In [1]:
```
import pandas as pd
import numpy as np
```

In [2]:
```
#part 1
```

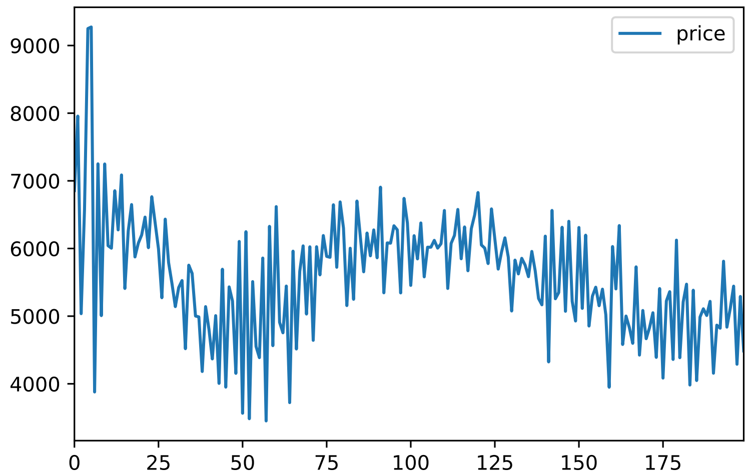
In [3]:
```
part1_train = pd.read_table(r"C:\Users\86187\Desktop\CUHKSZ\MAT2040\projects\project2\data\Part1_training_data.txt", names=["price"])
part1_test = pd.read_table(r"C:\Users\86187\Desktop\CUHKSZ\MAT2040\projects\project2\data\Part1_testing_data.txt", names=["price"])
part1_data = pd.concat([part1_train, part1_test])
print(part1_data.head())
print(part1_data.describe())
```

```
         price
0  6855.585318
1  7957.034758
2  5036.333172
3  6575.793236
4  9250.628680
             price
count   300.000000
mean   5840.664456
std    1029.776584
min    3411.350575
25%    5114.556628
50%    5851.293665
75%    6441.582334
max    9274.051623
```

In [4]:
```
print("Training data")
print(part1_train.head())
print(part1_train.describe())
print("="*78)
print("Testing data")
print(part1_test.head())
print(part1_test.describe())
```

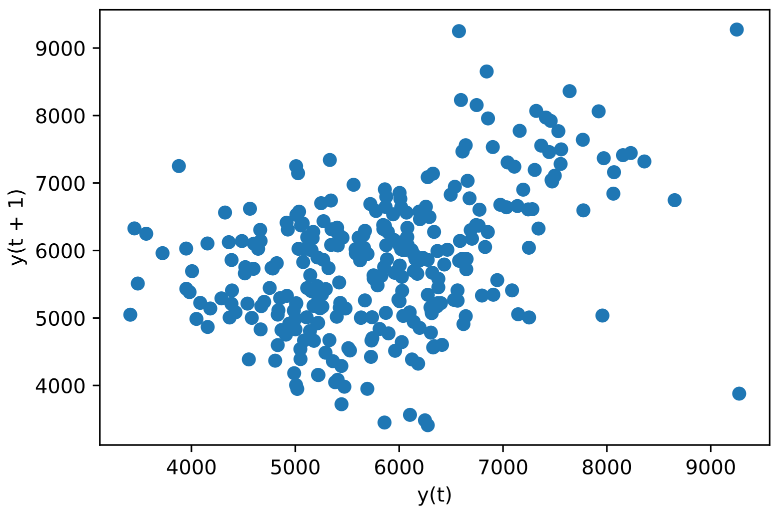
```
Training data
         price
0  6855.585318
1  7957.034758
2  5036.333172
3  6575.793236
4  9250.628680
             price
count   200.000000
mean   5591.679934
std     887.662119
min    3451.054975
25%    5035.011724
50%    5658.279165
75%    6161.805364
max    9274.051623
==============================================================================
Testing data
         price
0  6140.242914
1  4942.426720
2  4916.509842
3  6412.387005
4  4601.522714
             price
count   100.000000
mean   6338.633498
std    1114.979120
min    3411.350575
25%    5234.349113
50%    6532.408250
75%    7252.532070
max    8652.710184
```

In [5]:
```
import matplotlib.pyplot as plt
from datetime import datetime
%matplotlib inline
part1_train.plot()
plt.show()
```



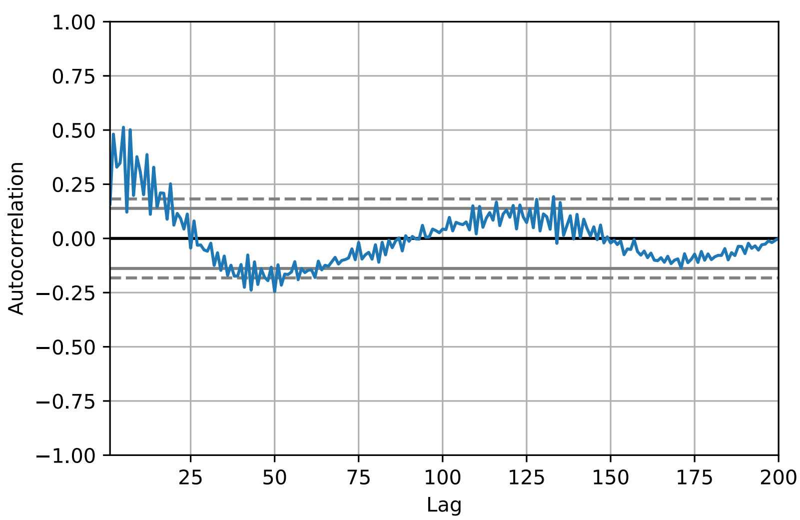
In [6]:
```
# Autoregression Model
```

In [7]:
```
pd.plotting.lag_plot(part1_data)
plt.show()
values = part1_train
lag1 = pd.concat([values.shift(1), values], axis=1)
lag1.columns = ['t-1', 't+1']
lag1_corr = lag1.corr()
print(lag1_corr)
```

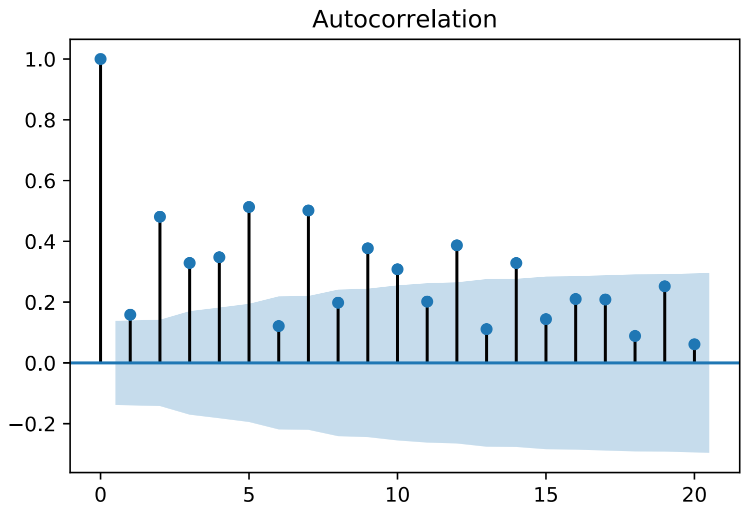



```
          t-1       t+1
t-1  1.000000  0.159974
t+1  0.159974  1.000000
```

In [8]:
```
pd.plotting.autocorrelation_plot(values)
plt.show()
```



In [9]:
```
""" ACF describes the autocorrelation between an observation and another observation at a prior time step that includes direct and indirect dependence information.
This means we would expect the ACF for the AR(k) time series to be strong to a lag of k and the inertia of that relationship would carry on to subsequent lag values, trailing off at some point as the effect was weakened."""
from statsmodels.graphics.tsaplots import plot_acf
plot_acf(values, lags=20)
plt.show()
" The ACF graphic seems not random. Thus, there exists a serial correlation in this series"
```




```
' The ACF graphic seems not random. Thus, there exists a serial correlation in this series'
```

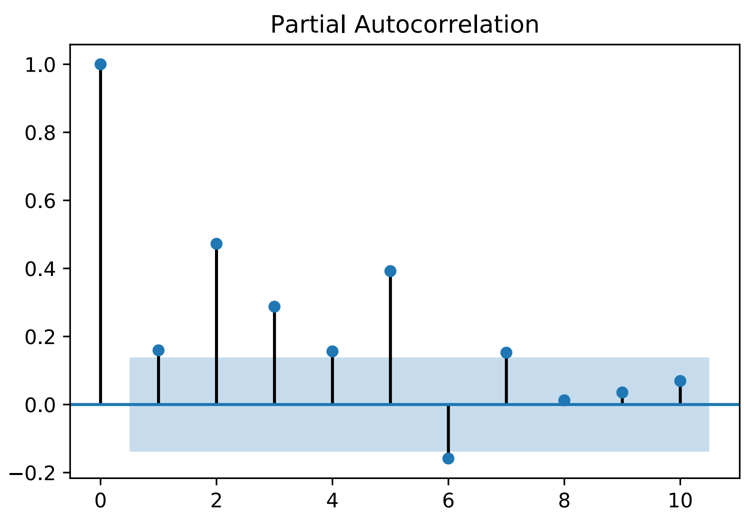
In [10]:
```
lags = values
for i in range(1,20):
    lags = pd.concat([lags, values.shift(i)], axis=1)
column = ["t"]
for i in range(1,20):
    name = "t"+"-"+str(i)
    column.append(name)
lags.columns = column
lags_corr = lags.corr()
print(lags_corr.head())
```

```
            t       t-1       t-2       t-3       t-4       t-5       t-6  \
t    1.000000  0.159974  0.494700  0.340624  0.361464  0.560440  0.141617   
t-1  0.159974  1.000000  0.158464  0.489977  0.340851  0.359632  0.558147   
t-2  0.494700  0.158464  1.000000  0.156829  0.489847  0.340264  0.358958   
t-3  0.340624  0.489977  0.156829  1.000000  0.156336  0.488772  0.336176   
t-4  0.361464  0.340851  0.489847  0.156336  1.000000  0.155970  0.489020   

          t-7       t-8       t-9      t-10      t-11      t-12      t-13  \
t    0.587108  0.236878  0.450445  0.377031  0.248909  0.475380  0.141545   
t-1  0.143835  0.584844  0.232469  0.444038  0.375767  0.245408  0.473881   
t-2  0.558744  0.142712  0.584646  0.231412  0.443621  0.375183  0.244719   
t-3  0.362793  0.556207  0.136889  0.578784  0.229188  0.441269  0.373055   
t-4  0.336440  0.362813  0.556622  0.136743  0.578792  0.229004  0.441272   

         t-14      t-15      t-16      t-17      t-18      t-19  
t    0.411049  0.188718  0.272315  0.275331  0.123799  0.334611  
t-1  0.137146  0.403580  0.187512  0.262437  0.275169  0.120753  
t-2  0.473474  0.135463  0.403296  0.186428  0.262198  0.274639  
t-3  0.240426  0.465219  0.133835  0.393414  0.185918  0.259738  
t-4  0.373111  0.241342  0.465157  0.134008  0.393353  0.185792  
```

In [11]:
```
"""The partial autocorrelation at lag k is the correlation that results after removing the effect of any corr
elations due to the terms at shorter lags."""
from statsmodels.tsa.stattools import pacf
PACF_array = pacf(values, nlags=9)
print("The Partial Auto Correlation Function")
name = ["lag"+str(i) for i in range(1,11)]
PACF_data = pd.DataFrame({
    "lags": name,
    "PACF": PACF_array
})
print(PACF_data)
```

```
The Partial Auto Correlation Function
    lags      PACF
0   lag1  1.000000
1   lag2  0.159280
2   lag3  0.472379
3   lag4  0.287595
4   lag5  0.156203
5   lag6  0.392081
6   lag7 -0.158633
7   lag8  0.152445
8   lag9  0.012361
9  lag10  0.035462
```

In [12]:
```
from statsmodels.graphics.tsaplots import plot_pacf
plot_pacf(values, lags=10)
plt.show()
"We choose lag1, lag2, lag3, lag4, lag5, lag6"
```




```
'We choose lag1, lag2, lag3, lag4, lag5, lag6'
```

In [13]:
```
from statsmodels.tsa.ar_model import AutoReg, ar_select_order
Lag_selected = ar_select_order(values,maxlag=10, ic="aic")
print("The lags selected are:")
print(Lag_selected.ar_lags)
print("="*78)
model = AutoReg(part1_train, lags=[1,2,3,4,5,6])
model_fit = model.fit()
print(model_fit.summary())
```

```
The lags selected are:
[1 2 3 4 5 6 7]
==============================================================================
                            AutoReg Model Results
==============================================================================
Dep. Variable:                  price   No. Observations:                  200
Model:                     AutoReg(6)   Log Likelihood               -1498.410
Method:               Conditional MLE   S.D. of innovations            547.206
Date:                Mon, 28 Dec 2020   AIC                             12.692
Time:                        16:39:55   BIC                             12.827
Sample:                             6   HQIC                            12.747
                                  200
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept   1530.6964    384.158      3.985      0.000     777.760    2283.633
price.L1      -0.2667      0.065     -4.079      0.000      -0.395      -0.139
price.L2       0.3117      0.056      5.534      0.000       0.201       0.422
price.L3       0.1812      0.058      3.100      0.002       0.067       0.296
price.L4       0.1340      0.057      2.367      0.018       0.023       0.245
price.L5       0.3812      0.055      6.924      0.000       0.273       0.489
price.L6      -0.0270      0.060     -0.447      0.655      -0.145       0.091
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           -0.9914           -0.5466j            1.1321           -0.4198
AR.2           -0.9914           +0.5466j            1.1321            0.4198
AR.3            1.0792           -0.0000j            1.0792           -0.0000
AR.4            0.2629           -1.3334j            1.3591           -0.2190
AR.5            0.2629           +1.3334j            1.3591            0.2190
AR.6           14.4765           -0.0000j           14.4765           -0.0000
-----------------------------------------------------------------------------
```

In [14]:
```
predictions = model_fit.predict(start=201, end=300)
for i in range(len(predictions)):
    print("The predicted value is: %f, The true value is: %f" % (predictions.iloc[i], part1_test.iloc[i]))
```

```
The predicted value is: 4983.622555, The true value is: 6140.242914
The predicted value is: 4866.230989, The true value is: 4942.426720
The predicted value is: 5250.548677, The true value is: 4916.509842
The predicted value is: 4828.916575, The true value is: 6412.387005
The predicted value is: 5331.739797, The true value is: 4601.522714
The predicted value is: 4973.855671, The true value is: 6101.448394
The predicted value is: 5165.161529, The true value is: 5083.614217
The predicted value is: 5186.888155, The true value is: 6019.416231
The predicted value is: 5072.137233, The true value is: 6641.271118
The predicted value is: 5299.286539, The true value is: 5026.656710
The predicted value is: 5082.507135, The true value is: 7145.518436
The predicted value is: 5275.830268, The true value is: 5055.215411
The predicted value is: 5185.606053, The true value is: 6371.772463
The predicted value is: 5216.748527, The true value is: 5219.661781
The predicted value is: 5275.994530, The true value is: 5320.111214
The predicted value is: 5190.684723, The true value is: 5738.951467
The predicted value is: 5305.006051, The true value is: 5010.511203
The predicted value is: 5223.221465, The true value is: 6528.440756
The predicted value is: 5287.453289, The true value is: 5268.262560
The predicted value is: 5275.860544, The true value is: 5863.291207
The predicted value is: 5265.350699, The true value is: 6364.072155
The predicted value is: 5311.104432, The true value is: 5172.825837
The predicted value is: 5267.868589, The true value is: 6275.509145
The predicted value is: 5316.896347, The true value is: 3411.350575
The predicted value is: 5291.073813, The true value is: 5048.829802
The predicted value is: 5307.844637, The true value is: 4539.328023
The predicted value is: 5316.139563, The true value is: 5213.108113
The predicted value is: 5303.327590, The true value is: 5897.364632
The predicted value is: 5328.765966, The true value is: 4771.718787
The predicted value is: 5310.571065, The true value is: 5741.713617
The predicted value is: 5329.232646, The true value is: 4700.492682
The predicted value is: 5324.186663, The true value is: 5239.244890
The predicted value is: 5326.352555, The true value is: 5143.980412
The predicted value is: 5335.188959, The true value is: 5633.851140
The predicted value is: 5327.470690, The true value is: 5007.921857
The predicted value is: 5339.604864, The true value is: 5210.223341
The predicted value is: 5333.427016, The true value is: 4919.314915
The predicted value is: 5339.604017, The true value is: 5328.442264
The predicted value is: 5340.505678, The true value is: 4672.542453
The predicted value is: 5339.516091, The true value is: 5175.716704
The predicted value is: 5345.186594, The true value is: 5179.486219
The predicted value is: 5341.674241, The true value is: 4662.084109
The predicted value is: 5346.841417, The true value is: 6305.674465
The predicted value is: 5345.440458, The true value is: 4783.230221
The predicted value is: 5347.146400, The true value is: 5733.434700
The predicted value is: 5348.908814, The true value is: 4666.063417
The predicted value is: 5347.916969, The true value is: 6142.034272
The predicted value is: 5350.916819, The true value is: 5941.667562
The predicted value is: 5349.681995, The true value is: 6536.375744
The predicted value is: 5351.690901, The true value is: 6944.512749
The predicted value is: 5351.806726, The true value is: 5561.126259
The predicted value is: 5352.154504, The true value is: 6973.971765
The predicted value is: 5353.466765, The true value is: 6677.650306
The predicted value is: 5352.963634, The true value is: 6773.084980
The predicted value is: 5354.384522, The true value is: 6605.031933
The predicted value is: 5354.123056, The true value is: 5875.646740
The predicted value is: 5354.849773, The true value is: 6797.187572
The predicted value is: 5355.255374, The true value is: 5332.532588
The predicted value is: 5355.289491, The true value is: 7341.629133
The predicted value is: 5356.058701, The true value is: 6325.420821
The predicted value is: 5355.897024, The true value is: 7138.595147
The predicted value is: 5356.524515, The true value is: 6659.069077
The predicted value is: 5356.585725, The true value is: 7033.047441
The predicted value is: 5356.840808, The true value is: 6640.796303
The predicted value is: 5357.176205, The true value is: 7560.003003
The predicted value is: 5357.179031, The true value is: 7498.319160
The predicted value is: 5357.580810, The true value is: 7109.068338
The predicted value is: 5357.575884, The true value is: 7242.593470
The predicted value is: 5357.843466, The true value is: 6609.108674
The predicted value is: 5357.964719, The true value is: 7467.232311
The predicted value is: 5358.060748, The true value is: 7044.196612
```

```
The predicted value is: 5358.273842, The true value is: 7305.184767
The predicted value is: 5358.292042, The true value is: 7194.278196
The predicted value is: 5358.489392, The true value is: 6901.533773
The predicted value is: 5358.532911, The true value is: 7532.881528
The predicted value is: 5358.648000, The true value is: 7768.366953
The predicted value is: 5358.747710, The true value is: 7641.091207
The predicted value is: 5358.792503, The true value is: 8361.203422
The predicted value is: 5358.913061, The true value is: 7319.498322
The predicted value is: 5358.939620, The true value is: 8068.635190
The predicted value is: 5359.034289, The true value is: 7160.151921
The predicted value is: 5359.080069, The true value is: 7773.406742
The predicted value is: 5359.132717, The true value is: 6594.887006
The predicted value is: 5359.198406, The true value is: 8229.874369
The predicted value is: 5359.225147, The true value is: 7444.203259
The predicted value is: 5359.289535, The true value is: 7458.978054
The predicted value is: 5359.314551, The true value is: 7920.826436
The predicted value is: 5359.360429, The true value is: 8062.287881
The predicted value is: 5359.394861, The true value is: 6842.567504
The predicted value is: 5359.421559, The true value is: 8652.710184
The predicted value is: 5359.460659, The true value is: 6745.983605
The predicted value is: 5359.478737, The true value is: 8154.877489
The predicted value is: 5359.512368, The true value is: 7412.705346
The predicted value is: 5359.531583, The true value is: 7969.846833
The predicted value is: 5359.554703, The true value is: 7367.698111
The predicted value is: 5359.577227, The true value is: 7554.404614
The predicted value is: 5359.592250, The true value is: 7282.347870
The predicted value is: 5359.614360, The true value is: 6612.212361
The predicted value is: 5359.626742, The true value is: 7472.846386
The predicted value is: 5359.644366, The true value is: 7023.201715
```

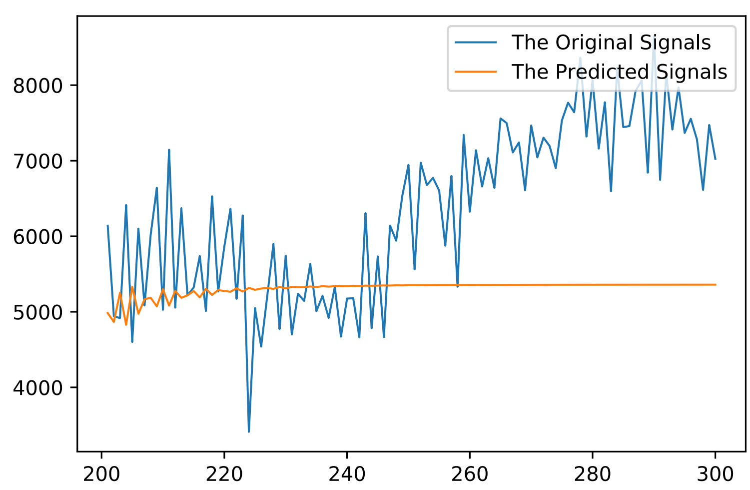
In [15]:

```
from sklearn.metrics import mean_squared_error
from math import sqrt
RMSE = sqrt(mean_squared_error(part1_test, predictions))
print("Test RMSE: %.3f" % RMSE)
```

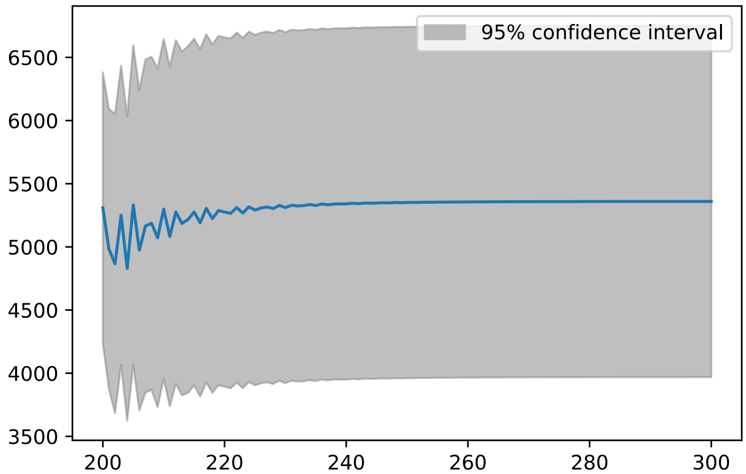
```
Test RMSE: 1498.986
```

In [16]:

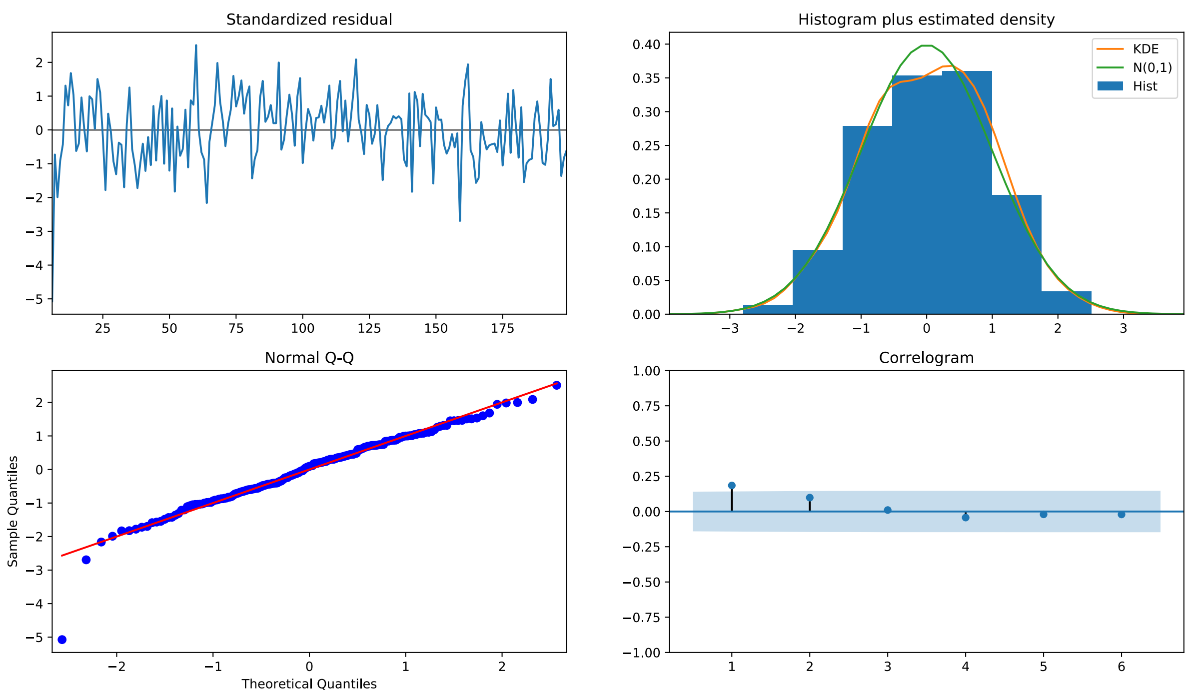
```
date = np.arange(201, 301)
plt.plot(date, part1_test, linewidth=1, label='The Original Signals')
plt.plot(date, predictions, linewidth=1, label='The Predicted Signals')
plt.legend(loc='upper right')
plt.show()
```



In [17]:
```
fig = model_fit.plot_predict(200, 300)
plt.show()
```



In [18]:
```
fig = plt.figure(figsize=(16, 9))
fig = model_fit.plot_diagnostics(fig=fig, lags=[1,2,3,4,5,6])
plt.show()
print("The first graph indicates that the residuals are not correlated with regressors\nThe Second and third graph indicate that the dependent variable and regressors have finite fourth moments, and it meets the normality(residuals are normally distributed) as well as homoskedasticity assumptions in regression\nThe fourth graph indicates that there are no new auto correlated regressors should be taken into consideration(The chosen regressors are comprehensive enough")
```

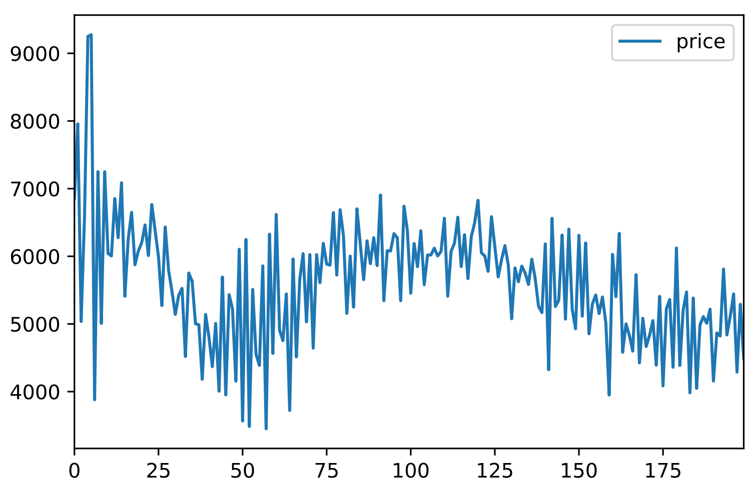



```
The first graph indicates that the residuals are not correlated with regressors
The Second and third graph indicate that the dependent variable and regressors have finite fourth moments, and it meets the normality(residuals are normally distributed) as well as homoskedasticity assumptions in regression
The fourth graph indicates that there are no new auto correlated regressors should be taken into consideration(The chosen regressors are comprehensive enough
```

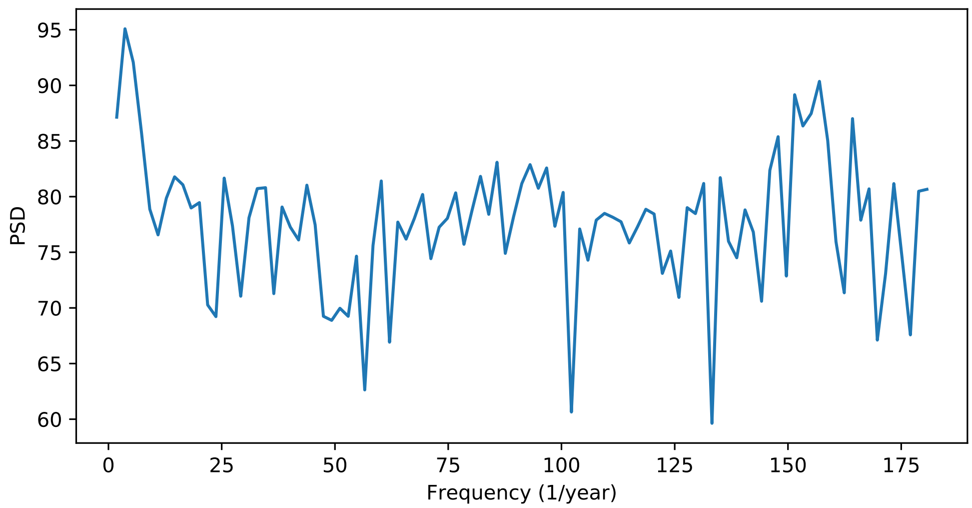
In [19]:
```
# Fourier Series
```

In [20]:
```
from scipy import pi
from scipy.fftpack import fft, fftfreq, ifft
values = part1_train
print(values.head())
print(values.describe())
values.plot()
plt.show()
```

```
         price
0  6855.585318
1  7957.034758
2  5036.333172
3  6575.793236
4  9250.628680
             price
count   200.000000
mean   5591.679934
std     887.662119
min    3451.054975
25%    5035.011724
50%    5658.279165
75%    6161.805364
max    9274.051623
```



In [21]:
```
# Suppose the data is measured in the unit of days
part1_train_ftf = fft(values["price"])
part1_train_psd = np.abs(part1_train_ftf) ** 2 # Get the power spectral density
part1_train_fftfreq = fftfreq(len(part1_train_psd), 1./365)
selected = part1_train_fftfreq > 0 #Because the target is price, we only focus on the positive frequencies
fig, ax = plt.subplots(1, 1, figsize=(8, 4))
ax.plot(part1_train_fftfreq[selected], 10*np.log10(part1_train_psd[selected]))
ax.set_xlabel("Frequency (1/year)")
ax.set_ylabel("PSD")
plt.show()
print("The first few Fourier Coefficients are: ")
print(part1_train_ftf[:10])
```




```
The first few Fourier Coefficients are:
[1118335.98687567     +0.j           -22752.28783461  -113.44252342j
   46058.09909923-33237.99808286j   13009.50384759-37988.165855j
   -4354.18593348-18857.20670224j    5251.47756782 -7045.53028751j
   -4275.62360805 -5201.49885916j    5890.06760783 -7876.95742182j
   10462.91044866 -6418.83296383j    1609.39307757-11194.43616001j]
```

In [22]:
```
print(part1_train_fftfreq[selected].argmax())
"We observe a peak for f = 98"
```

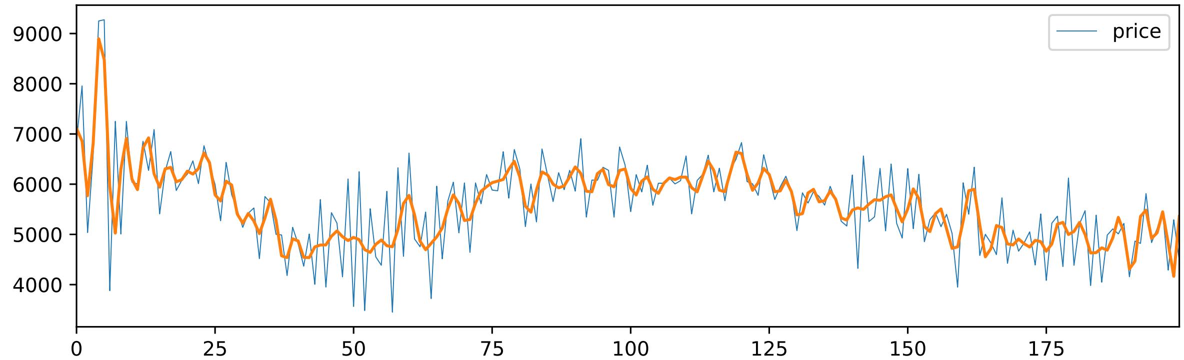
```
98
```

```
'We observe a peak for f = 98'
```

In [23]:
```
# cut out frequencies higher than the fundemental frequency
part1_train_bis = part1_train_ftf.copy()
part1_train_bis[np.abs(part1_train_fftfreq) > 98.1] = 0
```

In [24]:
```
part1_train_slow = np.real(ifft(part1_train_bis))
fig, ax = plt.subplots(1, 1, figsize=(10, 3))
values.plot(ax=ax, lw=.5)
ax.plot_date(range(200), part1_train_slow,"-", xdate=False)
# There exists some seasonally changes
```

```
[<matplotlib.lines.Line2D at 0x1cbd3e05cc8>]
```



In [25]:
```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
date = pd.DataFrame({"date":[i for i in range(200)]})
model.fit(date, values)
pred_linreg = model.predict(values)
residuals = values - pred_linreg
pred_linreg = pred_linreg.flatten()
data = pd.DataFrame({
    "pred_linreg": pred_linreg,
    "residuals": residuals["price"]
})
print(data.head())
print("R-squared: {:.2e}".format(model.score(date, values)))
```

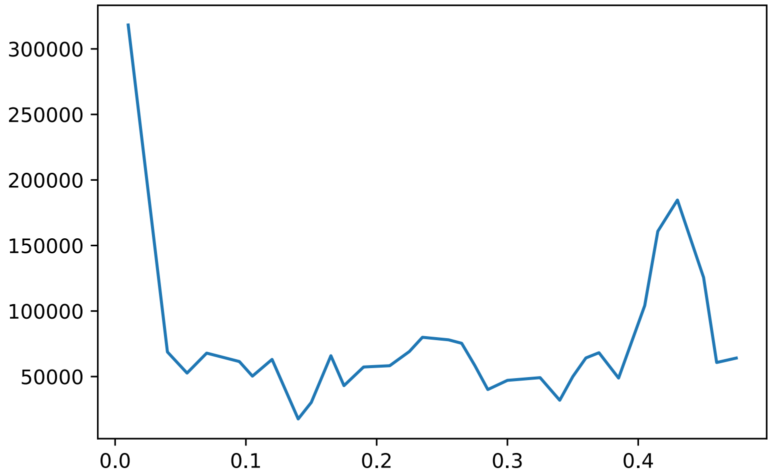
```
    pred_linreg     residuals
0 -25505.753164  32361.338482
1 -30575.590010  38532.624769
2 -17131.958396  22168.291569
3 -24217.904487  30793.697723
4 -36529.844368  45780.473049
R-squared: 9.01e-02
```

In [26]:
```
fft_output = fft(data['residuals'])
power = np.abs(fft_output)
freq = fftfreq(len(residuals))

print(fft_output[:10])
print(freq[:10])
```

```
[5055966.28918931      +0.j          -127478.28158225    -635.60456204j
  258057.8871362 -186228.43160376j    72890.65639394-212843.03973556j
  -24395.97039753-105654.61914496j    29423.38550655  -39475.24312352j
  -23955.79531203 -29143.3609243j     33001.32727342  -44133.62750722j
   58622.4055374   -35963.93478934j     9017.232263    -62721.0544862j ]
[0.    0.005 0.01  0.015 0.02  0.025 0.03  0.035 0.04  0.045]
```

In [27]:
```
from scipy.signal import find_peaks
peaks = find_peaks(power[freq>=0])[0]
peak_freq = freq[peaks]
peak_power = power[peaks]
plt.plot(peak_freq, peak_power, '-')
```

```
[<matplotlib.lines.Line2D at 0x1cbd433ecc8>]
```

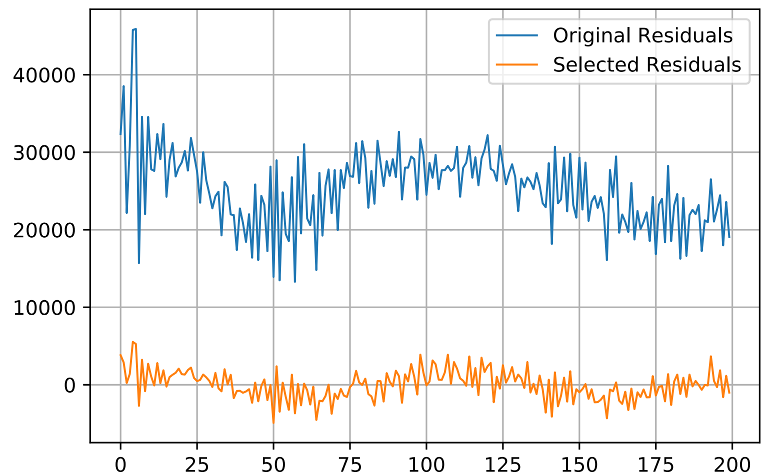


In [28]:
```
output = pd.DataFrame()
output['index'] = peaks
output['freq (1/365)'] = peak_freq
output['amplitude'] = peak_power
output['period (yeas)'] = 1/peak_freq/365
output['fft'] = fft_output[peaks]
output = output.sort_values('amplitude', ascending=False)
output.head()
```

	index	freq (1/365)	amplitude	period (yeas)	fft
0	2	0.010	318237.178612	0.273973	258057.887136-186228.431604j
28	86	0.430	184696.606582	0.006371	181340.309169-35050.374509j
27	83	0.415	160859.643804	0.006602	113871.798647-113617.949628j
29	90	0.450	125616.955195	0.006088	98288.763921-78223.642969j
26	81	0.405	104185.727180	0.006765	-71371.003425+75900.234638j

In [29]:

```
selected_fft_output = [value if value in list(output['fft']) else 0 for value in fft_output]
selected_residuals = ifft(selected_fft_output)

plt.plot(date, residuals, linewidth=1, label='Original Residuals')
plt.plot(date, selected_residuals, linewidth=1, label="Selected Residuals")
plt.legend(loc="upper right")
plt.grid()
plt.show()
```



In [30]:

```
from cmath import phase

fourier_terms = pd.DataFrame()
fourier_terms['fft'] = output['fft']
fourier_terms['freq (1/365)'] = output["freq (1/365)"]
fourier_terms['amplitude'] = fourier_terms.fft.apply(lambda x: abs(x))
fourier_terms['phase'] = fourier_terms.fft.apply(lambda x: phase(x))
fourier_terms.sort_values(by=['amplitude'], ascending=False)
fourier_terms['label'] = list(map(lambda x: 'F_{}'.format(x), range(1, fourier_terms.shape[0]+1)))
fourier_terms = fourier_terms.set_index('label')
fourier_terms_dict = fourier_terms.to_dict('index')

fourier_terms.head()
```

	fft	freq (1/365)	amplitude	phase
label				
F_1	258057.887136-186228.431604j	0.010	318237.178612	-0.625111
F_2	181340.309169-35050.374509j	0.430	184696.606582	-0.190931
F_3	113871.798647-113617.949628j	0.415	160859.643804	-0.784282
F_4	98288.763921-78223.642969j	0.450	125616.955195	-0.672209
F_5	-71371.003425+75900.234638j	0.405	104185.727180	2.325450

In [31]:

```python
# Create Fourier Series
import math

res = values
res['time'] = np.arange(0, 200)

for key in fourier_terms_dict.keys():
    a = fourier_terms_dict[key]['amplitude']
    w = 2*math.pi*(fourier_terms_dict[key]['freq (1/365)'])
    p = fourier_terms_dict[key]['phase']
    res[key] = res["time"].apply(lambda t: a*math.cos(w*t + p))

res['F_ALL'] = 0

for column in list(fourier_terms.index):
    res["F_ALL"] = res["F_ALL"] + res[column]

res
```

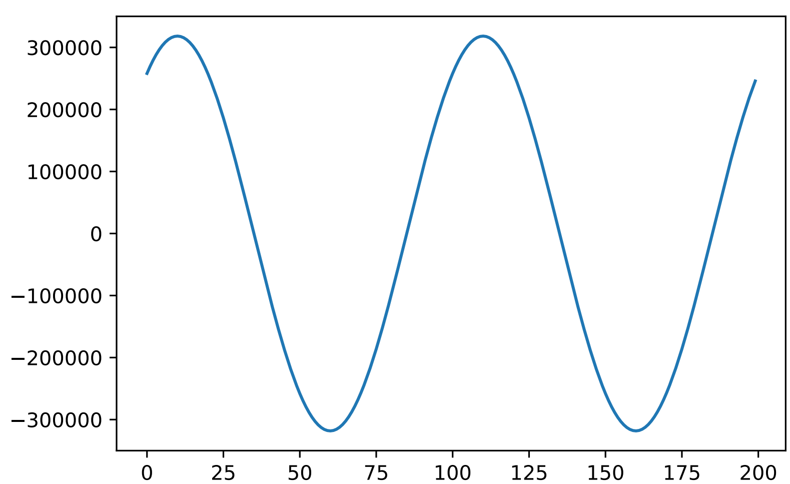
	price	time	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	6855.585318	0	258057.887136	181340.309169	113871.798647	98288.763921	-71371.003425	70800.137613	58677.008352
1	7957.034758	1	269242.048815	-149157.893811	-40178.000853	-69305.734367	16367.310235	43671.979898	49578.249103
2	5036.333172	2	279363.635132	88583.885649	-44706.010857	33538.576651	44296.834729	-62580.344878	-61791.58922
3	6575.793236	3	288382.700811	-11148.298485	117138.685661	5511.570625	-89641.413247	-55450.641305	-45696.40765
4	9250.628680	4	296263.651719	-68409.321533	-156946.368616	-44022.206966	103984.508365	52143.612226	64662.306932
...	...	...	...	...	...	...	...	...	...
195	5114.319541	195	187879.884941	78232.810739	49879.045784	-78223.642969	-58618.895338	-979.756386	-59992.43619
196	5443.779426	196	203637.396434	449.750494	34915.246802	104768.003782	69.936444	79512.994073	51766.338299
197	4288.908885	197	218591.244157	-79046.703566	-109985.086395	-121056.942423	58503.209189	15945.423905	56740.396231
198	5290.831477	198	232682.412103	142597.441096	154422.325606	125495.984086	-96843.672150	-76511.800176	-55330.85613
199	4486.011473	199	245855.288872	-179005.341066	-155850.484719	-117650.604445	101691.830765	-30346.216832	-53264.42784

200 rows × 35 columns

In [32]:

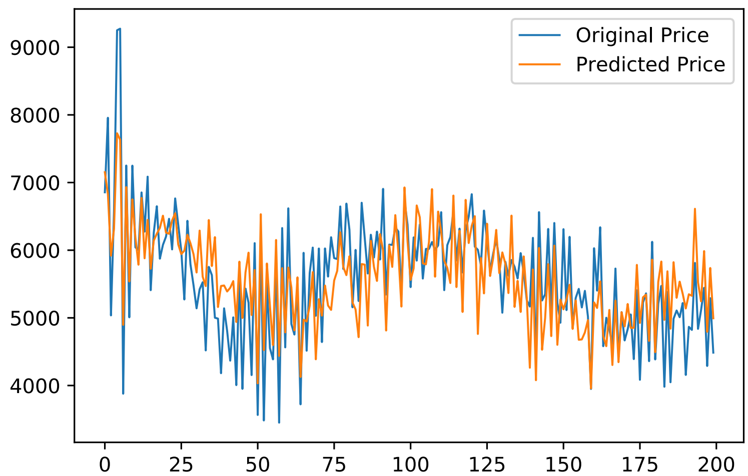
```python
plt.plot(res['time'], res['F_1'])
```

[<matplotlib.lines.Line2D at 0x1cbd3e05508>]



In [33]:
```
Regressor = pd.DataFrame(res[['time','F_ALL']])
y = pd.DataFrame(res['price'])

model = LinearRegression()
model.fit(Regressor, y)
print(model)
y_pred = model.predict(Regressor)

plt.plot(Regressor['time'], y, linewidth=1, label='Original Price')
plt.plot(Regressor['time'], y_pred, linewidth=1, label='Predicted Price')
plt.legend(loc='upper right')
plt.show()
```

```
LinearRegression()
```



In [34]:
```
print("R-squared: {:.2e}".format(model.score(Regressor, y)))
```

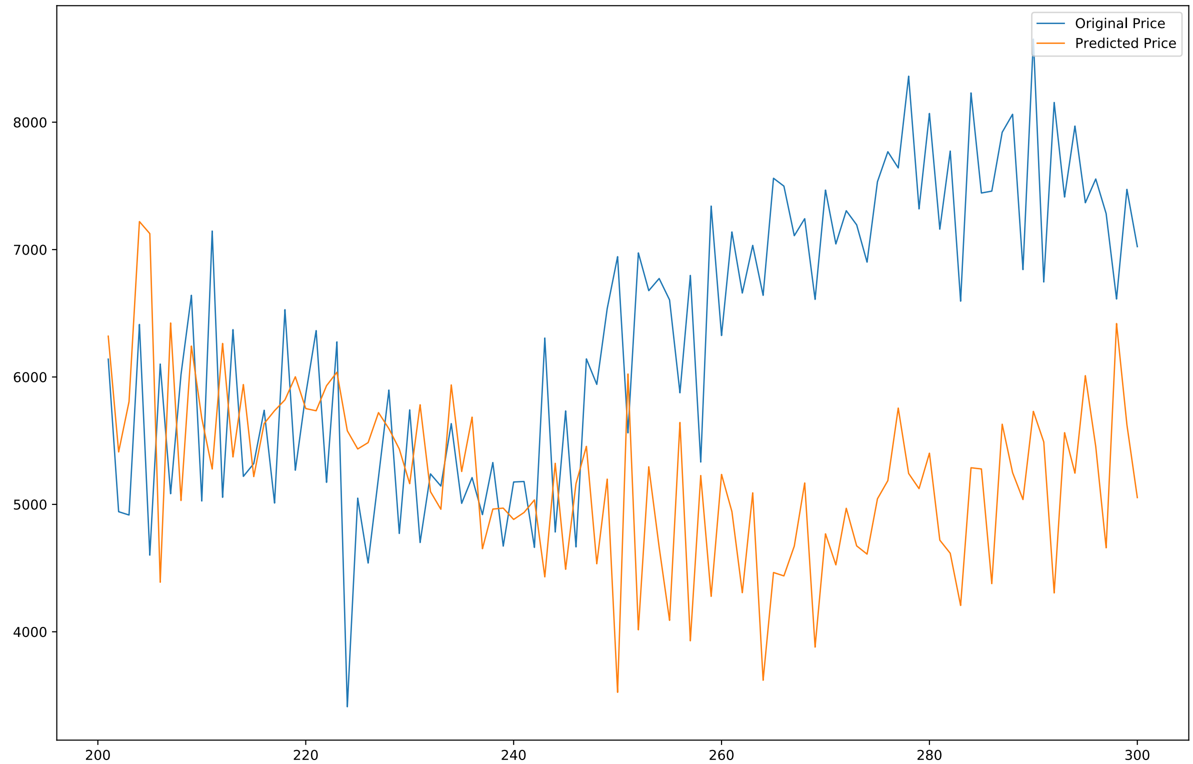
```
R-squared: 5.94e-01
```

In [35]:
```
test_data = pd.DataFrame()
test_data['time'] = np.arange(201, 301)

for key in fourier_terms_dict.keys():
    a = fourier_terms_dict[key]['amplitude']
    w = 2*math.pi*(fourier_terms_dict[key]['freq (1/365)'])
    p = fourier_terms_dict[key]['phase']
    test_data[key] = test_data["time"].apply(lambda t: a*math.cos(w*t + p))

test_data['F_ALL'] = 0

for column in list(fourier_terms.index):
    test_data["F_ALL"] = test_data["F_ALL"] + test_data[column]

test_data
```

	time	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F
0	201	269242.048815	-149157.893811	-40178.000853	-69305.734367	16367.310235	43671.979898	49578.249103	52900.13055
1	202	279363.635132	88583.885649	-44706.010857	33538.576651	44296.834729	-62580.344878	-61791.589228	-58468.4042
2	203	288382.700811	-11148.298485	117138.685661	5511.570625	-89641.413247	-55450.641305	-45696.407658	-41895.4047
3	204	296263.651719	-68409.321533	-156946.368616	-44022.206966	103984.508365	52143.612226	64662.306932	66353.81600
4	205	302975.385344	134945.508013	153041.985246	78223.642969	-82365.720542	65264.936099	41634.223369	29406.51332
...	...	...	...	...	...	...	...	...	...
95	296	203637.396434	449.750494	-34915.246802	104768.003782	-69.936444	-79512.994073	-51766.338299	-23856.2973
96	297	218591.244157	-79046.703566	109985.086395	-121056.942423	-58503.209189	-15945.423905	-56740.396231	-68964.0816
97	298	232682.412103	142597.441096	-154422.325606	125495.984086	96843.672150	76511.800176	55330.856131	36836.48414
98	299	245855.288872	-179005.341066	155850.484719	-117650.604445	-101691.830765	30346.216832	53264.427848	62030.84287
99	300	258057.887136	181340.309169	-113871.798647	98288.763921	71371.003425	-70800.137613	-58677.008352	-48511.7201

100 rows × 34 columns

In [36]:
```
X_test_pred = pd.DataFrame(test_data[['time', 'F_ALL']])
y_test_pred = model.predict(X_test_pred)
plt.figure(figsize=(15,10))
plt.plot(X_test_pred['time'], part1_test, linewidth=1, label='Original Price')
plt.plot(X_test_pred['time'], y_test_pred, linewidth=1, label='Predicted Price')
plt.legend(loc='upper right')
plt.show()
```



In [37]:
```
from sklearn.metrics import mean_squared_error
from math import sqrt
RMSE = sqrt(mean_squared_error(part1_test, y_test_pred))
print('The RMSE for Fourier Series Forcasting is %.3f' % RMSE)
```

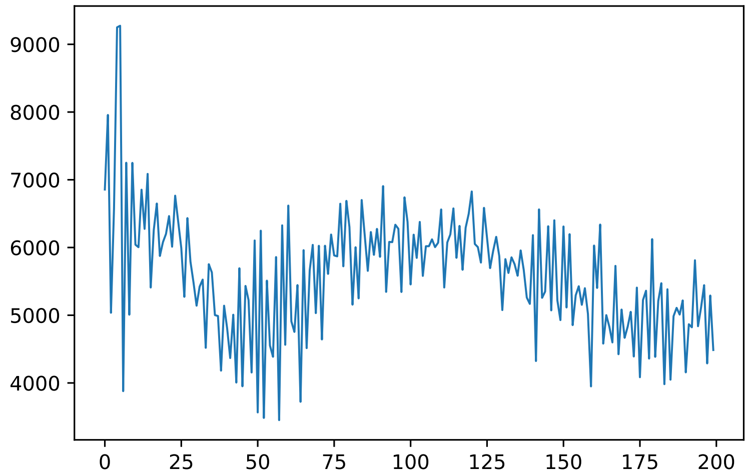
```
The RMSE for Fourier Series Forcasting is 1844.897
```

In [38]:
```
# Taylor Expansion
```

In [39]:
```
data_poly = pd.DataFrame()
data_poly['time'] = np.arange(0,200)
data_poly['price'] = part1_train["price"]
data_poly.head()
```

	time	price
0	0	6855.585318
1	1	7957.034758
2	2	5036.333172
3	3	6575.793236
4	4	9250.628680

In [40]:
```
plt.plot(data_poly['time'], data_poly['price'], linewidth=1)
plt.show()
```



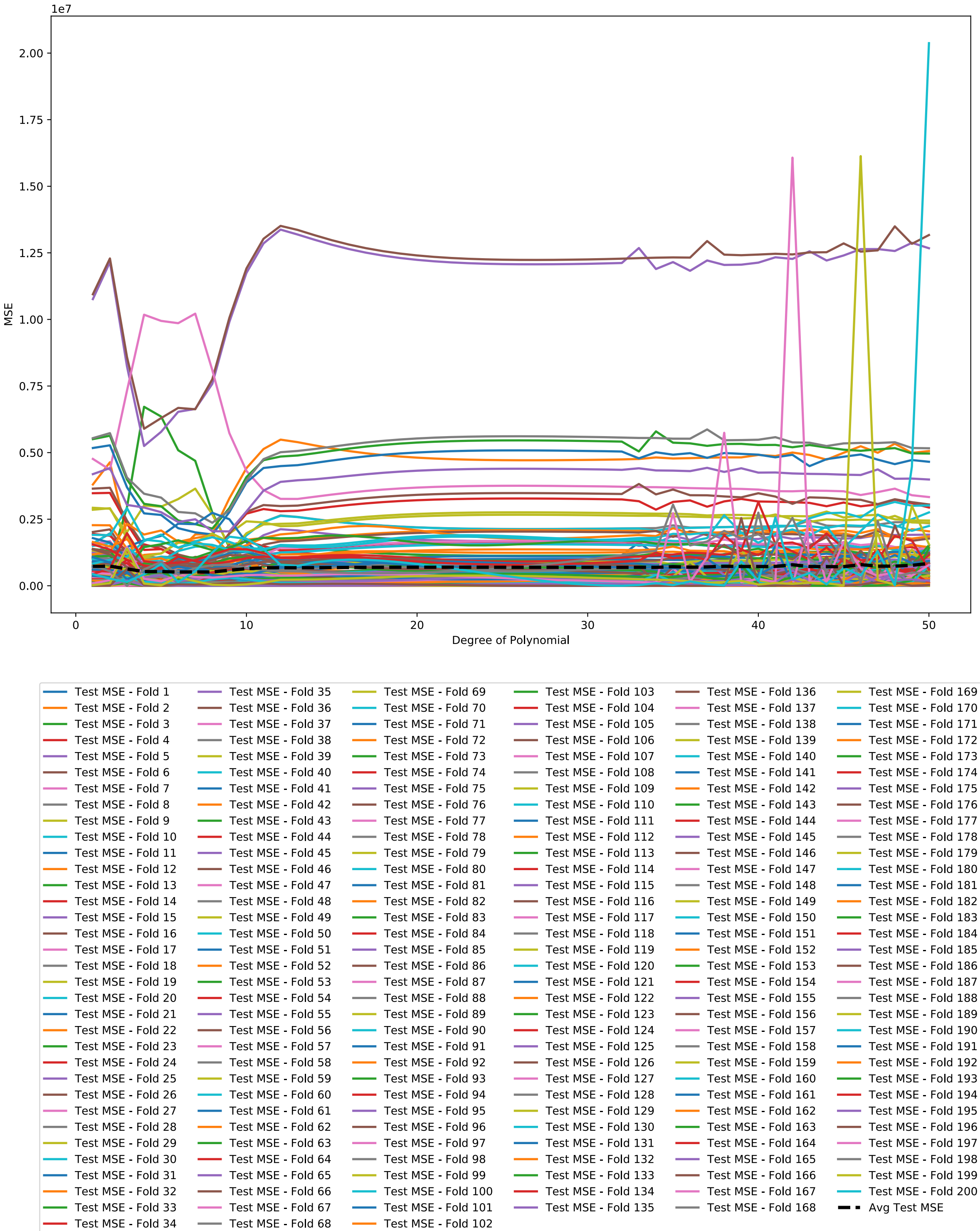
In [41]:
```
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import  train_test_split, KFold
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import Pipeline
```

In [42]:
```
def K_Fold_Val(K, degree, X_poly, Y_poly):
    n = len(X_poly)
    kf = KFold(n_splits=K)
    kf_dict = dict([("fold_%s" % i,[]) for i in range(1, K+1)])
    fold = 0
    for train_index, test_index in kf.split(X_poly):
        fold += 1
        # print("Fold %s" % fold)
        X_train, X_test = X_poly.take(train_index), X_poly.take(test_index)
        Y_train, Y_test = Y_poly[train_index], Y_poly[test_index]
        for d in range(1, degree+1):
            # print("Degree %s" % d)
            poly_features = PolynomialFeatures(
                degree=d, include_bias=False
            )
            linear_reg = LinearRegression()
            model = Pipeline([
                ("polynomial_features", poly_features),
                ("linear_regression", linear_reg)
            ])
            model.fit(X_train, Y_train)
            Y_pred = model.predict(X_test)
            test_mse = mean_squared_error(Y_test, Y_pred)
            kf_dict["fold_%s" % fold].append(test_mse)
        kf_dict["fold_%s" % fold] = np.array(kf_dict["fold_%s" % fold])
    kf_dict["avg"] = np.zeros(degree)
    for i in range(1, fold+1):
        kf_dict["avg"] += kf_dict["fold_%s" % i]
    kf_dict["avg"] /= float(K)
    return kf_dict

kf_dict = K_Fold_Val(len(data_poly["time"]), 50, data_poly[["time"]], data_poly["price"])
```

In [43]:
```
kf_dict['avg']
```

```
array([729987.10044714, 735789.06533774, 634240.38105162, 531186.45837885,
       534406.35781395, 517187.55361984, 518838.91321028, 517444.61861215,
       581579.40996651, 643996.22428986, 674518.68219609, 679823.62169838,
       676353.86709072, 679924.19043465, 684165.13341387, 688211.64467678,
       691568.49241919, 694044.91555859, 695650.43928423, 696500.01704585,
       696748.43277792, 696548.27821862, 696035.06032425, 695329.76155721,
       694518.21088778, 693689.30380981, 692887.42326621, 692199.11166302,
       691663.43100636, 691293.32922023, 691052.57325061, 691081.74470311,
       694309.88227646, 691026.72477679, 718417.43847833, 696846.12261151,
       708934.15627475, 734387.95658478, 724571.04453193, 721837.67048062,
       726243.70724617, 789352.07590604, 722937.15929711, 717225.43430862,
       717552.67420126, 795375.24821791, 736604.37838177, 737676.79057064,
       768124.0754853 , 840526.53745426])
```

In [44]:
```
def plot_test_error_curves(kf_dict, K, degree):
    fig, ax = plt.subplots(figsize=(15,10))
    ds = range(1, degree+1)
    for i in range(1, K+1):
        ax.plot(ds, kf_dict["fold_%s" % i], lw=2, label='Test MSE - Fold %s' % i)
    ax.plot(ds, kf_dict['avg'], linestyle='--', color='black', lw=3, label='Avg Test MSE')
    ax.legend(loc="lower center", ncol=6, bbox_to_anchor=(0.5, -1.05))
    ax.set_xlabel("Degree of Polynomial")
    ax.set_ylabel("MSE")
    plt.show()

plot_test_error_curves(kf_dict, len(data_poly["time"]), 50)
```



1e7
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
MSE
0
10
20
30
40
50
Degree of Polynomial
Test MSE - Fold 1
Test MSE - Fold 2
Test MSE - Fold 3
Test MSE - Fold 4
Test MSE - Fold 5
Test MSE - Fold 6
Test MSE - Fold 7
Test MSE - Fold 8
Test MSE - Fold 9
Test MSE - Fold 10
Test MSE - Fold 11
Test MSE - Fold 12
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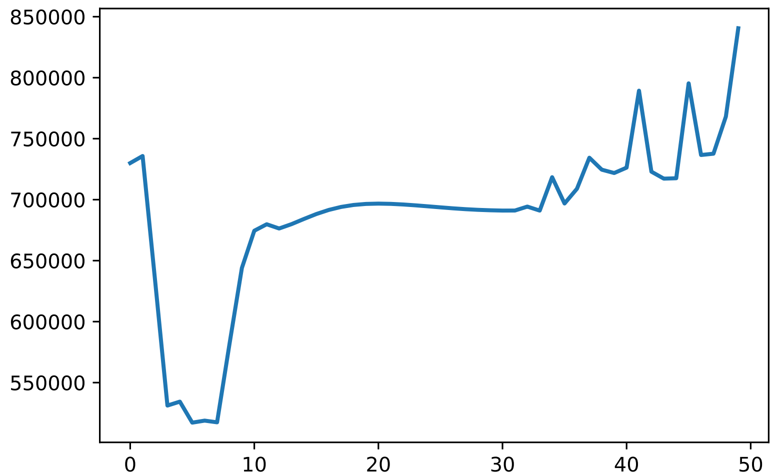
Test MSE - Fold 193
Test MSE - Fold 194
Test MSE - Fold 195
Test MSE - Fold 196
Test MSE - Fold 197
Test MSE - Fold 198
Test MSE - Fold 199
Test MSE - Fold 200
Avg Test MSE

In [45]:
```
plt.plot(kf_dict["avg"], linewidth=2)
plt.show()
```



In [46]:
```
print("The degree %s gives the minimum MSE" % kf_dict['avg'].argmin())
```

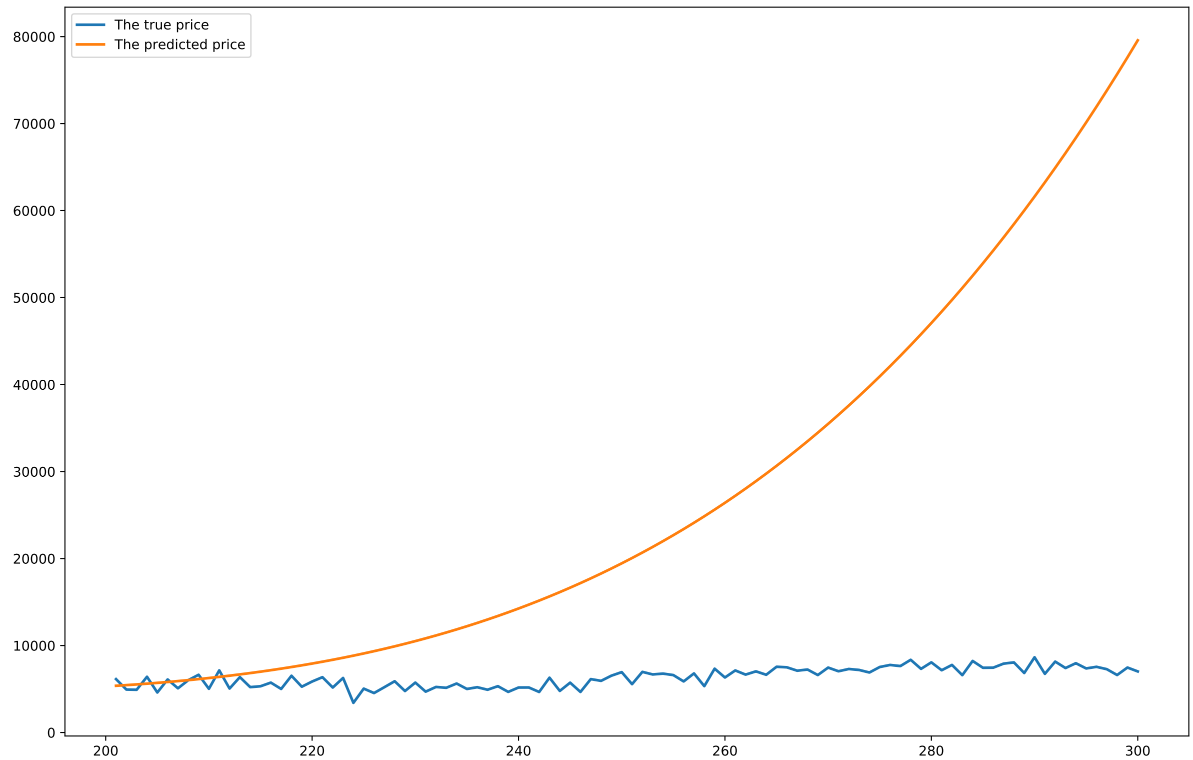
```
The degree 5 gives the minimum MSE
```

In [47]:
```
lin_poly = LinearRegression()
poly_feature = PolynomialFeatures(degree=5, include_bias=False)
model = Pipeline([
    ("polynomial_feature", poly_feature),
    ("linear_regression", lin_poly)
])
model.fit(data_poly[["time"]], data_poly["price"])
test_poly = pd.DataFrame()
test_poly["time"] = np.arange(201, 301)
test_poly["price"] = part1_test["price"]
price_poly_pred = model.predict(test_poly[["time"]])
```

In [48]:
```
import math
RMSE_poly = math.sqrt(mean_squared_error(test_poly["price"], price_poly_pred))
print("The RMSE of polynomial regression model is: %.3f" % RMSE_poly)
```

```
The RMSE of polynomial regression model is: 29320.724
```

In [49]:
```
fig, ax = plt.subplots(figsize=(15,10))
ax.plot(test_poly["time"], test_poly["price"], linewidth=2, label="The true price")
ax.plot(test_poly["time"], price_poly_pred, linewidth=2, label="The predicted price")
plt.legend(loc="best")
plt.show()
```



In [50]:
```
model.score(data_poly[["time"]], data_poly["price"])
```

0.37501197054586466

In [51]:
```
part1_result = pd.DataFrame()
part1_result["Methods"] = ["Autoregressive Model", "Fourier Series", "Taylor Formula"]
part1_result["RMSE"] = ["1498.986", "1844.897", "29320.724"]
part1_result
```

	Methods	RMSE
0	Autoregressive Model	1498.986
1	Fourier Series	1844.897
2	Taylor Formula	29320.724

In [52]:
```
plt.figure(figsize=(15,10))
plt.plot(test_poly["time"], test_poly["price"], linewidth=2, label="The True Price")
plt.plot(test_poly["time"], predictions, linewidth=2, label="The Price Predicted by Autoregressive Model")
plt.plot(test_poly["time"], y_test_pred, linewidth=2, label="The Price Predicted by Fourier Series")
plt.plot(test_poly["time"], price_poly_pred, linewidth=2, label="The Price Predicted by Taylor Formula")
plt.legend(loc="best")
plt.show()
```