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2019
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Summary Sheet

Plan of a New Energy Compact by Energy Profiles: Arizona, California, New Mexico, and Texas

Summary

A new energy compact of Arizona, California, New Mexico, and Texas is coming into being, devoted to improving clean energy usage.

With data from State Energy Data System, a static structure of energy usage in each state is constructed. This consists of pie charts showing energy consumption ratio of each source, and a column chart to compare energy consumed by the end-use sectors and the electric power sector. Four abstract indicators: energy consumption per capita, clean energy usage ratio, total consumption of all petroleum products, and the ratio of clean electricity generated, are then extracted from the structural variables, and together they constitute the energy profile.

By plotting the evolution of elements in energy profile over time, time series of all four indicators are derived. In addition, two more indicators: change rate of clean energy ratio and that of energy consumption per capita, are acquired. Indicators are then used to assess the energy profile. Weights of indicators are generated through *matrix-modified entropy method*. The weighted average of the scores for six indicators stands for the superiority in clean energy usage, and Arizona has the "best" energy profile by this criteria.

It turned out that energy consumption from clean energy matters most in the assessment of energy profile, so we focused on it in the following steps. By two approaches, *gray model* and *BP neural network*, predictions on the energy consumption from each clean energy source in each state are made respectively. Results show that the total clean energy consumption reaches 2.19 and 2.45 Million Billion Btu up to 2025 and 2050 respectively, and all four states' clean energy usage proportion level off to 10%.

To set the target of clean energy usage in 2025 and 2050, we first come up with an evaluation index defined as $I = \alpha \cdot \left(\sum_{i=1}^{4} \sum_{j=1}^{6} \eta_{ij}\sigma_{ij}\right)$, where α is the *flare factor*; η_{ij} is *utilization efficiency*; and σ_{ij} is the consumption quantity of the jth energy by the ith state bounded in a certain range. To optimize the index, we can either promote development of effeciency η_{ij}, or obtain the best σ_{ij} distribution through linear programming.

According to the approaches above, suggestions such as establishment of platforms for technology communication, and an energy deployment system are made to help the compact reach the optimum.

Keywords: clean energy; entropy method; time series forecast; linear programming

To: Governors of Arizona, California, New Mexico, and Texas
From: Team # 85434
Date: February 12, 2018
Subject: Prospect of New Energy Compact: Arizona, California, New Mexico, and Texas

State Profiles as of 2009

- Fossil fuels still dominate energy consumption. Clean energy usage is still not so advanced as to outdo traditional fossil fuels in all four states.
- States in descending order of energy consumption per capita are Texas, New Mexico, Arizona, California.
- States in increasing order of share of energy consumption from clean energy are Arizona, California, Texas, New Mexico.

Predictions Regarding Energy Usage Absent Policy Changes

- The share of energy consumption from clean energy of the four states tend to all converge towards 10% after year 2025, meaning that the four states will obtain an energy structure of 10% clean energy sources with respective to total energy exploitation.
- The four states may not make progress in the energy structure, if no interstate energy compact is formed and no change of policy is made.

Recommended Goals for the Energy Compact

- Improve the utilization efficiency of clean energy sources in the whole compact. An interstate public renewable energy and nuclear electricity database can be built, meanwhile a Border-State Clean Energy Technology Cooperation Board can be formed to accelerate communication of clean energy exploitation and utilization technologies among the four border states. They also make it possible for the four states to simultaneously focus on cutting-edge challenges in the field of energy.
- Increase subsidies for nuclear power industry, and hold back the stagnancy that the nuclear market in America is going through in recent years. This maintains the nonnegligible contribution of nuclear energy to overall clean energy usage, preventing it from going down far before it reaches the optimal quantity.
- For clean energy sources which are overconsumed now (exceeding optimal value) by one state and are under-utilized (below optimal value) by another state, establish a pairwise energy deployment system between states.

CONTENTS

1 Introduction

1.1 Background

Energy is what sustains human production and life. Burning fossil fuels, which are traditional energy sources, contributes to environmental issues, including global warming, acid rain, and haze [11]. Also, fossil fuels are said to run out in approximately 70 years [4]. Thus, increased exploit of clean, renewable energy sources are urged.

In the United States, energy production and consumption can be disparate even in adjacent states, due to geography, population, and other influential factors. It is reasonable that much authority to formulate energy policy is delegated to state governments in the United States. However, instead of managing energy sources alone, it might be a better choice to form an energy compact with other states.

1.2 Statement of the Problem

The four states, Arizona (AZ), California (CA), New Mexico (NM), and Texas (TX), adjacent but hold different geographic and demographic conditions, seek to increase usage of clean, renewable energy by forming an energy compact.

Data describing energy usage of the four states from 1960 to 2009 is retrieved from [12]. Energy profiles of each state are created to characterize energy production, consumption and flow. Furthermore, the evolution of the energy profile of each state is modeled to analyze the trend in clean energy usage. Comparing the trend of the four states, similarities and differences can be seen. Then, with uniform criteria for assessing energy profiles established, the four states are ranked in clean energy usage condition, and thus the "best" energy profile is determined. On the basis of the model of energy profile evolution, the energy profile of each state in 2025 and 2050 is predicted respectively assuming no policy changes, in other words, if the four states do not form the compact.

Again based on the criteria for "best" profile, together with the predictions before, clean energy usage targets for 2025 and 2050 is set and regarded as goals for the new four-state energy compact. Accordingly, suggestions about the execution of the energy compact is offered to the governors.

2 Preliminaries

2.1 Classification of Energy Sources and Codes

We examined the energy sources covered in the dataset, and selected the primary energy sources listed in Table 1, classified according to generally accepted principles [3]. In this paper, we focused on clean energy sources, consisting of nuclear power and renewable energy sources.

However, little data about crude oil had been provided, but consumption of it cannot be overlooked, so we used the total production of all petroleum products, mainly secondary energy sources, to estimate the total consumption of crude oil. Petroleum products and codes representing them are listed in Table 2.

Table 1: Classification of Primary Energy Sources and Corresponding Codes

Energy Source	Code	Clean	Renewable
Coal	CL	✗	✗
Crude Oil	CO	✗	✗
Natural Gas	NG	✗	✗
Nuclear Electric Power	NU	✔	✗
Hydroelectric Power	HY	✔	✔
Wood and Waste	WW	✔	✔
Geothermal	GE	✔	✔
Solar Thermal Energy	SO	✔	✔
Wind	WY	✔	✔

Table 2: Petroleum Products and Corresponding Codes

Petroleum Product	Code
Distillate Fuel Oil	DF
Jet Fuel	JF
LPG	LG
Motor Gasoline	MG
Residual Fuel Oil	RF
Asphalt and Road Oil	AR
Aviation Gasoline	AV
Kerosene	KS
Lubricants	LU
Other Petroleum Products	PO

2.2 Assumptions

To carry out the modeling steps in the rest of this paper, we made the following assumptions:

- The total production of all petroleum products is a good estimate of the total consumption of crude oil. As in subsection 2.1, we used the total production of all petroleum products to estimate the total consumption of crude oil, ignoring the yield rate.
- Ignore high-effect, low-probability incidents when predicting, like destructive disasters, political unrest, and so on.
- Ignore the consumption of energy sources not listed in subsection 2.3, which is actually too low to affect the models.

2.3 Static Structure

Before dealing with dynamic evolutionary models, we start with abstracting the static energy structure for a state in one year. The variables characterized and examined are listed in Table 3. The relationships between them are shown in Figure 1.

Table 3: Definitions of Variables

Symbol	Description
p	Total production of primary energy
s	Total consumption of all petroleum products
σ_{nc}	Total consumption of non-clean energy
σ_c	Total consumption of clean energy
E	Total production of electricity

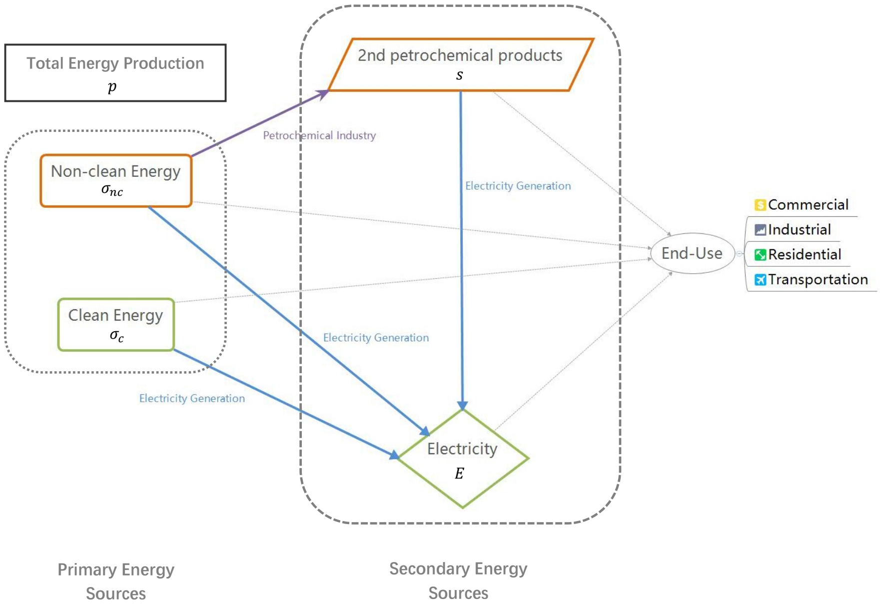


Figure 1: Relationship between the Variables in Table 3

3 Energy Profiles and Evolution

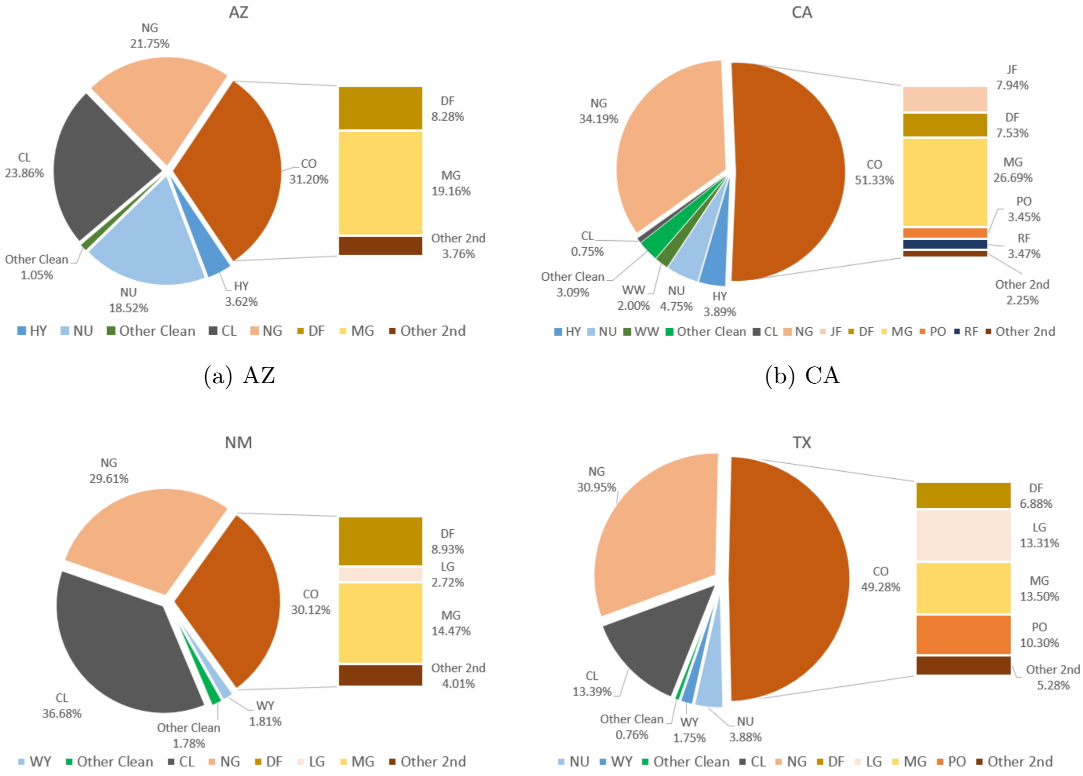
3.1 Energy Profiles for Each State

Concrete values of variables in the static structure (Table 3) at year 2009 are listed in Table 4.

Table 4: Other Variables Characterizing Energy Profiles by State (Unit: Billion Btu)

Variable	p	s	σ_{nc}	σ_c	E
AZ	570994.046	540273.348	1350820.172	103493.285	250553.154
CA	2605311.838	3590000.286	7292810.591	712704.460	885699.322
NM	2412219.049	251452.547	634459.123	35635.384	73860.028
TX	11914996.720	5512412.546	10940775.769	356634.821	1178148.454

Based on the variables in Table 3, we extracted several significant indicators. Indicator values in 2009 are provided below for instance. In Figure 2 are pie charts characterizing the share of consumption from different energy sources, where share of consumption from crude oil is shown as the aggregate of share of petroleum products. Energy consumed by the end-use sectors and the electric power sector respectively in each state are shown in Figure 3. Structural variables, together with indicators, made up a comprehensive profile of energy usage.



(a) AZ (b) CA

(c) NM (d) TX

Figure 2: Share of Energy Consumption by Energy Resource (2009)

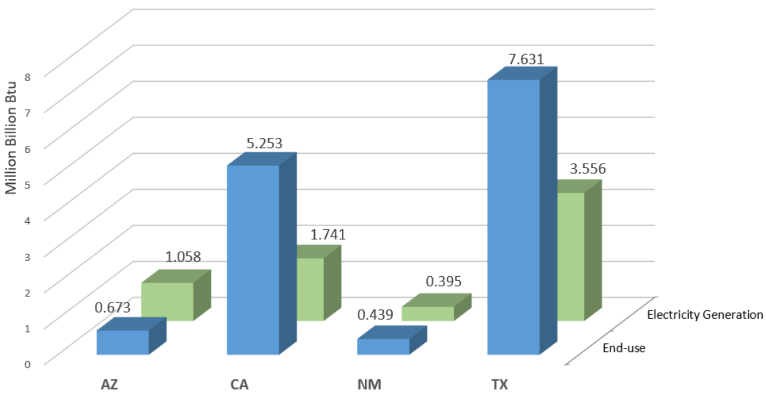


Figure 3: Energy Consumed by the End-use Sectors and the Electric Power Sector (2009)

3.2 Energy Profile Evolution

Indicators and their variation are plotted below to indicate the evolution of the energy profiles of each state: energy consumption per capita (Figure 4), energy outflow per capita (Figure 5), share of energy consumption from each clean energy source and all clean energy sources (Figure 6).

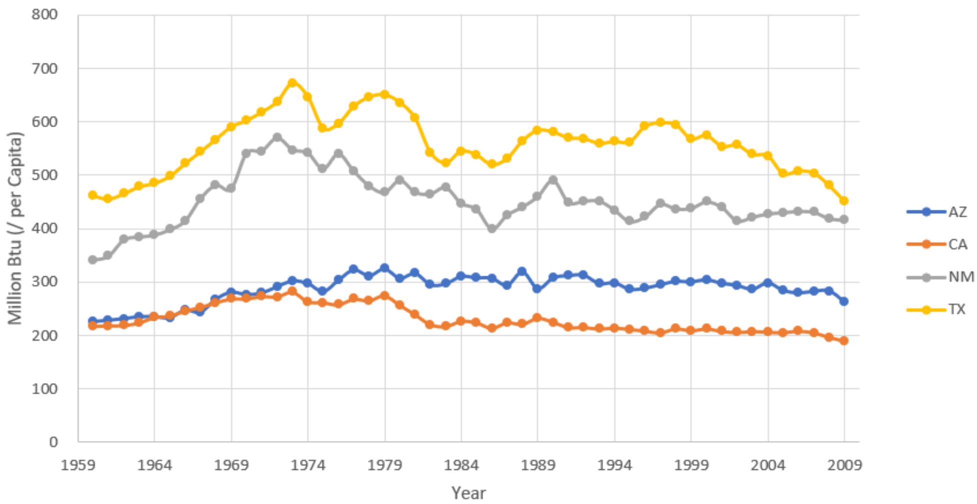
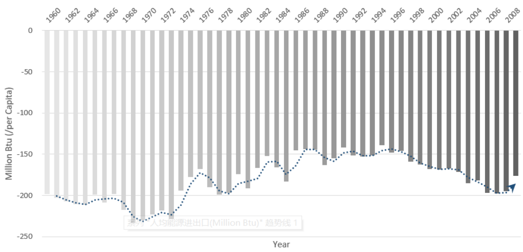
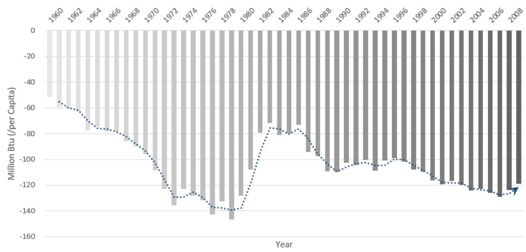


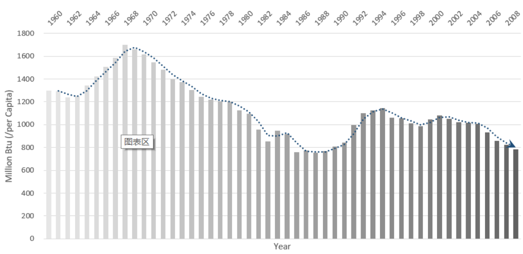
Figure 4: Time Series of Energy Consumption per capita by State



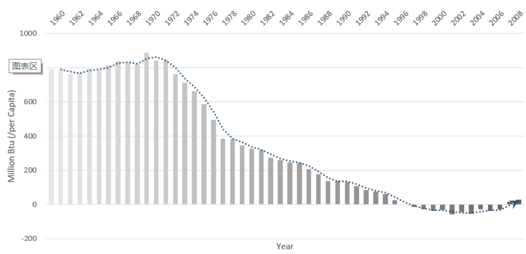
(a) AZ



(b) CA

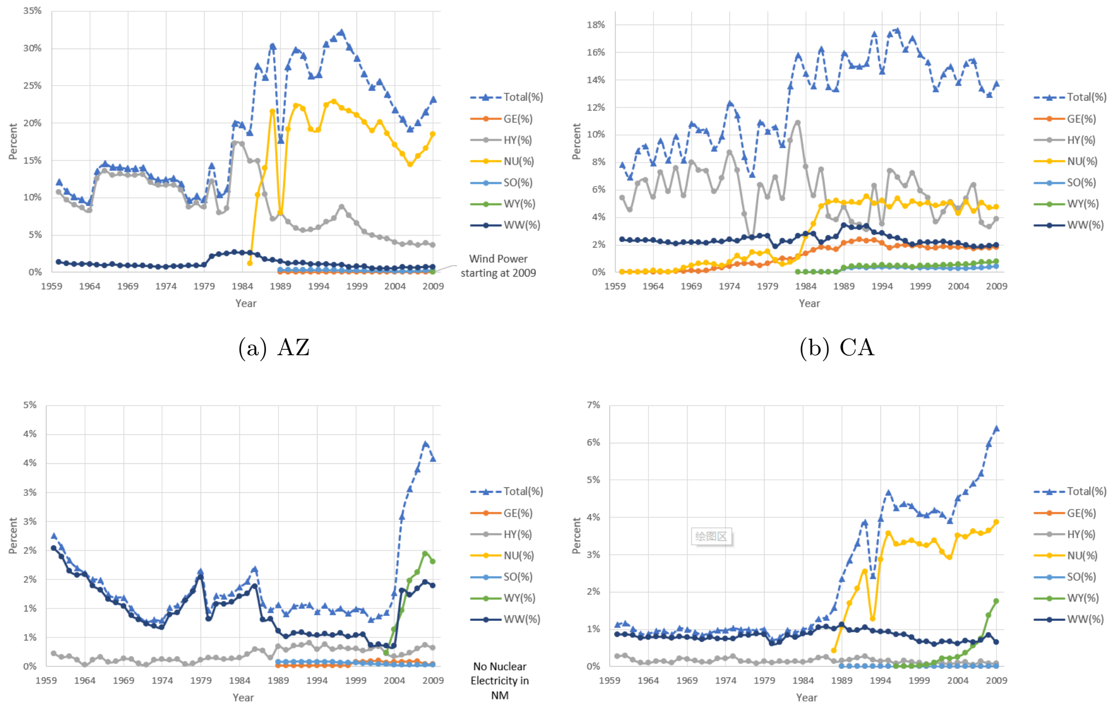


(c) NM



(d) TX

Figure 5: Time Series of Energy Outflow per capita (Negative Values Stand for Net Inflow)



(a) AZ

(b) CA

(c) NM

(d) TX

Figure 6: Time Series of Share of Energy Consumption from Clean Energy Sources

3.2.1 Interstate Similarities and Differences

After comparison among energy profiles of four states and their evolutions, data patterns started to reveal. The following similarities and differences among states are summarized.

Similarities:

- Fossil fuels still dominate energy consumption. Clean energy usage is still not so advanced as to outdo traditional fossil fuels in all four states.
- Solar power constitute little of energy consumption. Possible reason is the relatively low efficiency in solar power usage.

Differences:

- States in descending order of energy consumption per capita are Texas, New Mexico, Arizona, California. Climate extreme in Texas and New Mexico while mild in Arizona and California leads to difference in energy usage due to cooling and heating. Also, energy-intensive industries crowded in Texas and efforts in efficient energy usage in California contribute to the order.
- Arizona and California have energy flowing in, while New Mexico have net outflow of energy. Meanwhile, Texas transformed from a state with energy inflow to a state with energy outflow.

- New Mexico has no nuclear power due to absence of any nuclear plant.
- Arizona started to use wind energy in 2009 with the Dry Lake Wind Power Project into production. However, wind energy makes up only a tiny part of energy consumption. In New Mexico and Texas, share of wind energy increased notably after 2005.
- States in increasing order of share of energy consumption from clean energy are Arizona, California, Texas, New Mexico.

3.2.2 Influencing Factors

Similarities and differences among states are caused by different geographical conditions, population and other influencing factors. In Figure 7 is favorability for clean energy sources due to environmental factors of each clean energy source: geothermal energy [6], hydroelectricity [5], nuclear electricity [9], solar power [8], wind power [2], and forest cover (for wood and waste) [1].

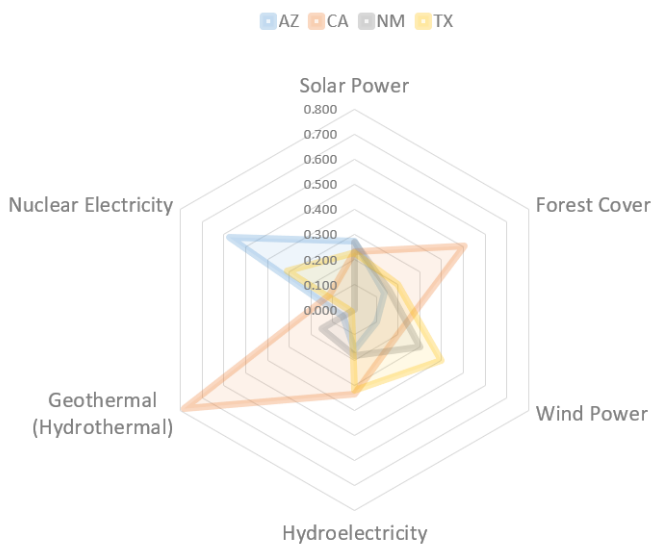


Figure 7: Favorability of Clean Energy Sources by State

Other factors are all changeable and could play significant roles in advanced modeling, thus will be stated detailedly in the following sections.

3.3 Assessment of Energy Profiles

We selected several indicators (and part of their rate of change) to assess the energy profiles, listed in Table 5. Among them, some are positive indicators (the higher, the better), and others are negative indicators (the lower, the better), denoted by + and − respectively. Rate of change of an indicator is estimated by the slope of the straight line fitting the time series of the corresponding indicator.

Table 5: Six Indicators Used in Assessment of Energy Profiles

Symbol	Description	+/−
σ	Energy consumption per capita	−
r_σ	Rate of change of σ	−
μ	Share of energy consumption from clean energy	+
r_μ	Rate of change of μ	+
s	Total consumption of all petroleum products	−
$E_c : E_{nc}$	Ratio of electricity from clean energy to that from fossil fuels	+

Thus an *indicator matrix* **A** can be formed:

	σ	r_σ	μ	r_μ	s	$E_c : E_{nc}$
AZ	262.898	-3.390	23.19%	-0.0059	540273.348	0.5731
CA	189.609	-1.742	13.73%	-0.0012	3590000.286	1.0004
NM	415.833	-2.057	3.59%	0.0026	251452.547	0.0483
TX	451.624	-12.155	6.39%	0.0039	5512412.546	0.2211

where each row stands for a state and each column stands for an indicator, and a_{ij} stands for the value of the jth indicator of the ith state, $i \in \{1, 2, 3, 4\}$, $j \in \{1, 2, \ldots, 6\}$.

Shannon first introduced the concept of information entropy in [10]. The larger the entropy of an indicator is, the more information the indicator provides, thus the weight of the indicator should be higher in assessment. The entropy method for deciding the weights of indicators is introduced in details in [13], which we mainly followed to compute an energy profile score for each state. The steps are as follows:

1. Standardize the indicators to get rid of different units and scales of the indicators and let $\mathbf{B} = (b_{ij})$ be the standardized indicator matrix. For positive indicators,

$$b_{ij} = \frac{a_{ij} - \min_{1\le i\le 4}\{a_{ij}\}}{\max_{1\le i\le 4}\{a_{ij}\} - \min_{1\le i\le 4}\{a_{ij}\}}$$

   For negative indicators,

$$b_{ij} = \frac{\max_{1\le i\le 4}\{a_{ij}\} - a_{ij}}{\max_{1\le i\le 4}\{a_{ij}\} - \min_{1\le i\le 4}\{a_{ij}\}}$$

2. Normalize the indicators and let $\mathbf{P} = (p_{ij})$ be the normalized indicator matrix.

$$p_{ij} = \frac{b_{ij}}{\sum_{i=1}^{4} b_{ij}}$$

3. Compute the entropy

$$e_j = -\frac{1}{\ln 4}\sum_{i=1}^{4} p_{ij}\ln(p_{ij})$$

   where $p_{ij}\ln(p_{ij})$ is defined to be 0 in case $p_{ij} = 0$.

4. Compute the redundancy $d_j = 1 - h_j$.

5. Compute the weight
$$w_j = \frac{d_j}{\sum_{j=1}^{6} d_j}$$
and store them in weight vector w.

The weight vector

$$\mathbf{w} = \begin{bmatrix}0.162138 & 0.288713 & 0.166425 & 0.108523 & 0.120374 & 0.153828\end{bmatrix}^{\mathrm{T}}$$

However, in the sense of the whole problem, we are mainly dealing with the usage of clean, renewable energy, thus indicators directly characterizing clean energy usage should have higher importance in the assessment of energy profiles. We claimed that the importance of the indicators as $\mu \gg \sigma > r_\sigma = r_\mu > E_c : E_{nc} > s$.

We construct a 6×6 *pairwise comparison matrix* $\mathbf{C} = (c_{ij})$, where c_{ij} stands for the relative importance of i to j. We assess the relative importance on a scale of 1 to 3. $c_{ij} = 1$ means that the importance of the ith indicator equal that of the jth indicator, and $c_{ij} = 3$ means that the ith indicator is far more important than the jth indicator.

According to the definition, $\mathbf{C}$ has the following properties: $c_{ii} = 1$, $c_{ij} > 0$, $c_{ij} = \frac{1}{c_{ji}}$.

$$\mathbf{C} = \begin{bmatrix} 1 & 11/10 & 2/3 & 1 & 1 & 113/100 \\ 10/11 & 1 & 5/6 & 1 & 1 & 1 \\ 3/2 & 6/5 & 1 & 9/5 & 27/10 & 14/5 \\ 1 & 1 & 5/9 & 1 & 11/10 & 1 \\ 1 & 1 & 10/27 & 10/11 & 1 & 10/11 \\ 100/113 & 1 & 5/14 & 1 & 11/10 & 1 \end{bmatrix}$$

Normalize the product $\mathbf{Cw}$, the original weight vector multiplied by the pairwise comparison matrix, in other words, scale the elements in the matrix-vector product to make the elements add to 1. The result is the updated weight vector

$$\mathbf{w}_{\text{new}} = \begin{bmatrix}0.164474 & 0.125255 & 0.318465 & 0.157746 & 0.106560 & 0.127500\end{bmatrix}^{\mathrm{T}}$$

The final scores of each state, the weighted average of standardized indicators, are computed by $\mathbf{Bw}_{new}$ and listed in Table 6.

Table 6: Scores for Energy Profiles of Each State

AZ	CA	NM	TX
0.627745271	0.571325779	0.269637427	0.351573283

We define the state with the highest score as the state with the "best" profile. Therefore, Arizona is decided to have the "best" profile.

4 Predictions of Energy Profiles of 2025 and 2050

According to the weights of the six indicators generated by the entropy method in subsection 3.3, it is apparent that the most significant influential factor in energy profiles is the consumption of clean energy σ_c. Based on this observation and the evolution of energy profiles acquired, we make predictions about consumption of the six clean energy sources (GE, HY, NU, SO, WY, WW) and the sum of them to represent the future trend of clean energy usage towards 2025 and 2050.

Since the exploitation and consumption of different energy sources are relatively independent of each other, the prediction of usage of each individual energy source can be achieved through Time Series Forecasting Analysis(TSFA) methods. Depending on the number of time steps to predict, different TSFA methods can vary in accuracy. Therefore, we adopt two different methods as follows for prediction to ensure higher reliability:

- Gray Model Forecast
- BP Neural Network Forecast

4.1 Gray Model Forecast

The algorithm of gray model forecast is based on the existing time series data from the year 1960 to 2009. However, the previous years contains too many empty data points, and have rather small influence on the results. Therefore, we truncate the consumption of each type of clean energy in each state at the year 1995, and select data of the last 15 years (from 1995 to 2009) as input of the algorithm. For series that still contain empty data points, we apply a shift-reshift procedure to create proper input.

For consumption of the jth clean energy source in the ith state σ_{ij}, $i \in \{1, 2, 3, 4\}$, $j \in \{1, 2, \ldots, 6\}$, denote the input array of length 15 as $x^{(0)}$. Overall procedure of gray model forecast contains the following steps:

1. Stationarity Test: the input array must be positive, and the stepwise ratios λ of input array $x^{(0)}$ must satisfy the condition

$$e^{\frac{-2}{15+1}} < \lambda_k = \frac{x_k^{(0)}}{x_{k-1}^{(0)}} < e^{\frac{2}{15+1}}$$

for $k \in \{2, 3, \ldots, 15\}$. It appears that all our data satisfies this property well, except for those containing empty data points, which indicates that most clean energy usage trends are stationary enough. For inputs containing zeros, we first shift the array by adding a constant to each element, until it satisfies the property above, then reshift the output by subtracting the same constant from each element.

2. Acquire the equally-weighted array using adjacent value generation

$$z_k^{(0)} = \frac{1}{2}x_k^{(0)} + \frac{1}{2}x_{k-1}^{(0)}$$

for $k \in \{2, 3, \ldots, 15\}$. Generate the accumulative sum series $x^{(1)}$ as

$$x_k^{(1)} = \sum_{1}^{k} x_k^{(0)}$$

for $k \in \{2, 3, \ldots, 15\}$, which weakens the randomness of original input and strengthens its regularity. Define $z^{(1)}$ similarly.

3. Construct discrete model as follows

$$x_k^{(0)} + az_k^{(1)} = b,\ \forall\, k \in \{2, 3, \ldots, 15\}$$

with unknown parameters a, b to optimize. Denote

$$\mathbf{p} = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_2^{(0)} \\ x_3^{(0)} \\ \vdots \\ x_{15}^{(0)} \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} -z_2^{(1)} & 1 \\ -z_3^{(1)} & 1 \\ \vdots & \vdots \\ -z_{15}^{(1)} & 1 \end{bmatrix}$$

the model can be denoted as $\mathbf{X} = \mathbf{Zp}$. By Least Square Estimation(LSE), the best parameters $\mathbf{p}$ is

$$\mathbf{p} = (\mathbf{Z^T Z})^{\mathbf{-1}} \mathbf{Z^T X}$$

4. Converting to continuous equation: $x_k^{(0)}$ is the "derivative" of $x_k^{(1)}$, and $z_k^{(1)}$ can be transferred to $x_k^{(1)}$, therefore the corresponding differential model is

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = b,\ \ t > 0$$

whose solution gives the estimated values

$$\hat{x}_k^{(1)} = (x_1^{(0)} - \frac{b}{a})e^{-a(k-1)} + \frac{b}{a}$$

for $k \geq 2$.

5. Conduct the inverse of cumulative sum operation to get the prediction values of original input σ_{ij}

$$\hat{\sigma}_{ij}(k) = \hat{x}_k^{(0)} = \hat{x}_k^{(1)} - \hat{x}_{k-1}^{(1)}$$

for $k \geq 3$.

Thus $\hat{\sigma}_{ij}(31)$ and $\hat{\sigma}_{ij}(56)$ are the predicted consumption of each clean energy source in each state in 2025 and 2050 respectively, presented in Table 7 and Table 8. $\sum_{j=1}^{6} \hat{\sigma}_{ij}(31)$ and $\sum_{j=1}^{6} \hat{\sigma}_{ij}(56)$ are the predicted total consumption of clean energy in each state in 2025 and 2050.

Table 7: Prediction of Clean Energy Consumption (Billion Btu) by Gray Model in 2025

	GE	HY	NU	SO	WY	WW
AZ	434.372	26098.199	257313.508	3712.093	222.581	10153.307
CA	126843.093	165575.161	343182.003	30411.362	92635.169	117782.536
NM	957.036	2408.791	0.000	115.408	44302.717	24693.037
TX	3951.316	7830.946	488826.776	930.599	382404.311	61322.386

Table 8: Prediction of Clean Energy Consumption (Billion Btu) by Gray Model in 2050

	GE	HY	NU	SO	WY	WW
AZ	619.236	7558.028	210773.677	3862.981	480.499	9168.612
CA	121892.880	68879.048	335095.974	37141.508	159708.679	82970.988
NM	1493.913	2526.629	0.000	100.732	109455.305	51130.020
TX	8232.423	3593.947	590450.738	1335.399	887781.970	37179.805

4.2 BP Neural Network Forecast

Back Propagation (BP) Neural Network algorithm is a handy tool which is widely used to help compute TSFA Autoregression (AR) forecast. The process of using BP algorithm to conduct autoregression forecast is demonstrated in Figure 8.

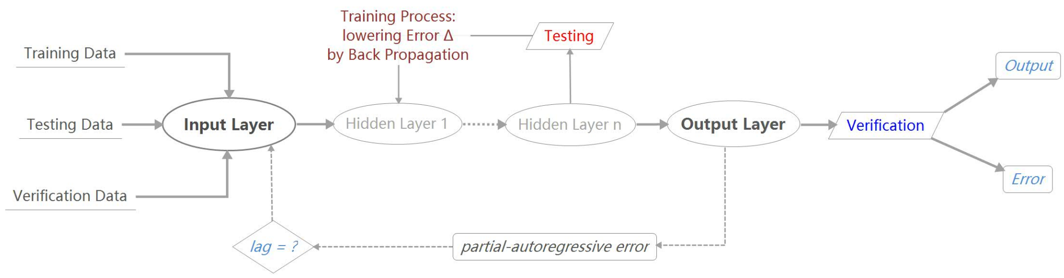


Figure 8: Demonstration of the Process of BP Algorithm

Choose the number of hidden layers to be 10 by convention. In order to avoid overfitting, we divide the input data into data for training, testing and verification with proportion of $70\%, 15\%$ and 15% respectively. According to our observation of partial-autoregressive error, we set autoregressive order $lag = 3$. Predicting results are presented in Table 9 and Table 10.

Table 9: Prediction of Clean Energy Consumption (Billion Btu) by BP in 2025

	GE	HY	NU	SO	WY	WW
AZ	299.191	88748.908	334268.685	5613.816	36.045	3509.403
CA	125512.060	266214.488	303793.034	30711.738	53399.153	150645.533
NM	768.620	2478.914	0.000	232.616	10935.414	3509.403
TX	2728.960	12335.324	436321.995	293.140	128857.172	102549.988

Table 10: Prediction of Clean Energy Consumption (Billion Btu) by BP in 2050

	GE	HY	NU	SO	WY	WW
AZ	346.047	86468.339	332834.906	4866.409	36.045	3507.691
CA	133144.593	333652.589	379532.719	30711.764	64173.484	156607.371
NM	-235.718	2478.908	0.000	281.094	13246.323	3507.691
TX	964.957	11712.142	418925.678	637.348	328434.425	45515.918

4.3 Evaluation of Prediction Results

4.3.1 Error Analysis

With input series of length 15, accuracy of gray model forecast is measured by residual errors between real values and predicted values from the third year 1997 to 2009:

$$e_{ij}(k) = \frac{\sigma_{ij}(k) - \hat{\sigma}_{ij}(k)}{\sigma_{ij}(k)}, \ k \in \{3, 4, \ldots, 15\}$$

Detailed statistics are included in Appendix A. Most of the residuals satisfy $|e_{ij}(k)| < 0.1$, with several points exceeding 0.1 but still smaller than 0.2. The result of gray model forecast is considerably accurate.

Compared to gray model forecast, the results of BP Neural Network Forecast reveal larger discrepancies and exaggerated fluctuation. Prediction of geothermal energy (GE) in Arizona is illustrated below in Figure 9 as an example, which reflects relatively larger error especially at the end of the input array. A possible cause of such phenomenon could be the small size of the dataset that does not satisfy requirements of the neural network algorithm.

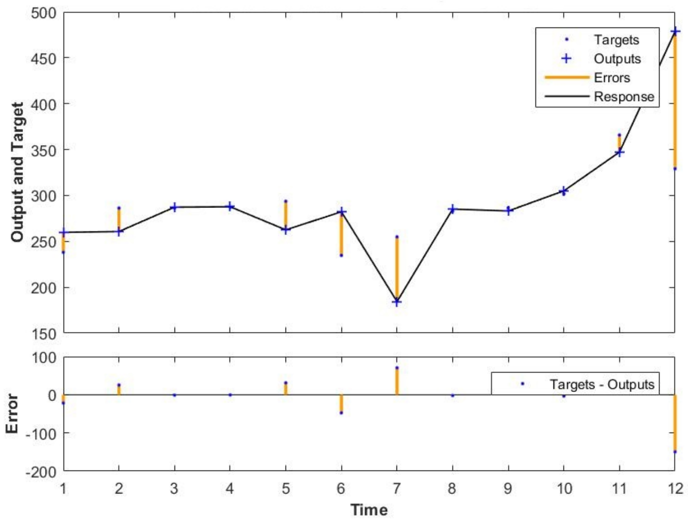


Figure 9: Response of Output for the Input Array

4.3.2 Insights

Gray model forecast predicts the trend of the sum of clean energy consumption of the four states towards the year 2050, as illustrated in Figure 10. The general expansion of consumption of clean energy thus can be defined as *flare factor* α, which in average equals 1.041 in the year of 2025 and 1.169 in 2050.

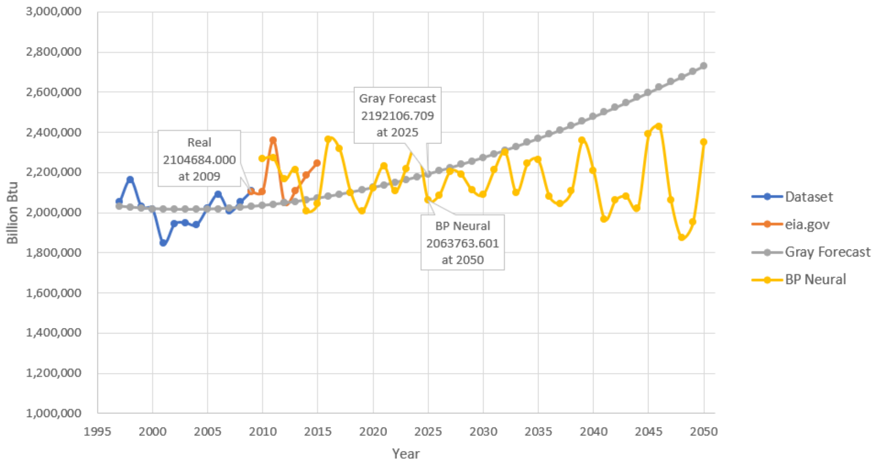


Figure 10: Sum of Clean Energy Consumption of the Four States

Both methods generate an approximately similar trend of share of energy consumption from clean energy $\frac{\sigma_c}{\sigma_c+\sigma_{nc}}$, as shown in Figure 11. They turn out to all converge towards 10% after year 2025, meaning that the four states will obtain an energy structure of 10% clean energy sources with respective to total energy exploitation. It indicates that the four states may not make progress in the energy structure, if no interstate energy compact is formed and no change of policy is made.

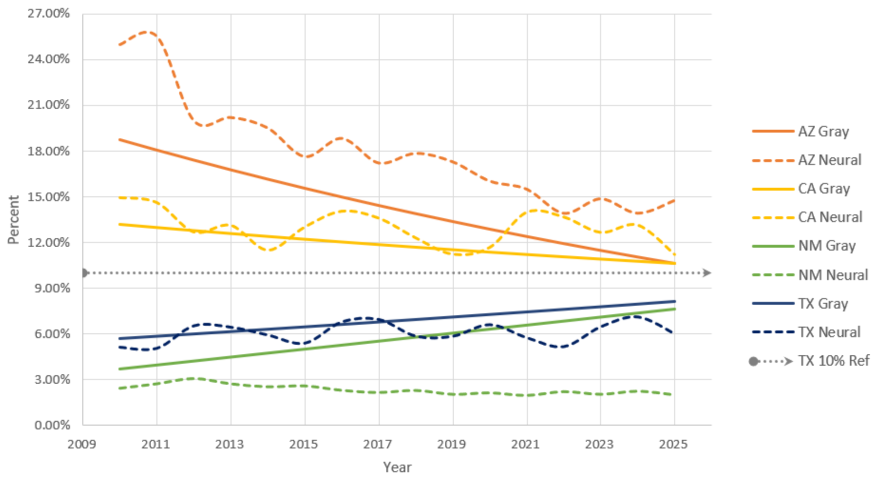


Figure 11: Trend of Share of Energy Consumption from Clean Energy Sources

5 Optimization

With the purpose of converting into an optimization problem, we define the evaluation index of clean energy usage $I(y)$ at year y as follows

$$I(y) = \alpha(y) \cdot \left(\sum_{i=1}^{4} \sum_{j=1}^{6} \eta_{ij} \sigma_{ij} \right)$$

for $y \geq 2009$, state $i \in \{1, \ldots, 4\}$ and clean energy type $j \in \{1, \ldots, 6\}$, where the variables involved are listed in Table 11.

Table 11: Variables Involved in the Evaluation Index

Symbol	Description
y	Year
$\alpha(y)$	Flare factor of total consumption of clean energy defined in subsubsection 4.3.2
η_{ij}	Utilization efficiency, defined in details in subsection 5.2
$\sigma_{ij}(y)$	Consumption of the jth clean energy source in the ith state in the target year y

5.1 Targets and Approaches

After defining a comprehensive evaluation index that quantifies usage of clean energy at a certain year y, our goal is to maximize the index $I(2025)$ and $I(2050)$. For convenience of demonstrating our optimization approaches, we take year 2025 as target year in the following sections. Optimization for year 2050 is similar, results in Appendix C.

Our target for further clean energy usage at year 2025 can be stated as the model below:

$$\max \left\{ \alpha(2025) \cdot \left(\sum_{i=1}^{4} \sum_{j=1}^{6} \eta_{ij} \sigma_{ij} \right) \right\}$$

Extracting the three variables inside this model gives a much clearer view about how we can make progress.

- Flare factor $\alpha(2025)$, the ratio of the sum of total consumption of energy by the four states as a compact in 2025 to that in 2009, is fixed.
- Utilization efficiency η_{ij}, which indicates state i's ability of utilizing energy source j, is possible to get enhanced jointly. With the common enhancement of efficiency, index I becomes larger.
- Energy consumption quantity σ_{ij} is the quantity of usage of energy j by state i, under the constraint of a fixed total exploitation quantity. Though their sum is fixed, with the influence of utilization efficiency η_{ij}, their distribution among the four states can be optimized to obtain a larger index I.

5.2 Improving Utilization Efficiency

Specific definition of utilization efficiency is $\eta_{ij} = l_{ij} \cdot k_i$, where l_{ij} comes from the environmental favorability of state i on energy source j in Figure 7, and k_i is scientific and technological strength of state i in energy industry, for $i \in \{1, \ldots, 4\}$, $j \in \{1, \ldots, 6\}$. Current scientific and technological strength is ranked in Table 12.

Environmental favorability can be viewed as an unchangeable factor embedded in each state itself, up to year 2050. However, scientific and technological strength of four states can get improved jointly by technical exchanges and concentrated scientific research, through building interstate energy compact.

Table 12: Score of Current Scientific and Technological Strength by State [7]

AZ	CA	NM	TX
78	81	67	84

5.3 Better Distribution: Multivariate Linear Programming

Optimization of distribution of σ_{ij} can be abstracted as a standard mathematical multivariate linear programming problem. Our detailed process goes as follows:

1. Assumptions: For state governments, policies take effects in a long time period (possibly years in the field of energy). For example, a plan for shutting down a nuclear power plant should be promulgated several years before the planned close time, and it takes several additional years for the plant to completely cut down its capacity. Therefore, usage of clean energy cannot make abrupt changes in one year. We set a limitation that σ_{ij} in the next year changes in the range of 95% to 105% of that in the current year, to model such limits.

2. Under the constraint of a fixed total, in addition to the limitation stated above, the constraint conditions needed for linear programming are acquired.

   - Consumption vector to solve: $\mathbf{X} = \begin{bmatrix}\sigma_{11} & \sigma_{21} & \sigma_{31} & \sigma_{41} & \sigma_{12} & \cdots & \sigma_{46}\end{bmatrix}^{\mathrm{T}}$
   - Equality constraint $\mathbf{AX} = \mathbf{b}$: Denote a 1×4 summing row vector $v = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$, then the 6×24 matrix $\mathbf{A}$ and the 6×1 vector $\mathbf{b}$ are

   $$\mathbf{A} = \begin{bmatrix} v & & & & & \\ & v & & & & \\ & & v & & & \\ & & & v & & \\ & & & & v & \\ & & & & & v \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \sum_{i=1}^{4} \sigma_{i1} \\ \sum_{i=1}^{4} \sigma_{i2} \\ \vdots \\ \sum_{i=1}^{4} \sigma_{i6} \end{bmatrix}$$

   - Linear inequality constraints: in year y, for every σ_{ij} in $\mathbf{X}$,

   $$\sigma_{ij}(2009) \cdot 0.95^{y-2009} \leq \sigma_{ij}(y) \leq \sigma_{ij}(2009) \cdot 1.05^{y-2009}$$

3. Linear programming results: Refer to Table 13 for linear programming optimal solution $\tilde{\mathbf{X}}$.

Table 13: σ_{ij}'s in the Optimal Solution $\tilde{\mathbf{X}}$ Maximizing $I(2025)$ (Unit: Billion Btu)

	GE	HY	NU	SO	WY	WW
AZ	158.721	30254.240	203220.125	10241.097	624.056	6205.721
CA	134197.307	304148.984	719042.246	26606.475	123349.954	199882.461
NM	152.917	5723.343	0.000	136.273	32670.153	5610.535
TX	992.103	21727.548	209343.588	1774.064	122172.768	34776.909

4. By comparing evaluation index $I(2025)$ with both consumption vectors with and without optimized distribution in Table 14, the progress made by linear programming optimization is obvious.

Table 14: Comparison between $I(y)$ with and without Optimized Distribution

	Without Optimization	With Optimization
$I(2025)$	452908	695824
$I(2050)$	571191	1036749

This verifies that it is an efficient way to obtain better usage of clean energy by adjusting each σ_{ij} towards its optimal value.

5.4 Summary of Actions to Take

Based on the approaches analyzed above, the following three actions are proposed to state governments as our advice.

- Build interstate public renewable energy and nuclear electricity database, meanwhile contribute to form a Border-State Clean Energy Technology Cooperation Board. These actions accelerate communication of clean energy exploitation and utilization technologies between the four border states. They also make it possible for the four states to simultaneously focus on cutting-edge challenges in the field of energy.
- Increase subsidies for nuclear power industry, and hold back the stagnancy that the nuclear market in America is going through in recent years. This maintains the nonnegligible contribution of nuclear energy to overall clean energy usage, preventing it from going down far before it reaches the optimal quantity.
- For clean energy sources which are overconsumed now (exceeding optimal value) by one state and are under-utilized (below optimal value) by another state, establish a pairwise energy deployment system between states. Suggested exchanges which are prior to be conducted is listed in Figure 12, based on the most prominent data from Table 13.

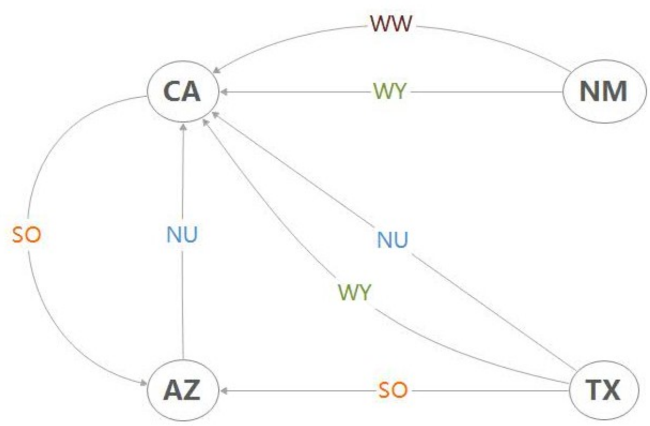


Figure 12: Prior Suggested Exchanges of Clean Energy

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Appendices

A Residuals of Gray Model Forecast (In the Order of AZ, CA, NM, TX)

	GE	HY	NU	SO	WY	WW
1997	-0.003	0.150	-0.002	0.014	0.018	0.037
1998	-0.006	0.110	0.005	0.011	0.012	-0.005
1999	0.014	0.051	0.006	0.007	0.006	0.003
2000	0.011	-0.057	0.006	-0.002	0.001	0.012
2001	0.008	-0.089	-0.002	-0.010	-0.005	-0.034
2002	0.008	-0.080	0.011	-0.017	-0.011	-0.036
2003	-0.024	-0.072	-0.001	-0.020	-0.017	-0.032
2004	-0.017	-0.058	-0.002	-0.022	-0.023	-0.029
2005	-0.006	-0.097	-0.014	-0.023	-0.029	0.005
2006	-0.008	0.007	-0.023	-0.014	-0.035	-0.003
2007	-0.004	0.023	-0.006	-0.004	-0.041	0.005
2008	0.022	0.156	0.008	0.021	-0.047	0.028
2009	0.003	0.080	0.017	0.039	0.123	0.024

	GE	HY	NU	SO	WY	WW
1997	0.000	-0.055	-0.015	0.011	0.008	-0.007
1998	0.001	0.155	0.005	0.006	-0.011	-0.016
1999	0.004	0.010	-0.001	0.002	-0.002	-0.004
2000	-0.005	-0.018	0.007	-0.004	0.001	0.005
2001	-0.004	-0.456	-0.002	-0.006	-0.005	0.004
2002	0.005	-0.171	0.004	-0.011	-0.004	0.011
2003	0.004	0.038	0.009	-0.015	-0.007	0.006
2004	0.003	-0.011	-0.016	-0.015	-0.003	0.008
2005	0.002	0.157	0.012	-0.018	-0.011	-0.001
2006	-0.002	0.323	-0.007	-0.013	0.000	-0.006
2007	0.000	-0.153	0.012	-0.001	0.013	-0.005
2008	-0.001	-0.264	-0.004	0.018	0.000	-0.002
2009	-0.003	-0.066	-0.007	0.027	0.005	-0.001

	GE	HY	NU	SO	WY	WW
1997	-0.063	0.020	-	-0.015	0.032	0.032
1998	-0.065	0.007	-	0.059	0.016	0.014
1999	0.043	0.011	-	0.106	-0.001	0.007
2000	0.048	-0.002	-	0.080	-0.018	0.000
2001	0.055	0.009	-	0.036	-0.035	-0.036
2002	0.053	0.022	-	0.004	-0.053	-0.048
2003	0.007	-0.033	-	-0.042	-0.046	-0.061
2004	0.016	-0.055	-	-0.179	-0.023	-0.072
2005	0.028	-0.039	-	-0.620	-0.008	0.038
2006	0.025	-0.020	-	-0.348	0.025	0.021
2007	0.029	0.018	-	-0.191	0.022	0.025
2008	-0.067	0.041	-	0.202	0.030	0.023
2009	-0.080	0.017	-	0.407	0.003	0.007

	GE	HY	NU	SO	WY	WW
1997	0.012	0.050	0.005	-0.002	0.024	0.021
1998	0.008	0.018	0.009	0.009	0.012	0.008
1999	0.002	-0.010	-0.001	0.010	0.002	-0.015
2000	-0.009	-0.038	0.000	0.002	-0.009	-0.008
2001	-0.013	0.003	0.001	0.000	-0.014	-0.025
2002	-0.018	-0.005	-0.011	-0.003	-0.012	-0.005
2003	-0.009	-0.025	-0.021	-0.006	-0.026	-0.008
2004	-0.011	0.014	0.006	-0.011	-0.034	-0.013
2005	-0.008	0.019	-0.004	-0.014	-0.036	-0.002
2006	-0.007	-0.047	0.006	-0.014	-0.025	-0.004
2007	-0.001	0.049	0.004	-0.009	-0.016	0.009
2008	0.007	-0.006	0.001	0.006	0.036	0.034
2009	0.025	-0.006	0.003	0.027	0.053	-0.009

B Utilization Factors η (η_{ij} is the element at the ith row, jth column)

	GE	HY	NU	SO	WY	WW
AZ	0.124	0.078	0.042	0.213	0.372	0.106
CA	0.273	0.162	0.635	0.188	0.202	0.41
NM	0.123	0.201	0.101	0.18	0.15	0.107
TX	0.269	0.336	0.01	0.191	0.043	0.167

C Optimal Solution for the Optimization in 2050 (Unit: Billion Btu)

	GE	HY	NU	SO	WY	WW
AZ	49.441	9424.107	48182.438	38641.136	2373.123	1933.063
CA	151755.948	292534.592	1157354.407	4716.796	279095.587	262268.315
NM	47.634	21764.376	0.000	42.445	2267.879	1747.666
TX	309.057	82624.149	65209.905	123.154	29363.308	10832.904