snakes

Alessandro Gentilini[*]

July 14, 2014

Abstract

snakes

1 Inference for Snakes

The likelihood is formula (17.3)

$$Pr(\mathbf{x}|\mathbf{W}) \propto \prod_{n=1}^{N} \exp\left[-(\text{dist}[\mathbf{x}, \mathbf{w}_n])^2\right] \tag{1}$$

The prior is formula (17.4)

$$Pr(\mathbf{W}) \propto \prod_{n=1}^{N} \exp\left[\alpha \text{space}[\mathbf{w}, n] + \beta \text{curve}[\mathbf{w}, n]\right] \tag{2}$$

The inference is

$$\underset{\mathbf{W}}{\arg\max}[Pr(\mathbf{W}|\mathbf{x})] = \underset{\mathbf{W}}{\arg\max}[Pr(\mathbf{x}|\mathbf{W})Pr(\mathbf{W})] \tag{3}$$

Since log is monotone I can write

$$\begin{aligned} \underset{\mathbf{W}}{\arg\max}[Pr(\mathbf{W}|\mathbf{x})] &= \underset{\mathbf{W}}{\arg\max}[\log\left[Pr(\mathbf{W}|\mathbf{x})\right]] \\ &= \underset{\mathbf{W}}{\arg\max}[\log\left[Pr(\mathbf{x}|\mathbf{W})Pr(\mathbf{W})\right]] \\ &= \underset{\mathbf{W}}{\arg\max}[\log\left[Pr(\mathbf{x}|\mathbf{W})\right] + \log\left[Pr(\mathbf{W})\right]] \end{aligned} \tag{4}$$

[*]alessandro.gentilini@gmail.com

And so for what regard the likelihood I can write:

$$\begin{aligned}\log\left[Pr(\mathbf{x}|\mathbf{W})\right] &\propto \log\left[\prod_{n=1}^{N}\exp\left[-(\text{dist}[\mathbf{x},\mathbf{w}_n])^2\right]\right] \\ &= \sum_{n=1}^{N}\log\left[\exp\left[-(\text{dist}[\mathbf{x},\mathbf{w}_n])^2\right]\right] \\ &= \sum_{n=1}^{N}-(\text{dist}[\mathbf{x},\mathbf{w}_n])^2\end{aligned} \tag{5}$$

And for what regard the prior I can write:

$$\begin{aligned}\log\left[Pr(\mathbf{W})\right] &\propto \log\left[\prod_{n=1}^{N}\exp\left[\alpha\text{space}[\mathbf{w},n]+\beta\text{curve}[\mathbf{w},n]\right]\right] \\ &= \sum_{n=1}^{N}\log\left[\exp\left[\alpha\text{space}[\mathbf{w},n]+\beta\text{curve}[\mathbf{w},n]\right]\right] \\ &= \sum_{n=1}^{N}\alpha\text{space}[\mathbf{w},n]+\beta\text{curve}[\mathbf{w},n]\end{aligned} \tag{6}$$

with $\alpha \geq 0$ and $\beta \geq 0$.

Finally the inference is:

$$\underset{\mathbf{W}}{\arg\max}[\log\left[Pr(\mathbf{W}|\mathbf{x})\right]] = \underset{\mathbf{W}}{\arg\max}[\sum_{n=1}^{N}-(\text{dist}[\mathbf{x},\mathbf{w}_n])^2+\alpha\text{space}[\mathbf{w},n]+\beta\text{curve}[\mathbf{w},n]] \tag{7}$$

The spacing term is formula (17.5)

$$\text{space}[\mathbf{w},n] = -\left(\frac{\sum_{n=1}^{N}\sqrt{(\mathbf{w}_n-\mathbf{w}_{n-1})^T(\mathbf{w}_n-\mathbf{w}_{n-1})}}{N}-\sqrt{(\mathbf{w}_n-\mathbf{w}_{n-1})^T(\mathbf{w}_n-\mathbf{w}_{n-1})}\right)^2 \tag{8}$$

and it is always nonpositive because it is a square and because of the leading minus.

The curvature term is formula (17.6)

$$\text{curve}[\mathbf{w},n] = -\left(\mathbf{w}_{n-1}-2\mathbf{w}_n+\mathbf{w}_{n+1}\right)^T\left(\mathbf{w}_{n-1}-2\mathbf{w}_n+\mathbf{w}_{n+1}\right) \tag{9}$$

and it is always nonpositive because o the leading minus and because $\mathbf{x}^T\mathbf{x} = \|\mathbf{x}\|^2$ and so it is always nonnegative since it is the square of the (ℓ^2)norm.

With these new definitions

$$\begin{aligned} \mathrm{d}[\mathbf{x}, \mathbf{w}_n] &= (\mathrm{dist}[\mathbf{x}, \mathbf{w}_n])^2 \\ \mathrm{s}[\mathbf{w}, n] &= -\alpha \mathrm{space}[\mathbf{w}, n] \\ \mathrm{c}[\mathbf{w}, n] &= -\beta \mathrm{curve}[\mathbf{w}, n] \end{aligned} \tag{10}$$

we have that $\mathrm{d}[\mathbf{x}, \mathbf{w}_n]$, $\mathrm{s}[\mathbf{w}, n]$ and $\mathrm{c}[\mathbf{w}, n]$ are all nonnegative and the inference formula can be rewritten as

$$\arg\max_{\mathbf{W}}[\log\left[Pr(\mathbf{W}|\mathbf{x})\right]] = -\arg\max_{\mathbf{W}}[\sum_{n=1}^{N} \mathrm{d}[\mathbf{x}, \mathbf{w}_n] + \mathrm{s}[\mathbf{w}, n] + \mathrm{c}[\mathbf{w}, n]] \tag{11}$$

and now the argument of $\arg\max$ is nonnegative.