Modern Physics

Laboratory Experiment

"Compton Scattering"

Figure 1: Compton Scattering

Boston University International Program

Technische Universität Dresden

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COMPTON SCATTERING

Determination of the Energy E$_\gamma$' of Scattered Photons and the Differential Scattering Cross Section (dσ/dΩ) of the Compton Scattering in Dependence on the Scattering Angle Θ

PURPOSE

In this experiment, the effects of Compton scattering shall be investigated, which are
- the shift in energy when the photon is scattered to an angle θ and

- the differential scattering cross section $\dfrac{d\sigma}{d\Omega}(\Theta)$ which is a measure for the probability of the process to occur.

INTRODUCTION

The collision of a photon with a free electron (Compton scattering) can be described as follows: a photon of an initial energy E$_\gamma$ = h·f is scattered off an electron (see fig. 1).
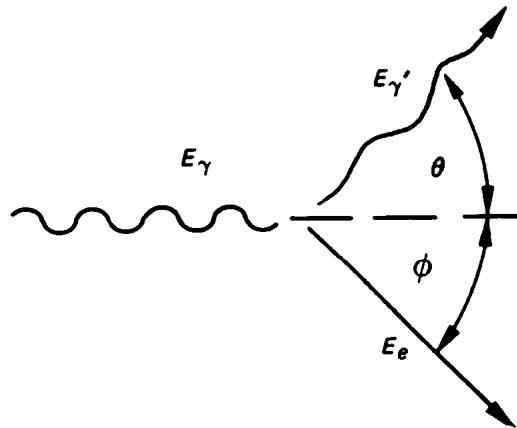
Fig. 1 **Scattering Caused by Compton Interaction.**

The energy of the impinging photon is shared after the collision among the interacting particles according to kinematic equations obtained from the laws of energy and linear momentum conservation. Then the energy of the scattered photon E$'_\gamma$ = h·f$''$ is given by eq. 1.

$$E'_\gamma = h \cdot f' = \frac{E_\gamma}{1 + \dfrac{E_\gamma}{mc^2}(1 - \cos(\Theta))} \tag{1}$$

where mc^2 = 511 keV is the electron rest energy and Θ is the scattering angle of the photon in the laboratory system. The probability of the photon being scattered to a particular angle Θ is proportional

to a quantity called differential cross section $\frac{d\sigma}{d\Omega}(\Theta)$. The rate $I_\gamma(\Theta)$ of scattered photons per second into a solid angle $d\Omega_{Det}$ subtended by the detector at angle Θ is

$$I_\gamma(\Theta) \cdot d\Omega_{Det} = I_{\gamma 0} \cdot N \cdot \frac{d\sigma}{d\Omega}(\Theta) \cdot d\Omega_{Det} \tag{2}$$

where N is the number of electrons in the scattering target and $I_{\gamma 0}$ is the number of incident photons per cm^2 and s at the scattering sample which is correlated to the activity of the photon source used.

A theoretical description of the differential cross section for Compton scattering was first proposed by Klein and Nishina. This formulation is given in eq. 3 and describes the scattering of a photon off a single electron

$$\frac{d\sigma}{d\Omega}(\Theta) = \frac{r_0^2}{2}(\frac{E_\gamma^{'}}{E_\gamma})^2(\frac{E_\gamma}{E_\gamma^{'}} + \frac{E_\gamma^{'}}{E_\gamma} - \sin^2(\Theta)) \tag{3}$$

with the so-called classical electron radius $r_0 = 2.82 \cdot 10^{-13}$ cm. According to eq. 2, the count rate $\dot{N}$ in the full energy peak (see below) at a scattering angle Θ is given by eq. 4

$$\dot{N}(\Theta) = \int I_\gamma(\Theta) \cdot \eta \cdot d\Omega_{Det} \tag{4}$$

where η is the registration efficiency of the photon detector.

If a given experimental condition (detector properties and scattering geometry set up, see fig.2) the count rate of the photo peak is kept fixed, the photo peak is directly proportional to the differential cross section for Compton scattering

$$\dot{N}(\Theta) \propto \frac{d\sigma}{d\Omega}(\Theta) \tag{5}$$

A multi-channel analyzer is used for photon spectrometry. It must be calibrated in order to determine the energy of the scattered photons at a given scattering angle. The count rate of the photo peak can be used to determine the differential scattering cross section.

An absolute measurement of the cross section requires an accurate knowledge of all experimental parameters, such as geometry, source strength (activity) and registration efficiency of the photon detector. Since these data are not well-known, normalized to data at $\Theta = 90°$ should be used for simplification. Then eq. 5 yields

$$\dot{N}(\Theta) / \dot{N}(90°) = \frac{d\sigma}{d\Omega}(\Theta) / \frac{d\sigma}{d\Omega}(90°) = \frac{d\sigma}{d\Omega}_{rel}(\Theta) \tag{6}$$

Inserting eq. 1 into eq. 3, the Klein-Nishina formula for the differential Compton scattering cross section becomes

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{r_0^2}{2}\left\{\frac{1+\cos^2\theta}{\{1+\varepsilon\cdot(1-\cos\theta)\}^2}\right\}\cdot\left\{1+\frac{\varepsilon^2\cdot(1-\cos\theta)^2}{\left(1+\cos^2\theta\right)\left[1+\varepsilon\cdot(1-\cos\theta)\right]}\right\} \tag{7}$$

with $\varepsilon = E_\gamma/(m \cdot c^2)$. This equation can be used to calculate the absolute differential cross section without knowledge of any experimental detail.

All experimental data obtained within the course of this experiment have to be compared to the theoretical predictions according eqs. 1, 3 and 7.

PROCEDURE

ENERGY CALIBRATION

The multi-channel analyzer sorts the pulses delivered by a photon detector, in this case a NaI(Tl) crystal, into channels according to the pulse height. This pulse height distribution reflects the energy distribution of the electrons which are produced in the detector by the incident photons. Using low energy photons (see table 1), the main photon interaction is the photo effect, producing the so-called full energy peak (or photo peak) in the pulse height distribution.

The power supply of the photo-multiplier tube should be set at about 600 to 800 V whereas the gain of the amplifier should be fixed at the maximum value. For these settings, the dynamic range of the multi-channel analyzer is optimally adjusted and the photo peak of a 60 keV photon corresponds roughly to channel 600 and, therefore, to a channel width of 0.1 keV per channel.

The energy calibration of the detector is performed by using 5 different sources of low energy photons (see table1). Obtain spectra of these sources and determine the channel number which corresponds to the center of gravity of the photo peak. Plot channel number vs. photo peak energy and fit the data by linear regression

$$E_\gamma = a + b \cdot C \tag{8}$$

in order to find the the relation between channel number C and photon energy E_γ. Take the spread of energy of the emitted photons into account and estimate the uncertainty of the peak channel number. Include the error bars in the plot and take these errors into account in the regression analysis.

ENERGY SHIFT

Measure the energy E'_γ of the scattered photons. A collimated beam of 59.54 keV photons is emitted from a 7 GBq ^{241}Am source.These photons are scattered off a 6 mm in diameter cylindrical sample of Aluminium (scattering target). As shown in fig. 2, the source should be turned around the axis where the scattering target is positioned. Set the detector at 5 different angles between 30^0 and 150^0 including 90^0 and take the spectra at each angle for a measuring time of 10 minutes.
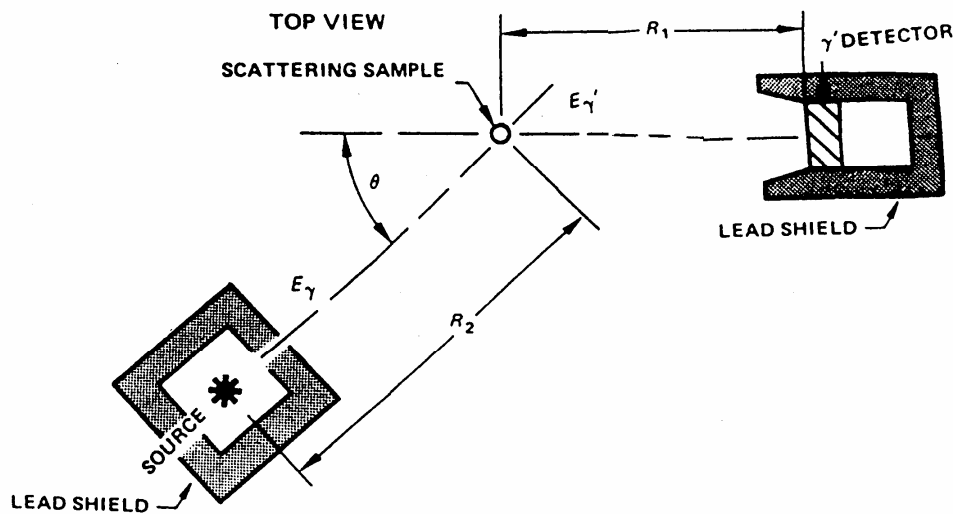

Fig. 2: Compton Scattering experimental set up

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Locate the center of gravity of the photo peak and determine its energy. Plot the photon energy vs. scattering angle and compare the experimental results with the theoretical one according to eq. 1.

DIFFERENTIAL CROSS-SECTION

At each angle, the scattering spectrum is composed of two components, the real effect from photon scattering in the scattering sample and contributions from photons scattered in air or back-scattered from the walls of the laboratory. So $\dot{N}_{E+B}(\Theta)$ is measured. Therefore, parallel to this spectrum a second spectrum at the same angle and measuring time should be taken with removed scattering target (background measurement, $\dot{N}_B(\Theta)$). These background spectra are necessary for correcting the effect spectra according

$$\dot{N}_E(\Theta) = \dot{N}_{E+B}(\Theta) - \dot{N}_B(\Theta) \tag{9}$$

Both count rates can be obtained from the photo peak area which has to be defined within a range of channels (region of interest ROI).

Normalize the corrected effect count rates to the one taken at 90^0 using eq. 6, plot these data and compare them with the respective normalized calculated ones using eq. 7. Take into account the spread in angle Θ caused by finite beam size, target extension and detector area and estimate the statistical errors for the net count rate. Include the error bars in the plot.

An independent method to determine the absolute differential cross section exists by using the measured energies of scattered photons and insert into eq. 3. These results can be compared with the theoretical prediction using eq.7 again and plot the data.

PREPARATIVE TASKS

- study the relevant chapter 2 of the text book "Modern Physics" by Serway, Moses and Moyer.
- derive eq. 1 in your lab report
- check eq. 2 for consistency
- derive eq. 7 proceeding from eq. 1and eq. 3
- calculate $E_\gamma'(\theta)$ according to eq. 1 for an incidence energy E_γ=59.54 keV
- calculate the differential cross-section for Compton scattering according to eq.7 and normalize to 90^0
- derive all equations (eqs. 10 to 13) in the attachment by yourself

Nuclide	$E_{\gamma i}$ (keV)	$v_{\gamma i}$ (%)	Weighted average $E_{\gamma i}$ (keV)
^{133}Ba	30,63	34,00	
	30,97	62,90	30,85
	81,01	32,75	-
^{137}Cs	31,82	1,92	
	32,19	3,70	32,06
^{152}Eu	39,52	16,00	
	40,12	30,00	39,91
^{210}Pb	10,80	24,30	-
	46,52	4,00	-
^{241}Am	59,54	36,3	-

Table 1: Photon source data for calibration

ATTACHEMENTS FOR PRE-LAB

A1   PHOTON DETCTION AND DETECTOR SIGNAL PRODUCTION

Study the operation of a scintillation detector with the components:
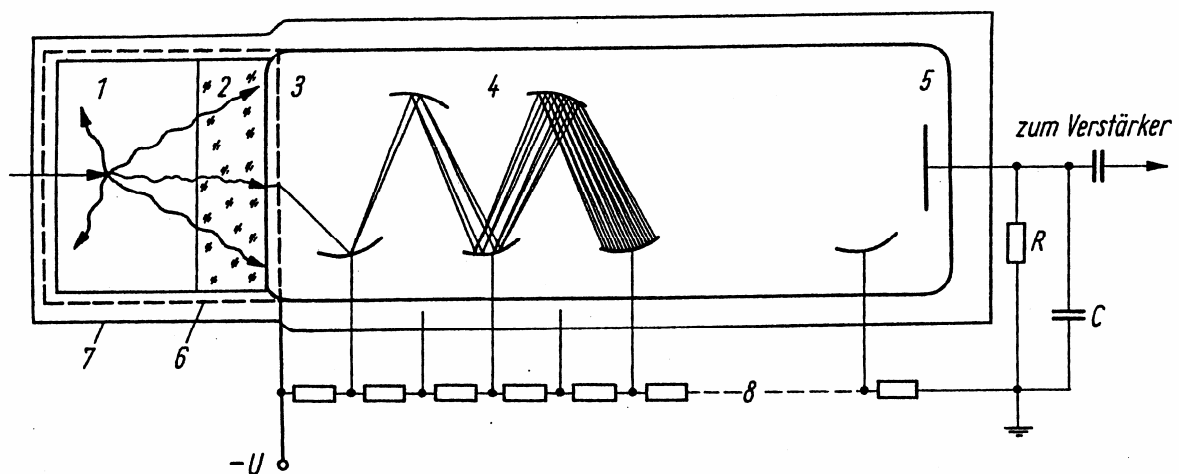
1.   scintillating crystal
2.   light pipe

the photo-multiplier tube with
3.   photo cathode
4.   dynodes
5.   anode

and the housing and electronic support
6.   light reflector
7.   cap impervious to light from environment
8.   power supply and R-C pulse shaping circuitry for signal processing by an amplifier

Fig. 3 Set up of a scintillation counter
(reproduced from W. Stolz, Radioaktivität, Teubner Verlag, 2005)

A2   SIGNAL PROCESSING BY MULTI-CHANNEL ANALYZER

Study the process of pulse height analysis and the sorting process into channels by a multi-channel analyzer
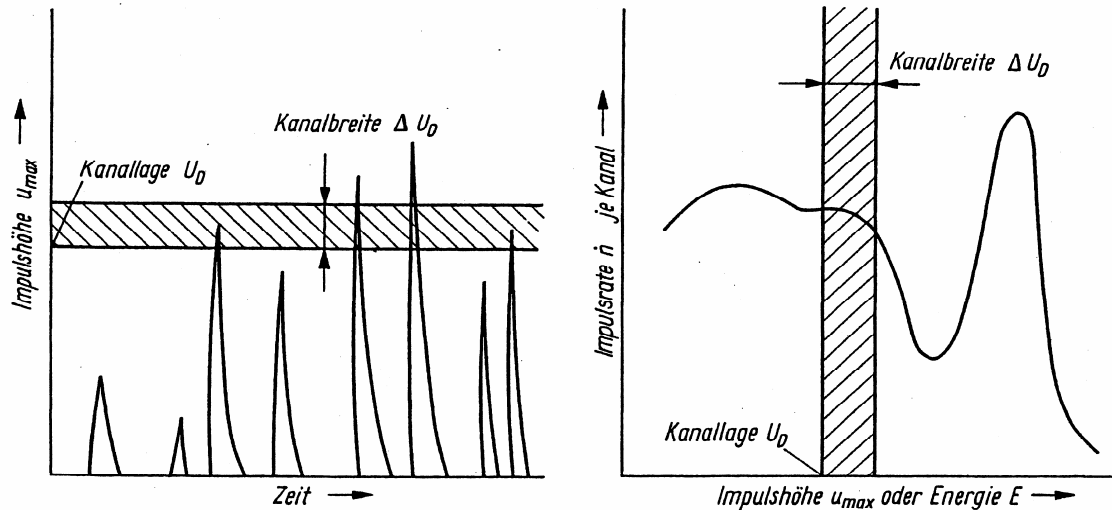
Fig. 4 Detector signal analysis by pulse height determination. The detector pulses occur randomly in time. Their pulse height corresponds to the photon energy released in the scintillation crystal. The channel width defines the energy resolution and can be adjusted by optimum settings of the scintillation detector parameters.
(reproduced from W. Stolz, Radioaktivität, Teubner Verlag, 2005)
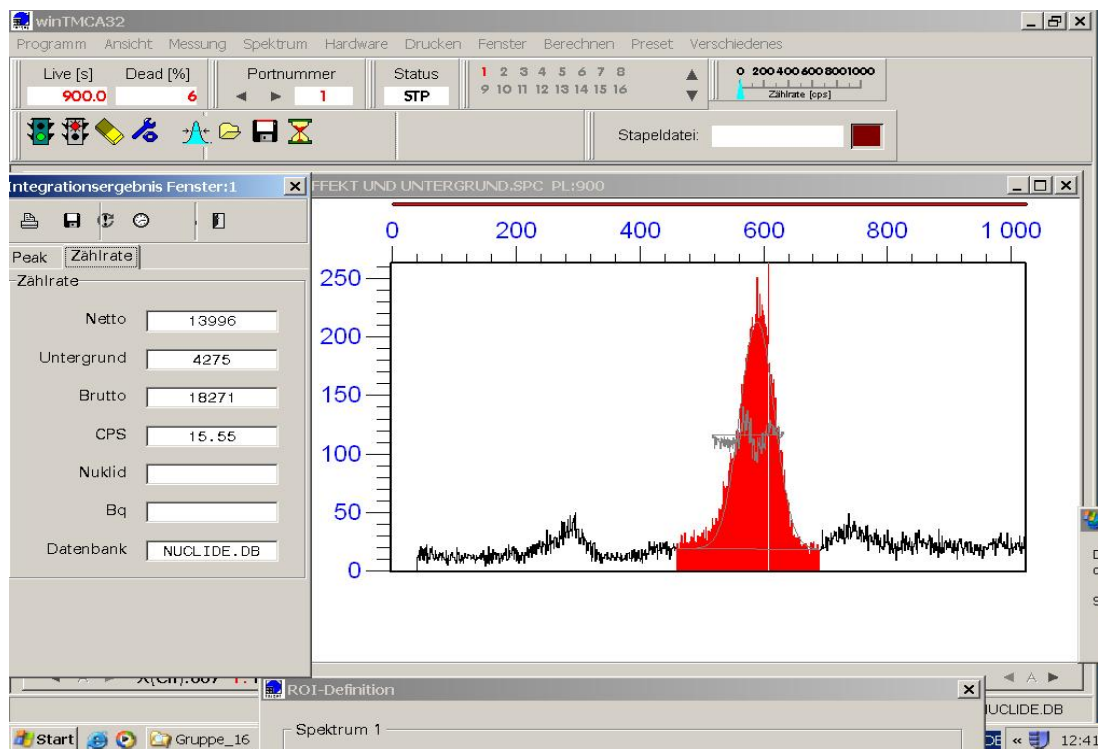
Fig. 5 Example for a pulse height distribution of photons from a ^{241}Am source taken with a multi-channel analyzer system The channel range for determination of the peak count rate (region of interest , ROI) is marked by the red area.
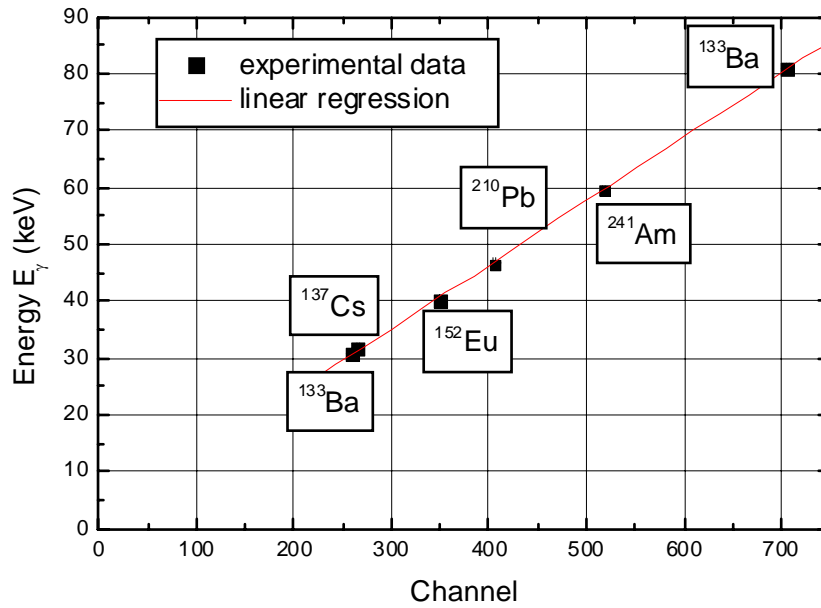
A3   ENERGY CALIBRATION

Fig. 6 Example for an energy calibration using photon sources from table 1

The regression analysis yields a linear relation between channel number of the peak center of gravity and the energy of the corresponding photons from different sources
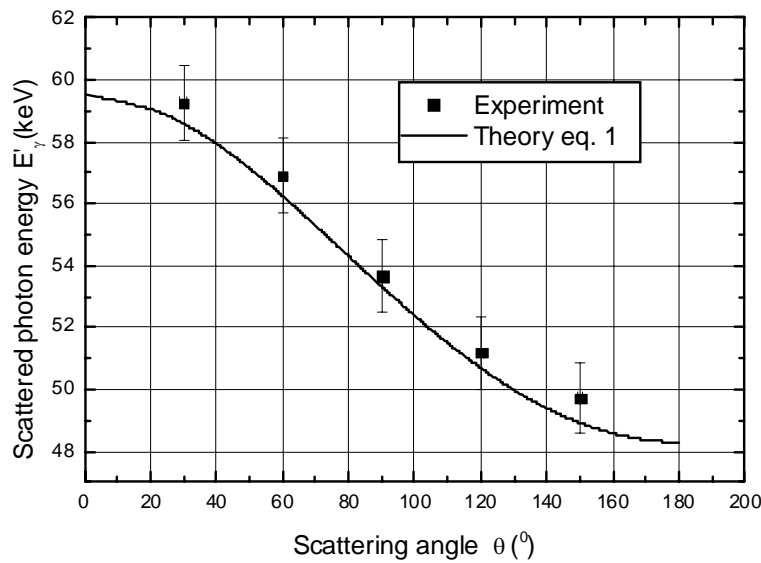
A4   ENERGY SHIFT

Fig. 7   Example for the measurement of the shift of incidence photon's energy after Compton scattering. The energy of scattered photons and their uncertainties have been calculated using eqs. 8 and 10 respectively. For comparison, the theoretically predicted angular dependence according eq. 1 is shown as smooth curve.

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A5   UNCERTAINTY ANALYSIS OF ENERGY DETERMINATION

According to eq. 8 the energy of a photon is determined by the linear relation $E_\gamma = a + b \cdot C$. The uncertainty of the measured energy ΔE_γ consists of two parts, the systematical and the random uncertainty $\Delta E_\gamma = \Delta E_{syst} + \Delta E_{rand}$. The systematical uncertainty is caused by the uncertainties of the coefficients a and b due to the numerical analysis of the calibration data set. Commonly, the uncertainty is estimated by the total differential of eq. 8 resulting in

$$\Delta E_\gamma = \Delta a + \Delta b \cdot C + b \cdot \Delta C \qquad (10)$$

with the uncertainty of peak maximum definition ΔC. For determining the regression coefficients a and b a numerically well-conditioned computer code should be applied. In the lab the code ORIGIN is recommended and available on the computers of the experimental equipment.

A 6   RELATIVE DIFFERENTIAL CROSS SECTION
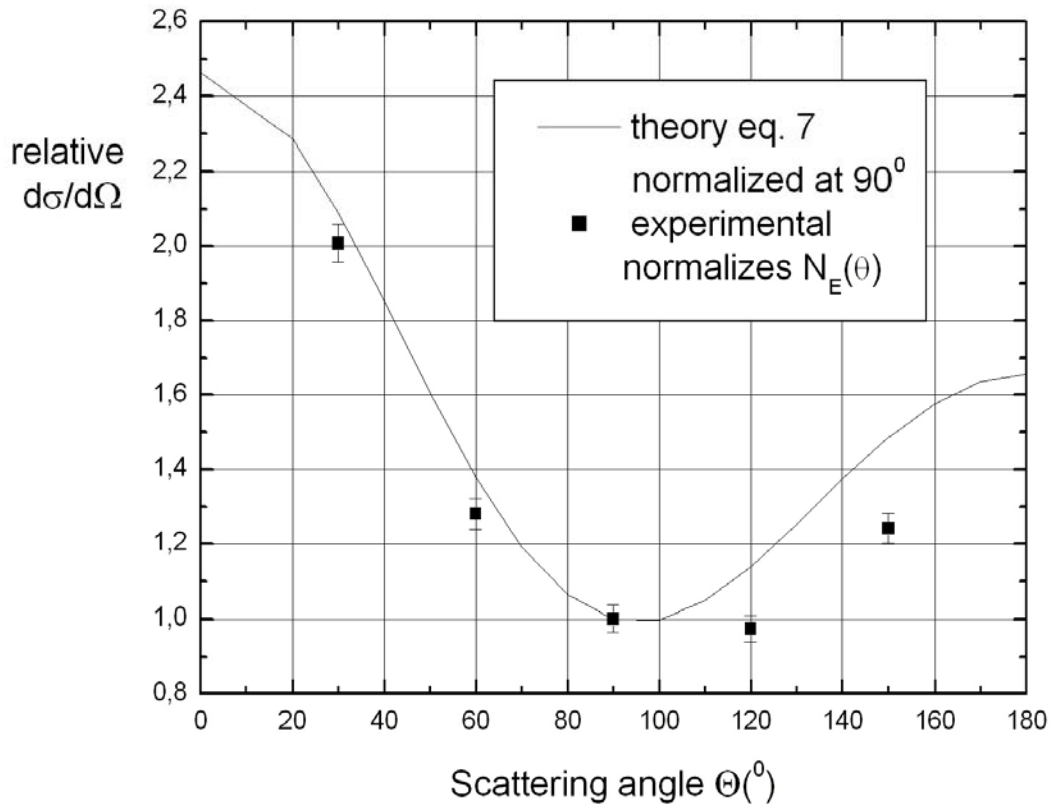
Fig. 8 Example for the measurement of the angular distribution of the relative differential scattering cross section using normalized effect count rates according eq. 6. The uncertainties have been calculated using eqs. 11 to 13. For comparison, the theoretically predicted angular dependence is calculated by eq. 7 and shown as smooth curve.

A7   ABSOLUTE DIFFERAL CROSS SECTION
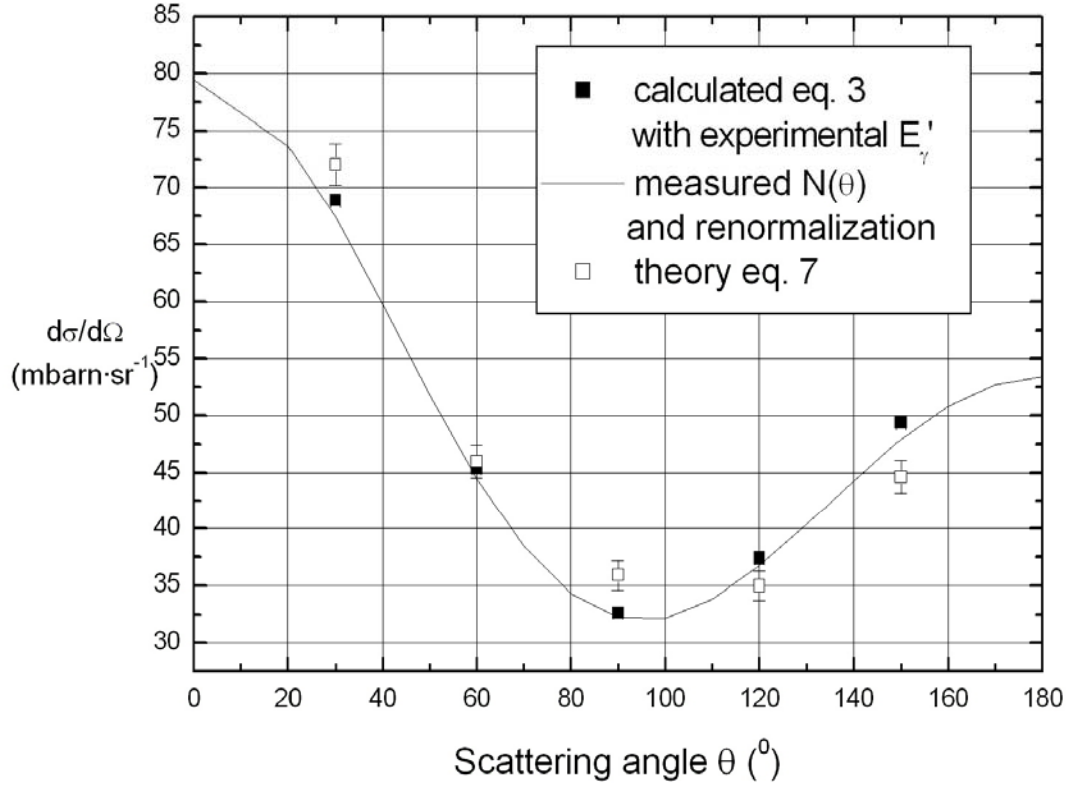
Fig. 9 Example for the measurement of the angular distribution of the absolute differential scattering cross section using both the energy of scattered photons and the re-normalized effect count rates according eqs. 3. The uncertainties have been calculated using eqs. 11 to 13. For comparison, the theoretically predicted angular dependence is calculated by eq. 7 and shown as smooth curve.

A8   UNCERTAINTY ESTIMATION OF CROSS SECTIONS

The estimation of the uncertainty of the differential cross section using the effect count rates $\dot{N}_E(\Theta)$ proceeds from the progression of uncertainties of statistical data (Gauss's error progression). Clearly, all count rates are randomly distributed data with statistical uncertainty. This yields an uncertainty $\Delta N = \sqrt{N}$ for a statistical quantity N. Because the effect count rate is a difference of two statistically distributed quantities $\dot{N}_E(\Theta) = \dot{N}_{E+B}(\Theta) - \dot{N}_B(\Theta)$ (eq. 9) then it follows with $\dot{N}_E(\Theta) = N_E / t$

$$\Delta \dot{N}_E(\Theta) = \sqrt{\Delta N_{E+B}^2 + \Delta N_B^2} / t \qquad (11)$$

with the assumption that the measuring time t is equal in both cases and has a vanishing uncertainty. Then we get

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$$\Delta \dot{N}_E(\Theta) = \sqrt{N_{E+B} + N_B} / t \qquad (12)$$

and finally for the incertainty of the relative cross section (eq. 6)

$$\Delta \left(\frac{d\sigma}{d\Omega}_{rel}(\Theta) \right) = \Delta \left(\frac{\dot{N}_E(\Theta)}{\dot{N}_E(90^\circ)} \right) = \frac{\Delta N_E(\Theta)}{N_E(90^\circ)} \qquad (13)$$

assuming again equal measuring times t for both spectra.

An estimation of the uncertainty of the absolute cross section calculated using eq. 3 and experimentally obtained energies of scattered photons is more difficult. It demands the derivation of the total differential of eq. 3 and the insertion of the errors $\Delta E_\gamma'$ calculated according eq. 10. For mathematically interested students this may be an useful exercise.

ATTACHEMENT REQUESTS FOR PROTOKOLL

Description of Experiment (Theory and Facility Set Up) (20 of 100)

1. Describe shortly the basic physics **(5)**
2. Describe shortly the gamma radiation detection method **(5)**
3. Describe shortly the experimental set up (source, scattering sample, detector and shielding) **(5)**
4. Describe shortly the electronics for photon spectroscopy **(5)**

Data Acquisition and Analysis (50 of 100)

1. Plot the photon energy versus the channel number C of each photon energy of each standard source; use data of Table 1;estimate the uncertainty in determination of the channel number and include the error bars in your plot. **(10)**
2. Determine the slope of the energy-channel relation by a linear regression of all data using a computer code or graphically. **(10)**
3. Plot the energy of the scattered photon versus scattering angle; estimate the uncertainties in energy and angle and include the error crosses in your figure; include your experimental results and compare with the theoretical prediction **(10)**
4. Plot the measured normalized cross sections versus the scattering angle; estimate the uncertainties of the normalized cross sections and the scattering angle and include both in your figure. Compare your results with the theoretical prediction **(10)**
5. Calculate the absolute cross sections by using experimental energies of scattered photons with eq. 3, plot your results and compare these with the theoretical predictions of eq. 7 **(10)**

Discussion of Results and Questions (30 of 100)

1. What is the meaning of the energy-intercept in your energy calibration plot **?(5)**
2. How can deviations between experimental results and the theoretical calculations be explained ? **(5)**
3. What are reasons for systematic errors in your experiment ? **(5)**
4. What are reasons for random (statistical) errors in the experiment ? **(5)**
5. How can the statistical error of a count number N be estimated (on a 1σ confidence level) ? **(5)**
6. Why can't the recoil electron be measured with this experimental set-up? **(5)**

Note:

Point reduction if there is no bound lab book **(-10)**
Total points: **100**