**POLITECNICO DI MILANO**
**Corso di Laurea (MAGISTRALE ?) in Ingegneria Informatica**
**Dipartimento di Elettronica e Informazione**

# TITOLO DELLA TESI

**AI & R Lab**
**Laboratorio di Intelligenza Artificiale**
**e Robotica del Politecnico di Milano**

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*A qualcuno...*

# Sommario

Il sommario deve contenere 3 o 4 frasi tratte dall'introduzione di cui la prima inquadra l'area dove si svolge il lavoro (eventualmente la seconda inquadra la sottoarea più specifica del lavoro), la seconda o la terza frase dovrebbe iniziare con le parole "Lo scopo della tesi è ..." e infine la terza o quarta frase riassume brevemente l'attività$\frac{1}{2}$ svolta, i risultati ottenuti ed eventuali valutazioni di questi.

NB: se il relatore effettivo è interno al Politecnico di Milano nel frontesizo si scrive Relatore, se vi è la collaborazione di un altro studioso lo si riporta come Correlatore come sopra. Nel caso il relatore effettivo sia esterno si scrive Relatore esterno e poi bisogna inserire anche il Relatore interno. Nel caso il relatore sia un ricercatore allora il suo Nome COGNOME dovrà$\frac{1}{2}$ essere preceduto da Ing. oppure Dott., a seconda dei casi.

# Ringraziamenti

Ringrazio ................

# Capitolo 1

# Focusing for X-rays

*"Terence: Rotta a nord con circospezione*
*Bud: Ehi, gli ordini li do io qui!*
*Terence: Ok, comante*
*Bud: Rotta a nord*
*Terence: Soltanto?*
*Bud: Con circospezione!"*

Chi Trova un Amico Trova un Tesoro

L'introduzione deve essere atomica, quindi non deve contenere nè sottosezioni nè paragrafi nè altro. Il titolo, il sommario e l'introduzione devono sembrare delle scatole cinesi, nel senso che lette in quest'ordine devono progressivamente svelare informazioni sul contenuto per incatenare l'attenzione del lettore e indurlo a leggere l'opera fino in fondo. L'introduzione deve essere tripartita, non graficamente ma logicamente:

## 1.1 Introduction

Image formation usually implies some form of focusing. How this focusing occurs depends on the way in which the radiation interacts with its surroundings. Thus for visible light the well-known laws of reflection and refraction are utilized, while, while electrons are caused to travel in curved paths in electromagnetic field. X-rays interact with matter in three ways (each causing attenuation of the X-ray beam): elastic scattering, inelastic scattering, and absorption via the photoelectric effect.

Elastic scattering (in which exchange of energy is involved) is caused by two process: Thomson scattering from single atomic electrons, and Rayleigh (or coherent) scattering, which occurs from strongly bound electrons acting cooperatively. The

scattered and incident beams have a ddefinite phase relationship and interference can occour (Bragg diffraction). Inelastic, incoherent, or Compton scattering occurs from loosely bound (essentialle free) electron and involves the transfer of a small fraction of the energy of an incident X-ray photon. The scattered and incident beams do not have fixed relationship, and the atom is raised to a different quantum state due to the excitation of the electron. Absorbtion (through the photoelectric effect) occurs when the X-ray photon transfer all its energy to an inner atomic electron, thereby releasing it from atom (ionization)

In Figure 1 the linear attenuation coefficient for these processes are plotted as functions of energy for two materials, carbon and gold, important in soft X-ray physics. For the soft X-ray regime

## 1.2   Interaction with Matter

When a beam of electromagnetic radiation passes through a material, the intensity is exponentially attenuated

$$I = I_0 exp(-\alpha x) \tag{1.1}$$

where $x$ is the thickness of the material, $\alpha$ is the linear attenuation coefficient, and $I_0$ is the intensity at $x = 0$. The amplitude of the electromagnetic wave at $x$ is

$$A = A_0 exp(\frac{-2\pi\beta x}{\lambda}) exp(\frac{-2\pi i(nx - ct)}{\lambda}) \tag{1.2}$$

where $\lambda$ is the vacuum wavelength of the radiation, $n$ is the refractive index of the material, and $\beta$ is its absorption coefficient. The complex refractive index of the material, which governs the propagation of the electromagnetic wave is

$$\bar{n} = n - i\beta \tag{1.3}$$

Since, at soft X-ray wavelengths, absorption is the dominant process, $\alpha$ may be identified with a linear absorption coefficient, where

$$\alpha = \frac{4\pi\beta}{\lambda} \tag{1.4}$$

Tabulation of X-ray absorption data usually give the mass absorption coefficient $\mu$, where

$$\alpha = \mu\rho \qquad (1.5)$$

and $\rho$ is the density of the material. The mass absorption of a compound is given by

$$\mu_{\text{com}} = \sum_j w_j \mu_j \qquad (1.6)$$

where $w_j$ is the weight fraction of the constituent with mass absorption coefficient $\mu_j$. The linear absorption coefficient of the compound is then

$$\alpha_{\text{com}} = \mu_{\text{com}}\rho_{\text{com}} \qquad (1.7)$$

where $\rho_{\text{com}}$ is the density of the compound.

In the X-ray region, the energies of individual photons are much larger than the binding energies of outer atomic electron (typically a few electron volts) and molecular binding energies. Absorbing atoms are therefore ionized by the radiation and most of the energy is transferred to the kinetic energy of ejected electron. The energy of an X-ray photon may only be absorbed by an atomic electron from the state. Thus, as the X-ray energy increase, the absorption coefficient will undergo several relatively sharp increases (absorption edges) at energies corresponding to binding energies of different atomic levels. as shown in Figure 1. In practice these increases are not so sharp as indicated, because of the finite energy widths of atomic states and because of the environment of the absorbing atoms.

The theoretical treatment of X-ray scattering and absorption has been given in detail by many authors. A brief summary is included here because the results have implication for the design of X-ray optical system. The starting point of the calculation is to consider the scattering of X rays by free electron (Thomson scattering). An electro-magnetic wave whose electric vector has amplitude $A_0$ causes such an electron (of charge $e$ and mass $m_e$) to be accelerated by an amount $A_0(e/m)$. Accelerated charges radiate, the amplitude of the electric vector at a distance $r$ from the charge being

$$A_T(\Phi) = \frac{e}{4\pi\varepsilon_0 c^2 r} a \sin\Phi \qquad (1.8)$$

where $\Phi$ is the angle between the direction $\mathbf{r}$ and the acceleration $\mathbf{a}$. Thus

$$A_T(\Phi) = A_0 \frac{e^2}{4\pi\varepsilon_0 c^2 r} \sin\Phi \qquad (1.9)$$

To describe the interaction of an electromagnetic wave with an electron bound in an atom, the Thomson amplitude $A_T(\Phi)$ is multiplied by a complex atomic scattering factor $f_1 + if_2$, so that the scattered amplitude is given by

$$A(\Phi, E) = A_T(\Phi)[f_1(E) + if_2(E)] \tag{1.10}$$

where the factor $f_1$ and $f_2$ depend on the energy $E$ of the incoming radiation but it is assumed that, to a first approximation, they do not depend on the scatting angle $\theta$ (i.e. the angle between the incoming and scattered radiation). This assumption is valid since the wavelengths of interest ($\sim 1 - 10nm$) are larger to typical dimension of the atomic electron distribution ($\sim 1 - 50pm$) so that the atomic electrons may be considered to scatter in phase. The factor $f_1$ and $f_2$ can be calculated in relativistic quantum dispersion theory and are given by

$$f_1(E) = Z + 4\frac{\varepsilon_0 m_e c}{he^2} \int_0^{+\infty} \frac{W^2 \sigma(W)}{E^2 - W^2} dW - \Delta_{(rel)} \tag{1.11}$$

and

$$f_2(E) = 2\frac{\varepsilon_0 m_e c}{h} E\sigma(E) \tag{1.12}$$

The first term in the equation 1.11 describes Thomson scattering ($Z$ is the atomic number of scatterer) and, to describe the angular dependence of the scattering, may be replaced by the angle-dependent form factor

$$f0 = \int_0^{+\infty} U(r) sinc[\frac{4\pi r}{\lambda} \sin\frac{\theta}{2}] dr \tag{1.13}$$

where $U(r)$ is the radical charge distribution and $sinc(x) = \frac{\sin x}{x}$. If the wavelength $\lambda$ is in nanometres, then for $\sin\frac{\theta}{2} \leq \frac{\lambda}{2}, f_0 = Z$, while for $\sin\frac{\theta}{2} = \lambda, f_0 \simeq 0.9Z$ for most elements.

The second term in 1.11, the anomalous dispersion integral, is the same as that given semiclassically by considering the electrons to be caused to oscillate by the incoming radiation, Because it neglects damping, this term results in imprecise values for $f_1$ close to absorption edges. The atomic photo ionization cross section $\sigma(E)$ ( in $m^2 atom^{-1}$) is related to the mass absorption coefficient by

$$\sigma(E) = A\frac{\mu}{N_0} \tag{1.14}$$

where $A$ is the atomic weight and $N_0$ is Avogadro's number. In order to calculate $\sigma(E)$ theoretically the atomic wave function must be know; these have to be obtained by approximation methods for all system excepts hydrogen, leading to to uncertainties in the expressions for $f_1$ and $f_2$.

The third term in equation 1.11 is a relativistic correction, which is negligible at X-ray energies (except near absorption edges) to assume that the solid state environment does nor greatly affect the ionization process, since it is the outer atomic levels that are most modified when an atom is bound in a solid. Then, the atomic parameters $f_1$ and $f_2$ may be related to macroscopic factors $n$ and $\beta$ by

$$\delta = 1 - n = \frac{e^2 \hbar^2}{2 \varepsilon_0 m_e E^2} \overline{f_1} \tag{1.15}$$

and

$$\beta = \frac{e^2 \hbar^2}{2 \varepsilon_0 m_e E^2} \overline{f_2} \tag{1.16}$$

where $f_1$ and $f_2$ are the average atomic scattering factors per unit volume,

$$\overline{f_1} = \sum j N_j f_{1j} \qquad \overline{f_2} = \sum j N_j f_{2j} \tag{1.17}$$

and $N_j$ is the number of atoms of type $j$ per unit volume. For energies well away from any absorption edges, equation 1.15 reduces to

$$\delta = \frac{N e^2 \hbar^2}{2 \varepsilon_0 E^2} = \frac{N e^2 \lambda^2}{8 \pi^2 \varepsilon_0 m_e c^2} \tag{1.18}$$

where $N$ is the total number of electrons per unit volume. This equation 1.18, is the same as that originality derived by Lorentz using classical ideas of absorption. In X-ray region, $\delta$ is small (typically $\sim 10^{-3}$) and positive, i.e., the refractive index for soft X rays is slightly less than unity. Tables of values for $f_1$ and $f_2$ have been published, and these were used to generate Figure 1, along with the experimentally observed variation of the absorption coefficient away from an absorption edge:

$$\beta \sim Z^2 \lambda^3 \tag{1.19}$$

## 1.3 Total External Reflection

The propagation of X rays in matter may be described by the complex refractive index

$$\bar{n} = n - i\beta = 1 - \delta - i\beta \tag{1.20}$$

where $\delta$ is small and positive. Thus the real part of the refractive index is, unlike the case for visible light, less than one and so, if a normal refractive lens were to be usef for focusing, it would habe ti ve concave to give a real focus for an incident plane wave. For a plano-concave lens with central thickness $d$ (Figure 1.19)

$$\frac{f}{\rho} = 1 + \frac{n}{\cos\phi - n\cos\phi'} \tag{1.21}$$

where $f$ is the focal length, $\rho$ is the radius of curvature of the concave surface, and $\phi$ and $\phi'$ are the anlges with respect to the normal to the concave surface as defined in Figure 2. For axial rays, this becomes

$$\frac{f}{\rho} = 1 + \frac{n}{1-n} = \frac{1}{\delta} \tag{1.22}$$

The depth of focus of such a lens is given by

$$\Delta f = \pm \frac{1}{2}\left(\frac{f}{r}\right)^2 \lambda \tag{1.23}$$

where $r$ is the radius of the lens aperture and $\lambda$ is the illuminating wavelength. The maximum thickness of lens is, form Figure 2 ,

$$t = \rho - (\rho^2 - r^2)^{\frac{1}{2}} + d \tag{1.24}$$

The closeness of the refractive index to unity, and the high absorption of soft X rays, means that lenses of the same sort of dimensions as those used for visible light would have impossibly long focal length ($\geq 10m$), very small depths of focus ($\sim$tens of micrometers), and they would absorb essentially all of the incident radiation. Thus this type of lens s impractical for X ray, as has been started by previous authors.

<center>X</center>

However, optical components currently in use for X rays can have focusing effective of about 10% and effective aperture radii of about $10-50\mu m$. To match this this efficiency, a refractive lens should have a mean thickness such that about 10% of the incident intensity is transmitted. Two possible lenses, for soft-X ray wavelength of about $3.5nm$, are shown in Table 1. These results show that such lenses may nor be unreasonable, although they have very large f-number and so very intense source would be needed to prevent long imaging times. Calculation for biconcave lenses give similar results. However, the exact focusing property of soft X-ray refractive lenses depend on a better knowledge than currently available for the optical constants, and to date no attempts have been made to manufacture them.

The other conventional method of focusing at visible wavelengths is to use refractive optics. The reflected amplitude, $a$, at an interface between vacuum and a material is given by the Fresnel equation. For radiation polarized so that the electric vector is perpendicular to the plane of incidence (s polarization)

$$a_\perp = \frac{\cos\phi - (\bar{n}^2 - \sin^2\phi)^{\frac{1}{2}}}{\cos\phi + (\bar{n}^2 - \sin^2\phi)^{\frac{1}{2}}} \tag{1.25}$$

where the angle of incidence $\phi$ is measured from the surface normal. In terms of the glancing angle, $\theta = 90° - \phi$, this becomes

$$a_\perp = \frac{\sin\theta - (\bar{n}^2 - \cos^2\theta)^{\frac{1}{2}}}{\sin\theta + (\bar{n}^2 - \cos^2\theta)^{\frac{1}{2}}} \tag{1.26}$$

For parallel polarized radiation (p polarization)

$$a_\parallel = \frac{\bar{n}^2\sin\theta - (\bar{n}^2 - \cos^2\theta)^{\frac{1}{2}}}{\bar{n}^2\sin\theta + (\bar{n}^2 - \cos^2\theta)^{\frac{1}{2}}} \tag{1.27}$$

The reflectivity is given by

$$R = \frac{I}{I_0} = aa^* \tag{1.28}$$

where $I_0$ is the incident intensity and $I$ is the reflected intensity. For radiation incident normally on a surface, $\theta = 90°$ and equations 1.26 and 1.27 both lead to the normal incident reflectivity

$$R_n = \left(\frac{1-\bar{n}}{1+\bar{n}}\right)^2 = \frac{\delta^2 + \beta^2}{(2-\delta)^2 + \beta^2} \tag{1.29}$$

Using the values given in Table 1, and equation 1.4, gives, for a wavelength of $3.5nm$, $R_n = 3.3x10^{-6}$ for carbon and $R_n = 4.6x10^{-5}$ for gold. Normal incidence reflectivity are very small for all material over soft X ray range, which means that conventional mirror used in this way are impracticable.

## 1.4 Enhancement of Reflectivity

The reflectivities of surfaces for X rays wavelength may be increased by using grazing angle incidence. If, in equation 1.26 and 1.27, the glancing angle is such that

$$(\bar{n}^2 - \cos^2\theta)^{\frac{1}{2}} = 0 \tag{1.30}$$

then the reflectivity is identically equal to unity. For a non absorbing medium $(\beta = 0)$, total reflection is obtained for glancing angles smaller than the critical angle $\theta_c$, where

$$\cos\theta_c = n = 1 - \delta \tag{1.31}$$

For real media, the reflectivity approaches unity as $\theta \to 0$. If $\beta << \delta$ a sharp increase in reflectivity is obtained as $\theta$ fall below $\theta_c$; for $\beta \sim \delta$, a more graduate transition occurs. For $\theta < \theta_c$ no wave can propagate in the mirror material and the incident energy is reflected ("total external reflection"). Calculated grazing incidence reflectivity (for s polarization) are shown for beryllium, carbon, and gold at $\lambda = 3.5nm$ in Figure 1. The p-polarization reflectivity not significantly different. A major problem with grazing incidence optics is their severe aberration, especially astigmatism that can be .

# Capitolo 2

# Mirrors for X-rays

*"Terence: Tu lo reggi il whisky?*
*Bud: Beh, i primi due galloni si, al terzo divento nostalgico e ci può scappare la lite...*
*E tu lo reggi?*
*Terence: Eh, che domande, io sono stato allattato a whisky!"*

I due superpiedi quasi piatti

A spherical surface is defined by only one parameter, the radius of curvature of the surface. A spherical surface has the property that the rate of change of the surface slope is exactly the same everywhere on the surface, and thus the aberration is inevitable. This shape bring an intrinsic aberration ("spherical aberration"). If a beam having rays parallel to the optical axis, with a big aperture, hits a concave mirror, as shown in Figure 3, the rays will not focus in the same point "focus", but they converge at the circumference of a circle. So, the image, is not any more a single point but a spot, beams at different position have different focal points. Consider ray AB parallel to the optical axis at a distance d from it

After the reflection of the mirror AB intercept the optical axis at the point $F'$ from the origin O. The relation between $F'$, the radius of the mirror R and the distance d is

$$\frac{f'}{R} = 1 - \frac{1}{2\sqrt{1 - \left(\frac{d}{R}\right)^2}} \tag{2.1}$$

Equation **??** can be obtained by Figure 4. $BF'$ is the reflected ray of the non-paraxial ray AB, $F'$ is the intersection point with the optical axis. Because the law of reflection

$$\alpha = \beta \Rightarrow \alpha = \gamma \tag{2.2}$$

Therefore $F'BC$ is isosceles and $F'D$ is both median and height, thus

$$DC = \frac{R}{2} \qquad (2.3)$$

From the right-angle triangle $F'DC$ we obtain

$$\cos\gamma = \frac{R}{2F'C} \Rightarrow F'C = \frac{R}{2\cos\alpha} = \frac{R}{2\sqrt{1-\sin^2\alpha}} \qquad (2.4)$$

From the right-angle triangle $CGB$

$$\sin\alpha = \frac{d}{R} \qquad (2.5)$$

The last two equation combined with

$$OF' = OC - F'C \Rightarrow f' = R - F' \qquad (2.6)$$

When $\frac{d}{R} \to 0$ then $\frac{f'}{R} = \frac{1}{2}$, therefore

$$f' = \frac{R}{2} \qquad (2.7)$$

and the mirror is ideal

If the slope in not any more constant all over the mirror but become flatten in the region surrounding the outer rays, it is possible to focus all the rays in the same point. While correction of spherical aberration is not the only application of aspherical surfaces, it is one of the major application areas. On the contrary from spherical surface, aspherical surfaces cannot be defined with only one curvature, this kind of aspherical surfaces are usually defined by an analytical formula. There exist different kind of aspherical surface with different properties, if there is not the rotational symmetry it is possible to have either a *biconic* surface with two basic curvature and two conic constant in two orthogonal direction or as an *anarmorphicasphere*, which has additional higher-order terms in two orthogonal directions. Another form of aspheric surface is a *toroidal* or *toric*. This shape is a surface of revolution with a hole in the middle, like a doughnut, forming a solid body. This shape is caraterized by two radii, the overall outer radius and the smaller cross-sectional radius. The good point of this geometry is the minimization of astigmatism that make it possible to focus on a small spot, differently form the spherical ones.

## 2.1 Conic Surfaces

A special kind of aspherical surfaces is those named "*conicsurfaces*" that can be defined as

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} \tag{2.8}$$

where $c$ is the base curvature at the vertex, $k$ is a conic constant, and $r$ is the radial coordinate of the point on the surface. In Table 2.1 it is shown how the different shape are obtained changing the conic constant $k$

| Conic Constant k | Surface Type |
|---|---:|
| 0 | Sphere |
| $k < -1$ | Hyperboloid |
| $k = -1$ | Paraboloid |
| $-1 < k < -0$ | Ellipsoid |
| $k > 0$ | Oblate Ellipsoid |

*Tabella 2.1: Parameter of different conic surfaces*

A good point that have the conical surfaces are the no-presence of spherical aberration. If the object is at the center pf curvature of the surface there are no aberration. Considering an ellipsoid, it is possible to form aberration-free-image for a pair of real image, similar to a hyperboloid mirror. For parabolic mirror there is only one point that make a perfect image of a point for an axial object at infinity. This parabolic behaviour is the main point that make those mirror widely used in astronomical optics. Moving axially the object from the free-aberration point induce a certain amount of spherical aberration: If the movement is laterally, different type of aberration are induced such as: coma, astigmatism.

## 2.2 Compound Optical system

A further step to obtain a better image is that to use a more than one mirror in order to have o perfect image at the focus. The system that whose invented which respect the sine-Abbe-conidtion are the wolter system that are widely used in astronomy, using combinations of coaxial and confocal conic section.A first approximation system that respect the sine Abbe condition are the Kirkpatrick-Baez system and Montel or nested-Kirkpatrik-Baez system, those compound optical system involves reflector whose meridian planes are at right angle (crossed).

### 2.2.1 Sine Abbe condition

### 2.2.2 Wolter System

In 1952 Wolter published a paper in which he discussed several disposition of two conical mirror in order to collect light for an astronoical use. Figure

show the different disposition discussed: Wolter I, Wolter II, Wolter III. Wolter I telescope consist of a coaxialparaboloid (primary mirror) and hyperboloid (secondary mirror). The focus of the paraboloid is coincident with the rear focus of the hyperboloid, and the reflection inside both mirrors. The Wolter II telescope use the same kind of mirror of Wolter I paraboloid and hyperboloid. But the focus of the paraboloid coincident with the front focus of the hyperboloid, and, the reflection, occurs internally for the paraboloid and externally for the hyperboloid. The Wolter III telescope consist in a paraboloid and an ellipse. In this system the first mirror is the paraboloid one, and the second is the ellipsoidal that have front focus coincident with that of the parabola, moreover the reflection is external for the paraboloid and internal for the ellipsoidal. The Wolter I have typical grazing angle of less than a degree and is used for hard X-rays. The Wolter II telescope has typical grazing angle of, approximate, 10 degree and is used for soft X rays and extreme ultraviolet (EUV). Because of circular symmetry, astigmatism and spherical aberration are eliminated but exhibit coma aberration. Other problem is the difficulty of fabrication , and require a huge area to achieve a very small collecting angle.

### 2.2.3 Kirkpatrick-Baez System

This kind of optics are used in the ESRF and consist, as shown in Figure, in two separated cylindrical surface conical mirror that focus the incident beam in both saggital and transverse thus astigmatism is removed. Although such system introduce another type of distortion, anamorphotism. Because of the different distance of the image plane with respect to the mirrors the magnification is different in the two direction. Another technical problem that face with system is the big volume that occurs to implement it. To overcome those two problem and obtain a system that conjugate the good behaviour of the KB system with an equal magnification of the two direction and compact system, it is possible to implement a system as it is showed in Figure, a system in which both mirrors are at the same distance from the object. This sort of arrangement is extremely difficult to manufacture and, consequently, very expensive. Despite these problem K-B system are very used in ESRF and in European synchrotron, on the contrary, in American synchrotron another type of optical system, named "Montel", is used that will be discussed in the next section.

# Capitolo 3

# Montel System

As discussed before KB system have some limitation that can be overcome with a different optical system named "Montel". This geometry bring four important advantages for high-precision focusing:

i) the optical system is more compact which allow greater working space;

ii) the focal distance of the two mirror are the same, this cancel out the anamorphotism;

iii) the alineation of the system is easier with respect to the KB system because, in this case, only one thing has to be aligned, on the contrary, in the KB there are two separated mirror that has to be aligned;

iv) the divergence that can be collected is larger which allows for greater flux and/or a lower diffraction limit.

## 3.0.1   Optical Design

The mirrors used in this Montel configuration are mirror that have a cylindrical shape in one direction and elliptical shape in the other direction. One approach to obtain the Montel system is that to use two pre-figured elliptical mirror and grind the cut site at 45° as shown in figure. After that it place the mirrors together makes a good fit with no gap requiring no contouring of the mirror side. Another way involves diveding pre-figured elliptical mirror into two part that, add them together, can form the Montel system. This approaches is primary driven by the fact that in a conventionally polished mirror, the clear aperture area has the best figure and

finish. As such uAs such, using two halves of a prefigured mirror cut in the middle has several advantages- including consistency and economy. There are major challenges however. First, the mirror surface must be protected against damage and deformation during cutting and subsequent figuring operations. After cutting into two, the cut sites must be treated (e.g., etched) to remove any subsurface damages that could alter a mirror's figure. Then the mating side of one of the mirrors must be contoured and polished such that when it is placed against the partner mirror, it makes a nearly perfect fit with good surface quality all the way to the contact edge.This last two-steps are crucial because if there is a significant gap or if the mirror surfaces in the vicinity of the interface are damaged, a significant part of the incident beam could be lost. As an example, we are developing a pair of Montel mirrors for polychromatic nanofocusing on Sector 33 at APS. This beam line will use 40 mm long elliptical mirrors for nano-focusing a 100 $\mu m$ beam to a 50 nm spot at 2000x demagnification. This concave elliptical mirror has a maximum depression of about 6 $\mu m$ at its center. If cut flat and placed against its mating mirror, a gap as large as 6 $\mu m$ is created which loses about 10% of the 100 $\mu m$ incident beam. Similarly, if the mirror surfaces near the intersection are damaged, then beam loss can be significant.

# Capitolo 4

# Mt script

*"Bud: No, calma, calma, stiamo calmi, noi siamo su un'isola deserta, e per il momento non t'ammazzo perché mi potresti servire come cibo ..."*

Chi trova un amico trova un tesoro

My thesis' work is the creation of a python script that simulate a ray-tracing of a beam. This tracing take in account the effect of different type of optical elements that that can find the beam in its way. To implement this ray-tracing it have to define three elements:
- source
- optical elements
- tracing system

## 4.1   Source

The beam source is characterized by two parameters: spot dimension divergence. For the geometry dimension is possible to choose between a spot of uniform ray in a rectangular or circular shape or a gaussian profile with a parameters setted by the users (radius for the circular case, two sides for rectangular shape and FWHM for Gaussian profile), there is the possibility to choose a point-wise shape for the source. Also the divergence profile have different possibilities, there is the Gaussian profile, with FWHM as parameter, rectangular flat divergence, with two side as parameters, and a collimate one. for the setting parameter of the source shape there are no limitation, but for the divergence parameters, because of the normalization of the velocity, be careful on the parameter's values. Figure 1 shows an example of circular shape for the source beam, and Figure 2 shows a gaussian profile. It is

choose a red marker for the plot of the real space, and blue marker for the plot of the divergence.

## 4.2 Optical Elements

There are implemented two kind of optical elements in the script: lens and mirror. Because, as discussed in Chapter 1, mirrors are the principal elements used in synchrotron, more attention is focused on them, on the contrary , for testing uses, only one kind of lens, an ideal lens, is implemented.

### 4.2.1 Mirrors

It is possible to choose a mirror that have shape of: plane, sphere, ellipse, parabola and hyperbola. All this mirror to have a conical shape in two dimension, or a conical shape in one dimension and cylindrical in the other, moreover is possible to set a dimension of the mirrors that, in the default case, are setted as infinite mirrors. The mirrors are expressed in a Cartesian defined as surface conical equation with a characteristic focal parameter depending on their nature, for plane mirror no parameter are needed, for spherical mirror only one focal distance is needed, for parabolic mirror is needed one focal distance plus a term that define the nature of the mirror and, for ellipsoidal and hyperbolic mirror, two focal distance are needed, moreover, apart for the plane mirror, also the incidence angle is needed to determine the shape of the mirror. The surface conical equation is determined such that the origin of the cartesian system correspond to the point where the optical axis hit the mirror and the normal correspond to the z-axis. To understand better the situation look at Figure 2, that report an ellipsoidal mirror characterised by two focal distance equal to p and q, and an angle of incidence equal to $\theta$, and also, it is possible to see the dimension of the mirror.
FIGURE
In the appendix A is reported the calculation of the changing from system solidal with the mirrors.

### 4.2.2 Lens

As said before the only lens defined is an ideal lens that bring the entrance beam and, following the typical equation 4.1 of a lens, do the correct transformation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{4.1}$$

As reported in equation 4.1, the parameter needed to characterize the lens is $f$, in the 3D system, a lens is defined by two parameter $f_x$ and $f_z$, that define the focal

distance for the two direction and, those parameter, are the parameter that the users have to define in order to use the ideal lens in the program.

## 4.3   Tracing System

Defined the source and the different optical element, to have a simulation, is needed a tool that put everything together and modify the property of the beam after the interaction with the optical elements.

Before to trace the system is important to define the system, in other word is important to locate the position of the optical elements. The program is written in such a way that the tracing system work in series one optical element after the other, and, for every element, it have to define the object plane distance and the image plane distance, that can be different from the focal distance which are defined as default. For example, if we want to simulate a system such that reported in figure 3, we have to define firstly the mirror and after there are different possibilities to define the tracing distance, one possibilities is that.

Figure Going deeper in the code, the algorithm that trace a single element is divided in 5 step

1. change the reference system with that soldal with the optical element after two rotation, one along x-axis, and second along y-axis, and a translation equal to the object distance of the optical element

2. free propagation up to the optical element

3. effect of the optical element

4. free propagation to the image plane

5. changing the Cartesian system in that one that have the optical axis equal to the y-axis

   FIGURE

## 4.4   Compound Optical Element

This program include also two different system composed by more mirrors. Starting from more surface conical mirror, combining them, is possible to have a compound optical elements that can simulate the behaviour of some typical instrumentation that characterize the facilities, in particular the synchrotron environment. The compound optical include are two of those mentioned in Chapter 2

- KirkPatrickBaez system (KB system)

- Montel

### 4.4.1   KirkPatrickBaez System

KirkPatrickBaez or, more simply, KB system are shown in Figure 2 are two cylindrical surfacing conic mirror placed one after the other with the two focal lens that converge in the same point. There are implemented two different kind of KB system, a one composed by elliptical mirrors and a second one composed by parabolic mirrors. The parameter parameter that needed to the program that define the system are two focal distances and the two incidence angle that define the two surface conic, in the default mode the two angle are identical.
FIGURE
Because this system is simply a system composed by two surface conical mirror in series the parameter that the system need to define the mirror are no the ones defined by the user but are the focal distance of the two mirrors that are, as shown in Figure, p1, q1, p2, q2. The parameter p and q are, respectively, the distance of the first focal point to the center of the system, and the distance between the center of the system to the second focal point. This system can do two different thing, the first one is to focalize a beam in a point, the second one is to collimate a beam, obviously this work if the system is defined with the correct parameters.

### 4.4.2   Montel System

FIGURE
This compound optical system is defined, as defined for KB, from the definition of two cylindrical conical surface rotating, one of them, of $90°$, in order to have a mirror in the xy plane, and another one in the zy plane. As shown in Figure 2 the center of the Cartesian system is setted in the point where the optical axis of the system hit the compound system having the normal of the first normal equal to the z-axis, and the second normal equal to the -x-axis. The system is defined by the following parameter p, q, $\theta_z$, $\theta_x$, where p and q are the focal distance of the two mirrors and $\theta_z$ and $\theta_x$ are the angle of incidence to define the correct mirrors (by default $\theta_z = \theta_x$).
Another parameter that can be setted to this system is the value of the angle that there is between the first optical element and the second one. As for the KB system also in this case there are the two cases, an ellipsoidal system (having the two mirror as ellipsoid), and parabolic system (having the two mirror as ellipsoid). The aim of the Montel, as for the KB, is that to work on the two dimension of the beam

for each mirror, such that to localize the system on the same point, or collimate the beam in the two direction.

## 4.5 Tracing system for compound optical elements

Because of the different definition, the tracing method of the rays' beam, need a different interpreter that can link the beam with the different optical elements that meet on his way. Because of the different nature, there are implemented two kind of tracing, a first one that trace the KB system, that is composed by a series of optical elements and so can be used for all the compound optical elements that are in series. And a second one that is specific for the Montel system, because it is not composed by mirrors in series rather than mirrors in parallel, having the two elements in a very small region of the space that have in which order the rays of the beam hit the different mirrors.

### 4.5.1 Tracing for KB

For KB system the situation is more or less the same as for a simple optical mirrors, with the only difference that there are more than one mirror. So the algoritm to simulate the tracing system is nothing else than a for loop, that use the tracing system of the simple optical element. In this case is needed the object and the image distance from the center of the system, as for the optical element tracing, that are set by default as the focal distances, and the incidence angle of the two mirror, that, by default, are the ones in which the mirrors are designed. As shown in Figure, having the object and image distances, and using the trace elements defined for one mirror, there is one degree of freedom, that is the
Figure (sistemare bene questa parte qui)

### 4.5.2 Tracing for Montel

Montel system is completely different from the KB system and all the series optical system, so it need a new trace system. This new trace system is divided as follow

1. Changing the reference frame in one having the center on the center of the mirrors, with a z-axis corresponding to the normal of one mirror and -x-axis equal to the normal of the second mirror. This transformation is done in a similar way of the normal tracing, two rotation of the beam, and one translation, differently from the normal tracing, the tw rotation are done such that the beam hit the mirrors with an incidence angle set by the user

2. Focus the attention on the travel time of each ray in order to know which is the nearest optical element of each ray

3. free propagation of each ray up to the nearest optical element

4. effect of the system for each ray

5. repeat the $2^{nd}$, $3^{rd}$ and $4^{th}$ passage two times, in order to consider the two reflection

6. Change the reference system solidale with the beam that is subject to two reflection, doing two rotation and one translation

What is reported above is the default tracing system that, because of its centrality on my thesis' work have many option. What is setted by the user is

1. focal distances and incidence angles, that define the two rotation and the translation of the tracing system

2. name of the File in which is saved the data of the simulation, by default no data is saved

3. there is the possibility to choose a different point, from the origin, in which the the optical axis hit the system

4. there is also the possibility to have a final output frame that is not solidal by the two-reflected beam, but with the non reflected beam or with the other two beam that are reflected only one time

5. It is also the possibility to figure out the footprint of the two reflected beam on the system. For clarity the beam that hit the first mirror and after the second is labelled with red point, the beam that hit the second and after first mirror is labelled by blue color

These options are added in order to study better the behaviour of a beam with a Montel system. The possibilities to change the angle of incidence and to hit different part from the origin can be used to study what happen to a beam when is not aligned, or not perfectly aligned, and use these result to align the system in the laboratories. The possibilities to save a File is useful in particular in those case where there is a huge computational effort that need a lot of time, in these cases is possible to work with the result of a big simulation without reappointing it, and so save time.

# Capitolo 5

# Results

*"Terence: Ma scusa di che ti preoccupi, i piedipiatti hanno altro a cui pensare, in questo momento stanno cercando due cadaveri scomparsi*
*Bud: Se non spegni quella sirena uno di quei due cadaveri scomparsi lo trovano di sicuro!"*

Nati con la camicia

A comparison of the simulation with respect to the software OASYS developed by Manuel Sanchez Del Rio and with a paper in order to demonstrate the correct operation of the program. The comparison with OASYS check the work of all the component apart from Montel system (mirrors, lens, KB ...), the paper is dedicate for the Montel system, because this particular kind of optical system is not implemented on OASYS.

## 5.1   Testing with OASYS

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# Capitolo 6

# Realizzazioni sperimentali e valutazione

*"Bambino: Questo ï¿½$\frac{1}{2}$ l'ultimo avviso per voi e i vostri rubagalline*
*Il pistolero si alza: Che avete detto?*
*Bambino: RUBAGALLINE*
*Il pistolero si risiede: Aaah."*

Lo chiamavano Trinità . . .

Si mostra il progetto dal punto di vista sperimentale, le cose materialmente realizzate. In questa sezione si mostrano le attività sperimentali svolte, si illustra il funzionamento del sistema (a grandi linee) e si spiegano i risultati ottenuti con la loro valutazione critica. Bisogna introdurre dati sulla complessità degli algoritmi e valutare l'efficienza del sistema.

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# Capitolo 7

# Direzioni future di ricerca e conclusioni

*"Terence: Mi fai un gelato anche a me? Lo vorrei di pistacchio.*
*Bud: Non ce l'ho il pistacchio. C'ho la vaniglia, cioccolato, fragola, limone e caffé.*
*Terence: Ah bene. Allora fammi un cono di vaniglia e di pistacchio.*
*Bud: No, non ce l'ho il pistacchio. C'ho la vaniglia, cioccolato, fragola, limone e caffé.*
*Terence: Ah, va bene. Allora vediamo un po', fammelo al cioccolato, tutto coperto di pistacchio.*
*Bud: Ehi, macchï¿$\frac{1}{2}$ sei sordo? Ti ho detto che il pistacchio non ce l'ho!*
*Terence: Ok ok, non c'ï¿$\frac{1}{2}$ bisogno che t'arrabbi, no? Insomma, di che ce l'hai?*
*Bud: Ce l'ho di vaniglia, cioccolato, fragola, limone e caffé!*
*Terence: Ah, ho capito. Allora fammene uno misto: mettici la fragola, il cioccolato, la vaniglia, il limone e il caffé. Charlie, mi raccomando il pistacchio, eh."*

Pari e dispari

Si mostrano le prospettive future di ricerca nell'area dove si è svolto il lavoro. Talvolta questa sezione può essere l'ultima sottosezione della precedente. Nelle conclusioni si deve richiamare l'area, lo scopo della tesi, cosa è stato fatto,come si valuta quello che si è fatto e si enfatizzano le prospettive future per mostrare come andare avanti nell'area di studio.

xxx

# Bibliografia

# Appendice A

# Documentazione del progetto logico

Documentazione del progetto logico dove si documenta il progetto logico del sistema e se è il caso si mostra la progettazione in grande del SW e dell'HW. Quest'appendice mostra l'architettura logica implementativa (nella Sezione 4 c'era la descrizione, qui ci vanno gli schemi a blocchi e i diagrammi).

# Appendice B

# Documentazione della programmazione

Documentazione della programmazione in piccolo dove si mostra la struttura ed eventualmente l'albero di Jackson.

# Appendice C

# Listato

Il listato (o solo parti rilevanti di questo, se risulta particolarmente esteso) con l'autodocumentazione relativa.

# Appendice D

# Il manuale utente

Manuale utente per l'utilizzo del sistema

# Appendice E

# Esempio di impiego

Un esempio di impiego del sistema realizzato.

# Appendice F

# Datasheet

Eventuali Datasheet di riferimento.