Project for MA-UY 2114
Parametrized Curves
Calculus III, Spring 2021: All Sections

This project is adapted from Matthias Kawski at Arizona State University.

1 Overview

The main purpose of this project is to

- ☐ Practice working with parameterized curves at more depth than usual text-book exercises.
- ☐ Introduce students to more extensive work with a computer algebra system (CAS). One can think of these plots as an equivalent of programming of a pen-plotter or of a welding robot.
- ☐ Cultivate teamwork and communication. For this project you will be working in assigned teams of 4 or 5.

The main task is to write a word as a single (broken) parameterized curve, and plot it using a CAS. Additional tasks involve the velocities, speed, components of the acceleration, and curvature.

2 Due Dates

Below are all the due dates for the project. One member of each team must submit the supporting documents via Gradescope no later than 11:59PM on the due date indicated below. All team members names and NetID must be atop the first page.

- ☐ 3/8 - Sketch of the word on graph paper with clearly labeled axes and marked times at end points.
- ☐ 4/5 - Your submission should include all of the tasks in the section below up to, but not including curvature. Do not worry about a detailed final write up at this time. Be sure to include clearly labeled work, calculations, and graphs.
- ☐ 5/10 - Final write up submitted via Gradescope .

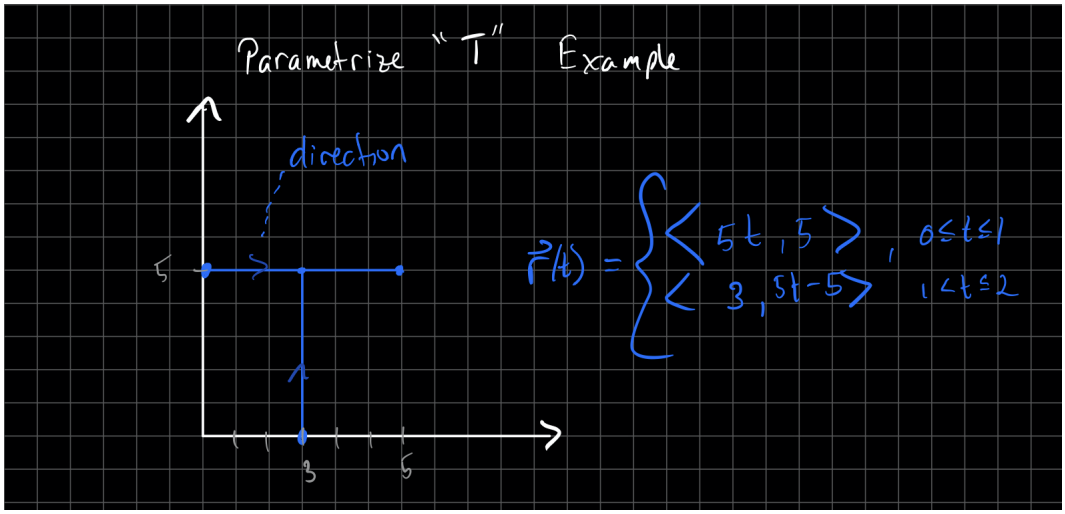
3 Tasks

Here you will find the tasks you are expected to complete with your team. Be sure to complete all of them to the best of your ability.

- ☐ Pick a word that has at least **four** characters. The word **must** include at least one horizontal, vertical, slanted and curved segment. For example, the word "HIKE" does not contain any curved segments. Therefore it would not be an acceptable choice. However, the word "CURVE" contains horizontal, vertical, slanted, and curved segments. Therefore it would be an acceptable choice.

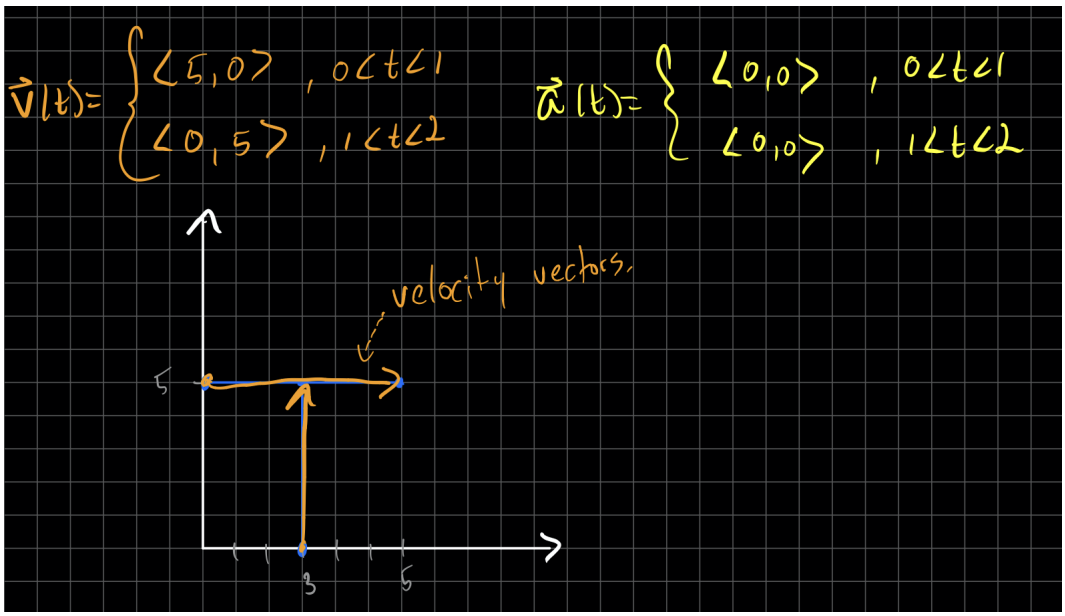
- ☐ Consider the word as a curve in the plane. Find a single parameterization for the entire curve. **The plotter can only be at one spot at each moment of time.** Different sections of word **must use different time intervals**. Write out very clearly what your parameterization is. Hence, you will end up with a **piecewise** function.

  As an example, see the following parameterization of the letter "T". Note that the choice of height and length could be different. The parameterization is not necessarily unique either–one could have found a different way to parameterize the letter, but you will still end up with a piece-wise function.



- ☐ Plot your parameterized curve. Include the source and the output. You can use sources such as (but not limited to)
  - ☐ Desmos
  - ☐ Matlab
  - ☐ Python

☐ On your plot, indicate and mark typical velocity vectors. You can do this by hand after creating your plot in the previous step. You Only have to draw a few velocity vectors for each letter. Included is a plot of my velocity vectors for the parameterized letter "T", be mindful that for curved segments of letters, the velocity vector may look a little different.



☐ Calculate and graph the speed as a function of time.

☐ Calculate and graph the perpendicular component of the acceleration vectors as a function of time.

☐ The **curvature** of a curve at a point measures how quickly the curve changes direction at that point. There are several equivalent ways to calculate curvature, but two of them are given by

$$\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}, \qquad \mathbf{T(t)} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

Calculate and graph the curvature as a function of time.

☐ **Bonus:** Now pick a parameterized surface in 3-space. Spell your word out on the surface in 3-space.

☐ Write up a clear summary of your work.

4 Advice

☐ Start early.

☐ Use portions of a circle or an ellipse for curved letters. It may not hurt to review chapter 10 in our text.

☐ Don't hesitate to communicate with your team members.

☐ Challenge yourselves.

5 Deliverables

The final write-up must be signed by all team members and must include:

- ☐ a brief narrative about the project.
- ☐ complete technical work (formulas, calculations, justifications and explanations)
- ☐ all graphs (clearly labeled).

The final write up should be well organized, checked for spelling errors and any other typos. The end product should be a single pdf file and will be submitted via Gradescope.

6 Evaluation

There are three milestones to reach. The total project is worth 12% of your total grade, and each milestone is worth 4% of your total grade. We are looking for complete work and clarity. No late work will be accepted. Each milestone will be graded as 0/4, 3/4 or 4/4. If your work is incomplete or contains numerous errors, no credit will be given. If you have a couple of small errors, 3/4 will be granted. If you satisfy all the criteria, then your team will receive the complete 4/4. Every member of the team receives one grade.