

Rubella Report

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1 Model

Let us define a k -age group compartmental SIRV model for the infection dynamics of the diseases covered by the MMR vaccine. This model takes the form of a system of four vectorised ordinary differential equations. Each of the k vector entries then corresponds to a given age subgroup with span m_i , $i \in 1 \dots k$. Each equation then governs the dynamics what proportion of the subpopulations is susceptible, infected, recovered, or has been vaccinated.

$$\begin{aligned}
 \frac{ds}{dt} &= \mathbf{B} - (\mathbf{V} - \mathbf{D})\mathbf{s} - \mathbf{s} * (\boldsymbol{\beta}(t)\mathbf{C}\mathbf{i}) + \delta_{t_{end}}(t)\mathbf{M}\mathbf{s} \\
 \frac{di}{dt} &= \mathbf{s} * (\boldsymbol{\beta}(t)\mathbf{C}\mathbf{i}) - (\mathbf{D} - \mathbf{d} - \gamma)\mathbf{i} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{i} \\
 \frac{dr}{dt} &= \gamma\mathbf{i} - \mathbf{D}\mathbf{r} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{r} \\
 \frac{dv}{dt} &= \mathbf{V}\mathbf{s} - \mathbf{D}\mathbf{v} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{v}
 \end{aligned}$$

$\mathbf{s} * \mathbf{i}$ Denotes the elementwise product

Where:

$\mathbf{s}, \mathbf{i}, \mathbf{r}, \mathbf{v} \in \mathbb{R}^k$	k -age group compartment vectors
$\mathbf{B} := \text{diag}(B, 0, \dots, 0)$	Birth rate
$\mathbf{V} := \text{diag}(V_1, \dots, V_k)$	Vaccination rates
$\mathbf{D} := \text{diag}(D_1, \dots, D_k)$	Death rates due to other causes
$\mathbf{d} := \text{diag}(d_1, \dots, d_k)$	Death rates due to disease
$\boldsymbol{\beta}(t) := \text{diag}(\beta_1(t), \dots, \beta_k(t))$	Age-based force of infection
$\mathbf{C} \in \mathbb{R}^{k \times k}$	Age-based <i>Who Interacts With Who</i> matrix
$\gamma \in \mathbb{R}$	Recovery rate
$\mathbf{M} \in \mathbb{R}^{k \times k}$	The age group transition matrix

where \mathbf{M} has the structure

$$\mathbf{M} = \begin{bmatrix} -m_1^{-1} & & & & \\ m_1^{-1} & -m_2^{-1} & & & \\ & m_2^{-1} & -m_3^{-1} & & \\ & & \ddots & \ddots & \\ & & & m_{k-1}^{-1} & 0 \end{bmatrix}$$

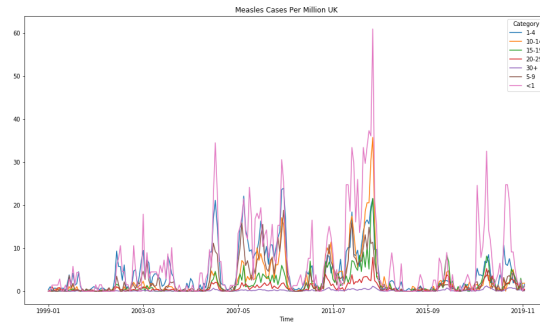
2 Data

Epidemiologic age structured data for Rubella can be obtained from either Health Protection Agency archives [1] or from ECDC Atlas of Infectious Diseases [4].

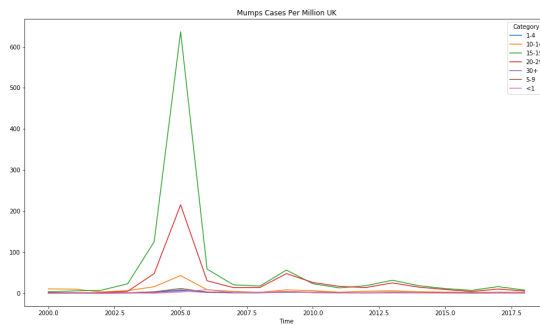
Immunisation is available from the NHS COVER reports [2].

As a starting point for *Who Interacts With Who* matrices, the POLYMOD study [3] can be utilised.

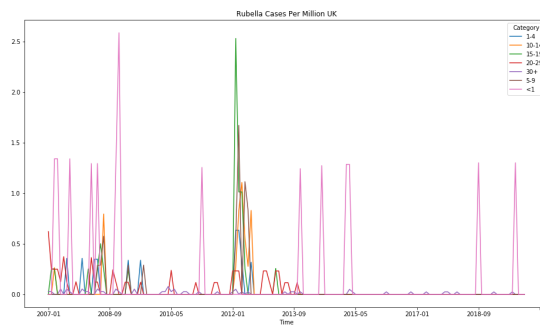
This data should enable us to parametrise and fit an age stratified SIRV model for years 1998-2019. Adding gender structure should be explored as well in the case of rubella as the primary concern is congenital rubella syndrome in newborns.



(a)



(b)



(c)

Figure 1: ECDC Age Standardised rates for the UK plotted against time 1a Measles data 1b Mumps data 1c Rubella data

References

- [1] Health Protection Agency. *Epidemiological Data*. Archive Location: UK Library Catalog: webarchive.nationalarchives.gov.uk Publisher: Health Protection Agency, 151 Buckingham Palace Road, London, SW1W 9SZ, 020 7811 7000, webteam@hpa.org.uk. URL: <https://webarchive.nationalarchives.gov.uk/20140505133414/http://www.hpa.org.uk/Topics/InfectiousDiseases/InfectionsAZ/Rubella/EpidemiologicalData/> (visited on 04/05/2020).
- [2] *Childhood Vaccination Coverage Statistics - England 2018-19*. NHS Digital. Library Catalog: digital.nhs.uk. URL: <https://digital.nhs.uk/data-and-information/publications/statistical/nhs-immunisation-statistics/england-2018-19> (visited on 04/02/2020).
- [3] Joël Mossong et al. "Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases". In: *PLOS Medicine* 5.3 (Mar. 25, 2008). Publisher: Public Library of Science, e74. ISSN: 1549-1676. DOI: 10.1371/journal.pmed.0050074. URL: <https://journals.plos.org/plosmedicine/article?id=10.1371/journal.pmed.0050074> (visited on 04/02/2020).
- [4] *Surveillance Atlas of Infectious Diseases*. URL: <https://www.ecdc.europa.eu/en/surveillance-atlas-infectious-diseases> (visited on 04/02/2020).