

Rubella Report

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1 Model

Let us define a k -age group compartmental SIRV model for the infection dynamics of the diseases covered by the MMR vaccine. This model takes the form of a system of four vectorised ordinary differential equations. Each of the k vector entries then corresponds to a given age subgroup with span m_i , $i \in 1 \dots k$. Each equation then governs the dynamics what proportion of the subpopulations is susceptible, infected, recovered, or has been vaccinated.

$$\begin{aligned}
 \frac{ds}{dt} &= \mathbf{B} - (\mathbf{V} - \mathbf{D})\mathbf{s} - \mathbf{s} * (\boldsymbol{\beta}(t)\mathbf{C}\mathbf{i}) + \delta_{t_{end}}(t)\mathbf{M}\mathbf{s} \\
 \frac{di}{dt} &= \mathbf{s} * (\boldsymbol{\beta}(t)\mathbf{C}\mathbf{i}) - (\mathbf{D} - \mathbf{d} - \gamma)\mathbf{i} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{i} \\
 \frac{dr}{dt} &= \gamma\mathbf{i} - \mathbf{D}\mathbf{r} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{r} \\
 \frac{dv}{dt} &= \mathbf{V}\mathbf{s} - \mathbf{D}\mathbf{v} + \delta_{t_{end}}(t)\mathbf{M}\mathbf{v}
 \end{aligned}$$

$\mathbf{s} * \mathbf{i}$ Denotes the elementwise product

Where:

$\mathbf{s}, \mathbf{i}, \mathbf{r}, \mathbf{v} \in \mathbb{R}^k$	k-age group compartment vectors
$\mathbf{B} := \text{diag}(B, 0, \dots, 0)$	Birth rate
$\mathbf{V} := \text{diag}(V_1, \dots, V_k)$	Vaccination rates
$\mathbf{D} := \text{diag}(D_1, \dots, D_k)$	Death rates due to other causes
$\mathbf{d} := \text{diag}(d_1, \dots, d_k)$	Death rates due to disease
$\boldsymbol{\beta}(t) := \text{diag}(\beta_1(t), \dots, \beta_k(t))$	Age-based force of infection
$\mathbf{C} \in \mathbb{R}^{k \times k}$	Age-based <i>Who Interacts With Who</i> matrix
$\gamma \in \mathbb{R}$	Recovery rate
$\mathbf{M} \in \mathbb{R}^{k \times k}$	The age group transition matrix

where \mathbf{M} has the structure

$$\mathbf{M} = \begin{bmatrix} -m_1^{-1} & & & & \\ m_1^{-1} & -m_2^{-1} & & & \\ & m_2^{-1} & -m_3^{-1} & & \\ & & \ddots & \ddots & \\ & & & m_{k-1}^{-1} & 0 \end{bmatrix}$$

2 Data

2.1 Immunisation Rates

Immunisation is available from the NHS COVER reports [4].

2.2 Between Group Contact Rates

As a starting point for *Who Interacts With Who* matrices, the POLYMOD study [6] can be utilised. This matrix will then further be weighted by force of infection for each age group, to be inferred from data.

2.3 Epidemiologic Data

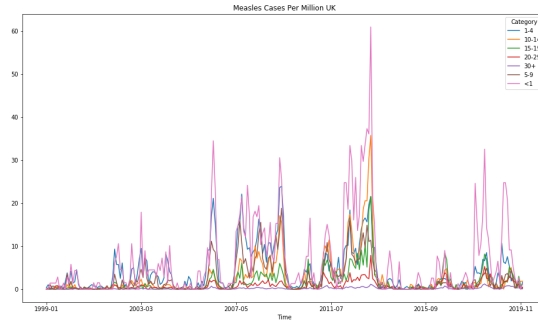
There are several sources of age structured epidemiologic data for MMR diseases available.

By far the highest temporal resolution for measles and rubella is offered by the ECDC Surveillance Atlas of Infectious Diseases dataset [9], which offers age standardised rates with monthly resolution, spanning years 1999-2019 for measles, and 2007-2019 for rubella. Unfortunately for mumps this dataset only offers yearly resolution for years 2000-2017.

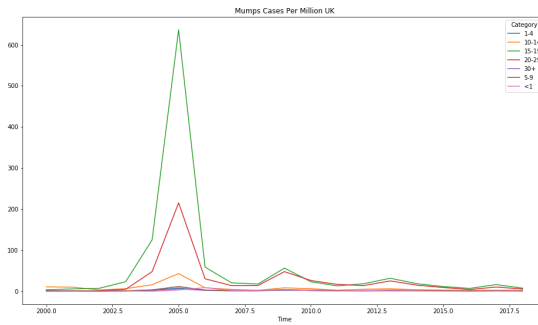
Further recent age structured data with yearly temporal resolution is available from Public Health England (PHE) reports [5, 7, 8]

Historic age-structured data can be obtained from the Health Protection Agency (HPA) archives [2, 3, 1]. These datasets span years 1989-2012, and offer yearly temporal resolution. This data offers an insight into situation when vaccination rate too low to confer herd immunity.

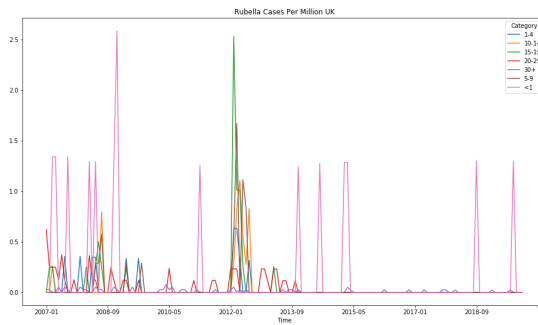
This data should be sufficient to parametrise and fit an age-stratified SIRV model for years 1998-2019, as well as to confirm that the compound behaviour of this model under low vaccination rates corresponds to real data.



(a)



(b)



(c)

Figure 1: ECDC Age Standardised rates for the UK plotted against time 1a Measles data 1b Mumps data 1c Rubella data

References

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- [4] *Childhood Vaccination Coverage Statistics - England 2018-19*. NHS Digital. Library Catalog: digital.nhs.uk. URL: <https://digital.nhs.uk/data-and-information/publications/statistical/nhs-immunisation-statistics/england-2018-19> (visited on 04/02/2020).
- [5] *Confirmed cases of measles in England and Wales by region and age: 2012 to 2019*. GOV.UK. Library Catalog: www.gov.uk. URL: <https://www.gov.uk/government/publications/measles-confirmed-cases/confirmed-cases-of-measles-in-england-and-wales-by-region-and-age-2012-to-2014> (visited on 04/07/2020).
- [6] Joël Mossong et al. "Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases". In: *PLOS Medicine* 5.3 (Mar. 25, 2008). Publisher: Public Library of Science, e74. ISSN: 1549-1676. DOI: 10.1371/journal.pmed.0050074. URL: <https://journals.plos.org/plosmedicine/article?id=10.1371/journal.pmed.0050074> (visited on 04/02/2020).
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- [8] *Rubella: confirmed cases in England and Wales by age and region, 2012 to 2019*. GOV.UK. Library Catalog: www.gov.uk. URL: <https://www.gov.uk/government/publications/rubella-confirmed-cases/rubella-confirmed-cases-in-england-and-wales-by-age-and-region-2012-to-2013> (visited on 04/07/2020).
- [9] *Surveillance Atlas of Infectious Diseases*. URL: <https://www.ecdc.europa.eu/en/surveillance-atlas-infectious-diseases> (visited on 04/02/2020).