

2021 STAT 5010 Final

Tony

1. Same as Q1 in 5010 final, fall 2020
2. $X_i \sim N(\theta_i, 1)$, $i = 1, \dots, n$, $Loss = \sum_{i=1}^n (\delta(X)_i - \theta_i)^2$, $\delta(X)_i = X_i$, why is δ minimax? Is it the unique minimax?
3. $X_i \sim Unif(1, 2)$, *i.i.d.*
 - (a). Prove that the harmonic mean H_n converges to a constant c in prob, identify c .
 - (b). Find the limiting distribution of $\sqrt{n}(H_n - c)$
4. (X_i, Y_i) *i.i.d.* mutually independent. $X_i \sim N(0, 1)$, $Y_i|X_i \sim N(x\theta, 1)$
 - (a). MLE $\hat{\theta}$
 - (b). Asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$
 - (c). (d). Complicated and limited time, didn't give a shot and don't remember.
5. (a). $Exp(\theta, 1)$ (location parameter θ), simple test θ_0 vs θ_1 . UMPT?
 - (b). Normal mean test, double sided, is there UMPT?
6. $EX = \mu$, $Var(X) = \sigma^2$ (or $N(\mu, \sigma^2)$). Prove that $k\bar{X}$, $k \in (0, 1)$ has smaller MSE. Drawbacks?