

5010 qualify.

- $X_i \stackrel{iid}{\sim} U(0,1)$ ; (1)  $E\left(\frac{X_i}{X_n}\right)^k \forall k \in \mathbb{N}$ . (2)  $\Sigma(X_i - X_{(1)})$  与  $X_{(1)}$  independent.
- $(X_i, Y_i)$  ind pairs of ind poi with mean  $(e^{\lambda_i}, e^{\lambda_i + \beta W_i})$   $\lambda_i, \beta$  未知,  $W_i$  为 observed covariates. 求  $\hat{\beta}^{MLE}$ , it is consistent and asymptotic normal.
- (1)  $X_i \stackrel{iid}{\sim} U(\theta-T, \theta+T)$   $\theta \in \mathbb{R}, T > 0$  未知, 求  $EX_i$  的 UMVUE 并证.  
 (2)  $X_i \sim iid F$  未知, 求  $EX_i$  UMVUE.  
 (3)  $\textcircled{1}_{(2)}$  is in  $\textcircled{1}_{(1)}$ , why (1) 中 UMVUE isn't UMVUE of (2).
- UMPT with size  $\alpha$ ,  $\mu_0: \theta_1 = \dots = \theta_n = 0$   $A_1: \theta_1 = \theta_{10}$   
 $X_i \sim N(\theta_i, \sigma^2)$ ,  $\sigma^2$  已知. Identify test statistic and rejection region.

5005 qualify.

- (a) Slutsky's Th (b)  $P(X_n = \pm 1) = \frac{1}{2}(1 - \frac{1}{n})$   $P(X_n = k) = \frac{1}{2k^2}$  求 lim dist of  $\frac{\sum X_k}{(\sum X_k^2)^{1/2}}$ .
- $X_i \sim U(-1,1)$  (a) 求 dist of  $W_n = \frac{1}{n} \sum_{1 \leq i \leq j \leq n} X_i X_j$ ; (b)  $\frac{S_n}{\sqrt{n} \log n} \rightarrow 0$  a.s.  
~~(c) Doob's inequality~~ (c)  $P(|S_n| > \epsilon \sqrt{n}) \leq 2e^{-\frac{\epsilon^2}{2}}$  (hint:  $e^t + e^{-t} \leq 2e^{t^2}$ ).
- $E(S_{n+T}) \leq ES_n$   $\textcircled{2}$   ~~$E(X)$~~   $\textcircled{3}$   $E\left(\max_{1 \leq i \leq n} S_i^2\right) \leq 4 \sum_{i=1}^n EX_i^2$