PAPER CODE
ACFI204

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UNIVERSITY OF LIVERPOOL

JANUARY EXAM 2021

Financial Management

TIME ALLOWED: 3 Hours

(2.5 hours to complete the exam, plus 15 minutes to set up and access the exam, and 15 minutes to submit the exam)

INSTRUCTIONS TO CANDIDATES

The use of pre-programmable calculators is not permitted during this exam.

There are TWO Sections in total. You should answer ALL questions in Section A. You should answer THREE questions from section B.

Section A is worth 20% and Section B is worth 80%

Students are required to word process their answers using the template provided and submit their answers via Turnitin. For Section A, i.e. the multiple-choice questions, it is sufficient to write the letter of your chosen answer next to each question number. For Section B, please answer only THREE out of the FIVE questions. If you answer all questions in Section B, only the first three will graded. You are given 2.5 hours to process your answers, the template provides tables where applicable so you can show your working out for calculation questions that appear in Section B.

Note: Formula sheet, normal distributions, present value and annuity tables are attached

Student declaration:
I confirm that I have read and understood the University's Academic Integrity policy. I confirm that I have acted honestly, ethically and professionally in conduct leading to assessment for the programme of study. I confirm that I have not copied material from another source nor committed plagiarism nor fabricated data when completing the attached piece of work. I confirm that I have not previously presented the work or part thereof for assessment for another University of Liverpool module. I confirm that I have not colluded with any other student in the preparation and production of this work. I confirm that I have not incorporated into this assignment material that has been submitted by me or any other person in support of a successful application for a degree of this or any other University or degree awarding body. Students who require sympathetic marking should ensure that they attach the Sympathetic Marking Indicator to the first page of the document prior to submission

UNIVERSITY OF LIVERPOOL

ACFI204 Financial Management

January Examinations 2021

SECTION A

Q1: Which of the following statements is true?

a) The Capital Market Line prices efficient portfolios in terms of total risk.
b) The Capital Market Line prices individual assets in terms of total risk.
c) The Capital Market Line prices efficient portfolios in terms of market risk.
d) The Capital Market Line prices individual assets in terms of market risk.

Q2: The Capital Asset Pricing Model (CAPM) expresses the equilibrium expected return of an asset as a function of:

a) The risk-free rate, alpha, and the expected market return.
b) The risk-free rate and the sensitivity of the stock to market and idiosyncratic risk.
c) Alpha and the expected market return.
d) The risk-free rate and the sensitivity of the stock to the market risk premium.

Q3: The NPV investment decision rule for mutually exclusive projects is to:

a) Accept all projects that yield NPV>0.
b) Accept the project that yields the lowest NPV as this minimizes risk taken by the firm.
c) Accept the project that yields the highest NPV as this is consistent with the objective of a firm.
d) Accept the project that the manager prefers to choose due to expertise in certain aspects of the project.

Q4: A firm undertakes a risky project that costs £600,000 and generates cashflows of £250,000 per annum for the next 5 years. What is the Payback of the project?

a) 2.2 years
b) 2.3 years
c) 2.4 years
d) 2.5 years

Q5: Using a discount rate of 10% the modified payback of the project in Q4 is:

a) 2.88 years
b) 2.68 years
c) 2.48 years
d) 2.58 years

Q6: ARTO PLC are offering a 5 for 1 rights issue to its existing shareholders. The current price of ARTO PLC stocks are £23. They are offering the 5 new shares at a 20% discount to the current stock price. Assuming the price of existing stocks does not change before the rights issue occurs, what is the theoretical ex-rights share-price?

a) £18.40
b) £19.17
c) £23.00
d) £21.47

Q7: Stock P has an alpha of 0.08, a Beta of 0.95, and residual variance of 17. The variance of the market return is 13.50. The proportion of market risk to total risk (i.e. the R^2) for Stock P is:

a) 40.75%
b) 41.75%
c) 42.75%
d) 43.75%

Q8: You are long 12,500 call options written on MOOTS INC stock with a strike price of $88.00; with each contract costing $4.70. If the stock price at maturity is $95, what is the profit on your position?

a) -$28,750
b) -$2.30
c) $2.30
d) $28,750

Q9: You are a smoothie manufacturer who anticipates purchasing 105,000lbs of orange juice on the ICE FUTURES US Market **(NOTE: lbs or pounds are an imperial measurement of mass)**. The current spot price of orange juice is $0.94/lbs and the futures price is $0.97/lbs. Each contract is for the delivery of 15,000lbs of orange juice at maturity. In order to hedge exposure of upward price pressure on orange juice on the spot market you must go long how many contracts?

a) 7
b) 6
c) 5
d) 4

Q10: Supposing the futures price of orange juice at today is $1.07/lbs, the profit on your position for the strategy in Q9 is:

a) $6000
b) $7500
c) $10,500
d) $9,000

SECTION B

Question One

You are a Financial Manager at ZIPP PLC, a bicycle wheel manufacturing firm located in the UK. Currently, the firm's ordinary shares have a market value of £25m. The firm's equity has a beta of 0.90, the expected market return is 12% per annum and the risk-free rate is 2% per annum. The firm's current market value of debt is £30m with 59,750 zero coupon bonds in issue that mature in 10 years. Each bond has a par value of £1000 and corporate taxes are 23% per annum.

ZIPP PLC is considering moving its manufacturing to a domestic warehouse and has the following three contract options for a piece of new machinery:

	Contract 1	**Contract 2**	**Contract 3**
Cost	£5m	£10m	£22.5m
Service Cost	£0.00 per annum	£500,000 per annum	£0.00 per annum
Reinvestment rate	0% per annum	0% per annum	30% of gross cash flows per annum for parts
Cash flows	£2m per annum	£2.5m per annum	£3.5m per annum
Length (years)	5	15	Infinite (so long as reinvestment continues)

Required:

a) Compute the Net-Present Value for the three contract alternatives and make a recommendation.

[40 marks]

First need to calculate WACC, then use WACC as the discount rate to appraise the three mutually exclusive contracts for the machine.

Use CAPM to get Ke:

$Ke = R_f + B \times [E(R_m)-R_f] = 2 + 0.9 \times [12-2] = 11.00\%$

For Kd, we need to first work out the price of the bond, and then solve out for the yield to maturity to get the cost of debt capital.

Market value of debt is £30,000,000 and the No. bonds in issue is 59,750. Therefore, the price of a bond is equal to £30,000,000/59,750 = £502.09.

Now we have the price we need to work out the yield to maturity on these bonds. We do this by re-arranging the zero-coupon bond pricing equation for the yield to maturity:

Kd = (Par/Price)^(1/T) – 1

Kd = (1000/502.09)^(1/10) – 1 = 7.13%

Now the after-tax cost of debt capital is: Kdat = Kd x (1 – T_c) = 7.13 x (1 – 0.23) = 5.49%

Now we need the weights:

E = 25, D = 30. Therefore E/(D+E) = 25/55 and D/(D+E) = 30/55 which means the weights associated to equity and debt capital are 45.45% and 54.55% respectively.

Kwacc = 0.4545 x 11 + 0.5455 x 5.49 = 8.00%

20 marks for this correct answer.

NPV of contract 1 is found by using annuity tables or computing the 5 year annuity factor at a r=7% discount rate. From annuity tables the 5-year 8% annuity factor is 3.993. Therefore, the NPV is

$NPV_{contract\ 1}$ = - 5+ 2 x 3.993 = 2.986

Which is £2.986m. 4 marks for correct answer.

NPV of contract 2 is found by computing the net cashflow using the information in the table, and recognising that these cash flows are received as an annuity for N=15 years. The net cash flow is £2.5m – £0.5m = £2m and from annuity tables we take the 15-year 8% annuity factor which is 8.559. Therefore, the NPV is

$NPV_{contract\ 2}$= -10 + 2 x 8.559 = 7.118

Which is £7.118m.

8 marks for correct answer. Also do not penalise if this is slightly off. Award full marks. Answer using excel is approximately £7.123m

NPV for contract 3 is found first by calculating the net cash flow which is Gross Cash Flow per annum multiplied by (1 – reinvestment rate). Therefore, we have £3.5m x (1 – 0.30) = £2.45m. Now we need to recognise that the present value of these perpetual cash flows can be found by applying the perpetual cash flow formula.

Here the NPV for contract 3 is actually the simplest one to calculate. All we need to do is:

$NPV_{contract\ 3}$ = - 22.5 + 2.45/0.08 = £8.125

Which is £8.125m. 8 marks for correct answer. Again, do not penalise for rounding. Full marks if approximately correct and process is right. Note Excel solution states NPV to be £8.141m.

If students use Kd in their WACC calculation they will get approximately 8.89%. If they multiply Ke by the weight for debt and vice versa they will get WACC of approximately 8.50%. In these cases award half marks for the WACC calculation, BUT DO NOT penalise the NPVs.

Mistakes more serious than this demonstrate clear lack of understanding and therefore the award should be no more than 8 marks for the WACC calculation. Again, if the NPV calculations are correct then do not penalise for using the wrong discount rate.

b) Calculate the Internal Rate of Return for Contract 1 and 2. Comment on whether you are able to make a choice between these contracts using only Internal Rate of Return.

[10 marks]

$$IRR = i_0 + (i_1 - i_0)\frac{NPV_0}{NPV_0 + |NPV_1|}$$

IRRs award 3 marks each for the correct IRRs. The IRR for contract 1 is 29.09% and the IRR for contract 2 is 19.00%. Allow for around +/-1.5% around these values. The students should show their working in terms of finding negative NPVs at a higher discount rate than the WACC and interpolate between these values.

Remaining 4 marks is for comment on why using only IRR we cannot distinguish between the two projects. This is because when plotting NPV as a function of the discount rate for these mutually exclusive projects, they cross over which means the recommendation changes depending on what discount rate the firm uses to appraise these projects. The IRR tells us nothing about the profitability of the projects at the appropriate discount rates which renders it difficult to appraise using only this method of investment appraisal.

c) Supposing the firm's bond price falls by £100. Compute the new weighted average cost of capital and comment on how this will change the NPV of the three contracts.

[20 marks]

If the bond price falls by £100, holding all else equal, then Kd will rise. In turn this pushes up Kdat and indeed the WACC. Therefore, the bond price is now $402.09 which means Kd is:

Kd = (1000/402.09)^(1/10)-1 = 9.54%, then Kdat = 9.54% x (1 – 0.23) = 7.34%

Note also that because the price of each bond has fallen, this means that the total market value of debt, D has also fallen. This will influence the weights in the following manner:

D = £402.09 x 59,750 = £24,025,000

Which means

E/(D+E) = 25/(25 + 24.025) = 0.5099
D/(D+E) = 24.025/(25 + 24.025) = 0.4901

The new WACC is

K_{WACC} = 0.5099 x 11 + 0.4901 x 7.34 = 9.21%

A higher WACC implies lower NPVs.

18 marks for the correct calculation and 2 marks for the interpretation.

d) If ZIPP issues 10 million 9% CUM IRRD preference shares with a notional value of £1.00 and a current market value of £15m, what is the firm's Weighted Average Cost of Capital? You may use the original data provided above for equity and debt.

[30 marks]

For this question, we use the original data in the question. Students should recognise here that we now need to account for the preference share capital in our WACC calculation. In understanding that WACC is just a weighted average of costs of capital, then this truly tests understanding of what WACC measures.

For Ke:

$K_e = R_f + B \times [E(R_m)-R_f]$ = 11.00% as before

For Kdat:

Kd = 7.13%, which means that Kdat = 5.49%

We now need to calculate Kp, the cost of preference share capital. We know that the preference shares are offer a 9% dividend rate with a notional value of £1.00. We also know that the firm issues 10,000,000 of these shares and the market value of preference shares is P= £15,000,000.

We need to find the price a preference share, then re-arrange the pricing equation for the discount rate

Price = 15/10 = £1.50

So we have £1.50 = £0.09 / Kp which implies that Kp = £0.09/£1.50 = 6.00%

Next we find the weights. Total firm value is equal to E + D + P = £25m + £30m + £15m = £70m

Now the weights are:

E/(D+E+P) = 25/70 = 0.3571

D/(D+E+P) = 30/70 = 0.4286

P/(D+E+P) = 15/70 = 0.2143

The WACC is K_{WACC} = 0.3571 x 11 + 0.4286 x 5.49 + 0.2143 x 6 = 7.57%

10 marks for finding price of preference share, 10 marks for weights, 10 marks for correct WACC calculation.

Do not penalise if student gets original WACC question wrong here.

Question Two

You are an Investment Analyst at MSC INC. On 06/01/2021 a new client has tasked you with investing $200m in the following two US corporate bonds holding $130m in AUTOSOL and the remaining $70m in BLOKU:

	AUTOSOL Bond	BLOKU Bond
Coupon	10%	12%
Par	$1,000	$1,000
T (years)	3	3
i	10%	14%
Moody's Rating	Aa1	Ba1

Your client instructs you that they wish to benefit from changes in the yield to maturity and insists that you actively manage the portfolio moving funds on a daily basis.

On the 07/01/2021 the Federal Reserve announces an imminent increase in the Federal Funds rate by 3%. Your analysis indicates that the respective yield to maturities on AUTOSOL and BLOOKU bonds will rise by 50% and 100% of the announced increase in the Federal Funds rate.

Required:

a) Compute the Price, Duration, Modified Duration and Convexity factor of AUTOSOL and BLOKU bonds using the information in the table above.

[30 marks]

Table below reports correct answers. For each bond: 5 marks for price, 5 for Duration and modified Duration, 5 marks for convexity. If student get coupon rate and yield to maturity mixed up, then award maximum of 15 marks depending on how students report their methods.

Bond A				
Year	**CF**	**PV(CF)**	**t x PV(CF)**	**t x (t+1) x PV(CF)**
1	$100.00	90.91	90.91	181.82
2	$100.00	82.64	165.29	495.87
3	$1,100.00	826.45	2479.34	9917.36
		1000.00	2735.54	10595.04
Price=	**1000.00**			
Duration=	**2.74**			
Dm=	**2.49**			
C=	**5.30**			
Bond B				
Year	**CF**	**PV(CF)**	**t x PV(CF)**	**t x (t+1) x PV(CF)**
1	$120.00	105.26	105.26	210.53
2	$120.00	92.34	184.67	554.02
3	$1,120.00	755.97	2267.90	9071.62
		953.57	2557.84	9836.16
Price=	**953.57**			
Duration=	**2.68**			
Dm=	**2.35**			
C=	**5.16**			

b) Calculate the Duration and Modified Duration of your bond portfolio. If you expect a large fall in interest rates, how would you change the amount invested in each bond and why?

[20 marks]

Portfolio Duration and Portfolio Modified Duration are a weighted average of individual bond Durations. We have total funds of $200m and invest $130m into Bond A with the remaining $70m into Bond B.

Weight A = 130/200 = 0.65
Weight B = 70/200 = 0.35

Portfolio Duration = 0.65 x 2.74 + 0.35 x 2.68 = 2.72 years
Portfolio Modified Duration = 0.65 x 2.49 + 0.35 x 2.35 = 2.44

5 marks for weights, 5 marks for correct portfolio Duration and modified Duration. The remaining 10 marks for their explanation around a large fall in interest rates. If we expect a large fall in interest rates then in order to benefit from the inverse relationship an investor may wish to increase holdings into high Duration bonds. This is because (modified) Duration acts as a scaling factor to the change in the yield to maturity. Top answers would relate their answer to how much of the

increase in the Federal funds rate is expected to pass through into the respective yield to maturities of each bond. Note here that the question tells us that 100% of the change in the Federal funds rate passes through to Bond B. Therefore, even though this has a slightly lower Duration, investors may well choose to increase their holdings into Bond B as a result of this. Top answers would also relate to the ratings of the bond. Depending on the attitudes of the investor toward risk, would also guide their decision as to how much of their funds they adjust into the above bonds. Bond B is riskier with a lower rating and those with a more relaxed attitude toward risk would place more funds into this bond relative to those who are more risk averse.

Students who do not mention the pass through or the ratings of the bonds in their answers should only receive a maximum of 5 of the 10 marks available for the explanation part of this question. This, of course, depends on the quality of their explanation also.

c) Accounting for convexity, produce forecasts of the respective price changes in each individual bond and compute the assumed portfolio value at the end of trading on 07/01/2021. Explain the importance of accounting for convexity in this situation.

[40 marks]

Here we need to utilise the information on the pass through of Federal funds rate changes to the changes in the respective yields of each bond.

FFR rise	3.00%
rise in A (50%)	1.50%
i_{A}=	11.50%
rise in B (100%)	3.00%
i_{B}=	17.00%

The Federal reserve announces an imminent increase in the Federal funds rate by 3%. We note that Bond A's yield to maturity will rise by half as much (i.e. 1.50%) and Bond B's yield will rise by an equal amount (i.e. 3.00%). The table above reports this.

We know that we have $200m available and invest $130m into bond A and $70m into bond B. This makes the proportions of wealth held in A and B 65% and 35% respectively.

We also need to compute the number of bonds held on January 6 2021. We need this because it will help us compute the portfolio value after the rise in the Federal funds rate.

Number of Bond A in our portfolio is equal to the weight x Funds/Bond Price. We have:

No Bond A = 0.65 x (200,000,000/1,000) = 130,000
No Bond B = 0.35 x (200,000,000/953.57) = 73409

Note here that we round these values to the nearest whole number because we cannot hold a fraction of a bond!

Now in order to forecast the price changes we need to use account for convexity as per the question.

$dP/P = -D_M \times (1+ di) + C \times [(1+di)/(1+i)] ^2$

The new price of the bond is

$P_{NEW} = P_{OLD} \times (1 + dP/P)$

We have all of this information from part a) for each bond.

For Bond A:

$dP/P = - 2.49 \times (0.015) + 5.30 \times [0.015/1.10]^2 = -3.63\%$

$P_{A\ NEW} = 1000 \times (1 – 0.0363) = \$963.68$

For Bond B:

$dP/P = - 2.35 \times (0.03) + 5.16 \times [0.03/1.14]^2 = -6.70\%$

$P_{B\ NEW} = 953.57 \times (1 – 0.067) = \$889.66$

Now the value of our holdings in each bond are equal to the new price multiplied by the number of bonds held in the portfolio. Once we have these we add together and this gives us the assumed portfolio value on 07/01/2021

V_A = No Bond A x $P_{A\ NEW}$ = 130000 x 963.68 = \$125,278,698.52
V_B = No Bond B x $P_{B\ NEW}$ = 73409 x 889.66 = \$65,309,172.95

The portfolio value is:

V_P = \$125,278,698.52 + \$65,309,172.95 = \$190,587,871.47.

Relative to the original value of \$200,000,000 we have a loss of \$9,412,128.53.

10 marks for correct number of bonds. 10 marks each for correct forecasted price changes (including new bond price). 10 marks for correct portfolio value as of 07/01/2021.

If students do not use convexity to account for changes in yield to maturity, award a maximum of 20 marks. Their answers will, if they are correct, look like:

	Number of Bonds
Bond A:	**130000**
Bond B:	**73409**

Forecast Price Changes

Bond A:	**Linear**
dP/P=	**-3.73%**
New price=	**$962.70**
Bond B:	**Linear**
dP/P=	**-7.06%**
New price=	**$886.26**

Value of Holdings 07/01/2021

	Linear
Value A	**$125,150,638.62**
Value B	**$65,059,151.25**
Total	**$190,209,789.87**
Difference from original Value=	**-$9,790,210.13**

If students show that prices rise when yields rise, then they demonstrate no understanding of the relationship between bond prices and yields. Award a maximum of 10 marks for the question if this is the case.

d) During financial crises bond prices fall and yield to maturities' fall. Comment on whether this statement is true and how financial crises affect bond ratings.

[10 marks]

Students need to realise here that the statement is false. There is an inverse relationship. If the financial crisis causes interest rates in the wider economy to fall, then in turn yields should fall which means that bond prices will rise. However, note that if the financial crisis only affects corporate debt markets and not the wider economy then it is likely that yields will rise and prices will fall. Financial crises can result in downgrading of corporate bond ratings which will also influence yield to maturities and bond prices.

Award marks for understanding and reasoning around the above points.

Question Three

VANNIC INC is a large multinational conglomerate operating predominantly in the USA, the UK, and Hong Kong. They have both ordinary and preference shares listed on each of the major stock exchanges. That is, the S&P500, the FTSE All Share, and the Hang Seng Indexes. VANNIC INC's financial statements for 2020 report earnings of $268m, £100m and HK$125m in the US, UK, and Hong Kong respectively.

VANNIC INC has 50m class A ordinary shares outstanding on the S&P500 has just paid total dividends on these stocks of $250m. This dividend payment is anticipated to remain constant for the next 4 years and then there will be no dividend payment from year 5 onward. The required rate of return demanded by investors in the US market is 6%.

VANNIC INC has 15m class A ordinary shares outstanding on the FTSE All Share has just paid a dividend of £1.00 per share, dividend growth is expected to be 6% per annum over the next 5 years. From year 6 onwards, dividend growth will change to 8% per annum indefinitely. The required rate of return demanded by investors in the UK market is 12%.

VANNIC INC has 12m ordinary shares outstanding on the Hang Seng is expected to pay a dividend next period of HK$12.50 with dividends growing at 4% per annum for the next 4 years. Thereafter, there will be no dividend payments and the cost of capital on this market is 10%.

Preference shares issued on these markets each have a notional value of 1 unit of currency. The dividend rates on stocks listed on the US, UK, and Hong Kong exchanges are 10%, 9% and 12% respectively. The required rate of return for preference shares are 3%, 4% and 4.50% on the US, UK and Hong Kong stock exchanges respectively.

Required:

a) For each stock market, calculate the price of VANNIC INC's ordinary shares, the price of preference shares, and their price-to-earnings ratios.

[60 marks]

First we consider the US stocks:

US stock	
No shares	50,000,000
Dividend payment	$250,000,000.00
DPS=	$5.00
N=	4
k=	6%
Price=	**$17.33**
Earnings=	$268,000,000.00
EPS=	$5.36
Price-to-earnings=	**$3.23**

The price of the stock is found by multiplying the Dividend per share (DPS) by the N=4 year r=6% annuity factor which may either calculated manually or using the annuity tables.

$$PV(\text{Annuity}) = CF \times \left[\frac{1}{k} - \frac{1}{k(1+k)^t}\right]$$

The price of the US preference share is calculated by finding the dividend payment per share. This is the dividend rate times the notional value $1.00 x 10% = $0.10

Price = $0.10/0.03 = $3.33

20 marks for all correct answers here. 10 for ordinary share, 5 for price to earnings, 5 for preference share price.

For the UK stock we need to use the three step method to get the stock price:

UK stock	
DPS=Dt=	£1.00
N=	5
g1=	6%
g2=	8%
k=	12%
Earnings=	£100,000,000
No shares=	15,000,000
Using GGM	
first term=	£4.25
second term=	£20.50
Price=	**£24.75**

Step 1

Year	div	PV(div)
1	£1.06	£0.95
2	£1.12	£0.90
3	£1.19	£0.85
4	£1.26	£0.80
5	£1.34	£0.76
		£4.25

Step 2

P5=	£36.13
PV(P5)=	£20.50

Step 3

Price=	**£24.75**
EPS=	£6.67
Price-to-earnings=	**£3.71**

The preference share price = £0.09/0.04 = £2.25.

Again, 20 marks for all correct answers broken down in the same way as above. 10 for ordinary share price, 5 for price-to-earnings ratio, 5 for the preference share price.

For the HK market:

The price of the ordinary share is found by applying the growing annuity formula to the dividend payments.

HK stock	
Dt+1=	$12.50
g=	4%
k=	10%
N=	4
Earnings=	$125,000,000
No. shares=	12,000,000
Price=	**$41.87**
EPS=	$10.42
Price-to-earnings=	**$4.02**

$$\frac{d_{t+1}}{k-g_1}\left[1-\left(\frac{1+g_1}{1+k}\right)^N\right]$$

Price of Ordinary Share = $12.50/(0.1-0.04) x [1 – (1.04/1.10)^4] = $41.87

Now for the preference share:

Price of Preference Share = $0.12/0.045 = $2.67.

Again, 20 marks for all correct answers broken down in the same way as above. 10 for ordinary share price, 5 for price-to-earnings ratio, 5 for the preference share price.

b) In the US, the market portfolio is well approximated by 4 sectoral index returns, with the following information.

Sector	σ_{iM}	**Weight**
ENGY	100	0.25
FINC	110	0.25
MANU	200	0.35
MATR	90	0.15

i) What is the standard deviation of the US market portfolio's return and the sectoral index betas?

ii) Specify the Capital Market Line if $E(R_M) = 5.40\%$ and $R_f = 0.75\%$.

iii) Provide an Interpretation of the Capital Market Line and distinguish it from the Security Market Line.

[40 marks]

i) We now need to use some algebra to figure out the standard deviation of the US market portfolio return. We assume that the market portfolio is made up of the four sectors here. We have the weights and the covariances. Here we exploit the fact that the beta of the market portfolio can be found as a weighted average of the sectoral betas; as well as the fact that the beta of the market portfolio is equal to 1.

Therefore, we have:

$1 = 0.25 \times beta_{ENGY} + 0.25 \times beta_{FINC} + 0.35 \times beta_{MANU} + 0.15 \times beta_{MATR}$

Now we use the definition of beta which is covariance between sector return and market return divided by the variance of market return

$Beta = COV(R_i,R_M) / VAR(R_M)$

Plugging in the definition of beta into the above expression and multiplying through by $VAR(R_M) = \sigma_M^2$ we have:

$\sigma_M^2 = 0.25 \times COV(R_{ENGY},R_M) + 0.25 \times COV(R_{FINC},R_M) + 0.35 \times COV(R_{MANU},R_M) + 0.15 COV(R_{MATR},R_M)$

$= 0.25 \times 100 + 0.25 \times 110 + 0.35 \times 200 + 0.15 \times 90$

$\sigma_M^2 = 25 + 27.5 + 70 + 13.5 = 136$

This means that the standard deviation of the market portfolio is equal to the square root of 136 which is 11.66 (i.e. 11.66%).

Now that we have the variance of the market return, we can also find the betas of each sector.

$Beta_{ENGY}$ =	0.74
$Beta_{FINC}$ =	0.81
$Beta_{MANU}$ =	1.47
$Beta_{MATR}$ =	0.66

Award 20 marks for the above correct answers.

ii) for the CML we use $R_p = R_f + [E(R_M)-R_f]/ \sigma_M \times \sigma_P$

Assuming risk-free rate is 0.75% and the expected market return of 5.40% we can specify the CML

$CML = 0.75\% + (5.40\%-0.75\%)/11.66\% \times \sigma_P$

$CML = 0.75\% + 0.3988 \times \sigma_P$

Award 10 marks for correct answer

For iii) the CML prices efficient portfolios in terms of total risk, and the SML prices individual assets in terms of market risk. They are inherently different because the SML ignores idiosyncratic risk. Students may also note that the SML comes from the CML.

Award marks for understanding here. 10 marks available.

Question Four

You are a portfolio manager working for Quahog Financial Solutions. You have computed the following information regarding four stocks listed on the New York Stock Exchange using 6 years of monthly data from January 2015-January 2021:

	NQ	FH	WX	MW
Expected Return	12%	10%	6%	8%
Standard Deviation	14%	8%	4.50%	6%

The correlation matrix is:

	NQ	FH	WX	MW
NQ	1			
FH	0.44	1		
WX	0.25	0.00	1	
MW	0.15	-0.25	0.66	1

One of your clients has recently provided you with $120m to invest into combinations of the above stocks.

Required:

a) Compute the portfolio expected return and risk of all possible equally weighted 2 asset portfolios using MW.

[30 marks]

	Return	Risk
MW and NQ	**10.00%**	**8.02%**
MW and FH	**9.00%**	**4.36%**
MW and WX	**7.00%**	**4.79%**

Expected return is weighted average of individual asset returns and portfolio standard deviation are found applying the portfolio variance equation for 2 asset portfolio and taking square root.

If students do not take square root they will get:

	Return	Risk
MW and NQ	**10.00%**	**0.00643**
MW and FH	**9.00%**	**0.0019**
MW and WX	**7.00%**	**0.0023**

Do not penalise here as they have just forgotten to take square root.

5 marks per correct answer.

b) Compute the minimum variance portfolio expected return and risk for a portfolio consisting of MW and FH.

[20 marks]

Minimum Variance Portfolio MW and FH	
X_{MW}	**61.29%**
X_{FH}	**38.71%**
R*p=	**8.77%**
sigma*p=	**4.17%**

If students forget to to take square root of portfolio variance, they will get risk of 0.00174. Again, do not penalise for this.

5 marks per correct answer above.

c) If you form an equally weighted portfolio of all four stocks and it makes a return of 2.5 percentage points above its expected return over the next month, what is the portfolio value?

[15 marks]

Equally weighted portfolio of all four assets yields an expected return of:

$E(R_P) = 0.25 \times (12 + 10 + 6 + 8) = 9.00\%$

If the portfolio return over the next month is 2.50% above its expected return then the return will be equal to 11.5%.

Now we know that funds invested are $120,000,000 which means an 11.5% rise in portfolio value results in:

$120,000,000 \times (1.115) = \$133,800,000.$

If students cannot find the value but recognise an 11.5% increase, award maximum of 6 marks.

d) Explain what is meant if a portfolio's return is "two standard deviations below the mean". Then, relate how diversification with reference to correlation can help mitigate large fluctuations in portfolio value.

[15 marks]

If the portfolio return is two-standard-deviations below the mean then this means that the portfolio return is two standard deviations lower than the expected return of the portfolio. In reality, two standard deviations below the mean may constitute a sizeable loss in portfolio value.

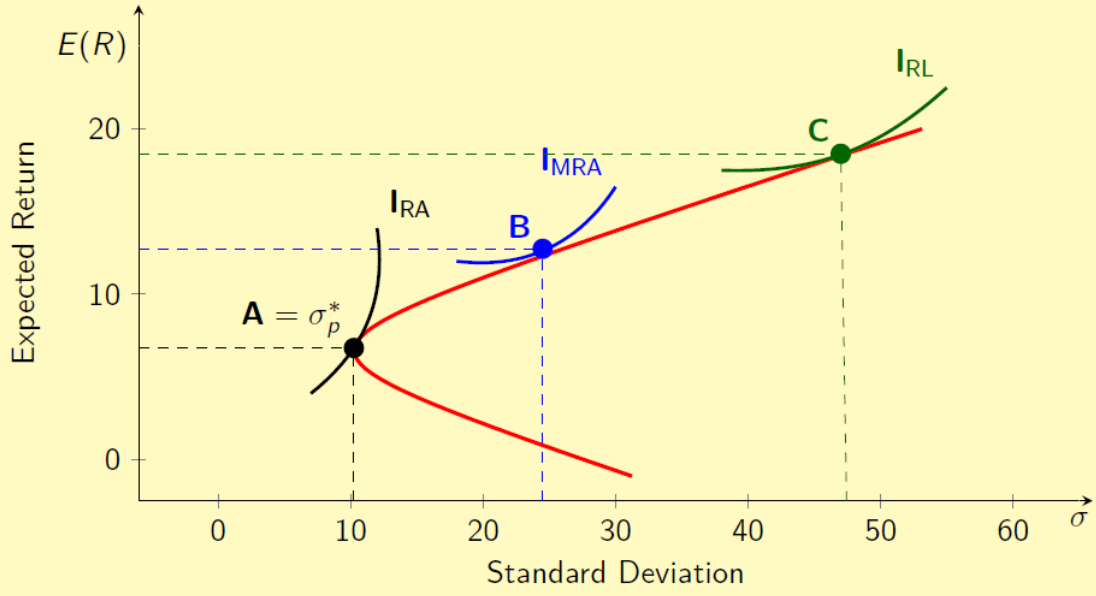
Regarding diversification, investors can exploit correlation between assets when forming portfolios in order to reduce exposure to individual asset risk. Even if correlation between assets is positive, we can still form portfolios that benefit from diversification. If we consider the 2-asset portfolio variance equation, we can see that the weighted covariance term accounts for correlation. It is this term that reduces portfolio risk. Students may also go on to talk about adding assets to an equally weighted portfolio benefits from diversification as we effectively get rid of individual asset risk. through the weighted covariance term, such that we are left with market risk (or the average covariance among stocks on the market).

e) Provide an explanation of how risk averse, moderately risk averse, and risk loving investors are likely to choose assets that make up portfolios. Also, provide an example of the two asset portfolios each investor would likely choose in relation to stocks NQ, FH, WX, MW.

[20 marks]

Explanation should include:

Discussion of indifference curve shape and slope for Risk Averse, Moderately Risk Averse, and Risk Loving investors.



Risk Averse:
Risk averse investors do not like risk. We can see this in the steepness of their indifference curves as above.
This means that they require high increases in expected return to compensate for small increments in portfolio risk exposure.

Their optimal portfolio in this case we assume is the minimum variance portfolio; note it may not be the case that a risk averse investor chooses this portfolio!

Moderately Risk Averse
These investors are more willing to take on risk than those investors that have a high degree of risk aversion.
We can see their indifference curves are flatter which shows this willingness to take on extra risk for smaller increases in portfolio expected return.
Their optimal portfolio is likely at point B on the efficient frontier.

Risk loving
These investors like risk! They are willing to take on high amounts of extra risk exposure for small increments in expected return.
We can see this behaviour in the flatness of their indifference curves relative to the other two types of investor.
Their optimal portfolio is likely point C which is even further away from the minimum variance portfolio.

Students are not required to draw graphs. Explanation is more than sufficient.

It is likely a risk lover would form a 2-asset portfolio using NQ and FH as these have the highest expected return and standard deviations. Moderately risk averse investors may choose a 2-asset portfolio of NQ and WX as these have the relatively high return and risk of the four stocks, but arguably less risky than portfolios using NQ and FH. Finally, the risk averse investor may choose a 2-asset portfolio of FH and MW to benefit from negative correlation, or maybe MW and WX as these have the lowest risk.

Award marks for understanding.

Question Five

TREXACO INC. is a US oil refinery that operates on a multinational scale in both producing and selling crude oil. It is currently January 31 2021, and the firm anticipates purchasing 600,000 barrels of crude oil to a US firm on May 31 2021. The current spot price for crude oil is $124/bl. TREXACO INC. are also expecting to receive £900m for the sale of 7,000,000 barrels of Jet Kerosene to a British Airways in six months. The current spot rate is $1.29/£.

In order to hedge against price exposure on the crude oil spot market, and foreign currency risk, the firm has the following Future and European Foreign Currency Option contracts available:

Crude Oil Futures		
Expiry	**Price**	**Term**
31/03/2021	$126/bl	1,000 Barrels
30/06/2021	$127/bl	1,000 Barrels
30/09/2021	$128/bl	1,000 Barrels

Foreign Currency Options			
Expiry	**Exercise Price**	**Call premium**	**Put premium**
31/03/2021	$1.27/£	$0.12	$0.04
30/06/2021	$1.29/£	$0.10	$0.07
30/09/2021	$1.33/£	$0.11	$0.06

Required:

a) Calculate the effective cost for the purchase of 600,000 barrels of crude oil, assuming that TREXACO INC used futures to fully hedge their exposure to price risk. The spot price and futures price on May 31,2021 are $136/bl and $138/bl, respectively.

[30 marks]

TREAXACO should go long N June 2021 June futures contracts to hedge exposure. The number of contracts is equal to 600 (Exposure/face value of 1 contract)

N=	600
s1=	$124.00
f1=	$127.00
s2=	$136.00
f2=	$138.00
Buy oil on spot @ s2=	**$81,600,000.00**
Profit Future=	**$6,600,000.00**
Net Cost=	**$75,000,000.00**

Using Basis we can work out by

f1+b2=	**$125.00**	/bl
No. Barrels=	**600,000**	
Effective Cost=	**$75,000,000.00**	

b) Compute the efficiency of the hedging strategy chosen in part a).

[10 marks]

Hedging Efficiency is equal to:

HE = 1 = (b1-b2)/(s2-s1)

b1=	-$3.00
b2=	-$2.00
b1-b2=	-$1.00
s2-s1=	$12.00
Efficiency=	**91.67%**

c) Calculate the net revenue from the receipt of £900m under the assumption TREXACO INC used a Foreign Currency Option to hedge against exchange rate risk. Your answer should provide two scenarios, one where the option is exercised, and another when the option is thrown away.

[30 marks]

TREXACO should take a long position in June 30 put options in order to hedge against foreign currency exposure. In particular, the put option gives them the right to sell £ at a predetermined exchange rate in the future. Therefore, the put will be exercised if the spot exchange rate at maturity is lower than the exercise price. TREXACO are hedging against an appreciation in the $ relative to the pound (the equivalent explanation is to hedge against a depreciation in the pound). If the $ appreciates relative to the £ this means that each pound is worth fewer dollars meaning TREXACO's revenue falls at lower $/£ exchange rates.

Assuming at maturity spot exchange rate is $1.25/£

Exercise put option and sell for $1.29/£	
X=	$1.29
Revenue=	$1,161,000,000
But paid a premium!	
Put premium per £=	$0.07
premium on £900000000	$63,000,000.00
Net Revenue=	**$1,098,000,000.00**

Assuming at maturity spot exchange rate is $1.35/£

Put option thrown away, but has cost Castroline INC	
Cost=	$63,000,000.00
Spot @ maturity=	1.35
Sell £900m on spot market for:	$1,215,000,000.00
Net Revenue=	**$1,152,000,000.00**

d) Define a short straddle and short strangle option strategy. Then, explain how and why an investor would conduct a short straddle and short strangle strategy.

[30 marks]

A short straddle consists of selling an at the money call and put option on the same underlying asset with the same time to maturity. A short strangle strategy sells an out of money call and put option on the same underlying asset and same time to maturity.

An investor must write these options and hope that another investor wishes to take long positions in the calls and puts. This is highly likely on options markets because they are highly liquid and traders have different objectives such as hedging and speculating.

An investor would conduct these strategies if they believed that the underlying asset would exhibit little to no volatility such that the price would be very close to the current price of the underlying. The maximum profit of the short straddle and short strangle are the option premia. The investor conducting the short straddle (strangle) loses out if the price of the underlying asset is very high or very low relative to the current price. This is because in these cases either the call or put are exercised and the investor writing the option must fulfil their end of the contract if an option is exercised.

Formula Sheet

Black and Scholes' option pricing model:

$$C = S_0 N(d_1) - \frac{X}{e^{rt}} N(d_2)$$

$$d_1 = \frac{\ln(S_0 / X) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Put Call Parity:

$$P + S_0 = C + \frac{X}{e^{rt}}$$

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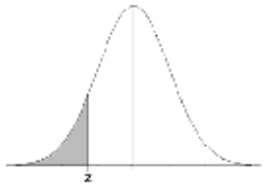
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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Standard Normal Cumulative Probability Table

Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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TABLES

Present value tables

Present value of £1 in n years at discount rate r: i.e. $\frac{1}{(1+r)^n}$

Discount rate (r)

Periods (n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	1
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	2
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	3
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683	4
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	5
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	6
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	7
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	8
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424	9
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386	10
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	11
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	12
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	13
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	14
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	15

	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694	2
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579	3
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482	4
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402	S
6	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335	6
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279	7
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233	8
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194	9
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162	10
11	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135	11
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112	12
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093	13
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078	14
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.074	0.065	15

UNIVERSITY OF LIVERPOOL

Annuity table

Present value of £1 receivable at the end of each year for n years at discount rate r:

i.e. $\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right]$

Discount rate (r)

Years (n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	1
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	2
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	3
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	4
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	5
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	6
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	7
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	8
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	9
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	10
11	10.37	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	11
12	11.26	10.58	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	12
13	12.13	11.35	10.63	9.986	9.394	8.853	8.358	7.904	7.487	7.103	13
14	13.00	12.11	11.30	10.56	9.899	9.295	8.745	8.244	7.786	7.367	14
15	13.87	12.85	11.94	11.12	10.38	9.712	9.108	8.559	8.061	7.606	15

	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	2
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	3
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	4
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	5
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326	6
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	7
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	8
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	9
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192	10
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	11
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	12
13	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533	13
14	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611	14
15	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675	15