Persistence of Geometric Structures in 2-Dimensional Incompressible Fluids

Jean-Yves Chemin

Abstract

In this paper, we study the properties of a solution of the incompressible Euler System for large time. We suppose that the initial vorticity is the characteristic function of a regular bounded domain. Then the vorticity remains, for ail time, the characteristic function of a bounded domain with the same regularity.

Keywords: Vector field(little regular), tangential regularity, flow, vortex(of patches).

Introduction

The principal results shown here are for primary motivation a classic problem from mechanics of the 2 dimensional perfect fluid: the problem of vortex patches. Remember the frame in which we work. The movement of such fluid is described by a vector field in the plan, depended on time, noting $v(t,x)$ and checking

$$\begin{cases} \partial_t v + v \cdot \nabla v = -\nabla p \\ \operatorname{div} v = 0 \\ v_{|t=0} = v_0 \end{cases} \tag{E}$$

where $p(t,x)$ denotes the pressure of fluid at point x and the instant t ant where $v \cdot \nabla = \sum_i v^i \partial_i$. And one may notice that the flow Ψ of the field of the vectors v, that is to say, the sure(checking) application of next differential equation:

$$\partial_t \Psi(t,x) = v(t, \Psi(t,x)) \quad \text{and} \quad \Psi(0,x) = x$$

The fundamental quantity in the study of this equation is the rotational field of speeds, also called vortex. As we are in dimension 2, this antisymmetric matrix is identifying in a real notation $w = \partial_1 v^2 - \partial_2 v^1$. The specific character of the dimension 2 is the conservation of w along the trajectory of the field of vectors v:

$$\partial_t w + v \cdot \nabla w = 0. \tag{0.1}$$

Considering the nullity of the divergence of the field of vectors v, we can, if we stick to the fields of marked vectors, recalculate v, into closed constant vector, from w, by the following well known formula, called the law of Biot-Savart:

$$v = \nabla^{\perp} \Delta^{-1} = \left(- \int \frac{x_2 - y_2}{|x-y|^2} w(y) dy, \int \frac{x_1 - y_1}{|x-y|^2} w(y) dy \right), \tag{0.2}$$

by letting $\nabla^{\perp} f = (-\partial_2 f, \partial_1 f)$.

It is clear that, if $w \in L^{\infty} \cap L^p$ with $p < 2$, above integral defines a field of marked vectors. Furthermore, it's well known (and trivial to verify) that if w satisfies (0.1) with the field of vectors v given by (0.2), so v itself is a solution to (E) with the initial data deducted from w_0 from the relation (0.2). We will always place ourselves on the frame and, in the statement of the theorems, we will not formulate the hypothesis in the vortex.