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An active opinion dynamics model: the gap between the voting result and group opinion

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ABSTRACT

Originally developed to simulate the evolution of public opinion, opinion dynamics models have also been successfully applied to market pricing and advertising. However, passive interactions initiated by locational or social relationships in these models are insufficient to characterize purposeful behaviours such as canvassing or trading, where people are driven by their specific intrinsic motivations. Here, we propose an active model in which people tend to communicate with someone who is more likely to be an ally and game theoretically decide whether to interact. Model simulations highlight the macroscopic development of opinion evolution, showing the ubiquitous gap between people's voting result and their collective opinion, and how it narrows with the stabilization of opinion evolution. Our results help explain why group opinion rarely reverses its initial stance and the significance of a level of inclusiveness that is neither too high nor too low. Additionally, we find and attest to the probability distribution of group opinion change, which contributes to predicting how much the collective opinion of a group will change after full discussion.

1. Introduction

Voting is one of the simplest methods to aggregate dispersed preferences in a group and is widely applied from corporate decisions to public referenda. However, a single vote is far from the whole story, because people's opinions about certain issues are neither static nor independent. Just as microparticles' actions collectively make macro performances emerge, in a social context, people are the basic actors whose behaviours are driven by their internal opinions and beliefs[1]. Therefore, when individuals constantly interact and update their opinions at the microscopic level, the group opinion evolves at the macroscopic level.

Since the first model simulating the development of contradictory opinions (Voter Model, VM) was proposed in 1973 [2], a series of models on opinion evolution have been developed and jointly referred to as opinion dynamics models [3]. These classic models, though place people's intercommunications in a passive and static condition initiated by locational or social relationships, have helped people understand many problems such as opinion stabilization [4-7] and consensus reaching [2, 8-11], and have been applied to many fields such as election, marketing, and public management [12]. However, due to the convenience of communication and the polarization of ideology, current opinion environment is becoming increasingly complex. Classic opinion dynamics models thus are gradually insufficient to meet today's situations, where purposeful interactions driven by people's inner motivations of win are active and dynamic.

In purposeful interactions, on the one hand, people are more willing to communicate with others who hold a similar opinion [13] because higher similarity means lower persuasion cost and thus higher attraction; on the other hand, when people decide to exchange views, they do not communicate just for the sake of communicating but seek a favourable payoff by evoking more support [14], especially in economic and political activities. Therefore, this article proposes an active opinion dynamics model to simulate purposeful interactions in opinion evolution, where agents prefer to connect with similar agents, decide whether to interact referring to historical experience and game theory, and then update their opinions according to the Bayesian rule. Numeric experiments of the model simulate both microscopic and macroscopic development of opinion evolution, yielding some unique results about the gap between the voting result and group opinion, as well as the change of group opinion after evolution stabilization.

The rest of the paper is organized as follows. Section 2 summarizes the progress of opinion dynamics models and existing research gaps. Section 3 defines the concepts involved and develops the active opinion

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dynamics model. Section 4 simulates the model and analyzes its results. Section 5 conjectures and attests to the probability distribution of group opinion change. In Section 6, main contributions of the research and application cases are discussed. Finally, Section 7 concludes the paper with its limitations and potential extensions.

2. Literature review

Opinion dynamics has been developing for nearly half a century, with most models simulating people as nodes and personal connections as edges between nodes, establishing a network context for point-to-point interactions. Based on the above, opinion dynamics models are composed of three elements: opinion expression formats, opinion fusion rules, and opinion dynamics environment [4, 15]. Through mathematical and computational methods, people can then observe and analyze the evolution process, stabilized results and influence factors, and make predictions.

2.1. Basic models

Widely regarded as a pioneer of opinion dynamics models, Clifford and Sudbury [2] introduced the Voter Model (VM) in 1973, and the deduction of its stable state was completed in the 1990s [16, 17]. Opinion in VM is discrete and has two states: positive and negative. However, for many issues in real life, people's opinions are not limited to support or objection. There could be several choices, as well as varying degrees of approval or disapproval. Therefore, during the same period, the DeGroot Model [8] was proposed. In terms of the evolution of continuous opinion, opinions in this model referred to individuals' subjective probabilities valued at [0,1].

Thereafter, opinion dynamics models were divided into two branches. On the discrete way, on the one hand, many successors adjusted VM to such models as the Constrained 3-state Voter Model [5], Vacillating Voter Model (VVM) [18], Non-conserved Voter Model (NVM) [19], Heterogeneous Voter Model (HVM) [20], and Confident Voter Model (CVM) [21]; on the other hand, researchers also introduced new models, including the Axelrod Model [22], Sznajd Model [9], Majority Rule (MR) [10], and Nonlinear q-voter Model [7]. While on the continuous way, after the DeGroot Model, the model proposed by Friedkin and Johnsen in 1990 [23] became another equally famous model for continuous opinion; then Deffuant et al. [24] and Hegselmann et al. [4] respectively proposed two bounded confidence models, supplementing a confidence limit to opinion interactions and pushing opinion dynamics models nearer to reality. In addition, some mixed models were later introduced. In 2008, Martins [11] proposed the Continuous Opinions and Discrete Actions model (CODA model), in which individuals updated their internal continuous opinions after observing others' external discrete actions. Because of its proximity to reality, the CODA model soon received attention from scholars [25-27]. In 2017, a public opinion dynamics model in the online-offline social network [28] was proposed to analyze the mechanism of opinion evolution based on social network data, which models online interactions with H-K model and offline interactions with DeGroot model.

2.2. Solved problems

In Table A.1 (see Appendix. A), we summarize 12 basic models, specifically introducing how they simulate opinion interactions and what their conclusions are. From 1973 to the present, these models were concerned mainly with three problems: **(i)** Will the agent's opinion stabilize? **(ii)** Will a consensus be reached? **(iii)** What influences opinion evolution?

The first question was answered by the Constrained 3-state voter model [5], the H-K model [6], and the Nonlinear q-voter model [7] respectively, and the answers were the same: Yes. Even with noise (in the Nonlinear q-voter model) or with different initial distributions of opinion (in the H-K model), agents' opinions would finally be stable.

The second question had different answers. The Voter Model [2], the DeGroot Model [8], and Majority Rule [10] argued that a consensus of all agents would always be reached; however, the Sznajd model [9] and CODA model [11] suggested that agents would evolve two opposite clusters and fail to take any common decision. To some extent, the two answers were both half-right, because other factors influenced the result. The Deffuant model [24] and H-K model [4] revealed that the confidence bound determined into how many clusters agents would converge, i.e., whether a consensus would be reached. The two models both assumed that people interact only when the difference between their own and the target's opinions was within an interval, and found that there existed a threshold of the interval. If the interval was large enough, a consensus would be reached; otherwise, two or more clusters would converge. Thus, they actually answered the third question at the same time.

Many other factors with greater social meaning were also studied to answer the third question. For example, the Axelrod model [22] indicated that cultural diversity and local interactions influence the degree of cultural polarization; the F-J model [23] discovered the impact of different structural contexts on interpersonal situations; and the DeGroot-Friedkin model [29] investigated the evolution of self-appraisal, social power, and interpersonal influences.

2.3. Improvements and applications

To deal with more complex problems, many researchers have made adaptive improvements based on these basic models, which are mainly from the following four perspectives. (i) Special agents in a group, such as stubborn agents who never update their opinions [30], mixed agents of minority and majority seekers [31], and psychologically stressed agents who take actions to reduce their cognitive dissonance [32]. To different extents, they influence the speed of opinion stabilization and the polarization of opinion evolution. (ii) Noise or disturbance, i.e., the sudden conversion of an individual's opinion [33, 34]. According to existing research, a lower level of noise could accelerate consensus achievement, while a higher level of noise generally leads to instability, thus trapping group opinion in disorder or fluctuation. (iii) Uncertain opinions. People always fail to formulate clear opinions but instead express vague attitudes or preferences [35], such as interval opinions [36], fuzzy opinions [37], and linguistic opinions [38]. (iv) Network environments such as complex networks [39], social networks [40], and media networks [41, 42]. Although in most studies this difference in the lattice, social network or complex network has little effect on the time required for consensus and the result of evolution, some studies indicate that opinion evolutions on the complex network could influence the fragmentation of opinion and lead to more clusters.

In addition to the improvements discussed above, there are several review papers studying applications of opinion dynamics. For example, Baumgaertner [43] applied results of opinion dynamics literature to theoretically demonstrate the Millian suggestion that increasing interactions between nonlike-minded individuals is a way to increase opinion diversity, which in turn improves society at large. Anderson and Ye [44] summarized advances in the modelling and analysis of opinion dynamics on influence networks, covering three social extensions of social power evolution, discrepancies between expressed and private opinions, and discussion of logically related topics. Dong et al. [12] reviewed the practical applications of opinion dynamics models in political elections, marketing, transportation, and public management.

2.4. Research gaps

Looking throughout the summary of extant models and their improvements, there are research gaps in the opinion dynamics models.

(i) With the development of the Internet and mobile Internet, distance is gradually becoming less of a barrier to interpersonal communication. People can intercommunicate with each other at any time and place, no longer passively depend on locational relationships. In turn, as communication becomes more convenient, people can be less passively constrained to neighbour or social acquaintances and more actively driven by potential interests and inner motivations in interacting with targets and exchanging their opinions.
(ii) In previous studies, people have been assumed to interact once they had the willingness to interact with others, or in some probability to do so. However, when confronted with practical problems, especially in economic and political contexts, people weigh possible costs and benefits after they have generated the willingness to communicate and before they decide to interact. There is thus an invisible decision-making process between the interactive willingness and behaviour, which makes the actual occurrence of interactions not as frequent as one might think and should not be ignored.
(iii) In developed models, people change their opinions through the uniform memoryless rules after communicating with others, that is, they update opinions based only on the current interaction in the same way. This is inconsistent with reality and does not take into account the dynamics of human awareness and the differences between people. In practice, because people's thoughts and thought-driven behaviours are constantly changing with their different experience accumulations, others' opinions affect people differently. Such dynamics and difference need to be considered in modelling opinion evolution and require an appropriate way to model.

Therefore, we present an active opinion dynamics model and simulate purposeful interactions in opinion evolution to fill these gaps.

3. Model development

In light of the activeness and dynamics of interactions and the experience and difference of interaction participants, the active opinion dynamics model proposed in this paper suits today's opinion interaction and evolution more. It is based on the CODA model that was introduced by Martins in 2008 [11], drawing on the thought of "continuous opinion and discrete action" and improving the CODA rule to dynamic CODA rule. In this paper's model, there are no fixed locational or social ties between people(agents), and the concept of a group is a collection of agents that can interact, rather than a collection of agents that are acquainted. Whether an agent in the collection will generate a willingness to interact with another one is determined by the similarity effect: they will be more willing to interact with other agents whom they think hold more similar opinions. Two agents that have had the willingness will then measure, through an imperfect game, whether potential gains can cover costs in the interaction. Such a process repeats in each possibly occurring interaction, allowing agents to learn from historical experience and constantly adjust their strategies to obtain better utility.

3.1. Preliminary

CODA model is the first opinion dynamics model that distinguishes people's opinions and actions, arguing that (i) agents in the model show discrete actions but have continuous opinions that are updated by interacting with others; (ii) agents can only observe others' external actions but not know their internal opinions when interacting; (iii) agents update opinions and choose actions by using a Bayesian description of how likely their interacting targets are to be correct.

In detail, suppose an issue has 2 choices A and B. If agent i considers choice A better than B with probability $P_i(A)=p$, and correspondingly considers choice B better than A with probability $P_i(B)=1-p$, then when $p>1-p$, agent i will act to choose A and $\sigma_i=+1$, conversely when $p<1-p$, agent i will prefer B and $\sigma_i=-1$, and when $p=0.5$, agent i will not make choice and σ_i is empty.

During the interaction of agent i and one of its neighbours j, set $\alpha=P(\sigma_j=+1|A)$ as the probability that neighbour j acts to choose A when A is the better choice than B, and $\beta=P(\sigma_j=-1|B)$ as the probability that neighbour j prefers B when B is better than A. Taking $P_i(A)=p$ as the prior opinion (subjective probability) of agent i, according to the Bayesian rule, then the posterior opinion of agent i after observing the action of neighbour j is

$$\begin{cases} P_i(A|\sigma_j=+1) \propto P_i(A)P(\sigma_j=+1|A) = p\alpha \\ P_i(B|\sigma_j=+1) \propto P_i(B)P(\sigma_j=+1|B) = (1-p)(1-\beta) \\ P_i(A|\sigma_j=-1) \propto P_i(A)P(\sigma_j=-1|A) = p(1-\alpha) \\ P_i(B|\sigma_j=-1) \propto P_i(B)P(\sigma_j=-1|B) = (1-p)\beta \end{cases}. \tag{1}$$

Due to the unknown constant $P_i(\sigma_j)$ in formulae (1), CODA model uses the odds in favour of A, $O(A)$, which is defined as the ratio between the belief in A and the belief in B, to get rid of the inconvenience of directly calculating the posterior opinion. Therefore, the prior odds $O_i(A)$ of agent i is given by

$$O_i(A) = \frac{P_i(A)}{P_i(B)} = \frac{p}{1-p}. \tag{2}$$

Then the posterior odds of agent i after observing that neighbour j acts to choose A is

$$O_i(A|\sigma_j=+1) = \frac{P_i(A|\sigma_j=+1)}{P_i(B|\sigma_j=+1)} = \frac{p}{1-p}\cdot\frac{\alpha}{1-\beta}, \tag{3}$$

and the posterior odds of agent i after observing that neighbour j acts to choose B is

$$O_i(A|\sigma_j=-1) = \frac{P_i(A|\sigma_j=-1)}{P_i(B|\sigma_j=-1)} = \frac{p}{1-p}\cdot\frac{1-\alpha}{\beta}. \tag{4}$$

Further, CODA model takes log-odds as the new form of opinion expression $l=\ln(O(A))$, assumes that agents have no specific preference towards choice A and choice B, i.e., $\alpha=\beta$, and makes $v=\ln[\alpha/(1-\alpha)]$ and $-v=\ln[(1-\alpha)/\alpha]$, so that the posterior opinion of agent i after its interaction with neighbour j can be updated through

$$\begin{cases} l_i(A|\sigma_j=+1) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{\alpha}{1-\beta}\right) = l_i(A)+v \\ l_i(A|\sigma_j=-1) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{1-\alpha}{\beta}\right) = l_i(A)-v \end{cases}. \tag{5}$$

In this way, the change of an agent's opinion is only related to v, and α becomes the most important factor influencing opinion evolution. Here, if $\alpha>0.5$, then $v>0$, making an agent's opinion change in the same orientation as the observed action of its neighbour; if $\alpha<0.5$, then $v<0$, making an agent's opinion change in the opposite orientation against the observed action of its neighbour; if $\alpha=0.5$, then $v=0$, the agent's opinion remains constant. In this process, the size of α determines the extent of opinion change: the larger $|\alpha-0.5|$ is, the faster agents' opinions polarize.

Combined with the research gaps we analyzed in Section 2.4, the original CODA model leaves four specific places that are not practical enough and need to be improved: (i) agents do not necessarily just interact with their neighbours, nor do they interact without preference, and there is no reason for them to devote time and energy to interactions that provide no expected gains; (ii) during the interaction, what an agent acquires when observing another one is not necessarily the true choice of action, but may only be extrinsic information displayed; (iii) the probability $\alpha=P(\sigma_j=+1|A)$ that an agent acts to choose A when A is better than B and the probability $\beta=P(\sigma_j=-1|B)$ that an agent chooses B when B is a better choice than A are different for each agent, because the degree that an agent acts in line with its opinion varies; (iv) the two

Table 1
Symbols and definitions in the active opinion dynamics model.

Symbol	Definition
G	A group of agents who take part in the discussion of an issue.
T	Number of times an agent interacts with other agents.
n	Size of group G, i.e., number of agents in the group.
x_i	An agent i in the group G, i=1, 2, …, n.
I	Issue discussed, having two orientations $\{I^+, I^-\}$ when referring to an agent's opinion or action, adding the neutral state I^0 when referring to an agent's faction.
P	Opinion of an agent, i.e., the subjective probability that an agent supports I^+, an invisible attribute known only to the agent alone.
P_G	Group opinion, the average of agents' opinions in a group.
A	Action of an agent, i.e., the vote choice that an agent makes based on its opinion, a half-visible attribute not known by others unless they interact with the agent.
A_G	Group voting result, the percentage of agents who vote for I^+ in a group.
F	Faction of an agent, synthesized reflection of the agent's opinion orientation based on its historical opinions, a visible(public) attribute known by everyone at any time.
c	Conservative degree of agents on a certain issue, embodying the amount of interaction memory that is contained in agents' historical opinions and then reflected in agents' factions.
h	Historical opinion of an agent, i.e., accumulated opinions in the most recent c times of interactions.
r	Agents' level of inclusiveness for different opinions, i.e., agents' rejection level of factions in the group.
u	Utility of an agent's interaction with another one, embodying the interaction benefit that exceeds cost.
e	Empirical parameter for an agent in the interaction game, embodying the agent's rational level, calculated by the frequency of successful interactions that confirm the other agent's support.
a	Adequate communication level of the group, i.e., the proportion of agents that have experienced at least one opinion interaction and updated their opinions.
δ	A minimum constant as the threshold of the difference between group opinions in several successive cycles to judge whether the group opinion has stabilized.
run	Number of model runs.
cycle	Number of cycles in the group opinion evolution, the current cycle of evolution moves to the next cycle after reaching the adequate communication level.

probabilities α and β should not be fixed but should dynamically change as each agent's experience of interaction accumulates and deepens.

3.2. Active opinion dynamics model

Learning strengths and improving flaws of CODA model, this paper builds an active opinion dynamics model below, where all parameters and variables used in the formulation can be referred to in Table 1.

3.2.1. Opinion expression

Suppose that we have a population of n agents represented by x_i (i=1, 2, …, n). Each agent has 3 attributes: opinion, action, and faction. Given an issue $I=\{I^-, I^+\}$, opinion $\boldsymbol{P_i}$ is the subjective probability of agent x_i backing I^+, $0\leq P_i\leq 1$. Accordingly, the probability backing I^- is $1-P_i$. Group opinion P_G is thus naturally defined as the average of n agents' opinions.

Action $\boldsymbol{A_i}$ is the choice that agent x_i makes voting for I^+ or I^- based on P_i. When $P_i>0.5$, we have $P_i>1-P_i$, which makes $A_i=I^+$; otherwise, when $P_i<0.5$, we have $A_i=I^-$. While if $P_i=0.5$ and agent x_i is hesitating, A_i will be randomly chosen from either of I^+ and I^- without a waiver. Then, the voting result could be represented by A_G, the percentage of agents who vote for I^+ in a group. The ratio of votes thus is A_G:$1-A_G$.

Faction $\boldsymbol{F_i}$ is an attribute that reflects the dynamic influence of time on agents. A faction usually is referred to a group of individuals within a larger entity, such as a party, a trade union, or simply a political climate, where members of the faction band together as a way of achieving their common goal. More generally, faction in this model is not an actual choice made by some agent, but more of bias or inclination.

Faction $\boldsymbol{F_i}$ here is acquired from x_i's historical opinion h_i that represents the agent's accumulative opinion during the most recent c times of interactions. Here, c embodies the amount of historical opinions reflected in agents' factions and could be understood as the population's degree of conservativeness on a certain issue. The smaller the c, the more radical and unstable the population's attitude; in turn, a larger c means a more conservative attitude relying on long-time observation. Faction split is related to the population's inclusiveness for different opinions r ($0<r<0.5$) that is equivalent to people's rejection of factions. A smaller r means that people in the population are less willing to embrace other's different opinions, making faction split in the group more distinct and fewer neutral agents. Conversely, a larger r suggests more neutral agents and the border between different factions is vaguer. If $h_i\geq 0.5+r$, meaning x_i's accumulative opinion inclines to I^+, then $F_i=I^+$; if $h_i\leq 0.5-r$, then $F_i=I^-$; otherwise, when $0.5-r<h_i<0.5+r$, x_i stands neutrally and $F_i=I^0$. As a result, $F_i(t_0)$, the faction of agent x_i in t_0th interaction, is defined through 2 steps.

Step 1. $h_i(t_0)$ calculation

$$h_i(t_0)=\begin{cases}\dfrac{1}{t_0}\displaystyle\sum_{t=1}^{t_0}P_i(t),1\leq t_0<c;\\[2ex] \dfrac{1}{c}\displaystyle\sum_{t=t_0-c+1}^{t_0}P_i(t),t_0\geq c.\end{cases} \tag{6}$$

Step 2. $F_i(t_0)$ identification

$$F_i(t_0)=\begin{cases}I^+,h_i(t_0)\geq 0.5+r;\\ I^0,0.5-r<h_i(t_0)<0.5+r;\\ I^-,h_i(t_0)\leq 0.5-r.\end{cases} \tag{7}$$

Among these three attributes, one's opinion is an invisible variable known by the agent alone, one's action is a half-visible variable not known by others until they have interacted with the agent, and one's faction is a visible(public) variable that could be seen by everyone at any time.

3.2.2. Interaction conditions

With whom do agents prefer to communicate and will they finally interact? Originally, opinion dynamics models ascribed both questions to local neighbours in the lattices or networks. Following bounded confidence, the occurrence of interaction then became limited, while agents' interaction preference continued to rely on neighbours and acquaintances. Exchanging opinions could be understood as chatting with friends in some situations, such as opinion propagation and cultural evolution. However, when in economic and political contexts, the activeness and dynamics of interactions should be taken into account.

(1) *Similarity effect*

Numerous studies have demonstrated that the similarity among individuals prompts interaction attraction [45, 46]. This occurs in various population scales and is amenable not only in laboratories but also in real relationships [47]. One study [48] further shows that perceived similarity, rather than actual similarity, predicts attraction in no-interaction (i.e., participants are unacquainted with each other but know a certain amount of information about their targets), short-interaction (i.e., the participant and a previously unacquainted target meet for 5~10 minutes before assessing the attraction), and existing relationships (i.e., partners have interacted at great length and in a variety of contexts). Therefore, we assume that one agent would have a greater desire to interact with another seemingly possessing a more similar opinion (a higher perceived similarity).

The level of perceived similarity is derived from faction, the only information seen by other agents before interactions. It means that two agents with the same faction are the most likely to generate the interaction willingness, two agents with one neutral and the other non-

Table 2
Probabilities that two agents generate the interaction willingness.

F_i	F_j	Probability 1	Probability 2
I^+	I^+	$0.8+\frac{\arctan(10r)}{5\pi}$	$0.8+0.2r$
I^-	I^-		
I^0	I^0		
I^+	I^0	$0.5+\frac{7\arctan(10r)}{20\pi}$	$0.5+0.4r$
I^-	I^0		
I^0	I^+		
I^0	I^-		
I^+	I^-	$0.2+\frac{\arctan(10r)}{2\pi}$	$0.2+0.6r$
I^-	I^+		

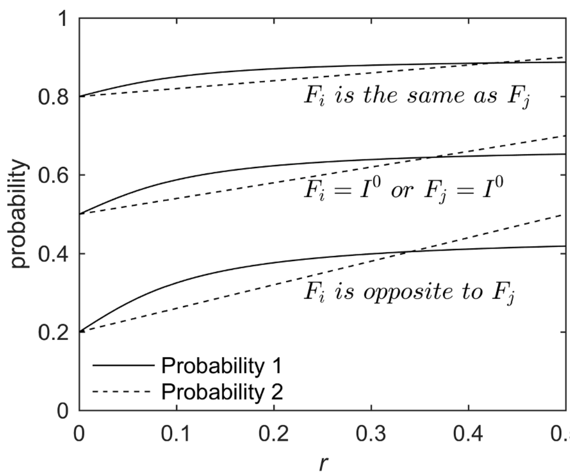

Fig. 1. Probabilities that two agents generate the interaction willingness.

neutral are the second least likely, and agents with opposing factions are the least likely. Moreover, how distinct the factions in a population are depends on *r*, the population's inclusiveness for different opinions. If *r* is relatively small, agents will be less likely to interact because the inclination difference embodied by faction will be enhanced. But if *r* is relatively large, even two agents with opposing factions will be more likely to interact because the border between different factions is vaguer. Therefore, besides perceived similarity, how likely two agents wish to interact with each other is also positively correlated to *r*.

In this paper's model, the probability that two agents generate the interaction willingness is set by probability 1 given in Table 2, which is nonlinearly related to *r*. Alternative designs are acceptable if they suffice required characteristics, such as probability 2 in Table 2 that is linearly related to *r*, which does not change simulation results of the model at a confidence level of at least 99.9% (for details see Appendix. B). The two probabilities are visualized in Fig. 1, in which probability 1 is enhanced at a decreasing speed as *r* increases, and probability 2 is enhanced at a constant speed with increasing *r*, especially when factions of the agents involved are opposite.

When agent x_i and x_j are selected and show a willingness to interact with each other, they could then economically consider the possibility and come to a decision.

(2) *Interaction game*

Many studies have game theoretically analyzed evolutionary decisions, such as [49, 50]. Similarly, each decision is deemed as a game in this model, where agents with interaction willingness evaluate whether potential benefits of the interaction could match their costs. Both agents x_i and x_j carry out the game simultaneously; when their equilibrium is reached, interaction occurs.

Let G_{ij} represent the game whereby agents x_i and x_j consider whether to interact or not. S_i={*Yes, No*} is x_i's strategy set; likewise, x_j's is S_j={*Yes, No*}. Their strategy profile set is then $S=\{S_i, S_j\}$, and one specific strategy profile is $s=\{s_i, s_j\}$.

Table 3
Utility of interaction for agent x_i.

Utility	Requirement(s)
$u_i>0$	$A_j'=A_i$,
$u_i\ll 0$	$A_j'=-A_i$ & $A_i'=-A_i$
$u_i<0$	$A_j'=-A_i$ & $A_i'=A_i$

The utility of interaction u_i and u_j are measured by their actions following the interaction, A_i' and A_j'. For agent x_i, as shown in Table 3, if x_j's post-interaction action A_j' is the same as x_i's current action A_i, i.e., x_i confirms an ally no matter if x_j is persuaded or originally stands on the same side with x_i, x_i will regard the interaction worthy and $u_i>0$; if A_j' is opposite to A_i, or, even worse, x_i is drawn to the other side and its later action A_i' belies current action A_i, x_i will regard the interaction counterproductive and $u_i\ll 0$; if A_j' is opposite to A_i but x_i's action does not convert, x_i will deem the interaction ineffectual and $u_i<0$. In the last situation, although agent x_i does not gain its expected benefit and the paid cost of communication could not be recovered, it is still better than discarding x_i's own stance. Here we do not compare the post-interaction actions to A_j because A_j is invisible information for x_i.

This process is the same for x_j if replacing the subscripts *i* and *j*. Therefore, for agents x_i and x_j, $G_{ij}=\{(S_i;u_i), (S_j;u_j)\}$ and the game matrix is

$$\begin{array}{ccc} & & x_j \\ & & \begin{array}{cc} Yes & No \end{array} \\ x_i & \begin{array}{c} Yes \\ No \end{array} & \begin{pmatrix} u_i, u_j & 0,0 \\ 0,0 & 0,0 \end{pmatrix} \end{array}. \tag{8}$$

In the case of complete information, a dichotomous game such as this is usually a zero-sum game with $u_i+u_j=0$, because only $A_i\neq A_j$ could leave "persuasion" space. However, here, $x_i(x_j)$ does not know $x_j(x_i)$'s current opinion or action. Therefore, the zero-sum condition is broken by the uncertainty that comes from incomplete information.

Moreover, due to incomplete information, solving the equilibrium of the game requires both agents to estimate their own and the other's utility, which is unknown but needed. According to the definition of

Table 4
Symbols and definitions used in presentation and analysis of the simulation results.

Symbol	Definition	Calculation
P_{G0}	initial group opinion	the average of agents' initial opinions.
A_{G0}	initial voting result	the initial percentage of agents who vote for I^+.
P_G(end)	eventually stabilized group opinion	the last item of $P_G=\{P_G(1), P_G(2), \ldots\}$.
A_G(end)	eventual voting result	the last item of $A_G=\{A_G(1), A_G(2), \ldots\}$.
ΔP_G	group opinion change, the amount that stabilized group opinion changes from initial group opinion	$\Delta P_G=P_G(\text{end})-P_{G0}$.
ΔA_G	voting result change, the amount that eventual voting result changes from initial voting result	$\Delta A_G=A_G(\text{end})-A_{G0}$.
C_P	group opinion conversion, representing how group opinion converts ($C_P=\pm 1$) or maintains (C_P=0) its initial stance	$C_P = \begin{cases} -1, & P_{G0}>0.5 \text{ and } P_G(\text{end})\leq 0.5; \\ 1, & P_{G0}<0.5 \text{ and } P_G(\text{end})\geq 0.5; \\ 0, & \text{otherwise.} \end{cases}$
C_A	voting result conversion, representing how voting result converts ($C_A=\pm 1$) or maintains (C_A=0) its initial stance	$C_A = \begin{cases} -1, & A_{G0}>50\% \text{ and } A_G(\text{end})\leq 50\%; \\ 1, & A_{G0}<50\% \text{ and } A_G(\text{end})\geq 50\%; \\ 0, & \text{otherwise.} \end{cases}$

Table 5
Evolutionary algorithm of the active opinion dynamics model.

Input: initial value of each attribute and parameter
Output: group opinions in each cycle $P_G(cycle)$ and voting results in each cycle $A_G(cycle)$

Generate a group of agents G, where each agent x_i ($i=1, 2, \ldots, n$) respectively owns three attributes: opinion $P_i \in [0, 1]$, action $A_i = \{-1, 1\}$, and faction $F_i = \{-1,0,1\}$, and two intermediate variables: historical opinion $h_i \in [0, 1]$ and actual utility $u_i = \{-1, -0.1, 1\}$
Assign *initial values to each agent's attributes and variables*
Set number of model runs *run* = 0 and number of cycles in group opinion evolution *cycle* = 0
while *run* *0* **do**
 Randomly select two agents x_i and x_j, j i
 # Interaction condition 1: Similarity effect
 Determine whether x_i and x_j both generate the interaction willingness based on the probability of two agents wishing to interact calculated according to Table 2
 if x_i and x_j both generate the interaction willingness **then**
  # Interaction condition 2: Interaction game
  ## For agent x_i : x_i*'s* expected game
  ### From x_i*'s own perspective*
  Predict post-interaction actions A_i' and A_j' of the two agents and then estimate x_i*'s expected utility* u_i' referring to Table 3

(continued on next page)

  ### From x_j*'s perspective*
  Predict post-interaction actions A_i' and A_j' of the two agents and then estimate x_j*'s expected utility* u_j' referring to Table 3
  if $u_i' > 0$ and $u_j' > 0$ (equilibrium realization of x_i*'s* expected game) **then**
   Calculate x_i*'s empirical parameter* e_i
   Determine whether x_i chooses to interact based on e_i
   if agent x_i chooses *Yes* **then**
    ## For agent x_j : x_j*'s* expected game
    ### From x_j*'s own perspective*
    Predict post-interaction actions A_j' and A_i' of the two agents and then estimate x_j*'s expected utility* u_j' referring to Table 3
    ### From x_i*'s perspective*
    Predict post-interaction actions A_j' and A_i' of the two agents and then estimate x_i*'s expected utility* u_i' referring to Table 3
    if $u_j' > 0$ and $u_i' > 0$ (equilibrium realization of x_j*'s* expected game) **then**
     Calculate x_j*'s empirical parameter* e_j
     Determine whether x_j chooses to interact based on e_j

Table 5 (*continued*)

```
                **if** agent x_j chooses Yes **then**
                    ## Equilibrium realization of actual interaction game
                    Update opinion P, action A, faction F, historical opinion h, and actual utility u of x_i and
                    x_j following the dynamic CODA rule and opinion expressions
                **end**
            **end**
        **end**
    **end**
  **end**
  # Calculate group opinion and voting result by cycle
  **if** over a % agents have experienced at least one opinion interaction in the current cycle **then**
    Calculate and record group opinion in the current run for the current cycle as P_G(cycle)
    Calculate and record voting result in the current run for the current cycle as A_G(cycle)
    ## Stop condition for opinion evolution
    **if** P_G(cycle) = 1, or P_G(cycle) = 0, or the differences between group opinions in several successive cycles
    are less than a minimum constant **then**
        break
    **end**
    cycle = cycle + 1
  **end**
  run = run + 1
**end**
Output P_G = {P_G(1), P_G(2), …} and A_G = {A_G(1), A_G(2), …}
```

utility, because one's current action is known, to estimate utility is to estimate one's own and the other's post-interaction actions. Taking x_i as the instance, x_i needs to

(i) view from x_i's own perspective: since x_i's own historical record is known, x_i could first estimate x_j's action A_j with F_j, with which then calculate x_i's post-interaction opinion P_i' through the opinion fusion rule (dynamic CODA rule, see section 3.3.3) and predict its post-interaction action A_i'; while A_j' could only be estimated by x_j's current faction F_j because x_j's opinion and action are both invisible now.
(ii) view from x_j's perspective: for x_i who puts itself in the scenario of x_j, all the information x_j could use is merely their factions; therefore, x_j in x_i's supposition has to directly estimate A_i' and A_j' using F_i and F_j, respectively.

From this, and referring to Table 3, x_i estimates its own and x_j's expected utilities u_i and u_j. To solve the game G_{ij} and find its Nash equilibrium $s^*=(s_i^*, s_j^*)$, it should satisfy

$$\begin{cases} u_i\left(s_i^*, s_j^*\right) \geq u_i\left(s_i, s_j^*\right) \\ u_j\left(s_j^*, s_i^*\right) \geq u_j\left(s_j, s_i^*\right) \end{cases} \text{is true for } \forall s_i \in S_i \text{ and } \forall s_j \in S_j. \quad (9)$$

According to the game matrix, the solution is $u_i \geq 0$ and $u_j \geq 0$ at the same time.

In addition, it should be noted that this game process is not perfectly rational. On the one hand, both players in the game make their decisions based on estimated utility rather than actual utility, which weakens the reliability of the obtained equilibrium; on the other hand, although the payoff of *Yes* is larger than that of *No*, one may still choose *No* guided by nonconscious biases [51] stemming from past experience. Therefore, here this model adds an empirical parameter e_i for agent x_i to count the frequency of actual utility that is larger than zero (the frequency of successful interactions that confirm x_j's support) as the probability of x_i insisting *Yes* after the game, i.e., x_i's rational level.

To summarize, only when x_i's estimated utilities of both $u_i \geq 0$ and $u_j \geq 0$, meaning an equilibrium realization, would x_i believe saying *Yes* to the possible interaction might bring benefits and then decide based on its empirical parameter. The same operation of estimation, measurement, and decision occurs to x_j simultaneously, which only requires subscripts *i* and *j* to be replaced. Though it may appear complicated, this is quite a natural process.

3.2.3. Dynamic CODA rule

How do agents change their opinions after communicating with others? Many simple methods have been considered, such as following or reversing the target agent's opinion, taking the (weighted) average of certain or all other agents' opinions, or multiplying and/or adding parameters to the previous opinion. Based on the Bayesian rule and considering the experience and difference of interaction participants, this model derives dynamic CODA rule as the opinion fusion rule.

When satisfying interaction conditions, the interaction between two agents x_i and x_j occurs. According to the Bayes theorem, x_i's updated (posterior) opinion $P_i(I|A_j)$ after observing x_j's action A_j is

$$P_i(I|A_j) = \frac{P_i(A_j|I)P_i(I)}{P_i(A_j)}. \tag{10}$$

To eliminate $P_i(A_j)$ in formula (10), which is inconvenient for understanding and computing, we at first split issue I to I^+ and I^-, obtaining

$$\begin{cases} P_i(I^+|A_j) = \dfrac{P_i(A_j|I^+)P_i(I^+)}{P_i(A_j)} \\ P_i(I^-|A_j) = \dfrac{P_i(A_j|I^-)P_i(I^-)}{P_i(A_j)} \end{cases} \text{and } P_i(I^+|A_j) + P_i(I^-|A_j) = 1 \tag{11}$$

and then let $P_i(I^-|A_j)$ divide $P_i(I^+|A_j)$ to obtain $O_i(I|A_j)$, the odds in favour of x_i's posterior probability backing I^+ or against x_i's posterior probability backing I^-:

$$O_i(I|A_j) = \frac{P_i(I^+|A_j)}{P_i(I^-|A_j)} = \frac{P_i(A_j|I^+)P_i(I^+)}{P_i(A_j|I^-)P_i(I^-)}. \tag{12}$$

As defined previously, P_i=$P_i(I^+)$ and $P_i(I^+)$+$P_i(I^-)$ =1, thus formula (12) equals

$$O_i(I|A_j) = \frac{P_i(A_j|I^+)P_i(I^+)}{P_i(A_j|I^-)[1-P_i(I^+)]} = \frac{P_i(A_j|I^+)}{P_i(A_j|I^-)}\cdot\frac{P_i}{1-P_i}. \tag{13}$$

Here, $P_i(A_j|I^+)$ and $P_i(A_j|I^-)$ can be estimated by the frequency of agent x_i's historical actions and factions. To be more specific, computing $P_i(A_j{=}I^+|I^+)$ is equivalent to counting how many times agent x_i's faction had inclined to I^+ in the previous interaction and its action voted for I^+ likewise in this interaction, of all experienced interactions. See Appendix. C for a detailed computation.

Finally, when the value of $O_i(I|A_j)$ is obtained, through $P_i(I^+|A_j)$+ $P_i(I^-|A_j)$=1, the updated opinion of x_i could be solved as

$$P_i(I|A_j) = P_i(I^+|A_j) = \frac{O_i(I|A_j)}{1+O_i(I|A_j)}. \tag{14}$$

3.2.4. Opinion evolution and stop condition

In each run of the model, two agents x_i and x_j are randomly selected as potential interaction subjects. Through the similarity effect, if they generated willingness of interaction, x_i and x_j would become players in the later interaction game; otherwise, two other agents would be reselected. When agent x_i and x_j both chose *Yes* in the interaction game, their opinion interaction would occur. Then the two agents could know each other's actions, update their opinions P_i and P_j through the dynamic CODA rule, and in turn update their actions A_i and A_j, historical opinions h_i and h_j, factions F_i and F_j, and actual utilities u_i and u_j.

The model is run repeatedly in the opinion evolution. During the process, it could be considered appropriate to stop when the group opinion gradually stabilizes. Here, it is important to note that an interaction does not necessarily occur in each run. By assuming a constant a in the interval of (0, 100] as the adequate communication level, this model defines evolution cycle that differs from the run. This means that the opinion evolution in the current cycle will move to the next cycle after over a% of agents have experienced at least one opinion interaction and updated their opinions.

Therefore, stability of the group opinion comprises two situations: (i) the group opinion P_G polarizes to one of the ends, P_G=1 or P_G=0 (because once P_G reaches 0 or 1, it will not recover to other numbers and further interactions will change nothing); (ii) the group opinion P_G fluctuates in each cycle but tends to be stable on the whole, where the difference of P_G between successive cycles is less than δ, a small enough constant.

Of course, the state whereby the group opinion continuously fragments and changes could not be excluded completely. At this time, a fixed number of runs or cycles for the evolution should be set to artificially cease the procedure.

4. Simulation experiments

Individuals in the active opinion dynamics model are independent and can communicate with and learn from each other. Therefore, it is well suited to discuss the opinion evolution through multi-agent simulation, especially when random and complex interactions make it hard to derive the process by differential equations.

Except for symbols in Table 1, additional notations used in subsequent presentation and analysis of the simulation results are summarized in Table 4.

4.1. Evolutionary algorithm

The evolution process of the model is described in Table 5, which omits some computational details but presents corresponding operations to avoid the table being too long. To adapt to computer implementation, take agent x_i as the example, we write its action A_i={I^-, I^+} as A_i={-1, 1}, faction F_i={I^-, I^0, I^+} as F_i={-1, 0, 1}, and utility {$u_i \ll 0$, $u_i<0$, $u_i>0$} as u_i={-1, -0.1, 1}.

4.2. Settings, methods, and results

4.2.1. Variable and parameter settings

Simulations in this paper were carried out in two ways. The first was to observe once simulation with certain parameters (Fig. 3), and the second was to simulate the model for 1000 groups where parameters were randomly chosen and combined (Fig. 2, Fig. 4~Fig. 7, and Fig. 8b). In the latter way, parameters that decided the evolution environment, including group size n, conservative degree c, inclusiveness for different opinions r, and adequate communication level a, were randomly selected from the range of $n\in${50, 51, ..., 300}, $c\in${1, 2, ..., 10}, $r\in$(0.1, 0.5), and $a\in$(0.1, 2/3) in each simulation, respectively.

Agents' initial opinions in this paper's simulations were set as n normally distributed random numbers with a mean of m and a standard deviation of $m/3$ when m<0.5 or $(1-m)/3$ otherwise. Then actions, factions and historical opinions were calculated according to opinion expressions. Actual utilities were uniformly set to 1 to ensure that the first run can begin.

The constant δ for stop condition (ii) that limited the difference of group opinion between successive cycles was set to 10^{-15}. For various group sizes and initial opinions, the disparity between stabilized group opinions under δ=10^{-15} and that under δ=0 was at least 0 and at last 0.0008. In addition, the set of δ=10^{-15} helped opinion evolutions save 12.27% runs on average without any result distortion.

4.2.2. Analysis methods

For the convenience of analysis, group opinion change ΔP_G and initial group opinion P_{G0} were transformed in some situations. ΔP_G was taken with the absolute value when considering its correlations with variables without directions (variables other than initial group opinion P_{G0}) to avoid missing or misunderstanding possible correlations. And P_{G0} was transformed to $0.5-|P_{G0}-0.5|$ when describing its relationship with ΔP_G (Fig. 7, Fig. 8) because ΔP_G distributed symmetrically with respect to P_{G0}=0.5 and ΔP_G=0.

In this paper, correlations of continuous variables were tested using the Pearson method and correlations involving discrete variables were tested using contingency tables.

4.2.3. Numeric results

Since opinions as subjective probabilities should belong to [0, 1], normally generated initial opinions outside the range of [0, 1] were eliminated. This made the average of these n initial opinions closer to 0.5 than m when $m\neq$0.5 and rarely fell within [0, 0.25) and (0.75, 1]. To ensure readability and the convenience of analysis, we adjusted the generation of initial opinions to spread P_{G0} (initial group opinions) evenly between 0.25 and 0.75 in 1000 simulations.

Table 6 presents the numerical results of a portion of the 1000-group model simulations, where group opinion change ΔP_G, voting result change ΔA_G, group opinion convert C_P, and voting result convert C_A are indirect results calculated from raw outputs of group opinions P_G and voting results A_G (see Table 4 for calculation methods and detailed explanations).

4.3. Gaps between the voting result and group opinion

What happens when agents' individual opinions and their collective opinion become stable? At the beginning of the opinion evolution, its initial group opinion P_{G0} is not necessarily equivalent to its initial voting result A_{G0}, since there is a gap of 0% to 21.86% between agents' continuous preference and the statistical result of their votes (blue marks in Fig. 2). Then, as interactions progress and opinions update, every agent's opinion gradually polarizes to near 0, 0.5 or 1 and the gap gradually narrows as displayed in Fig. 3, where the former is the microscopic cause of the latter. In 6 redly marked situations in Fig. 3, gaps between voting results and group opinions even fade away. After the evolution has stabilized, the gap eventually decreases to less than 12.66% (red marks in Fig. 2). Among these cases, over 92.0% of the gaps diminish to less than 5%, 73.2% to less than 3%, and 46.3% to less than 1%.

Thus, especially in the early days of an election or decision, people might feel that the voting result does not fit the actual atmosphere well. This incompatibility would remain until an increasing number of people's opinions stabilize. In the end, the group opinion and voting result would be consistent, reaching the eventual agreement.

In addition to the reconcilable gaps between voting results and group opinions, no sign of the gap between voting result conversion and group opinion conversion being filled emerges.

In our simulations, it happens infrequently that the group opinion converts against its initial stance after the opinion evolution. Group opinions convert only when their initial group opinions are close to 0.5, and are rarely accompanied with voting result conversions (specific 17 cases where $C_P \neq 0$ are summarized in Fig. 4). Out of the 17 cases where group opinions convert, the voting result does not convert ($C_P \neq 0$ and $C_A = 0$) in 15 cases. Furthermore, in the only 2 cases where voting results convert, they convert to the opposite directions against the group opinions' ($C_P = +1$ but $C_A = -1$, marked as green). So in general, in none of our simulation cases does the voting result convert in the same way as

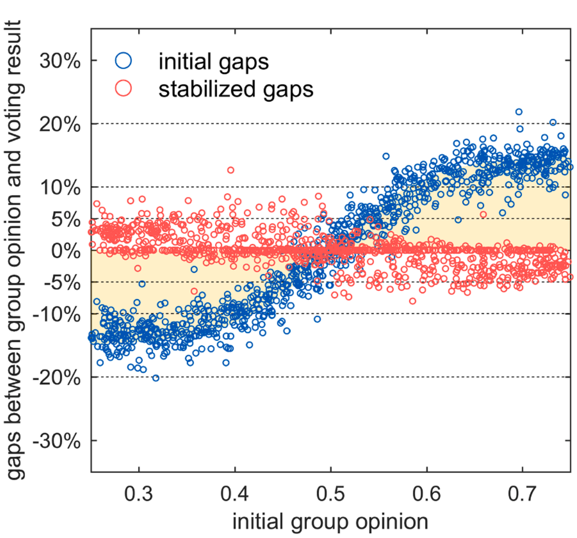

Fig. 2. Gaps between the voting result and group opinion.

the group opinion ($C_P = C_A = \pm 1$, being marked as red). That is, when people's collective opinion initially inclined to one side but transferred to another side after evolution, their voting results hardly reflect the conversion.

Therefore, due to both the gap between voting results and group opinions, and the gap between voting result conversion and group opinion conversion, a voting result neither necessarily represents the collective opinion of a group nor necessarily reflects the group opinion conversion.

4.4. Decisive factors in opinion evolution

How much will group opinion change after stabilization and what influences the change? After correlation tests between group opinion changes ΔP_G and different parameters of group size n, initial group opinion P_{G0}, conservative degree c, and inclusiveness for different opinions r, the following specifically analyzes the only two correlated factors whose statistical significance level are below 0.001.

4.4.1. Initial group opinion

Initial group opinion P_{G0} influences the direction of group opinion change.

As shown in Fig. 5, in the left diagram, group opinions change merely negatively when $P_{G0} \leq 0.43$ and merely positively when $P_{G0} \geq 0.57$. This suggests that if agents in a group initially support one side with an average probability less than 0.43, the opinion evolution will only strengthen its initial orientation. Certainly, the bound is not necessarily the same as 0.43 in this paper, given different parameter designs; however, what can be concluded is that the group opinion changing to the opposite side only occurs in a confined space when interactions between people are active.

As for the voting result change, in the right diagram of Fig. 5, even when the initial group opinion is 0.35 or 0.65, individual cases whereby voting results change to the other side remain. Similar to group opinion changes, the voting result will not change its direction if the initial collective opinion in a group is lower than a bound of 0.35. But different from group opinion changes, because the voting result change requires a certain accumulation of group opinion change, voting result changes in most scenarios are close to zero.

That is, if one side was much stronger than the other at the beginning, the initial losing side could hardly reverse the situation.

4.4.2. Inclusiveness for different opinions

The population's inclusiveness for different opinions r influences the consistency of stabilized group opinions and voting results, and the intensity of group opinion changes.

On the first aspect, the inclusiveness r influences how much the gap between the voting result of a group and its group opinion will narrow after the opinion evolution is stabilized (Fig. 6). When $r<0.15$, i.e., at least 70% of decision-makers in the group have their clear preferences on the issue discussed and are unwilling to embrace different opinions, over 93.14% of gaps between the voting result and group opinion will narrow to less than 3% and over 79.41% will narrow to less than 1% after the opinion stabilization; when $r>0.45$, i.e., over 90% of agents are willing to listen to and accept others' different opinions, gaps that eventually narrow to less than 3% will account for 86.36% and gaps of less than 1% will account for 43.63%; while if r is between 0.15 to 0.45, the average proportions of the two eventual states are 58.56% and 24.05%, respectively.

On the second aspect, the group's inclusiveness for different opinions r influences how much the collective opinion of a group will change over the opinion evolution (Fig. 7, where absolute values of group opinion changes are taken and P_{G0} are transformed through the method in Section 4.2.2, the same in Fig. 8). As shown in Fig. 7a, when $r<0.2$, i.e., more than 60% of agents in the group held unwilling-to-change preferences, stabilized group opinions change no more than 0.2205, and the

Table 6
Part of numeric results of the model simulations.

P_{G0}	n	runs	cycles	c	r	a	ΔP_G	ΔA_G	C_P	C_A
0.2502	125	12470	227	6	0.1303	0.5357	-0.1750	-0.0080	0	0
0.2701	79	28771	953	6	0.4550	0.2463	-0.1815	-0.0759	0	0
0.2913	60	4811	178	10	0.1305	0.4494	-0.1861	0.0000	0	0
0.3122	300	72326	1023	10	0.2536	0.2328	-0.1815	-0.0233	0	0
0.3343	273	119421	1117	4	0.3997	0.2932	-0.2198	-0.0623	0	0
0.3531	257	55593	318	3	0.2921	0.5502	-0.2178	-0.0233	0	0
0.3716	81	60148	800	6	0.4429	0.5081	-0.2328	-0.1358	0	0
0.3912	210	80733	642	5	0.3678	0.3305	-0.2521	-0.0762	0	0
0.4121	73	98262	1475	6	0.4986	0.2373	-0.2621	-0.1918	0	0
0.4351	269	167142	1625	10	0.2461	0.2634	-0.0955	-0.0149	0	0
0.4537	134	204483	1060	4	0.4758	0.6623	-0.3208	-0.2537	0	0
0.4728	67	41261	652	9	0.2678	0.3414	0.0261	-0.0299	0	-1
0.4860	208	136127	480	4	0.2856	0.6168	0.0176	-0.0337	1	-1
0.5000	167	474644	2047	7	0.4910	0.2382	0.4062	0.4192	0	1
0.5203	257	429487	834	10	0.4091	0.6578	0.2386	0.1751	0	0
0.5305	158	14495	832	1	0.0606	0.0643	-0.0305	0.0000	-1	0
0.5595	123	170709	1314	2	0.4815	0.4554	0.3291	0.2683	0	0
0.5709	141	170344	7998	5	0.4974	0.0257	0.2942	0.1844	0	0
0.5921	211	184956	643	8	0.4571	0.6591	0.2695	0.1611	0	0
0.6134	291	158176	1724	7	0.2014	0.2972	0.1397	0.0206	0	0
0.6377	220	103641	1123	2	0.1816	0.3915	0.0918	0.0045	0	0
0.6534	89	32422	1380	5	0.3955	0.1660	0.2391	0.0674	0	0
0.6729	141	110910	1708	10	0.4816	0.2180	0.2217	0.0993	0	0
0.6933	228	42131	2214	9	0.2860	0.0737	0.2062	0.0132	0	0
0.7136	269	38717	576	2	0.1633	0.2867	0.1547	0.0000	0	0
0.7317	60	14280	618	3	0.3933	0.3962	0.2292	0.0333	0	0
0.7496	135	15226	208	5	0.2001	0.6258	0.1741	0.0000	0	0

change is increasingly smaller to 0.0001 as the initial group opinion moves towards 0.5; in Fig. 7b, when r>0.4, i.e., only less than 20% of agents among the population identified their stances clearly (which usually happens when the issue discussed is very controversial or the discussion is not mature), group opinions after the evolution change more than 0.1650, growing increasingly larger to 0.4062 as the initial group opinion approaching 0.5; and in Fig.7c, when r was less than 0.4 but greater than 0.2, as the initial group opinion increases, the group opinion change have two possible results: it will be either less intense or more intense. This uncertainty implies diversity, which brings more possibilities but also more risks for opinion evolution.

In sum, the inclusiveness for different opinions r is a double-edged sword to opinion evolutions. Though a lower level of inclusiveness highly and effectively ensures the consistency between stabilized voting results and group opinions, it limits the development space and solidifies the group opinion evolution. And when a higher level of inclusiveness helps in adequately eliminating the gap of voting difference from real opinion, it at the same time generates intense changes that would not necessarily be desirable because polarizations might cause unintended consequences. An intermediate level of inclusiveness does not benefit both sides, too. Even if it injects more diversity into the opinion evolution and allows the original preference to be overturned, it is inevitable to be criticized for the relatively large discrepancy between the population's voting result and their group opinion after evolution.

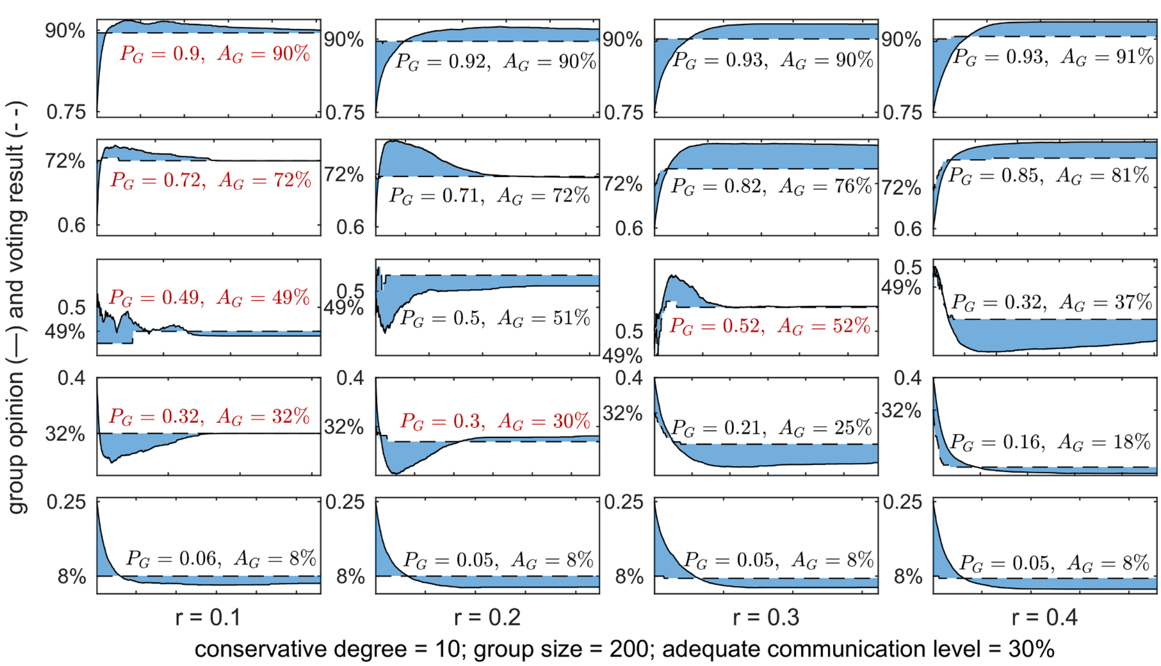

Fig. 3. Gradual decreases in gaps between voting results and group opinions.

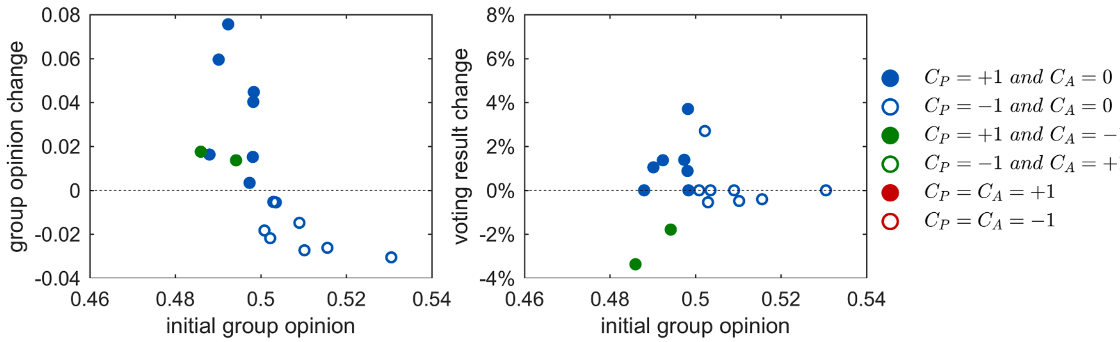

Fig. 4. 17 cases where group opinion converted.

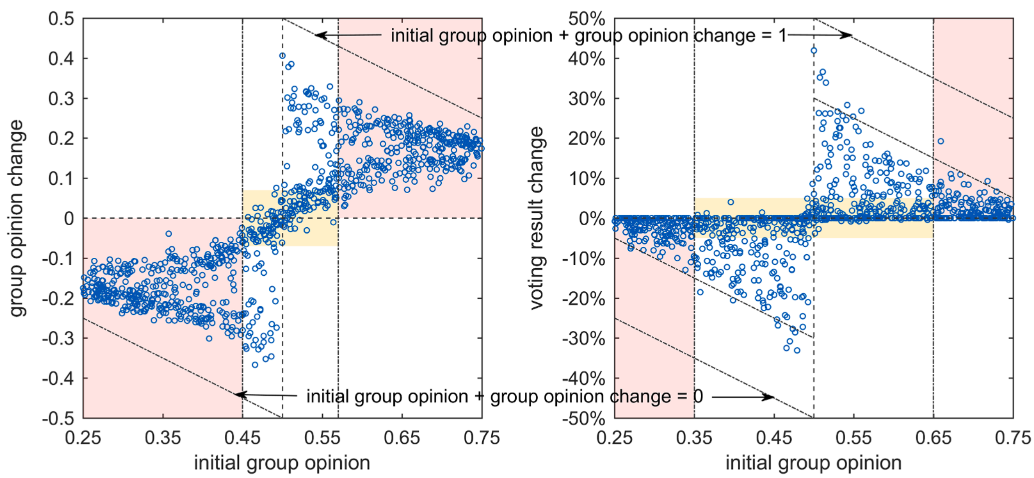

Fig. 5. Group opinions and voting results change with initial group opinion.

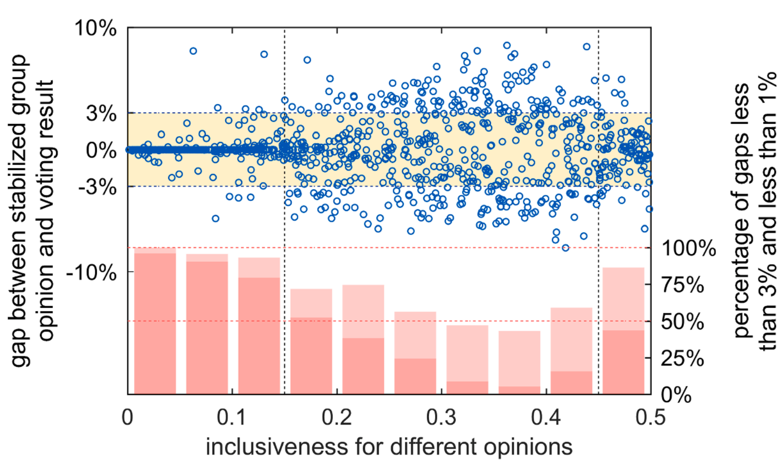

Fig. 6. Stabilized gaps influenced by the inclusiveness for different opinions.

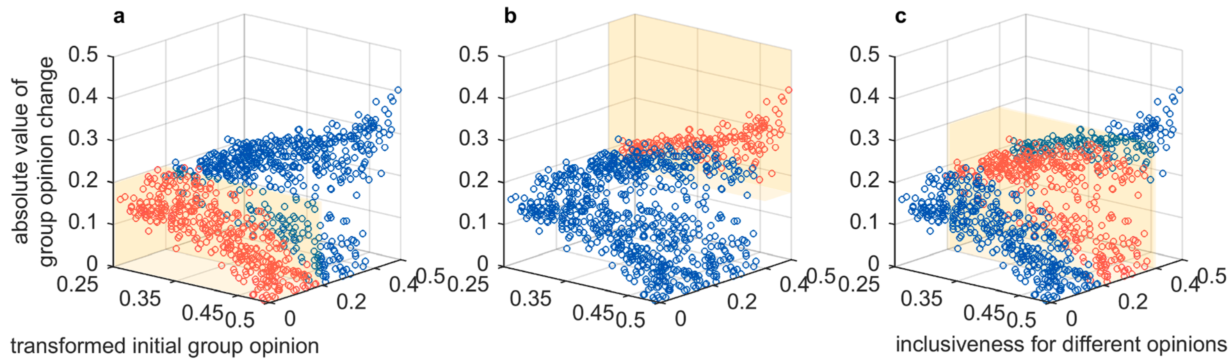

Fig. 7. Group opinions change with the population's inclusiveness and initial group opinions.

5. Probability distribution of group opinion change

Can we predict changes in group opinion? Unfortunately, even if all factors and how they influence opinion evolution were known, given the butterfly effect, an exact prediction of group opinion change is never obtainable. However, the good news is that we can still predict how likely a certain change is to occur if we know its probability distribution.

Intuitively, simulation results of the active opinion dynamics model show a beautiful regularity rather than randomness. Looking at Fig. 7 again, it can be seen that changes of group opinions roughly cluster into two components when the initial group opinion approaches 0.5, and concentrate around 0.2 when the initial group opinion approaches 0.25. Therefore, we speculated that the probability of group opinion changes might obey a bimodal distribution.

Let ΔP_G denote the group opinion change. According to the definition of bimodal distribution, the probability density function can be written as

$$p(\Delta P_G) = \frac{\alpha_1}{\alpha_1+\alpha_2}\cdot\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(\Delta P_G-\mu_1)^2}{2\sigma_1^2}\right) + \frac{\alpha_2}{\alpha_1+\alpha_2}\cdot\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{(\Delta P_G-\mu_2)^2}{2\sigma_2^2}\right). \tag{15}$$

The distribution is composed of two separate Gaussian distributions. $\alpha_1/(\alpha_1+\alpha_2)$ and $\alpha_2/(\alpha_1+\alpha_2)$ are coefficients of the two parts and are complementary. μ_1 and σ_1, μ_2 and σ_2 are means and variances of the two sub-distributions, respectively. As analyzed in Section 4.4, two factors determine how much the group opinion will change after stabilization: the initial group opinion P_{G0} and the population's inclusiveness for different opinions r. Therefore, on the basis of the simulation results, we conjectured parameters as follows:

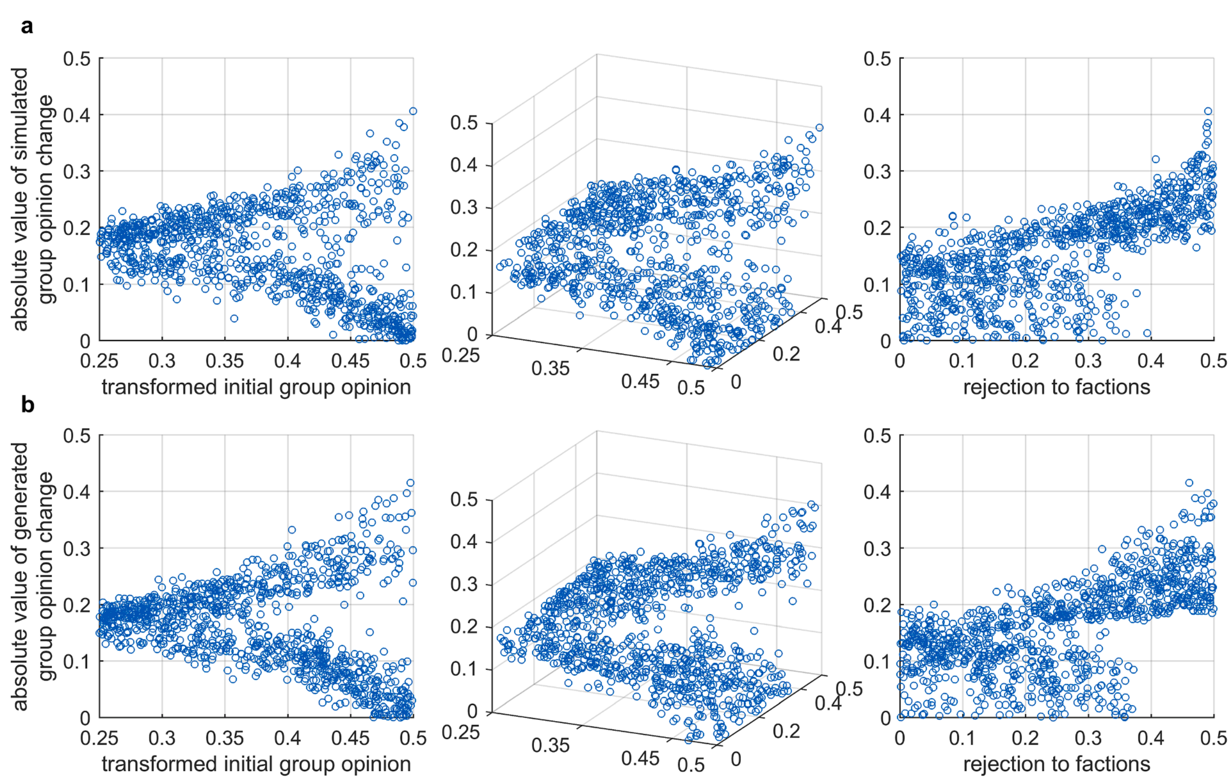

Fig. 8. Comparison of simulated and generated group opinion changes.

$$\begin{cases} \alpha_1 = \dfrac{0.5 - r}{1 + \exp(100(r - (P_{G0} - 0.1)))}; \\ \alpha_2 = \dfrac{r}{1 + \exp(100(-r + (1.8P_{G0} - 0.45)))}; \\ \mu_1 = -0.0004\exp(12P_{G0}) + 0.1614; \\ \mu_2 = 0.09\exp(2P_{G0})\cdot(0.65r + 1); \\ \sigma_1 = 0.06P_{G0}; \\ \sigma_2 = 0.16P_{G0} - 0.035. \end{cases} \tag{16}$$

To verify our conjecture and the efficiency of formulae (16), we performed the Kolmogorov-Smirnov (K-S) test for ΔP_G gained from simulations and ΔP_G generated according to the calculated probability by formulae (15) and (16). We divided the domain of $P_{G0}\times r$ into 5×10 grids and repeated data generations and K-S tests 20 times in each space, respectively. The results showed that data in 83% of grids on average passed the K-S test at a significance level of 0.05. Compared with simulation results in Fig. 8a, Fig. 8b shows one of the 20 generations with the highest percentage of grids passing the test (92%), which indicates that the conjecture worked.

To determine how likely it is that the collective opinion of a group will change to a certain extent after evolution, the cumulative probability needs to be computed. Since the probability distribution is a weighted sum of two Gaussian distributions, utilizing the cumulative distribution function of the standard normal distribution (formula 17), the cumulative distribution function of formula (15) could be obtained as formula (18). Hence, the probability of a group opinion change from its initial state to a stable state being between ΔP_{G1} and ΔP_{G2} could be calculated by $F(\Delta P_{G2}) - F(\Delta P_{G1})$.

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \tag{17}$$

$$\begin{aligned} F(\Delta P_G) &= \int p(\Delta P_G) d\Delta P_G \\ &= \frac{\alpha_1}{\alpha_1 + \alpha_2} \Phi\left(\frac{\Delta P_G - \mu_1}{\sigma_1}\right) + \frac{\alpha_2}{\alpha_1 + \alpha_2} \Phi\left(\frac{\Delta P_G - \mu_2}{\sigma_2}\right) \end{aligned} \tag{18}$$

Here, the two Gaussian distributions imply two trends of the group opinion evolution. The first one represents stabilization, and the closer the initial group opinion is to 0.5, the more stable the opinion evolution is; while the second one represents destabilization, and the closer the initial group opinion is to 0.5, the more unstable the opinion evolution is. The population's inclusiveness for different opinions determines the priority of both trends. When the inclusiveness is relatively low ($r<0.2$), α_1 is larger and the trend in stabilization dominates; when the inclusiveness is higher ($r>0.4$), α_2 is larger and the trend in destabilization predominates; and when the inclusiveness is between 0.2 and 0.4, the two trends coexist. This is consistent with previous analysis based on simulations, indicating that an appropriate level of inclusiveness for different opinions is important to the opinion evolution. In addition, this allows for a flexible regulation, because sometimes we want the evolution of group opinion to strengthen a certain state, sometimes we want it to break the stalemate, and at more times, we hope to see more possibilities.

6. Discussion

The main contribution of the article lies in presenting a new model of opinion dynamics that simulates purposeful interactions in opinion evolution, in finding that a group's voting result does not necessarily reflect its collective opinion, and in obtaining the probability distribution formulae of group opinion change, which could be used to predict and regulate the opinion evolution.

6.1. Model contributions

Based on the activeness and dynamics of interactions and the experience and difference of interaction participants, the active opinion dynamics model proposed in this article fills the research gaps analyzed in Section 2.4 and differs from extant models mostly in four aspects: (i) independence from fixed passive connections considering interactions are actively driven by people's internal motivations; (ii) attributes with different perceptibility considering the invisible generation of interaction willingness; (iii) game theoretical and experience-based decision-making of interactions; (iv) Bayesian and dynamic opinion update for different people. These pragmatic considerations enable the model to go beyond a toy model and yield practical results.

Compared with the focus of existing models on, as summarized in Section 2.2, whether individual opinions will coincide or how many clusters of different opinions will be formed, we are more concerned with overall results after the opinion evolution at the macro level. Through multi-agent simulations, we find that:

(i) The voting result of a group neither necessarily represents its group opinion nor necessarily reflects the group opinion conversion. On the one hand, there is a gap from 0% to 21.86% between the initial collective opinion of a group and its first voting result, which narrows as people's discussions further and will eventually narrow to less than 5% in most (92.0%) situations. On the other hand, group opinion conversions seldom (2 out of 17 cases) are accompanied with the voting result conversion and never are reflected by the voting result conversion.

(ii) Too biased initial group opinion and voting result predestine the outcome of opinion evolution. From the view of group opinion, if the initial group opinion is less than a threshold (0.43 in this paper's simulations), opinion evolution will only strengthen the initial dominant side. From the view of voting, if the initial voting result is less than a threshold (0.35 in this paper's simulations), opinion evolution will have no chance to reverse the voting situation.

(iii) Different levels of the population's inclusiveness for different opinions adjust the opinion evolution. The consistency between stabilized voting result and group opinion is ensured well by lower or higher (<0.2 or >0.4 in this paper's simulations) levels of inclusiveness, but is relatively weak when the level of inclusiveness is in the middle. And the group pinion evolution is solidified by a lower level of inclusiveness, intensified by a higher level of inclusiveness, and diversified by the middle level of inclusiveness.

(iv) Changes from initial group opinions to stabilized group opinions obey bimodal distribution in this paper's opinion evolutions. The distribution comprises two Gaussian distributions representing the tendency of stabilization and destabilization respectively, and which of them dominates is determined by the population's inclusiveness for different opinions.

These discoveries can help people answer such questions as: Why sometimes people's voting result is not consistent with their overall preference? Is there any chance for people who support the weaker side to win back and how? How much will the group opinion change throughout an opinion evolution? What measures can be taken to influence the outcome of an opinion evolution? By knowing the answers, for products in the market, students in a discussion, candidates in an election, governments in a referendum, and each of us in public life, the model could be used to understand, predict, and regulate opinion evolutions where interactions are purposeful.

6.2. Potential applications

To specifically illustrate how the model proposed in this paper and its

Table A.1
Classification and summary of basic opinion dynamics models.

Name	Ref.	Category	Environment	Opinion form	Target selection	Interactive rule	Result
Voter Model (VM)	[2]	Discrete opinion	Agents in regular lattices	$\{-1, +1\}$	Randomly select an agent i	Agent i reverses opinion in a probability of $w_i = \frac{1}{2}\left(1 - \frac{\sigma_i}{z}\sum_j \sigma_j\right)$. σ_i is agent i 's opinion, σ_j is the opinion of the neighbour j selected, z is the number of neighbours	Researchers have proven that for any dimension of the lattice, VM leads to one of the two possible consensus states (0 or 1). The probability of which one will be reached depends on the initial distribution of opinions.
DeGroot model	[8]	Continuous opinion	Agents fully connected in a group	subjective probability of agent i, $F_i \in [0, 1]$	No selection, all agents update simultaneously	Take the weighted average of others' opinions, $F_i = \sum_{j=1}^{k} p_{ij} F_j$	The consensus in the DeGroot model is a linear process whereby the initial opinion vector multiplies weight matrix P whose row sum equals 1 again and again. Because this is basically a Markov chain, a consensus will always be reached.
Friedkin-Johnsen model (F-J model)	[23]	Continuous opinion	Agents fully connected in a group	Given numbers	Opinion updated collectively	Round 1: determine the group opinion based on the external variable X and coefficient B, $Y_1 = XB$ Round t ($t>1$): considering the opinion in the previous round $Y_{t\text{-}1}$ and internal influence W (the row sum of W is 1) and utilizing weight α and β, collectively determine the group opinion $Y_t = \alpha W Y_{t-1} + \beta XB$	The F-J model encompasses various situations in which interpersonal influence has been studied. It suggests that a single process of interpersonal influence is involved in each situation and different outcomes arise as a result in different structural contexts.
Axelrod model	[22]	Discrete opinion	Sites in two-dimension lattices	Culture in each site has F features and each feature has several traits	Randomly select a site	Randomly choose a neighbour; if no feature of the site's culture is consistent with its neighbour's, nothing will happen; if there are n attributes being consistent ($0<n<F$), then randomly make one of them consistent with the probability of n/F	This model discusses culture clustering beyond binary variables, which shows that the degree of polarization (measured by the number of different cultural regions) after evolution increases when there are few features to the culture, when there are many traits of each feature, and when interactions are only with adjacent sites.
Sznajd model	[9]	Discrete opinion	Agents in one-dimension lattices	$\{-1, +1\}$	Randomly select a pair of agents (i, $i+1$)	If opinions of the pair of agents are the same ($S_i S_{i+1}=1$), their neighbors take the opinion ($S_{i-1}=S_i=S_{i+1}=S_{i+2}$); if opinions of the pair of agents are opposite ($S_i S_{i+1}=-1$), their neighbors take the second adjacent agent's opinion ($S_{i-1}=S_{i+1}$, $S_{i+2}=S_i$)	Different from other models, the closed (isolated) community described in this model evolves two opposite clusters and is unable to take any common decision, which means that reaching a consensus must rely on a dictatorship.
Deffuant model	[24]	Bounded confidence model	–	[0, 1]	Randomly select a pair of agents (x, x')	If the difference between the pair of agents' opinions is within the confidence interval ($\lvert x-x'\rvert<d$), according to fusion parameter μ ($\mu\in[0, 0.5]$) adjust their opinions as $\begin{cases} x = x + \mu(x' - x) \\ x' = x' + \mu(x - x') \end{cases}$, otherwise they remain unchanged	Agents' stabilized opinions in this model will always maintain the average initial opinion because of its updating rule. For individuals, however, when the confidence condition $d\geq 0.5$, all agents will keep the same opinion of 1/2 (their initial opinions obey a standard normal distribution), which is a complete consensus; when $d<0.5$, two or more clusters survive and the number is suggested as $1/(2d)$.
Majority Rule (MR)	[10]	Discrete opinion	Cliques of agents in regular lattices	$\{-1, +1\}$	Extract $G=2d+1$ cliques (d is dimension of the lattice)	Update to the opinion held by the majority of groups. (Note: the model approximates the Ising model with the temperature of	On a one-dimensional lattice, the boundary between neighbouring opposite opinion voters

(continued on next page)

Table A.1 (*continued*)

Name	Ref.	Category	Environment	Opinion form	Target selection	Interactive rule	Result
						0 when opinions are updated individually)	decays in time; on hypercubic lattices, the consensus is always achieved.
Hegselmann-Krause model (H-K model)	[4]	Bounded confidence model	–	[0, 1]	No selection, all agents update simultaneously	Take the average of others' opinions that are within the interval of confidence ($\lvert x-x'\rvert \leq d$)	Similar to the Deffuant model, a larger d will yield fewer clusters in the H-K model, and when a large enough d is taken, the consensus will be reached. One key feature about the model proven later in [6] is that agents with any initial opinion profile will reach the stable state.
Constrained 3-state voter model	[5]	Discrete opinion	Agents in regular lattices	{−1, 0, +1}	Randomly select an agent	Evenly change opinion from (−1, 0) to (−1, −1) or (0, 0); from (0, +1) to (+1, +1) or (0, 0); opinion (−1, +1) maintains	The evolution result of this model is either an ultimate consensus or a frozen state comprising non-interacting leftists and rightists, and as the centrist density approaches 100%, the probability of reaching a frozen state drops to zero.
Continuous opinions and discrete actions model (CODA model)	[11]	Mixed model	Agents in square lattices	Probability of agent i preferring view A is $P_i(A)=p$ (preferring view B is $P_i(B)=1–p$); action of agent i voting for view A is $\sigma_i=+1$ (voting for view B is $\sigma_i=-1$)	Randomly select an agent i	Randomly choose an agent j; according to the Bayesian theorem calculate the posterior probability $P_i(A\lvert\sigma_j)$ of agent i against view A and $P_i(B\lvert\sigma_j)$ against view B, then decide the corresponding action $\sigma_i=\mathrm{sgn}(l_i)$ based on the logarithmic probability $l_i = \ln\left(\frac{P_i(A\lvert\sigma_j)}{P_i(B\lvert\sigma_j)}\right)$; specially, keep the previous action when $\sigma_i=0$	The CODA rule is applied to both the VM and Sznajd model and clear domains with different opinions are observed, showing that the two opinions will have to live with each other for a long time. In addition, inside those domains and to a lesser extent in the boundaries, opinions become very extreme.
Nonlinear q-voter model	[7]	Discrete opinion	Agents in a fully connected network	{−1, +1}	Randomly select an agent	Randomly choose q neighbours; if these q agents hold the same opinion, agent i keeps consistent with them; otherwise, it reverses with a probability of ε.	The q-voter model shows that voters are not allowed to break the absorbing state even with a noise effect controlled by ε, and exhibits a disordered phase for high ε and an ordered one for low ε with three possible ways of going from one to the other.
DeGroot-Friedkin model	[29]	Mixed model	–	Agents' opinions to event s is $y(s)\in[0, 1]$	No selection, all agents update simultaneously	Weighted average all agents' opinions on event s in the previous round (round t), $y(s, t+1)=W(x(s))\,y(s, t)$. The composition of the weight matrix $W(x(s))=\mathrm{diag}(x(s)+(I_n–\mathrm{diag}(x(s)))C$ embodies self-appraisal and social power*	The model investigates the evolution of self-appraisal, social power, and interpersonal influences in a group of individuals discussing and forming opinions.

*As for weight matrix W, when agent i expresses its opinion on event s, $w_{ii}(s)=x_i(s)$ represents the individual's self-appraisal, $w_{ij}(s)=(1-x_i(s))c_{ij}$ represents its interpersonal weights, and C is the relative interaction matrix with a diagonal value of 0. Through constructing the influence network $W(x(s))$, agent i forms its social power $w(x(s))$.

simulation results can be used to deal with practical problems, some simple application examples are provided below.

In June 2016, immediately following the announcement of the Brexit vote, over 4 million people signed a petition demanding a second referendum [52]. In November of the same year, the US presidential election ended up with a reversal against previous polls [53]. The both events had been discussed nearly a year before the final vote, meaning that people's opinions had evolved for a long time. However, why did the results still seemingly fail to embody public will?

(i) Understanding the gap between the voting result and group opinion.

The final voting result of Brexit was 51.9% to 48.1%. It was a narrow victory of the controversial issue whose initial and stabilized group opinions were both close to 0.5. The close competition, referring to Fig. 4, is exactly responsible for why the demand for a second referendum obtained over 4 million signatures. Under this situation, group opinion of people might had converted, while their voting result did not reflect that, leading discontent of some people.

The US presidential election was another story. As we know, as one of the ways to investigate people's opinions, polls are inevitably more or less biased by sample selection or survey methods [54]. However, according to the results summarized in section 4.3, because of the gap between group opinion and voting result, even if a polling did accurately reflect people's opinion, it is not necessarily a good barometer for the voting result. Therefore, though there are other political causes, the rooted gap helps justify why the voting result violated previous polling

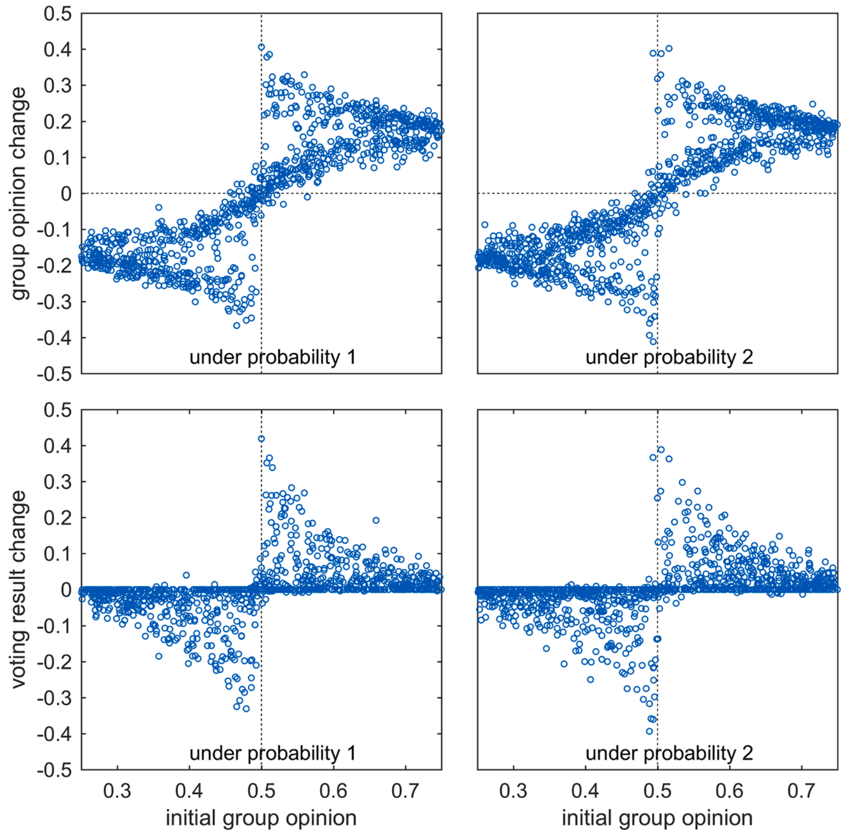

Fig. B.1. Simulation comparison under two possibilities of interaction willingness generation

indications.

(ii) Predicting the evolutionarily stabilized group opinion(voting result).

Even though the probability distribution of group opinion change proposed in this article requires corresponding adjustments in parameters according to different situations, it is still useful in making predictions. Since the voting result will eventually be consistent with group opinion, if we knew the initial opinions in a group that satisfied the normal distribution, and to what extent people in the decision group were willing to accept different opinions, the probability of a stabilized group opinion and voting result being in a certain interval can be calculated.

For instance, assume there was a group of decision-makers with an inclusiveness level of 0.3 (i.e., 60% of them were willing to listen to and might adopt opposite opinions) and their average initial opinions was 0.4 (i.e., the probability of their supporting one side or the other on average is 0.4). After the stabilization of their opinion evolution, referring to formula (18) with parameters set as formulae (16), the group's collective opinion and voting result will change to 0.3~0.4 with a probability of 7.70%, to 0.2~0.3 with a probability of 24.69%, to 0.1~0.2 with a probability of 66.26%, and to 0.1~0.2 with a probability of 1.35%.

(iii) Measures that can be used to regulate opinion evolution.

According to the analysis of decisive factors (Section 4.4) and the probability distribution of group opinion change (Section 5), there are two aspects influencing opinion evolution, whose general impacts are summarized in Section 6.1. They can be the basis of opinion regulation to prevent undesirable results such as extremization and fragmentation, or to achieve desired results such as stability and diversity, helping us be more flexible when dealing with group decisions.

On the first aspect of altering initial group opinion to influence the outcome of opinion evolution, measures that can be taken are to modify the group's structure or intervene inclinations of people in the group at the beginning, such as adjusting the composition of decision-makers, or earlier activities of campaigning and advertising. On the second aspect of altering people's inclusiveness for different opinions to influence the intensity of opinion evolution and the consistency between the voting result and group opinion, measures that can be taken are to affect people's firmness in their opinions or change the environment's opinion diversity, such as propaganda through news and influential people, or different incentive methods for opinion spread based on quantity or quality.

But it should be noted that practical opinion regulation is complex because of different purposes and multiple interferences.When facing a particular situation, the management departments of a government or an enterprise must take measures in line with the priorities of practical requirements.

7. Conclusion

In this paper, we propose an active opinion dynamics model whereby people do not intercommunicate passively based on their locational or social connections but are driven by their internal motivations of win, which simulates a complicated but stably evolving process of group opinion development. According to multi-agent simulations,we identify the reconcilable gap between the voting result and group opinion that gradually narrows as opinion evolution furthers, and the irreconcilable gap between group opinion conversion and voting result conversion; analyze the impact of initial group opinion on opinion evolution trends, the impact of people's inclusiveness for different opinions on narrowing the voting-opinion gap, and the impact of people's inclusiveness for different opinions on intensifying the change from initial to stabilized

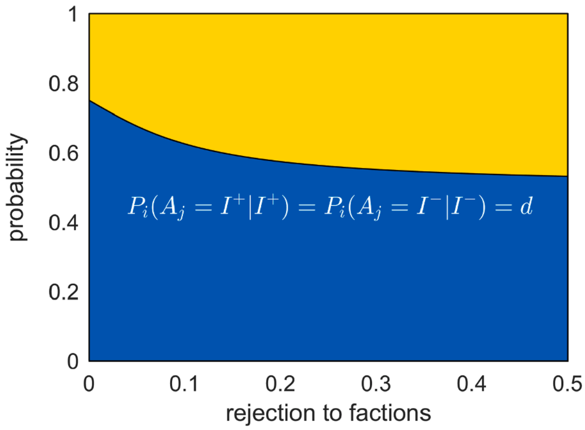

Fig. C.1. The primitive probability of the consistent action with one's faction

group opinion; conjecture and validate the bimodal distribution of group opinion change, which suggests two mutually restricted tendencies of stabilization and destabilization.

We summarize these theoretical findings and provide application examples to illustrate how they can contribute to understanding practical opinion problems, predicting opinion development, and taking measures to regulate opinion evolution, in response to today's gradually polarizing and fragmenting public opinion environment.

However, the model we introduced is relatively general and the simulations lack enough analysis in more specific conditions. For instance, how the opinion evolution will be given a different distribution of agents' initial opinions or in different community structures, how other purposeful group behaviours with more than two states of actions such as commenting in social media and marketing for competing products will evolve? In addition, our work needs detailed empirical research to further attest to the model's practical efficiency and to find appropriate parameters for different situations. Finally, the idea is expected to be extended from group decision-making to other dynamic systems, developing and studying more phenomena from an active perspective.

Data availability

No public data were used, and the experimental data are available at https://github.com/Yiru-Jiao/An-active-opinion-dynamics-model/tree/master/Experimental%20data

Code availability

https://github.com/Yiru-Jiao/An-active-opinion-dynamics-model

Author statement

Y. Jiao and Y. Li conceived and designed the study, Y. Jiao ran simulations, Y. Jiao and Y. Li wrote and revised the manuscript, and Y. Li supervised the work.

Declaration of Competing Interest

The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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Appendix. A. Basic opinion dynamics models

Appendix. B. Parameter sensitivity analysis

In model simulations of this paper, the values of parameters such as group size n, conservative degree c, inclusiveness for different opinions r, and adequate communication level a are randomized. Their effects on model results, therefore, can be directly compared within the 1000 simulation groups. While the probability that two agents generating the interaction willingness (introduced in Section 3.2.2), as one of the key parameters, requires a separate analysis to evaluate its effect.

Here, to inspect whether there is a difference in model results under different probabilities, and if so, what kind of the difference is, 1000 simulations of the model with other parameters randomized are performed under the probability 1 and probability 2 in Table 2, respectively. We then performed the Kolmogorov-Smirnov (K-S) test, which is widely used to detect whether two samples of data come from the same continuous distribution, on the outcome variables of group opinion change and voting result change under the two different probabilities of interaction willingness generation.

The result of the K-S test shows that the hypothesis of opinion changes and voting changes under the two probabilities being from the same distribution could not be rejected at the significance level of at least 0.001 (p-value=0.6785). Intuitively, taking the initial group opinion as the horizontal axis, the scatter diagrams of changes in group opinions and voting results under probability 1 and probability 2 are drawn in Fig. B.1, which shows that their distributions are almost identical.

Appendix. C. Estimation of $P_i(A_j|I^+)$ and $P_i(A_j|I^-)$

The two probabilities occurring in formula (13), $P_i(A_j|I^+)$ and $P_i(A_j|I^-)$, are intermediate steps of the dynamic CODA rule. They mean the possibility of one's action being consistent with his or her faction, and could be estimated by the frequency of agent x_i's historical actions and factions. Specifically, in t_0th interaction, if action $A_i(t_0)=I^+$, let auxiliary parameter $a_i^+(t_0)=1$ and $a_i^-(t_0)=0$; if $A_i(t_0)=I^-$, let $a_i^+(t_0)=0$ and $a_i^-(t_0)=1$. If faction $F_i(t_0)=I^+$, let auxiliary parameter $f_i^+(t_0)=1$ and $f_i^-(t_0)=0$; if $F_i(t_0)=I^-$, let $f_i^+(t_0)=0$ and $f_i^-(t_0)=1$; and if $F_i(t_0)=I^0$, let $f_i^+(t_0)=f_i^-(t_0)=0.5$. Then, $P_i(A_j|I^+)$ and $P_i(A_j|I^-)$ could be obtained as follows:

$$\begin{cases} P_i(A_j = I^+ | I^+) = \dfrac{\sum_{t=1}^{t_0} [a_i^+(t) | f_i^+(t-1) = 1] + 0.5 \sum_{t=1}^{t_0} [a_i^+(t) | f_i^+(t-1) = 0.5]}{\sum_{t=1}^{t_0} f_i^+(t)} \\ P_i(A_j = I^+ | I^-) = \dfrac{\sum_{t=1}^{t_0} [a_i^+(t) | f_i^-(t-1) = 1] + 0.5 \sum_{t=1}^{t_0} [a_i^+(t) | f_i^-(t-1) = 0.5]}{\sum_{t=1}^{t_0} f_i^-(t)} \end{cases} \tag{C.1}$$

$$\begin{cases} P_i(A_j = I^- | I^+) = \dfrac{\sum_{t=1}^{t_0} [a_i^-(t) | f_i^+(t-1) = 1] + 0.5 \sum_{t=1}^{t_0} [a_i^-(t) | f_i^+(t-1) = 0.5]}{\sum_{t=1}^{t_0} f_i^+(t)} \\ P_i(A_j = I^- | I^-) = \dfrac{\sum_{t=1}^{t_0} [a_i^-(t) | f_i^-(t-1) = 1] + 0.5 \sum_{t=1}^{t_0} [a_i^-(t) | f_i^-(t-1) = 0.5]}{\sum_{t=1}^{t_0} f_i^-(t)} \end{cases} \tag{C.2}$$

It should be noted that, when t_0=1 or the value of any item in the fraction is 0 (happening at the beginning of the evolution) the probability cannot be calculated. At this time, we could set a value d for $P_i(A_j=I^+|I^+)=P_i(A_j=I^-|I^-)$, and $P_i(A_j=I^-|I^+)=P_i(A_j=I^+|I^-)=1-d$. As the primitive probability of the consistent action with one's faction, here d should be inversely related to the population's inclusiveness for different opinions r, because a smaller r means lower inclusiveness and fewer neutral agents, which suggests agents' weaker ambivalence and higher consistency of their actions with factions.

In this paper, d is defined as $0.75-\arctan(10r)/(2\pi)$, whose visual relationship with r is shown in Fig. C.1. It is also encouraged to define d in other ways if it suits the trend described above.

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