray and Chu (2015), with the flying-sidekick traveling salesman problem. The vehicle is able to serve all customers, and only a subset of customers is served by the drone. Throughout the tour, the drone is able to separate to make a single delivery. The vehicle is restricted to the road network but in the case of the drone, road restrictions are ignored. The authors explore heuristic solutions to the problem. Agatz et al. (2018) consider a similar problem called the traveling salesman problem with drones. They are able to solve to optimality for up to 12 customers with well-known integer programming solution techniques. Otherwise, Agatz et al. (2018) explore a route-first, cluster-second approach. They find a TSP tour that visits all customers and then assign the delivery mode by partitioning heuristics followed by improvement techniques. For the CAVADP, we present a route-first, cluster-second optimal solution for a grid of customers.

3. CAVADP

The CAVADP considers a set of n customers to be served by an autonomous vehicle assisted by a delivery person. The vehicle leaves and returns to the depot with the delivery person on board. The vehicle transports the delivery person at a speed of d blocks per minute as well as has the ability to remain in continuous use while the delivery person is delivering packages. Because the vehicle does not have to find parking, the delivery person will save at least the amount of time it takes to find parking, which is p minutes per parking spot. The delivery person has a carrying capacity servicing sets of up to q packages at one time. Servicing of each set requires the delivery person to be dropped off by the vehicle at a determined location, load the packages to then serve the specified set, and be picked up by the vehicle at a location that is potentially different from the drop-off location. The delivery person walks to serve each customer at a speed of w blocks per minute. The fixed time for loading packages, f, is independent of the number of customers being served in a set. We denote the possible sets of customers to be served as S and $m := |S|$. For every set $\sigma_j \in S$, we have $|\sigma_j| \le q$. The objective of the CAVADP is to minimize the completion time of the delivery tour. The parameters of the CAVADP are summarized in Table 1.

3.1. Assumptions

For the purposes of this analysis, we make the following assumptions in the model:

- Each customer being considered has a single delivery.
- Every customer set of size less than or equal to capacity q is considered.
- The vehicle is available to pick up and drop off the delivery person at all customers.
- The speed of driving d is faster than the speed of walking w. Therefore, the vehicle is able to meet the delivery person when needed.

3.2. Service Times

Let $D(i,k)$ denote the distance in blocks from customer i to customer k. We define d_{ik} to be the time to drive from customer i to customer k. Given the driving speed d, it follows $d_{ik} = D(i,k)/d$ for $i,k \in \{0,\dots,n\}$ such that $i \neq k$, and the depot is customer 0. Similarly, we define w_{ijk} to be the time to walk from customer i to the first customer to be served in set σ_j, walk between customers in set σ_j, and return to customer k. For a given set σ_j, define $v := |\sigma_j|$. For the triple (i,j,k), let $(j_1, j_2, \dots, j_v)$ be the optimal order to serve set σ_j when loading at i and returning to k. Note that customers i and k may or may not be in set σ_j. Notice that this problem is equivalent to finding the shortest path starting at i and ending at k through the points of σ_j. The time to walk between customers in set σ_j when loading at customer i and returning to customer k is

$$w_{ijk} = \frac{1}{w} \cdot \big(D(i,j_1) + D(j_1,j_2) + D(j_2,j_3) + \cdots + D(j_{v-1},j_v) + D(j_v,k)\big).$$

Table 2 summarizes the service times of the CAVADP.

3.3. Model

We define the binary decision variables x_{ik}, z_{ik}, and y_{ijk} as well as the integer valued variable u_i in Table 3. For each $i \in \{1,\dots,n\}$ and $j \in \{1,\dots,m\}$, the parameter $I_{ij} = 1$ if customer i is a member of set σ_j.

Table 1. Set of Parameters

Notation	Description
n	Number of customers
m	Number of customer sets
d	Speed of driving (blocks/minute)
w	Speed of walking (blocks/minute)
f	Fixed time for loading packages from vehicle (minutes)
q	Capacity of delivery person (number of packages)
p	Expected time to park (minutes)

Table 2. Definition of Service Times in the CAVADP

Notation	Description
d_{ik}	Time to drive from customer i to customer k (minutes)
w_{ijk}	Time to walk from i to set σ_j, walk between customers in set σ_j, and return to k (minutes)

The CAVADP can then be modeled with the following integer program:

$$\min \sum_{i=0}^{n} \sum_{\substack{k=0\\k\neq i}}^{n} x_{ik} d_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}(w_{ijk} + f) \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{n} x_{0i} = 1, \tag{2}$$

$$\sum_{i=1}^{n} x_{i0} = 1, \tag{3}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} I_{lj} y_{ijk} = 1 \quad \text{for each } l \in \{1,\dots,n\}, \tag{4}$$

$$\sum_{\substack{k=0\\k\neq i}}^{n} x_{ki} + \sum_{\substack{k=1\\k\neq i}}^{n} z_{ki} = \sum_{\substack{k=0\\k\neq i}}^{n} x_{ik} + \sum_{\substack{k=1\\k\neq i}}^{n} z_{ik} \quad \text{for each } i \in \{1,\dots,n\}, \tag{5}$$

$$y_{ijk} \le \sum_{\substack{l=0\\l\neq i}}^{n} x_{li} + \sum_{\substack{l=1\\l\neq i}}^{n} z_{li} \quad \text{for all } i,k \in \{1,\dots,n\}, \; j \in \{1,\dots,m\}, \tag{6}$$

$$z_{ik} = \sum_{j=1}^{m} y_{ijk} \quad \text{for all } i,k \in \{1,\dots,n\} \text{ such that } i \neq k, \tag{7}$$

$$u_0 = 0, \tag{8}$$

$$u_k \le \sum_{i=1}^{n}\sum_{\substack{l=1\\l\neq i}}^{n} (x_{il} + z_{il}) + 1 \quad \text{for all } k \in \{1,\dots,n\}, \tag{9}$$

$$u_i - u_k + 1 \le n(1 - x_{ik}) \quad \text{for all } i \in \{0,1,\dots,n\}, k \in \{1,\dots,n\} \text{ such that } i \neq k, \tag{10}$$

$$u_i - u_k + 1 \le n(1 - z_{ik}) \quad \text{for all } i,k \in \{1,\dots,n\} \text{ such that } i \neq k, \tag{11}$$

$$x_{ik} \in \{0,1\} \quad \text{for all } i,k \in \{0,1,\dots,n\} \text{ such that } i \neq k, \tag{12}$$

$$z_{ik} \in \{0,1\} \quad \text{for all } i,k \in \{1,\dots,n\} \text{ such that } i \neq k, \tag{13}$$

$$y_{ijk} \in \{0,1\} \quad \text{for all } i,k \in \{1,\dots,n\}, j \in \{1,\dots,m\}, \tag{14}$$

$$u_i \in \mathbb{Z}_{\ge 0} \quad \text{for all } i \in \{0,1,\dots,n\}. \tag{15}$$

Table 3. Set of Decision Variables in the CAVADP

Notation	Description
x_{ik}	$x_{ik} = 1$ if the vehicle drives from customer i to customer k with the delivery person on board for $i,k \in \{0,1,\ldots,n\}$ such that $i \neq k$
z_{ik}	$z_{ik} = 1$ if the vehicle drops the delivery person off at customer i and meets the delivery person at customer k for $i,k \in \{1,\ldots,n\}$ such that $i \neq k$
y_{ijk}	$y_{ijk} = 1$ if the delivery person loads at customer i, serves set σ_j, and meets the vehicle at customer k for $i,k \in \{1,\ldots,n\}$ and $j \in \{1,\ldots,m\}$
u_i	Gives the position of the vehicle visiting customer i in the tour given that the customer is visited for $i \in \{0,1,\ldots,n\}$

The objective function (1) minimizes the completion time of the delivery tour for the delivery person. The time spent driving when the delivery person is not on the vehicle does not need to be accounted for in the objective value. Under the assumption that driving from customer i to k is faster than loading at customer i, walking through some set σ_j, and returning to customer k, the driving associated with z_{ik} can be disregarded. Constraints (2) and (3) ensure that the delivery person is with the vehicle from and to the depot, respectively. Constraints (4) ensure that all customers are served. When the vehicle visits a customer, Constraints (5) verify that the vehicle will also leave that customer. Given that the delivery person loads at customer i, serves set σ_j, and returns to customer k (i.e., $y_{ijk} = 1$), we must verify that the vehicle will visit customer i to load there. Constraints (6) ensure that the vehicle drives from some $l \in \{0,\ldots,n\} \setminus \{i\}$ to customer i with the delivery person on board (i.e., $x_{li} = 1$) or without the delivery person on board (i.e., $z_{li} = 1$ if $l \neq 0$). In the case where the delivery person is not on board the autonomous vehicle (i.e., $z_{ik} = 1$), constraints (7) ensure that between the vehicle's visits to customers i and k, the delivery person is serving a set by loading at customer i and returning to the vehicle at customer k. The subtour elimination constraints are adapted from Miller-Tucker-Zemlin constraints and given in constraints (8), (9), (10), (11), and (15). Finally, the binary constraints on variables x_{ik}, z_{ik}, and y_{ijk} are given in constraints (12), (13), and (14), respectively.

3.4. Structure of the Optimal Solution on a Grid

In this paper, we aim to understand the value of autonomous vehicle assisted delivery in urban areas. To that end, in this section, we consider a rectangular grid of customers, a reasonable assumption for an urban area (Boeing 2020), and characterize the optimal solution of the CAVADP. To characterize the optimal solution on a grid, we create a solution that minimizes each component of the delivery tour. In addition to the assumptions of Section 3.1, we make the following assumptions:

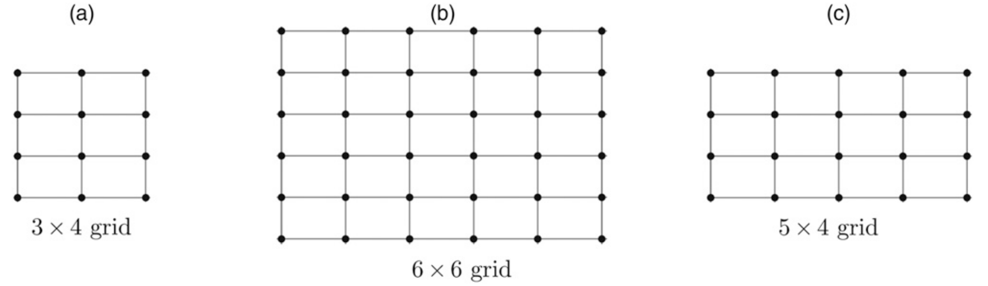
- There is a customer at every intersection of the grid (i.e., a solid rectangular grid), and the distance between every adjacent customer is one unit (representing one city block). Examples of solid rectangular grids of customers can be seen in Figure 1.
- There is drop-off/pickup capability at every intersection of the grid.
- The depot is located in the exterior of the customer grid. The results still hold when the depot is located on the interior, but it is more general to assume the depot is located on the exterior of an urban area.
- Distance is measured with the Manhattan metric.

We begin the analysis with the objective function given in Equation (1). The objective function can be decomposed as in Equation (16):

$$\underbrace{\sum_{i=1}^{n}\sum_{\substack{k=1\\ i\neq k}}^{n} x_{ik}d_{ik}}_{\text{(a)}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}w_{ijk}}_{\text{(b)}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}f}_{\text{(c)}} + \underbrace{\sum_{i=1}^{n} x_{0i}d_{0i} + \sum_{i=1}^{n} x_{i0}d_{i0}}_{\text{(d)}}. \tag{16}$$

In Equation (16), terms (a) and (b) describe the time spent driving and walking within the grid of customers, respectively; term (c) is the time spent loading packages; and term (d) describes the time spent

Figure 1. Examples of Solid Rectangular Grids of (a) 12, (b) 36, and (c) 20 Customers



driving on the exterior of the grid to and from the depot.

We now demonstrate how to minimize each term individually on the grid. We will later show that the sum of these individual minimizations is the optimal objective value for the problem on the grid. Without loss of generality, consider a $g \times h$ solid rectangular grid of customers with the bottom left corner at $(1,1)$, bottom right corner at $(g,1)$, top left corner at $(1,h)$, and top right corner at (g,h). Let (c_x, c_y) denote the coordinates of customer c, and let st denote the edge between adjacent customers s and t. We begin by identifying the closest customers to the depot. Traveling to these customers from the depot minimizes term (d) of Equation (16), but these customers also characterize the solution to minimize terms (a) and (b). Let c_1 be the closest customer (i.e., $D(c_1, 0) = MinDistance = \min_{c \in \{1,\ldots,n\}} D(c, 0)$), and let $\mathcal{C}_i$ be the set of ith closest customers to the depot (for $i = 2, 3$). We show that c_1 and the members of the sets $\mathcal{C}_2$ and $\mathcal{C}_3$ can be identified using Theorem 1. This and all other proofs not presented in this section can be found in Online Appendix B.

Theorem 1. *The closest customer $c_1 = (i_1, j_1)$ to the depot will be located on the boundary of the grid and is unique.*

The set of second closest customers to the depot $\mathcal{C}_2$ can include the following forms:

$$\mathcal{C}_2 \subset \{(i_1, j_1+1), (i_1, j_1-1), (i_1+1, j_1), (i_1-1, j_1)\}.$$

For all $c_2 \in \mathcal{C}_2$, $D(0, c_2) = MinDistance + 1$. If c_1 is located on the corner of the grid, $|\mathcal{C}_2| = 2$. Otherwise, $|\mathcal{C}_2| = 3$. In addition, there exists $c_2 \in \mathcal{C}_2$ such that c_1c_2 is a boundary edge.

The set of third closest customers to the depot $\mathcal{C}_3$ can include the following forms:

$$\begin{aligned}\mathcal{C}_3 \subset \{&(i_1-1, j_1+1), (i_1-1, j_1-1), (i_1+1, j_1-1),\\ &(i_1+1, j_1+1), (i_1-2, j_1), (i_1+2, j_1),\\ &(i_1, j_1-2), (i_1, j_1+2)\}\end{aligned}$$

For all $c_3 \in \mathcal{C}_3$, $D(0, c_3) = MinDistance + 2$. If c_1 is located on the corner of the grid, $1 \le |\mathcal{C}_3| \le 3$. Otherwise, $2 \le |\mathcal{C}_3| \le 5$.

To minimize terms (a) and (b) of Equation (16), we note that the minimum number of arcs that can be traversed is $n-1$. This minimum number is possible if there exists a Hamiltonian path through the grid. Itai et al. (1982) prove the existence of such a path given conditions on the starting and ending locations of the path. We will first consider the case when n is even in discussing the existence of the Hamiltonian path. Defining specific customers as the entrance point s and exit point t of the grid in traveling to and from the depot, Theorem 2 shows there exists a Hamiltonian path through the grid.

Theorem 2. *Assume n is even. For $c_2 \in \mathcal{C}_2$, there exists a Hamiltonian path from $s = c_1$ to $t = c_2$ through the solid rectangular grid.*

If the grid is of size $2 \times g$, $g \times 2$, $3 \times g$, or $g \times 3$ for some $g \in \mathbb{N}$, choose c_2 such that c_1c_2 is a boundary edge. Otherwise, choose any $c_2 \in \mathcal{C}_2$.

When n is odd, demonstrating the existence of a Hamiltonian path is more complicated. We must first characterize the vertices in the grid, a process Itai et al. (1982) refer to as coloring. The coloring of the graph allows us to define the starting and ending locations of the Hamiltonian path when n is odd. In this case, the grid is a bipartite graph, and Itai et al. (1982) provide the following coloring of vertex $v = (v_x, v_y)$:

$$\text{color}(v) = \begin{cases} \text{black} & \text{if } (v_x + v_y) \mod 2 = 0, \\ \text{white} & \text{otherwise.} \end{cases} \tag{17}$$

Note that the majority of vertices are colored black when n is odd. Theorem 3 defines the existence of the Hamiltonian path in this case.

Theorem 3. *Assume n is odd and the solid rectangular grid of customers is colored using Equation (17). Then,*

if c_1 is colored by the majority color, let $s := c_1$ and $t \in \mathcal{C}_3$;

if c_1 is not colored by the majority color, let $s, t \in \mathcal{C}_2$ such that $s \neq t$.

Then there exists a Hamiltonian path from s to t through the solid rectangular grid.

Table 4 summarizes the conditions necessary for the existence of a Hamiltonian path in the case of a solid rectangular grid and a depot on the exterior of the grid. When the depot is located on the interior of the grid, Umans and Lenhart (1997) show the existence of a Hamiltonian cycle in the solid rectangular grid.

The Hamiltonian path gives the route of the delivery person minimizing the number of arcs traveled. We now need to show which of the arcs are to be driven and which are to be walked. We show there exists a solution to the CAVADP along this path that minimizes the time spent by the delivery person within the grid. This time is represented by terms (a), (b), and (c) of Equation (16). Theorem 4 states how to optimally partition the customers to minimize the time the delivery person is within the grid.

Table 4. Existence of Hamiltonian Paths Between s and t Through the Grid in the CAVADP

n	Cases	s	t
Even	N/A	c_1	$c_2 \in \mathcal{C}_2$[a]
Odd	c_1 is the majority color	c_1	$c_3 \in \mathcal{C}_3$
	c_1 is not the majority color	$c_2 \in \mathcal{C}_2$	$c_2' \in \mathcal{C}_2$ such that $c_2' \neq c_2$

Note. N/A, Not applicable.
[a]If the grid is of size $2 \times g$, $g \times 2$, $3 \times g$, or $g \times 3$ for some $g \in \mathbb{N}$, choose c_2 such that c_1c_2 is a boundary edge.

Theorem 4. *Assume a Hamiltonian path through the solid rectangular grid. We minimize terms* (a), (b), *and* (c) *of Equation* (16) *by traversing the Hamiltonian path and serving*

- *n sets if $f \leq 1/w - 1/d$ and*
- *$\lceil n/q \rceil$ sets if $f \geq 1/w - 1/d$.*

Figure 2 provides an example of each case. The solid curved arrows indicate the delivery person being in the vehicle, whereas the dashed arrows indicate autonomous use of the vehicle. Each color indicates a set of customers to be served together. In case (a), where $f \leq 1/w - 1/d$, the vehicle drives the delivery person to all customers following the Hamiltonian path. Otherwise, in case (b), the delivery person walks to serve the $\lceil n/q \rceil$ sets on foot and is picked up by the vehicle along the Hamiltonian path. In this case, we can choose to serve $\lfloor n/q \rfloor$ sets of size q and, if needed, one set of size less than q.

We can now construct a solution of the CAVADP on a grid as follows. The algorithm is summarized in Algorithm 1. First, use Table 4 to determine the first customer s and last customer t that the vehicle will visit. Then, the delivery person follows the Hamiltonian path from customer s to customer t. The reloading points where the vehicle synchronizes with the delivery person are determined based on the values of f, w, and d (Theorem 4). The customers between these reloading points on the Hamiltonian path determine the sets of customers to be served. These results are extended to customers on a line in Online Appendix C.

Algorithm 1 (CAVADP on Solid Rectangular Grid).

1: **Input:**
2: Size of solid rectangular grid $g \times h$ with n customers
3: Location of the depot
4: Capacity of delivery person q
5: Driving speed of vehicle d
6: Walking speed of delivery person w
7: Fixed time for loading packages f
8: **Output:** Optimal Solution S
9: Find the closest customers to the depot.
10: Use Table 4 to determine the points of entrance s and exit t in the grid.
11: Find the Hamiltonian path from s to t through the grid.
12: Determine the structure of the optimal solution (whether $f \leq 1/w - 1/d$ or $f \geq 1/w - 1/d$).
13: Determine the reloading points and sets served for the delivery person along the Hamiltonian path based on the structure of the solution.

Theorem 5 establishes that the solution constructed in Algorithm 1 is an optimal solution to the CAVADP.

Theorem 5. *Construct a solution using Algorithm 1 and call the resulting solution S. Table 5 presents the value of solution S for each case, and these are the optimal solution values for the CAVADP.*

To define the complexity of Algorithm 1, Itai et al. (1982) provide an order $O(n)$ algorithm for Step 11 to find the Hamiltonian path in a solid rectangular grid. Similarly, when the depot is located on the interior of the grid, Umans and Lenhart (1997) provide an $O(n)$ algorithm to the Hamiltonian cycle problem. Step 9 requires at most $O(n)$ computations of distance relative to the depot. Steps 10 and 12 require a constant number of operations. Finally, Step 13 is a counting procedure to determine n or $\lceil \frac{n}{q} \rceil$ sets that follow the Hamiltonian path, and is completed in $O(n)$ operations. Thus, the cost of computing the optimal solution to the CAVADP is linear in the number of customers. Theorem 6 formalizes this result.

Figure 2. (Color online) Optimal Solutions for the CAVADP When (a) $f \leq 1/w - 1/d$ and (b) $f \geq 1/w - 1/d$ with Capacity $q = 4$ Packages

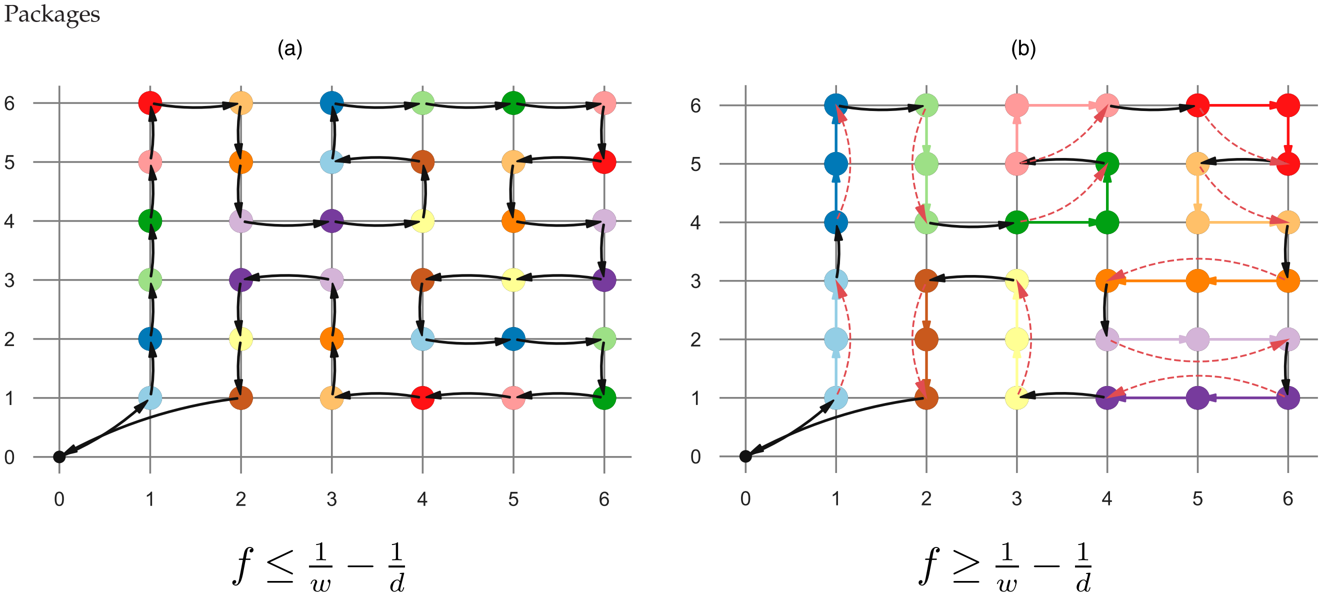


Table 5. Optimal Solution for the CAVADP on a Solid Rectangular Grid

n	d, w, f	Sets served	Optimal objective value
Even	$f \geq \frac{1}{w} - \frac{1}{d}$	$\lceil \frac{n}{q} \rceil sets$	$\frac{1}{d}(2 \cdot MinDistance + \lceil \frac{n}{q} \rceil) + \frac{1}{w}(n - \lceil \frac{n}{q} \rceil) + \lceil \frac{n}{q} \rceil \cdot f$
	$f \leq \frac{1}{w} - \frac{1}{d}$	n sets	$\frac{1}{d}(2 \cdot MinDistance + n) + nf$
Odd	$f \geq \frac{1}{w} - \frac{1}{d}$	$\lceil \frac{n}{q} \rceil$ sets	$\frac{1}{d}(2 \cdot MinDistance + \lceil \frac{n}{q} \rceil + 1) + \frac{1}{w}(n - \lceil \frac{n}{q} \rceil) + \lceil \frac{n}{q} \rceil \cdot f$
	$f \leq \frac{1}{w} - \frac{1}{d}$	n sets	$\frac{1}{d}(2 \cdot MinDistance + n + 1) + nf$

Theorem 6. *The solution algorithm of the CAVADP for a solid rectangular grids is* $O(n)$.

3.5. Extensions

We present multiple extensions to the CAVADP on a complete grid in Online Appendix D. In Online Appendix D, Section D.1, we characterize the optimal solution when the loading time depends on the number of packages being delivered in the service set. Online Appendix D, Section D.2, considers the CAVADP with pickups and deliveries. If we assume the fixed time for loading picked up packages is independent of the number of pickups and every customer has at most a single pickup, we characterize the optimal solution to the CAVADP with pickups and deliveries.

Then, we relax the assumptions regarding single package delivery, driving speed being faster than walking speed, and constant driving speeds to explore the implications of these assumptions on the solution of the CAVADP on a complete grid. Online Appendix D, Section D.3, considers multiple packages per customer. In Online Appendix D, Section D.3.1, we characterize the optimal solution to the CAVADP with multiple packages on a solid rectangular grid when $f \leq 1/w - 1/d$. In Online Appendix D, Section D.3.2, we consider higher loading times (i.e., $f \geq 1/w - 1/d$) and provide a counterexample to show that, in this case, the delivery person does not necessarily follow the Hamiltonian path in the optimal solution. Online Appendix D, Sections D.4 and D.5, explores the challenges of solving the CAVADP when walking speed is greater than driving speed and when driving speeds are heterogeneous, respectively. In both cases, we introduce counterexamples that show that the optimal solution characterized in this section may not hold under these two conditions.

4. Benchmark: CDPP

To understand the amount of time saved by the delivery person working with an autonomous vehicle, we introduce the benchmark problem CDPP. Like the CAVADP, the CDPP also considers a set of n customers to be served by a delivery person on foot, but this person must park his or her delivery vehicle before serving customers. The existing vehicle routing literature does not account for this need to park. The delivery person can serve multiple sets from the same parking location but must return to the vehicle to reload packages. The delivery person starts and ends his or her tour at the depot. The objective of the CDPP is also to minimize the completion time of the delivery tour. For details on the model formulation, see Online Appendix E.

The assumptions outlined in Section 3.1 for the CAVADP hold in the CDPP. We assume p is the time to find parking at a desired location. Any solution to the CDPP is a feasible solution to the CAVADP. Therefore, the optimal solution to the CDPP is an upper bound to the optimal solution of the CAVADP.

When solving the CDPP on a grid, we make the same assumptions outlined in Section 3.4 regarding the grid structure. The optimal solution of the CAVADP relies on the flexibility of the autonomous vehicle to meet the delivery person at an alternative location to the loading point. Then, the solution is able to minimize units traveled by the delivery person. In the CDPP, this solution is not feasible, as the delivery person must return to the parking spot. We cannot characterize the optimal solution to the CDPP or provide a lower bound to estimate savings.

To solve the CDPP, the integer programming model of Online Appendix E.2 is implemented in the Gurobi solver version 8.1 using the Python version 2.7.15 interface. The thread count is set to 32 threads. Because of the number of sets that must be considered, we add additional assumptions on the structure of the sets that we believe are reasonable for solutions to the CDPP on a solid rectangular grid. We make the following simplifying assumptions:

- Use contiguous sets of customers, that is, for every customer i in set σ_j such that $|\sigma_j| > 1$, there exists a customer $k \in \sigma_j$ such that $D(i,k) = 1$.
- Of the contiguous sets, eliminate sets covered by other sets. A set σ_j is covered by another set σ_l with respect to parking spot i if $\hat{w}_{ij} = \hat{w}_{il}$ and $\sigma_j \subset \sigma_l$. where $\hat{w}_{ij}$ is the time to walk from customer i to the first customer to be served in set σ_j, walk among customers in σ_j, and walk back to customer i where the vehicle is parked.

Still, some instances could not be solved to an optimality gap of 5.1%, and thus, in these cases, the IP was modified by further limiting the potential sets of customers to be served. In these cases, the sets available to the solution were restricted based on the shape of the individual set. The solution to this modified IP is a heuristic solution to the full problem. Thus, the heuristic provides an upper bound on the CDPP.

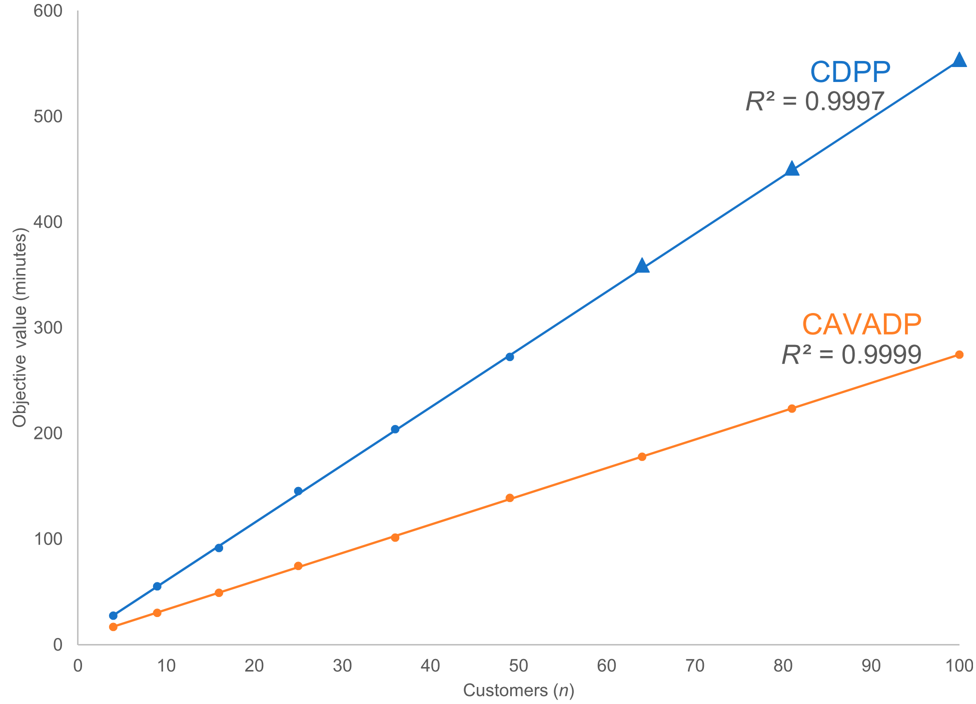
To verify the quality of the proposed heuristic, Figure 3 shows the potential existence of a linear relationship in the number of customers for the CDPP with parameter values of $p = 9$ minutes, $f = 2.8$ minutes, and $q = 3$ packages. This trend is consistent when varying the parameter values discussed in Section 5 and is further explored in a working paper on the CDPP (Reed et al. 2021). The heuristic solutions to the CDPP are represented as triangle markers. The fact that the solution values for the CDPP are on the linear trend formed by those solutions solved to optimality also gives us confidence in the quality of these heuristic solutions. As shown in Section 3.4, the optimal objective value of the CAVADP is linear in the number of customers.

5. Experimental Design

In this section, we discuss the parameters used to test the value of autonomous vehicles in a variety of urban environments. All of our experiments will be on square grids with the bottom left corner at $(1, 1)$ and the top right corner at $(\sqrt{n}, \sqrt{n})$ for $n \in \{4, 9, 16, \ldots, 100\}$. The depot is located on the exterior at $(0, 0)$. We set the length of the grid block to be 0.1 miles (Boeing 2020).

We set the parameters of d, w, and f based on discussion in the literature. Recall, d is the speed of driving (blocks per minute), w is the speed of walking (blocks per minute), and f is the fixed time for loading packages (minutes). According to Allen et al. (2018b), the average vehicle speed within the delivery area of London is 7 kph (ranging from 2.6 kph to 12.3 kph). Based on the vehicle speed and the length of a block, we assume $d = 0.8$ blocks per minute. According to Finnis and Walton (2008), the average walking speed is 80–95 meters per minute. Based on this walking speed and length of a block, we assume $w = 0.5$ blocks per minute. According to Allen et al. (2018b), the average time to deliver a package to a customer in London was 4.1 minutes, and the average walking distance per customer was 105 meters (0.065 miles). Assuming $w = 0.5$ blocks per minute, we estimate it to take approximately 1.3 minutes to walk to each customer. Removing the walking time of 1.3 minutes from the average delivery time of 4.1 minutes, we estimate the loading time to be 2.8 minutes. We use $f = 2.8$ minutes for our baseline. The structure of the optimal solution in the CAVADP differs when $f \le \frac{1}{w} - \frac{1}{d}$. Thus, we also experiment with $f = 0.5$ minutes to represent potentially smaller loading times.

Figure 3. (Color online) The Optimal Objective Value and Linear Trends for the CDPP and CAVADP When $p = 9$ Minutes, $q = 3$ Packages, and $f = 2.8$ Minutes



Recall that q is the capacity constraint on the delivery person (number of packages). According to Allen et al. (2018b), drivers deliver, on average, 118 items at 37 parking stops in London, which gives an average of 3 items per disembarkation. As a result, we will use a capacity of 3 packages for our baseline, but also consider capacities of 1 to 4 packages.

In the CDPP, we must also consider p representing the expected time to park (in minutes). We explore $p = 0$ to understand the best case for the CDPP. According to INRIX Research, in U.S. cities, the average time a driver takes to find a parking spot is 9 minutes, but search time ranges from 6 minutes in Detroit to 15 minutes in New York City (Cookson and Pishue 2017). Therefore, we also consider more realistic parking scenarios for urban environments by considering nonzero parking times ($p = 6$, 9, and 15 minutes).

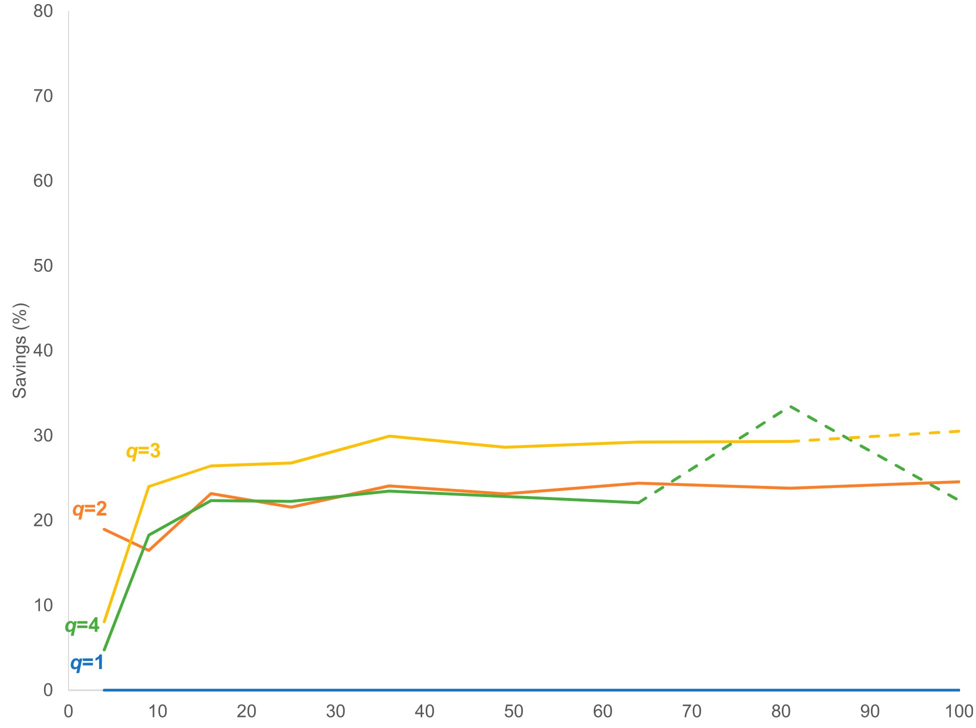
6. Experimental Comparison

In this section, we analyze the solutions to the CAVADP and CDPP to determine modeled savings from the use of autonomous vehicles in urban areas. Detailed results are provided in Online Appendix F. To make the fairest comparisons, the results compare solutions of the CAVADP and CDPP with the same parameter values. In Section 6.1, we ignore parking time to determine baseline savings from using an autonomous vehicle. Then, Section 6.2 considers nonzero times to find parking to model realistic parking scenarios in urban environments.

6.1. Ignoring Time to Find Parking, $p = 0$

In this section, we explore the $p = 0$ case. In response to emerging technologies, cities are considering infrastructure developments such as designated pickup and drop-off locations that may make $p = 0$ a feasible scenario (City of Boston 2019). Figure 4 shows the savings for the delivery person across all capacities when $p = 0$ and $f = 2.8$ minutes. Savings computed using heuristic solutions for the CDPP are represented as dashed lines here and in the graphs of the remaining sections. When $q = 1$ package, the optimal solutions to the CAVADP and CDPP are equivalent. Because of capacity constraints, the delivery person must serve every customer individually. Under the assumption that driving is faster than walking, in this case, the optimal solution to the CAVADP and CDPP reduces to finding the shortest path through all the customers. We show this path is achieved by following the Hamiltonian path in Table 4. The optimal solutions to the CAVADP and CDPP are also equivalent for greater capacities as long as the loading time f is sufficiently low so that the optimal solution is to drive to every customer in both problems. If $p \neq 0$, the CDPP is an upper bound on the CAVADP. Insight 1 summarizes these results.

Figure 4. (Color online) The Percentage of Savings for $q = 1, 2, 3$, and 4 Packages When $p = 0$ and $f = 2.8$ Minutes



Insight 1. *When $p = 0$, the optimal solutions to the CAVADP and CDPP are equivalent when $q = 1$ or f is sufficiently small that each customer is served individually. Otherwise, the optimal value of the CDPP is an upper bound on the optimal value of the CAVADP.*

When the loading time is sufficiently high that serving sets of greater than one customer is advantageous in reducing the completion time of the delivery tour, the CAVADP sees significant savings for $p = 0$ when $q > 1$. If the delivery person serves a set of greater than one customer in the CDPP, he or she must walk back to the vehicle. However, the autonomous vehicle can meet the delivery person at the end of service in the CAVADP, reducing the walking time. When $q = 2$ or 4 packages, Figure 4 shows savings of 22% on average. For $q = 4$, we see higher savings for $n = 81$ customers. We expect this observation is due to the heuristic solution, and if the CDPP could be solved to optimality, we would see a similar trend. Overall, savings increase to 26% for $q = 3$ packages. Insight 2 summarizes these results.

Insight 2. *Under certain capacity and loading conditions, even when there is no cost to parking, the CAVADP achieves significant savings.*

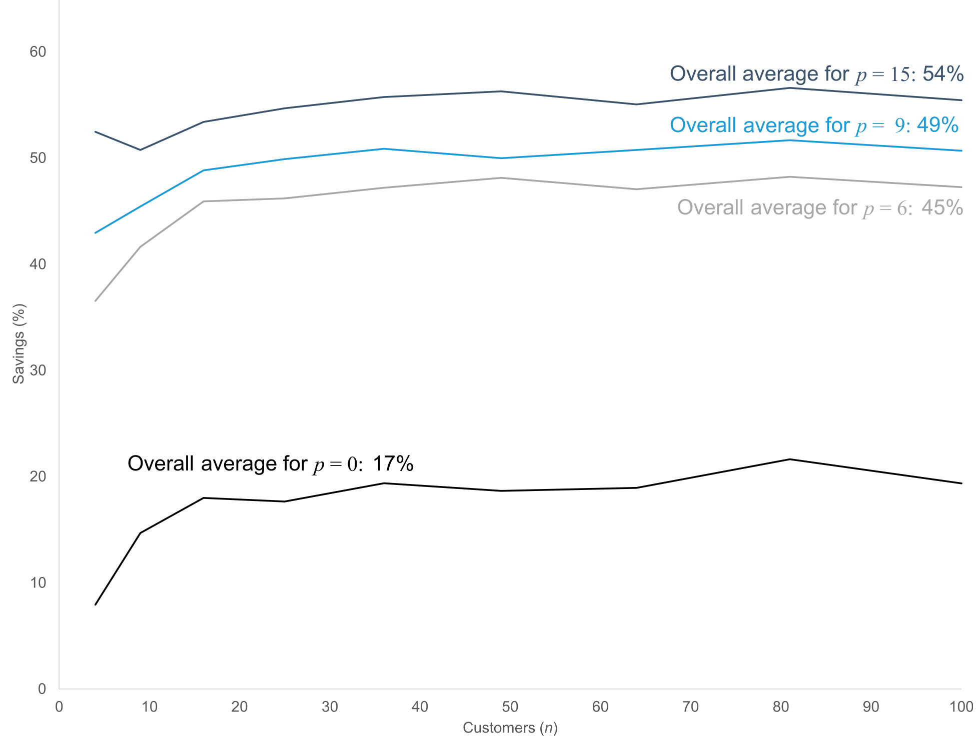
We want to understand why higher savings occur on average for $q = 3$. When $p = 0$, the delivery person can park in each service set at no cost. Due to Manhattan distance, when the delivery person services a contiguous set of size two or four while parking within the set, the delivery person walks 2 or 4 blocks, respectively, equating to 1 block per customer. However, when servicing three customers, the delivery person must walk at least 4 blocks, increasing the average walking distance to 1.3 blocks per customer. The delivery person may reduce loading time by servicing sets of three customers while simultaneously increasing the amount of walking per customer. This increased walking time is not realized in the CAVADP because the delivery person does not need to return to the parking spot. Therefore, the objective value of the CAVADP has a higher percentage decrease than the CDPP, resulting in higher savings for $q = 3$ packages.

6.2. Nonzero Times to Find Parking

In this section, we consider nonzero times to find parking. Figure 5 presents average savings for each parking time across all capacities and $f = 2.8$ minutes. On average, the level of savings is significantly greater than any optimality gap demonstrating realized model savings from the use of an autonomous vehicle. Insight 3 summarizes the results.

Insight 3. *When $p = 0$, a delivery person saves, on average, 17% of time in the delivery tour by utilizing autonomous vehicles. When nonzero parking times are considered,*

Figure 5. (Color online) The Average Savings of Time in the Delivery Tour from Use of an Autonomous Vehicle with Nonzero Parking Times ($p = 6$, 9, and 15 Minutes) and Ignoring Parking Time ($p = 0$) Across All Capacities and $f = 2.8$ Minutes



the delivery person saves, on average, 50% *of time in the delivery tour.*

To represent an average setting as described in Section 5, consider $f = 2.8$ minutes, $p = 9$ minutes, and $q = 3$ packages. Figure 6 illustrates the different solution structures of the CAVADP and CDPP on a six-by-six grid of customers. On the left is the optimal solution to the CAVADP, described in Section 3.4. In this solution, the delivery person enters the grid at the closest customer, exits the grid at a second closest customer, and serves $\lceil n/q \rceil = 12$ sets following the Hamiltonian path through the grid.

In the CDPP solution shown on the right, there is a different structure to the solution. Because of the parking search time of 9 minutes, the delivery person chooses to park the vehicle four times, at a total time of 36 minutes, and serve multiple sets from the same parking spot. The difference in the completion times of the tours is greater than the total parking search time of 36 minutes. Therefore, we observe that there are significant additional savings of using autonomous vehicles in addition to not having to park. As the delivery person serves multiple sets from the same parking spot, this additional savings includes the additional units needed to be walked to serve all sets. In the CAVADP, the number of units traveled is minimized by utilizing the Hamiltonian path.

6.2.1. Impact of Parking Time p. In this section, we consider the impact of time to find parking p on savings. For all p values, the savings decompose into the saved time from parking and the saved time in service (walking, driving, and loading packages) during the delivery tour. Figure 7 shows this decomposition for the solutions of $n = 25$ customers, $q = 4$ packages, and $f = 2.8$ minutes. The modeled savings from the CAVADP relative to the CDPP is much greater than the time to find parking across all parking times. In fact, we observe 22% savings for the delivery person when ignoring parking time (i.e., $p = 0$). These savings are realized by the autonomous vehicle eliminating the need for the delivery person to walk back to the parked vehicle. Furthermore, the CDPP solutions change with respect to parking time. The service time of the CDPP solutions when $p = 6$ and $p = 9$ are the same, and the vehicle parks two times in each case. When $p = 15$, the service time differs, as the vehicle parks only once at this high level of parking time. Thus, there exists a different underlying structure at these high parking times, resulting in even higher savings. Insight 4 summarizes this observation.

Insight 4. *The use of autonomous vehicles realizes additional savings aside from parking time because of different solution structures in the service of customers.*

Figure 8 demonstrates higher savings when the estimated time to find parking is high for $q = 3$ packages and $f = 2.8$ minutes. Recall that when $p = 0$, the delivery person saves, on average, 23% of time in the delivery tour by utilizing autonomous vehicles. For $p = 9$ minutes, we see savings of around 50%. When $p = 15$, savings increase to 53%. Then, at lower levels of parking time, such as $p = 6$, we see savings of 47%. There is similar behavior in savings for each value of q, which can be summarized by Insight 5.

Insight 5. *For every one minute increase in parking, there is, on average, a 1% increase in savings.*

Figure 9 shows an example of increasing and decreasing the time to find parking relative to the example of $p = 9$ minutes and $q = 3$ packages in Figure 6. In panel (a) of Figure 9, the vehicle parks six times when $p = 6$ minutes. When the time to find parking increases to $p = 15$ minutes as shown in part (b), the vehicle parks only three times, therefore, making the delivery person walk more to deliver packages. Insight 6 summarizes the changes in solution structure.

Figure 6. (Color online) Optimal Solutions for the (a) CAVADP and (b) CDPP Where $f > 1/w - 1/d$ with Capacity $q = 3$ Packages and Parking Search Time $p = 9$ Minutes

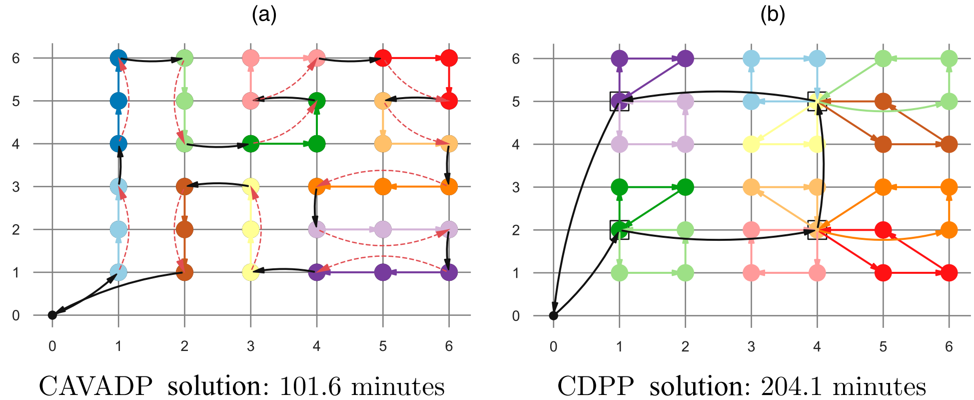
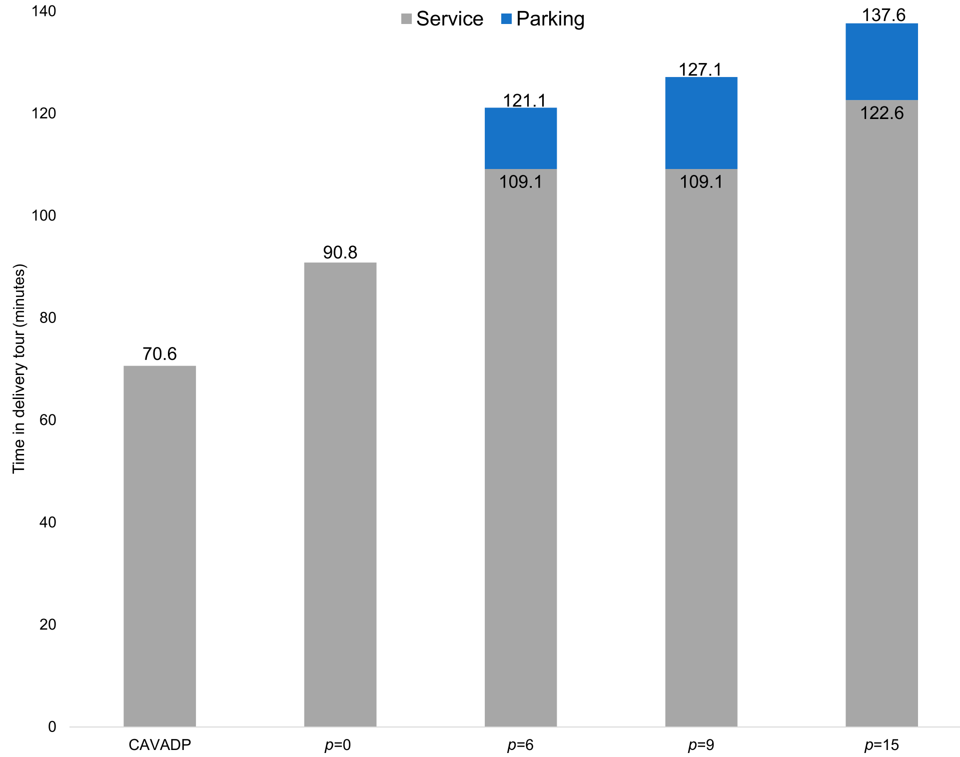


Figure 7. (Color online) Decomposition of CAVADP and CDPP Solutions into the Service Time and Parking Time for $p = 0, 6, 9$, and 15 Minutes for $n = 25$ Customers and $q = 4$ Packages



Insight 6. *As parking time increases, the vehicle parks fewer times in the optimal solution to the CDPP.*

Decreasing the number of times a vehicle parks will increase the amount of walking done by the delivery person. Thus, as parking time increases, either the amount of time to find parking increases or the number of units walked by the delivery person increases in the optimal solution. Because the autonomous solution

Figure 8. (Color online) The Percentage of Savings Across All Parking Times for $q = 3$ Packages and $f = 2.8$ Minutes

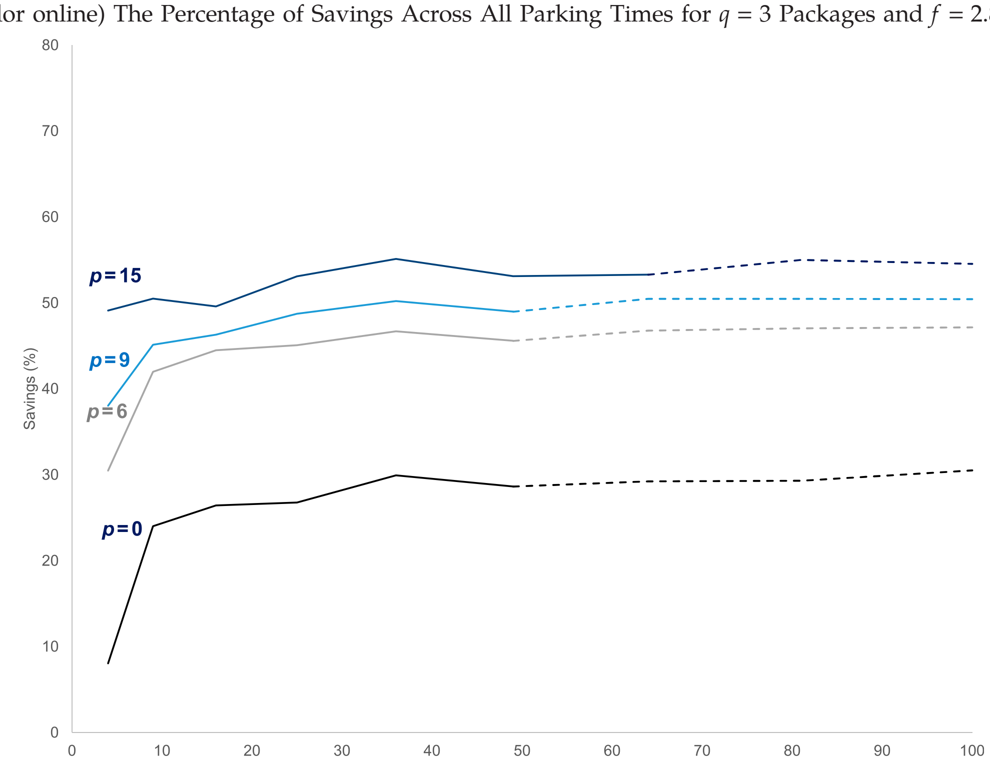
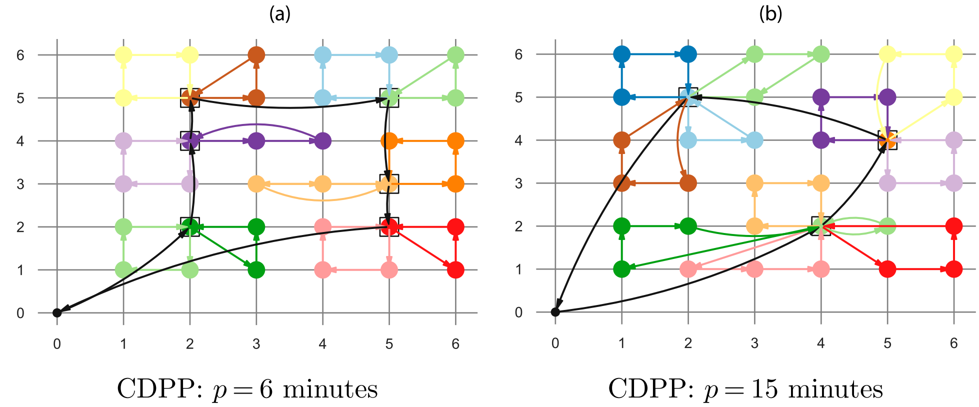


Figure 9. (Color online) Optimal Solutions for the CDPP Where $f = 2.8$ with Capacity $q = 3$ Packages and Parking Search Times of (a) 6 Minutes and (b) 15 Minutes



does not change with respect to parking time, the percentage of savings will increase as parking time increases.

6.2.2. Impact of Capacity *q*. In this section, we consider the effect of different capacities for the delivery person for nonzero parking times. Figure 10 demonstrates higher savings when capacities are small for $f = 2.8$ minutes and $p = 9$ minutes. This result differs from the conclusions of Figure 4 when $p = 0$. With nonzero parking times, the delivery person cannot park at any location without incurring the time to find parking. Parking in fewer locations to minimize time spent looking for parking results in the delivery person walking more to service customers, particularly with small capacities. When $q = 3$ packages, there are savings of 50%. At lower capacities of $q = 1$ and 2 packages, the savings hover around 54%. At a higher capacity of $q = 4$, savings decrease to around 46%. Similar behavior in savings is observed for other parking times.

Figure 10 shows that there are times when the percentage savings for $q = 2$ are higher than for $q = 1$. By Theorem 4, the optimal solution to the CAVADP is dependent on the capacity when f is large (i.e., $f > 1/w - 1/d$). Therefore, higher savings for $q = 2$ than $q = 1$ are observed when the marginal percentage change for the CAVADP is greater than the marginal percentage change of the CDPP. Figure 11 summarizes the average marginal percentage change in the objective value for the CAVADP and CDPP when increasing capacity q. When increasing from one to two packages, the objective values of the CDPP and CAVADP decrease by approximately the same percentage,

Figure 10. (Color online) The Percentage of Savings Across All Capacities for $p = 9$ Minutes and $f = 2.8$ Minutes

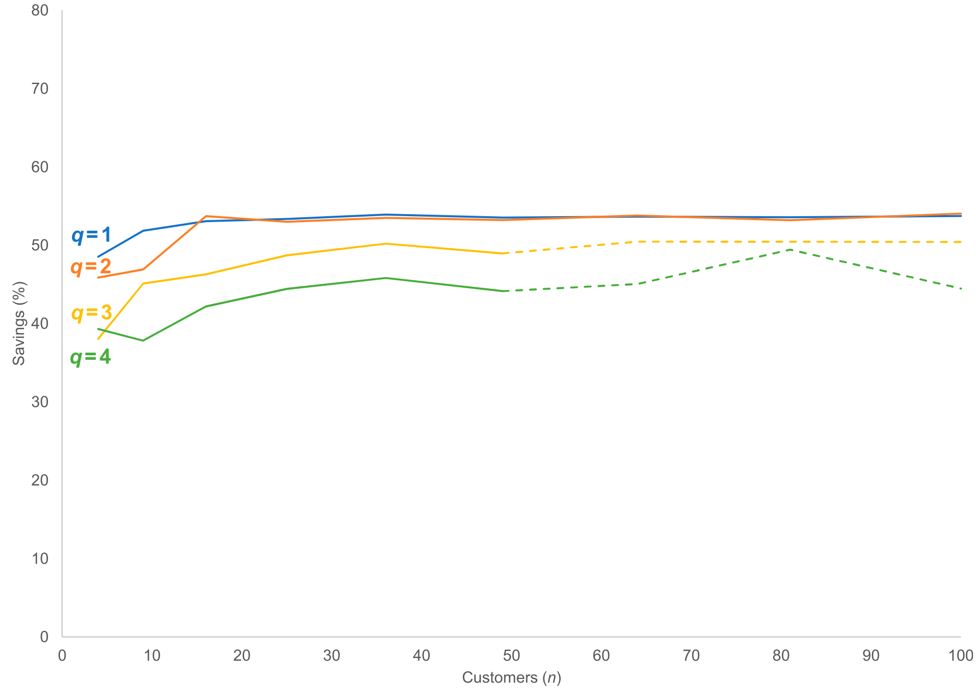
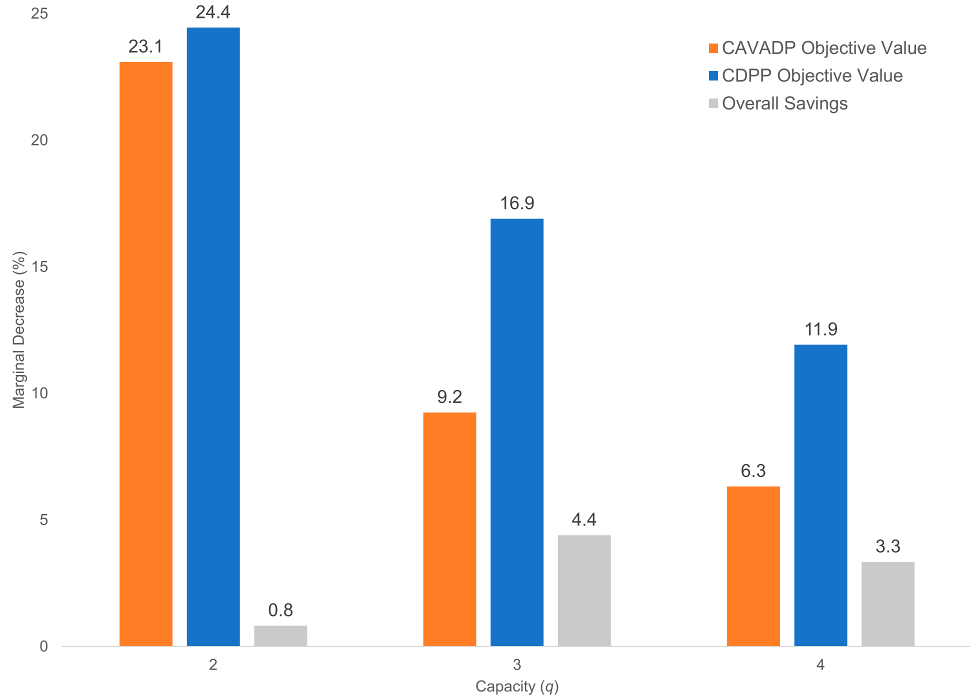


Figure 11. (Color online) The Average Marginal Decreases in Objective Value for the CAVADP and CDPP When Increasing to Capacity q at a Fixed Time for Loading of $f = 2.8$ Minutes



resulting in approximately the same level of savings between capacities. When increasing from $q = 2$ to 3, the objective value of the CDPP has a higher percentage decrease than the CAVADP. Thus, overall savings is expected to decrease. Similar behavior is seen when increasing to $q = 4$, however, the difference in the marginal changes is less than in the previous case, therefore resulting in a smaller impact on the change in overall savings. Insight 7 summarizes the effect of these marginal changes on overall savings.

Insight 7. *When $f = 2.8$ minutes, the savings decrease, on average,*

- 0.8% *from $q = 1$ to 2 packages,*
- 4.4% *from $q = 2$ to 3 packages,*
- 3.3% *from $q = 3$ to 4 packages.*

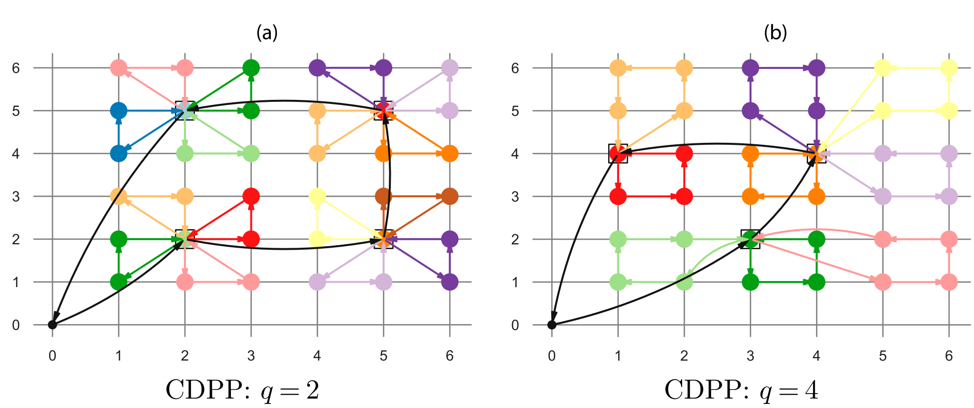
This trend is observed over all nonzero parking times.

We realize higher modeled savings when capacities are small. At small capacities, the delivery person must park more to avoid walking further to serve smaller sets of customers. Figure 12 shows examples of varying capacities with $n = 36$ customers, $f = 2.8$ minutes, and $p = 9$ minutes. We observe that as capacity increases, the optimal solution parks less and utilizes sets of full capacity. Because the CAVADP minimizes the number of units traveled, there will be higher savings when the delivery person has to walk more units in the CDPP, which occurs with smaller capacities.

6.2.3. Impact of Fixed Time for Loading Packages f.

Theorem 4 identifies that the optimal solution to the CAVADP is dependent on the capacity when f is large (i.e., $f > 1/w - 1/d$). In this section, we explore a lower

Figure 12. (Color online) Optimal Solutions for the CDPP Where $p = 9$ and $f = 2.8$ with Varying Capacities of (a) Two and (b) Four Packages



level of f (i.e., $f < 1/w - 1/d$) to understand how the change in the structure of the optimal solution to the CAVADP impacts savings. Now, we consider $f = 0.5$ minutes as this value of f satisfies $f < 1/w - 1/d$. Theorem 4 concludes that n service sets of size 1 are served in the optimal solution to the CAVADP. The delivery person is on board the vehicle for all units traveled and serves each customer individually with a low time for loading packages. Figure 13 shows that across all nonzero parking times and capacities, there are 12% higher savings on average when $f = 0.5$ than when $f = 2.8$ minutes. Online Appendix G further explores the effects of parking time and capacity constraints when $f = 0.5$ minutes. When $q = 1$, we observe around 18% higher savings at $f = 0.5$ relative to $f = 2.8$ at all nonzero parking times. Similarly, we see about 13% higher savings for $q = 2$, 11% higher savings for $q = 3$, and 10% higher savings for $q = 4$ at all nonzero parking times. Insight 8 summarizes this observation.

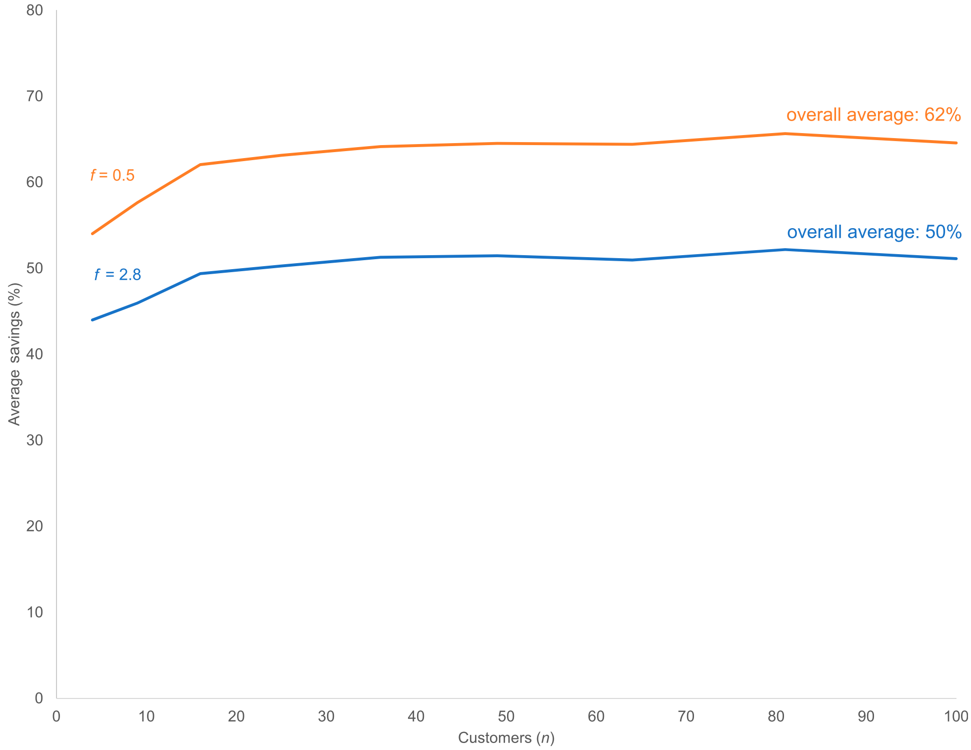
Insight 8. *There are higher savings when $f \leq 1/w - 1/d$. The difference in savings is convex in the capacity.*

7. Managerial Implications

The results show significant potential savings from pairing a delivery person with an autonomous vehicle. When considering implementation, a company must consider the trade-offs between the investment in this technology and the increased productivity of the delivery person. There are also externalities such as additional driving time for the autonomous vehicle and the availability of parking spaces for public use. In this section, we consider a specific example to gain insight into the trade-offs and externalities and to demonstrate how our results can be used to help companies explore the implications of using autonomous-assisted delivery in their operations.

Consider an example with the service of 360 customers in an urban area and vehicle capacity of 36 packages. In this example, we assume a limiting vehicle capacity of 36 packages to remain consistent with the analysis in Section 6 and show the advantages of completing more trips per day. Recall Figure 3 shows that savings grow linearly in the number of customers. In this analysis, we generalize this result to larger customer service regions and vehicle capacities. To service all customers with this vehicle capacity, the company must design 10 routes that divide total service into 10 six-by-six grids. We will consider our average urban setting parameters ($q = 3$ packages, $p = 9$ minutes, and $f = 2.8$ minutes). Figure 6 shows that the delivery person for the CDPP delivers packages to 36 customers in 3.4 hours. The completion time of the delivery tour reduces to 1.7 hours in the CAVADP. In a work day, we assume the delivery person can complete two routes in the CDPP and four routes in the CAVADP. If the return trip to the depot becomes a significant distance, this analysis may need

Figure 13. (Color online) The Average Savings from the Use of Autonomous Vehicles for $f = 0.5$ and 2.8 Minutes Across All Nonzero Parking Times and Capacities



to be adjusted to account for the return trips to the depot in both the CDPP and CAVADP. A personal conversation with a principal scientist at one major package express carrier revealed that vehicle capacity is often not a limiting factor. During the holiday season, delivery vehicles often carry twice as many packages as during normal delivery periods. Therefore, in some companies, additional productivity gains by the delivery person may be realized by servicing more customers per route.

Table 6 compares fleet analysis for various models to service these 360 customers with our average urban setting parameters. First, we focus on the second and third columns of Table 6 to compare fleets of traditional and autonomous vehicles, respectively. Because of the increased productivity of the delivery person in the CAVADP, the completion time of the individual routes reduces by 50%. A principal scientist at a major package express carrier sees advantages in shorter-in-time routes to reduce the length of the individual driver's work day, leading to reduced driver turnover. This productivity also reduces the number of employees by 40% reducing overall wage costs. Let h_d be the hourly wage for the delivery driver. The U.S. Bureau of Labor Statistics (2019) reports $h_d = \$26.01$ as the hourly mean wage for a light truck driver in the Couriers and Express Delivery Services class, so we use that value in our analysis. The wage for the CAVADP may be further reduced if the delivery person no longer needs a commercial driver's license. We also see that the use of autonomous vehicles reduces the fleet size by 40% (five vehicles down to three). In the worst case, we assume the autonomous vehicle drives continuously between pickup and drop-off locations, as opposed to finding parking elsewhere. Under this assumption, we observe that the fleet of autonomous vehicles spends 1.5 additional hours driving, which equates to an additional 0.3 minutes of driving per customer. As vehicles electrify, the cost of the extra driven miles will be reduced, as will concerns about emissions from these miles (U.S. Department of Energy 2020). This increased driving also means that the delivery person walks 3.1 fewer minutes per customer, potentially reducing the risks of injury and related costs.

The societal cost of autonomous vehicles includes the 15% increase in driving time per route. This increase in driving time may result in additional maintenance costs and depreciation in vehicles for the company. However, there are fewer vehicles needed to provide the same volume of services. In addition, the autonomous vehicles do not necessarily need to be parked while the delivery person is in service, opening up 18.7 hours of parking in this urban area. This reduction in parking may free up space for other drivers or reduce congestion by decreasing the time vehicles spend double parked or looking for parking.

In this paper, we focus on the use of an autonomous vehicle to demonstrate the potential of an emerging technology, but the results may also be achieved by using a team of a driver and a delivery person. The driver would simply drop off and pick up the delivery person and not need to park the vehicle during service. This model is similar to current practices at UPS, where temporary workers are hired to assist drivers on the road during peak delivery times (Nuggehalli 2019). The fourth column of Table 6 reflects this two person model. Adding a delivery driver to the CAVADP model increases the total wages while reducing route duration and fleet size. Let h_p be the hourly wage of the delivery person. The U.S. Bureau of Labor Statistics (2019) reports the hourly mean wage for a courier in the Couriers and Express Delivery Services class is \$15.50. If we focus only on driver wages, the two-person model is advantageous to the CDPP model if the time savings is greater than $h_p/(h_d + h_p) = 37\%$. In Section 6, we show these savings are achieved for all realistically sized instances when $p = 6, 9$, and 15 minutes.

Table 6. Fleet Analysis for a Company Servicing 360 Customers with a Vehicle Capacity of 36 Packages in the Average Urban Setting

	CDPP (each traditional vehicle has one driver)	CAVADP (each autonomous vehicle has one delivery person)	CAVADP (each traditional vehicle has one driver and one delivery person)
Route duration (hours)	34	17	17
Employees	5 delivery drivers	3 delivery people	3 delivery drivers 3 delivery people
Total wages (\$)	884.34	442.17	705.67
Fleet size (vehicles)	5	3	3
Vehicles driving (hours)	9.8	11.3	11.3
Vehicles parked (hours)	18.7	0	0
Loading time (hours)	5.6	5.6	5.6
Walking time (hours)	18.7	8	8

8. Conclusions and Future Work

In this paper, the model of the CAVADP was simplified to understand the potential value of autonomous vehicles in urban delivery. The CDPP was introduced as a benchmark to account for the need to park in today's model of delivery. Overall, we conclude high levels of savings at 30%–77% dependent on the values of time to find parking, capacity of the delivery person, and fixed time for loading packages. High estimated times to find parking and low capacities result in high overall completion time of the delivery tour for the CDPP. If the delivery person parks few times because of the high parking time, then he or she must walk more units when restricted to smaller capacities. On the other hand, if the delivery person parks more in hopes of minimizing units walked, the objective value becomes high because of the high parking time. At low loading times (i.e., $f < 1/w - 1/d$), the CAVADP has a solution of driving every unit traveled and serving one package at a time. By minimizing the number of units traveled and driving every unit, the CAVADP produces an optimal solution with a low completion time of the delivery tour. In summary, these factors cause a high completion time for the CDPP but a low completion time for the CAVADP resulting in high savings for the delivery person.

With this level of savings, the delivery person has significantly higher levels of productivity, showing the potential benefit from investing in this technology. This increased productivity can also help decrease the number of vehicles needed in urban environments, offsetting any increase in time on the road due to the autonomous driving. We may also consider having multiple delivery people in the autonomous vehicle with the opportunity to further eliminate vehicle use in urban areas. Managing overall interactions of a fleet of autonomous vehicles and delivery persons may offer increased savings over the single vehicle analysis in this paper. This analysis could include additional externalized costs, such as traffic congestion, fuel consumption, refueling time, and overall emissions.

In the simplified model of the grid, we allow the vehicle access to pick up the delivery person or park at any customer location. As autonomous vehicles become more prevalent in urban environments, city infrastructure may change to adapt to the new technology. For example, Barcelona has created superblocks that only allow local residents as well as public and delivery vehicles at a limited speed in an effort to reduce traffic congestion (Bausells 2016). The current model allows for restrictions on the vehicle's stopping locations. In future work, we look to find results on a general geography of customers to understand the value of autonomous vehicles in various settings.

This work also provides a foundation for future exploration of operationalizing autonomous vehicle delivery. The results in Online Appendix D, Sections D.4 and D.5, show that future work is needed to extend to the case where walking time can be faster than driving time or when there are heterogeneous driving speeds. Implementation may require the relaxation of key assumptions that allowed for theoretical insights into the solution. We are not able to operationalize until we know policies about how autonomous vehicles will be managed. Simulations with real-world data may provide additional insight into necessary modifications of city infrastructure to achieve these high levels of savings. Similarly, simulations with different use policies for autonomous vehicles may also provide additional insights. We may also consider stochastic elements, such as traffic patterns and the need to interact with the customer for delivery operations. In addition, the value of autonomous vehicles may be enhanced in combination with other innovations in last-mile delivery, such as on-foot porters, cycling couriers, drones, or delivery robots (Carlsson and Song 2017, Allen et al. 2018a, Boysen et al. 2018, McLeod et al. 2020).

Finally, the CDPP is also rich in potential for future research. The existing vehicle routing literature does not account for the need to park in current delivery practices. Advancing the solution methods for the CDPP will create a more realistic model of existing delivery operations.

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