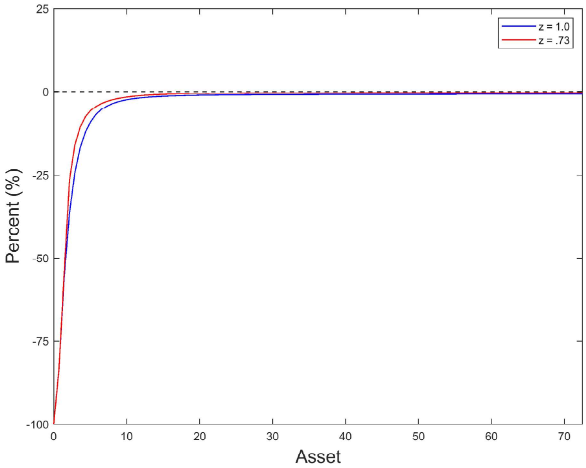
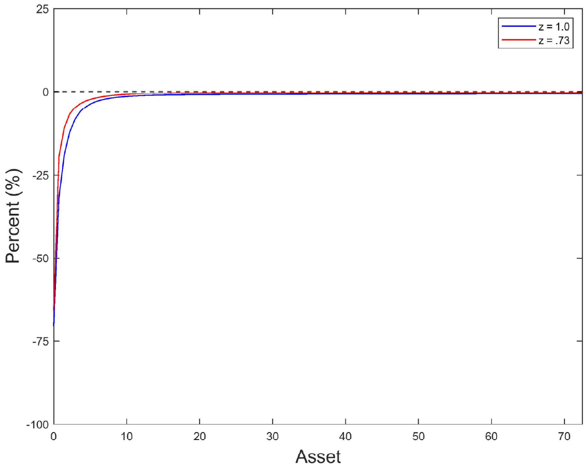
A. Figures

A.1. Figures related to Subsection 4.1

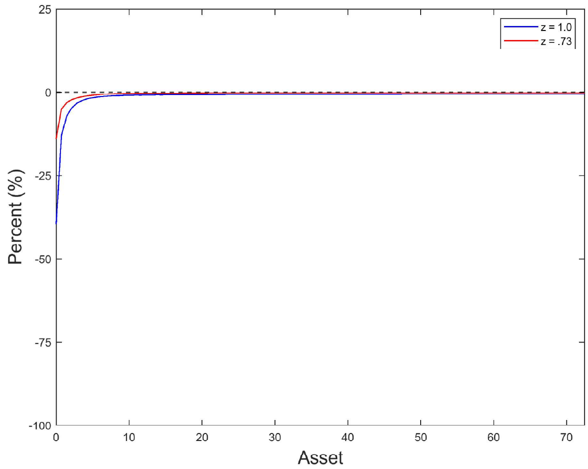
A.1.1. Difference in Consumption



(a) Scenario **without** tax



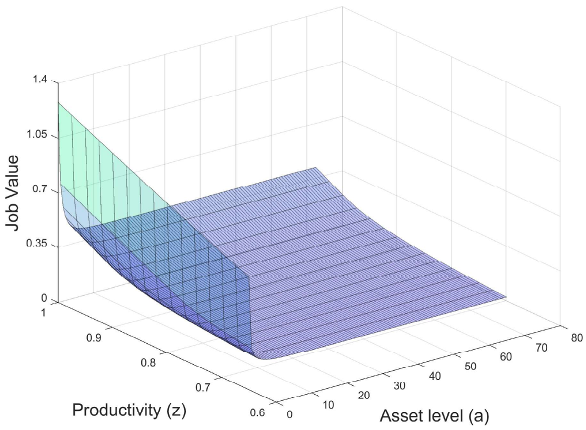
(b) Baseline with **1%** payroll tax



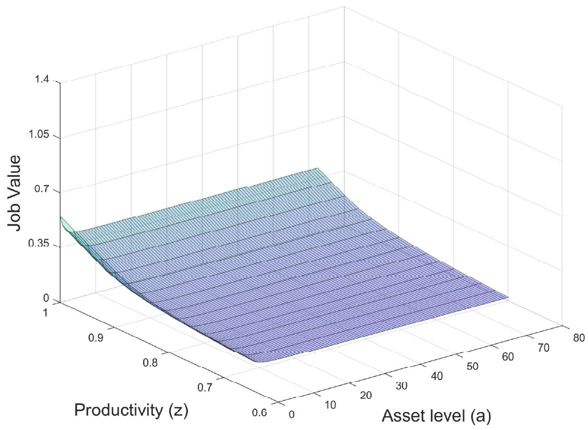
(c) Scenario with **2.5%** payroll tax

Figure 6: Distribution of consumers by asset level under different UB programs

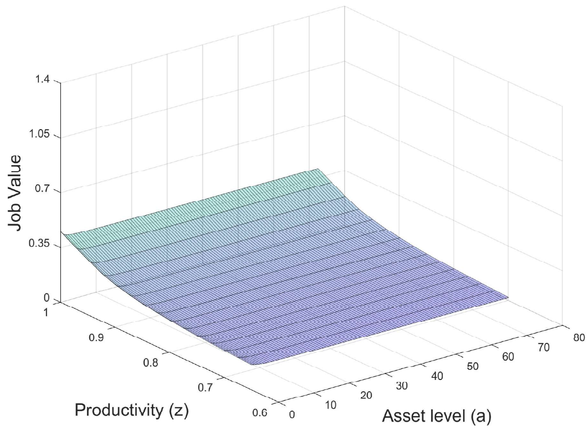
A.1.2. Filled Job Value



(a) Scenario without tax



(b) Baseline model with 1% payroll tax

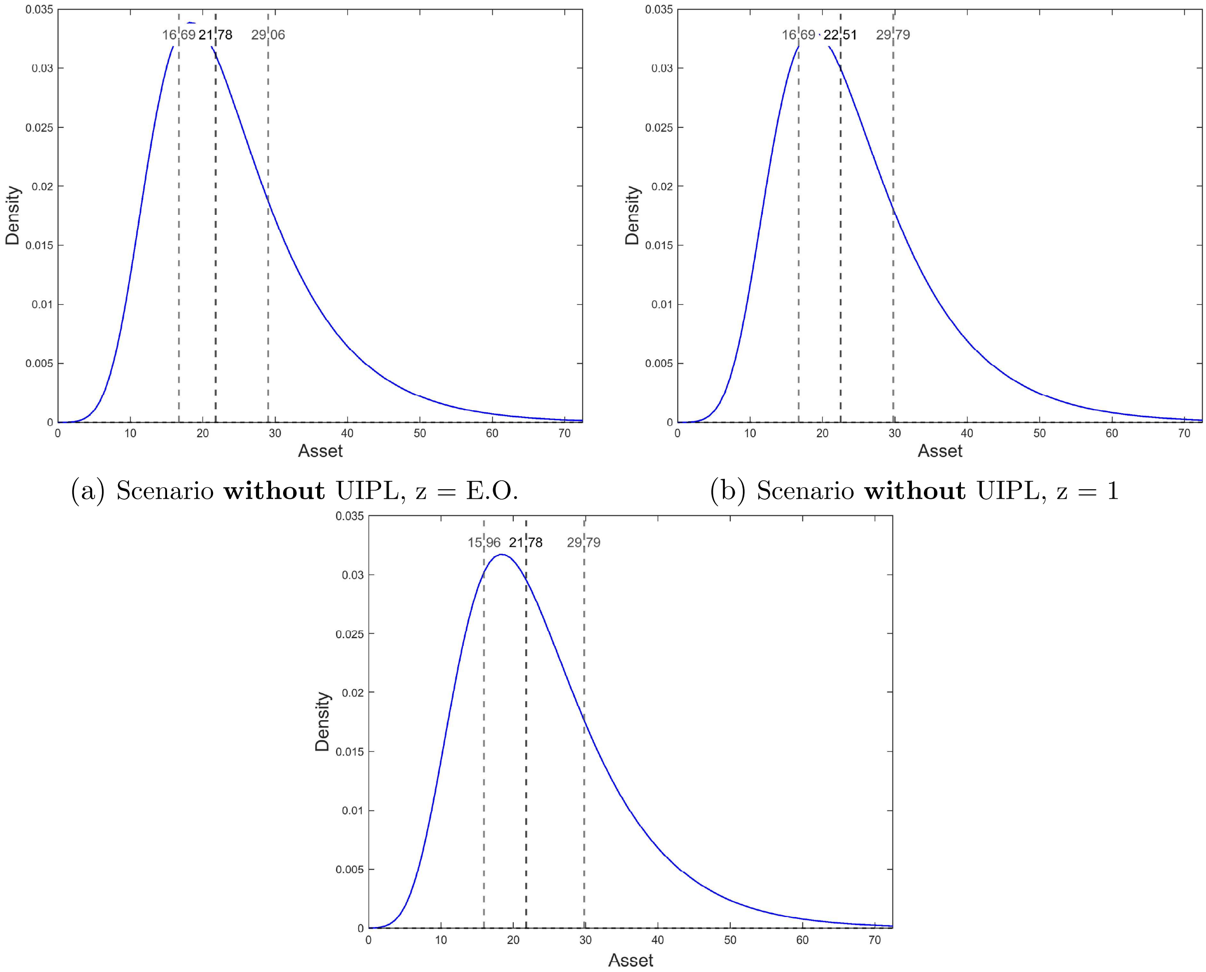


(c) Scenario with 2.5% payroll tax

Figure 7: Filled job values under different UB programs: 0%, 1%, and 2.5% payroll taxation

A.2. Figures related to Subsection 4.2

A.2.1. Distribution of Consumers by Asset Level

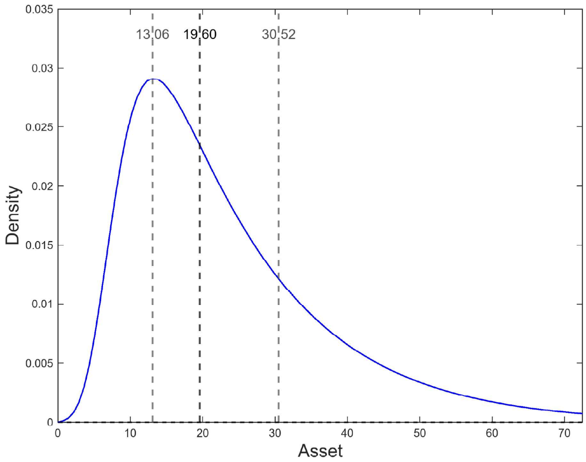


(a) Scenario **without** UIPL, z = E.O.

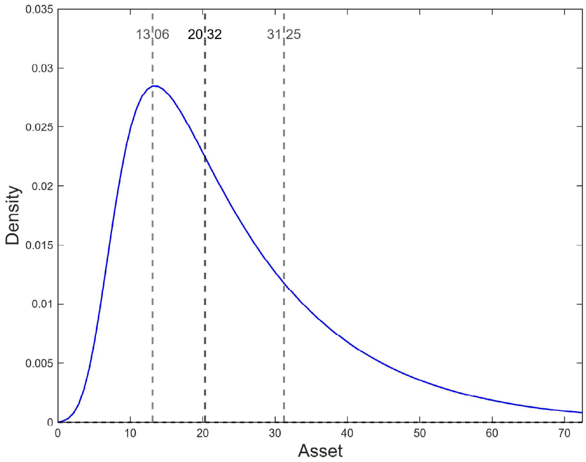
(b) Scenario **without** UIPL, z = 1

(c) Scenario **with** UIPL

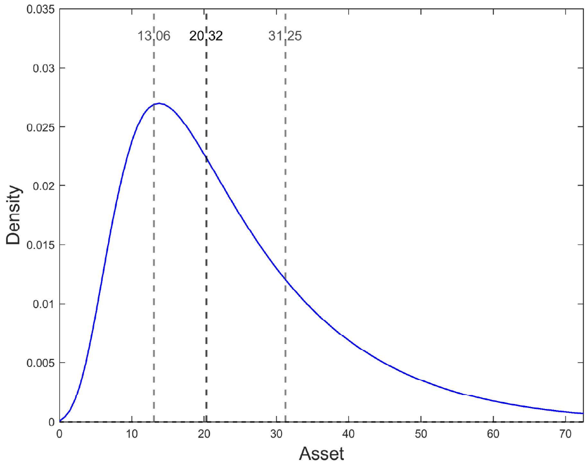
Figure 8: Distribution of consumers by Asset Level: Scenario without Tax and UB



(a) Scenario **without** UIPL, z = E.O.



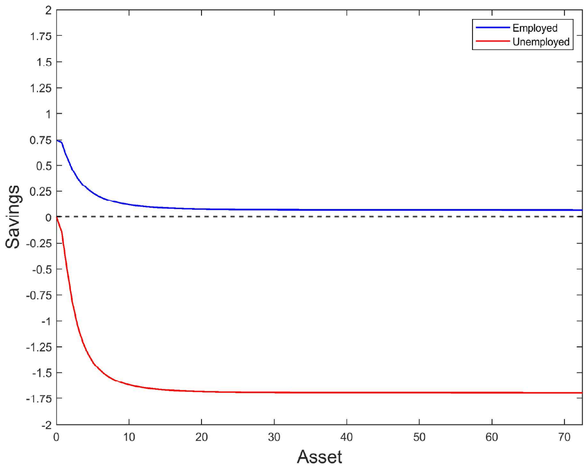
(b) Scenario **without** UIPL, z = 1



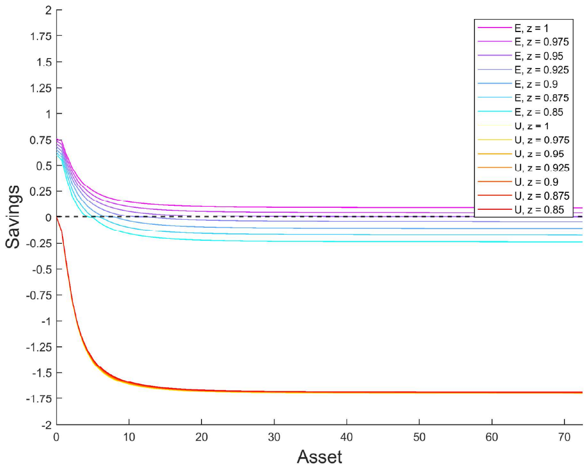
(c) Scenario **with** UIPL

Figure 9: Distribution of Consumers by Asset Level: Scenario with 2% Payroll Tax

A.2.2. Savings Behavior

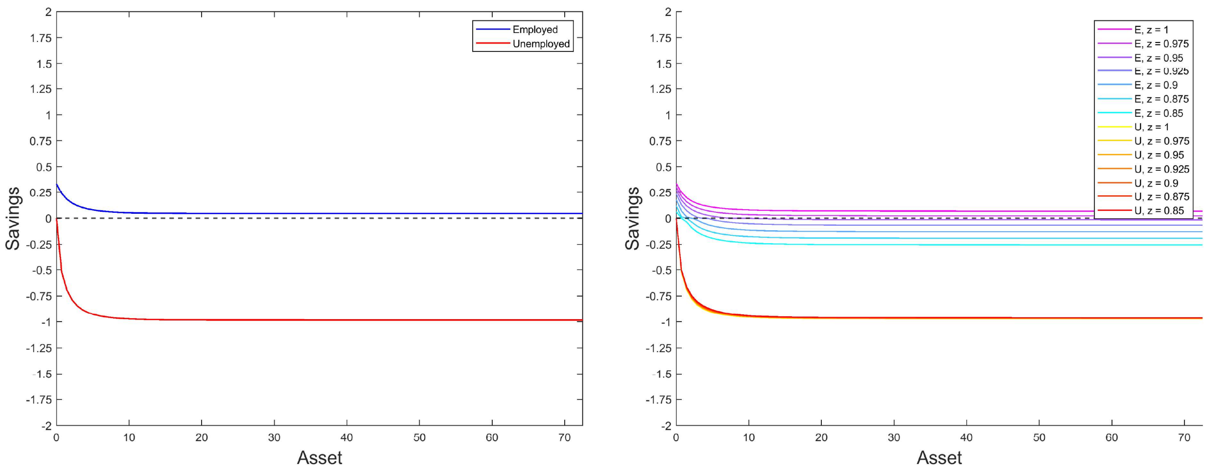


(a) Scenario **without** UIPL



(b) Scenario **with** UIPL

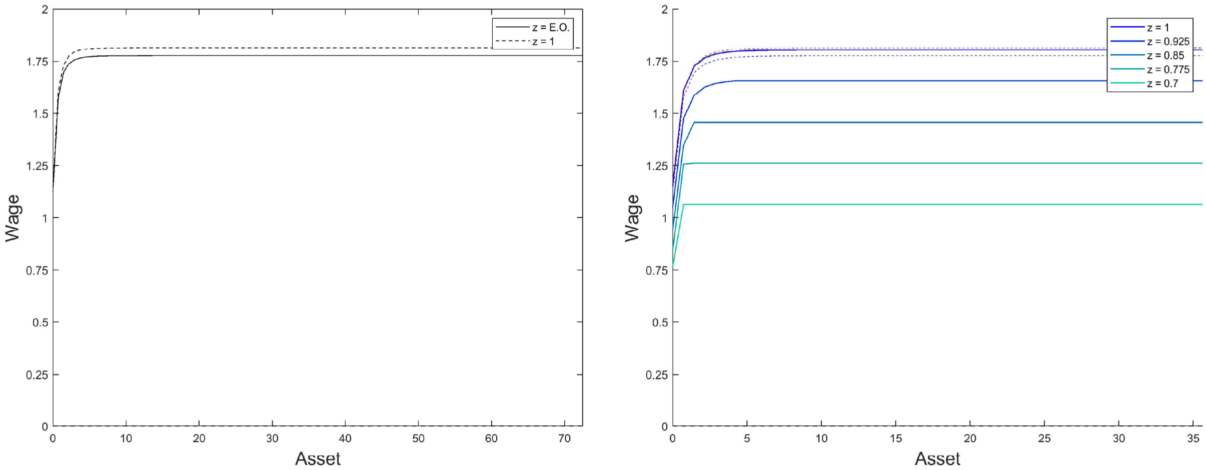
Figure 10: Savings Behavior: Scenario without Tax and UB



(a) Scenario **without** UIPL (b) Scenario **with** UIPL

Figure 11: Savings Behavior: Scenario with 2% Payroll Tax

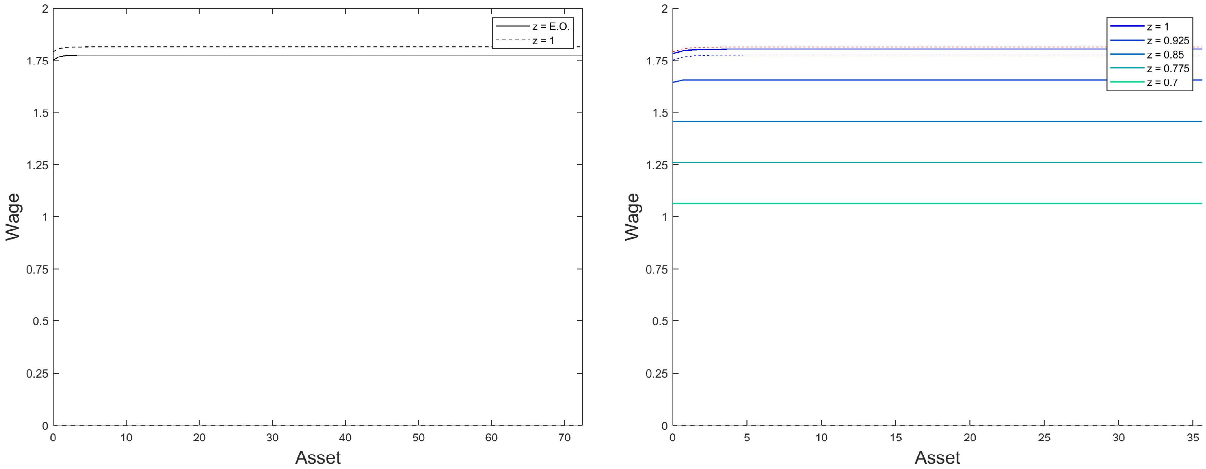
A.2.3. Wage Schedule



(a) Scenario **without** UIPL (b) Scenario **with** UIPL

The red and blue dashed lines just represent the wage schedules with z = E.O. and z = 1 in the left scenario without UIPL

Figure 12: Wage Schedule: Scenario without Tax and UB



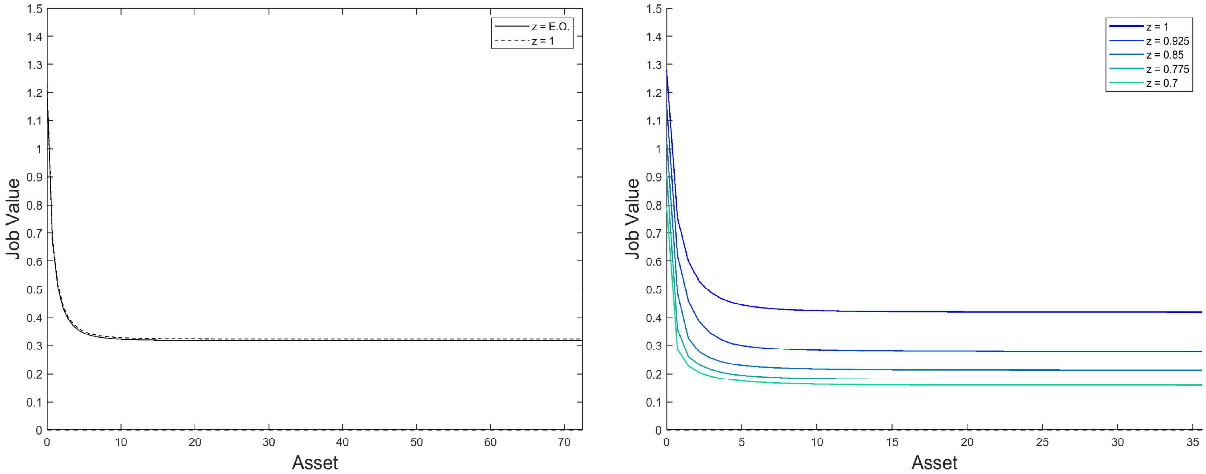
(a) Scenario **without** UIPL

(b) Scenario **with** UIPL

The red and blue dashed lines just represent the wage schedules with z = E.O. and z = 1 in the left scenario without UIPL

Figure 13: Wage Schedule: Scenario with 2% Payroll Tax

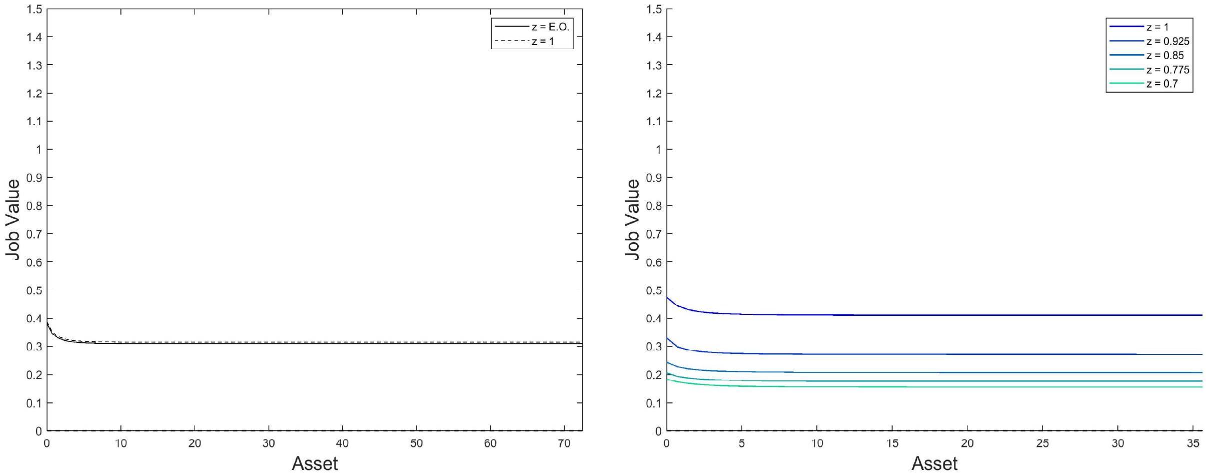
A.2.4. Filled Job Value



(a) Scenario **without** UIPL

(b) Scenario **with** UIPL

Figure 14: Filled Job Value: Scenario without Tax and UB



(a) Scenario **without** UIPL

(b) Scenario **with** UIPL

Figure 15: Filled Job Value: Scenario with 2% Payroll Tax

B. Mathematical Derivations

Notations:

1. The 1° partial derivative uses the notation: $\frac{\partial f(x_1,x_2)}{\partial x_1} \equiv \Big(f(x_1,x_2)\Big)'_{x_1}$.
2. The parentheses, $(\cdot)$, are used exclusively for functions, i.e., $f(x)$; the brackets, $(\cdot)$ are used for calculations.

B.1. Consumer's HJB Equations

B.1.1. Employed Consumer

The employed consumer solves the optimization problem:

$$W_{\mathbf{e}}(a_t, z_t) = \max_c \left\{ u(c)\Delta + (1-\rho\Delta)\Big((1-\lambda_{\mathbf{e}}\Delta)\, W_{\mathbf{e}}(a_{t+\Delta}, z_{t+\Delta}) + \lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta}, z_{t+\Delta}) \Big) \right\}$$

Perform a first-order Taylor expansion of $W_{\mathbf{e}}(a_{t+\Delta}, z_{t+\Delta})$ around $W_{\mathbf{e}}(a_t, z_t)$:

$$W_{\mathbf{e}}(a_t, z_t) = \max_c \left\{ u(c)\Delta + (1-\rho\Delta) \left(\begin{array}{l} (1-\lambda_{\mathbf{e}}\Delta)\Big(W_{\mathbf{e}}(a_t,z_t) + \Big(W_{\mathbf{e}}(a_t,z_t)\Big)'_a \Big(\omega(a_t,z_t) + (r-\delta)\,a_t - c\Big)\Delta + \Big(W_{\mathbf{e}}(a_t,z_t)\Big)'_z \Big(\theta\,(z_{\mathbf{e}} - z_t)\,\Delta\Big)\Big) \\ +\ \lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta}, z_{t+\Delta}) \end{array} \right) \right\}$$

Subtract $(1-\rho\Delta)\, W_{\mathbf{e}}(a_t, z_t)$ from both sides:

$$\rho\Delta W_{\mathbf{e}}(a_t,z_t)=\max_c\left\{u(c)\Delta+(1-\rho\Delta)\left(\begin{array}{c}+(1-\lambda_{\mathbf{e}}\Delta)\left(\Big(W_{\mathbf{e}}(a_t,z_t)\Big)_a'\Big(\omega(a_t,z_t)+(r-\delta)\,a_t-c\Big)\Delta+\Big(W_{\mathbf{e}}(a_t,z_t)\Big)_z'\Big(\theta\,(z_{\mathbf{e}}-z_t)\Big)\Delta\right)\\-\lambda_{\mathbf{e}}\Delta W_{\mathbf{e}}(a_t,z_t)+\lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta},z_{t+\Delta})\end{array}\right)\right\}$$

Divide by Δ and take $\Delta\to 0$:

$$\rho W_{\mathbf{e}}(a_t,z_t)=\max_c\left\{u(c)+\Big(W_{\mathbf{e}}(a_t,z_t)\Big)_a'\Big(\omega(a_t,z_t)+(r-\delta)\,a_t-c\Big)+\Big(W_{\mathbf{e}}(a_t,z_t)\Big)_z'\Big(\theta\,(z_{\mathbf{e}}-z_t)\Big)+\lambda_{\mathbf{e}}\Big(W_{\mathbf{u}}(a_t,z_t)-W_{\mathbf{e}}(a_t,z_t)\Big)\right\} \tag{18}$$

B.1.2. Unemployed Consumer

From the same logic of 19 , the HJB for the unemployed consumer can be written as:

$$\rho W_{\mathbf{u}}(a_t,z_t)=\max_c\left\{u(c)+\Big(W_{\mathbf{u}}(a_t,z_t)\Big)_a'\Big(\;h(z_t)\;+(r-\delta)\,a_t\;-c\Big)+\Big(W_{\mathbf{u}}(a_t,z_t)\Big)_z'\Big(\theta\,(z_{\mathbf{u}}-z_t)\Big)+\lambda_{\mathbf{u}}\Big(W_{\mathbf{e}}(a_t,z_t)-W_{\mathbf{u}}(a_t,z_t)\Big)\right\} \tag{19}$$

B.2. Firm's HJB Equation

The representative firm solve the maximization problem:

$$J(a_t, z_t) = \max_k \left\{ \underbrace{\Big(z_t F(k_t) - r_t \cdot k_t - \omega(a_t, z_t) \Big)}_{\pi_t} \Delta + \Big(1 - (r_t - \delta)\Delta \Big) \cdot \underbrace{(1 - \lambda_e \Delta) \cdot \Big(J(a_{t+\Delta}, z_{t+\Delta}) \Big)}_{\substack{\text{the expected value of this position,} \\ \text{taking separation into consideration}}} \right\} \tag{20}$$

Comparing to the consumer's problem, this is relatively easier to solve. The derivation is done by taking $\Delta \to 0$ and rearranging the terms.

C. Numerical Solutions

C.1. Solving Consumer's HJB Equation

To solve the consumers' optimization problem, I follow the semi-implicit method as outlined in the appendices of Achdou et al. (2022) and in Bardóczy (2017).
Rewrite the problems into numerical formulation for the following finite difference iteration:

$$\frac{W_{\mathbf{s}}^{n+1}(a,z)-W_{\mathbf{s}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{s}}^{n+1}(a,z)=u(c_{\mathbf{s}}^{n})+\left(W_{\mathbf{s}}^{n+1}(a,z)\right)_a^{\prime}\left(y_{\mathbf{s}}(a,z)+(r-\delta)\,a-c_{\mathbf{s}}^{n}\right)+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\left(z_{\mathbf{s}}-z\right)\right)+\lambda_{\mathbf{s}}\left(W_{\mathbf{s}'}^{n+1}(a,z)-W_{\mathbf{s}}^{n+1}(a,z)\right) \tag{21}$$

where $\quad s\in\{e,u\}\quad$ and $\quad y_{\mathbf{s}}(a,z)=\begin{cases}\omega(a,z) & \text{, if } \mathbf{s}=\mathbf{e},\\ f+h(z) & \text{, if } \mathbf{s}=\mathbf{u}.\end{cases}$

To have a clear expression, the HJB equations for the employed and the unemployed are as follows:

1. The employed:

$$\frac{W_{\mathbf{e}}^{n+1}(a,z)-W_{\mathbf{e}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{e}}^{n+1}(a,z)=u(c_{\mathbf{e}}^{n})+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_a^{\prime}\left(\omega(a,z)+(r-\delta)\,a-c_{\mathbf{e}}^{n}\right)+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\left(z_{\mathbf{e}}-z\right)\right)+\lambda_{\mathbf{e}}\left(W_{\mathbf{u}}^{n+1}(a,z)-W_{\mathbf{e}}^{n+1}(a,z)\right) \tag{22}$$

   where $c_{\mathbf{e}}^{n}=\left(W_{\mathbf{e}}^{n}\left(a,z\right)\right)_a^{\prime}$

2. The unemployed:

$$\frac{W_{\mathbf{u}}^{n+1}(a,z)-W_{\mathbf{u}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{u}}^{n+1}(a,z)=u(c_{\mathbf{u}}^{n})+\left(W_{\mathbf{u}}^{n+1}(a,z)\right)_a^{\prime}\left(f+h(z)+(r-\delta)\,a-c_{\mathbf{u}}^{n}\right)+\left(W_{\mathbf{u}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\left(z_{\mathbf{u}}-z\right)\right)+\lambda_{\mathbf{u}}\left(W_{\mathbf{e}}^{n+1}(a,z)-W_{\mathbf{u}}^{n+1}(a,z)\right) \tag{23}$$

   where $c_{\mathbf{u}}^{n}=\left(W_{\mathbf{u}}^{n}\left(a,z\right)\right)_a^{\prime}$

Numerically, given asset level, productivity and employment status, (a_i, z_j, s), the problem can be reformulated as:

$$\left(\frac{1}{\Delta}+\rho\right) W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)-\frac{1}{\Delta} W_{\mathbf{s}}^{n}\left(a_i, z_j\right)=u\left(c_{\mathbf{s}}^{n}\left(a_i, z_j\right)\right)+\left(\begin{array}{l} \quad \frac{W_{\mathbf{s}}^{n+1}\left(a_{i+1}, z_j\right)-W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)}{\Delta a} \cdot \left(\underbrace{y_{\mathbf{s}}\left(a_i, z_j\right)\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)}_{\xi_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)}\right)^{+} \\ +\frac{W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)-W_{\mathbf{s}}^{n+1}\left(a_{i-1}, z_j\right)}{\Delta a} \cdot \left(\underbrace{y_{\mathbf{s}}\left(a_i, z_j\right)\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)}_{\xi_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)}\right)^{-} \\ +\frac{W_{\mathbf{s}}^{n+1}\left(a_i, z_{j+1}\right)-W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)}{\Delta z} \cdot \theta\left(\underbrace{\bar{z}_{\mathbf{s}}-z_j}_{\mu_{\mathbf{s}, j}}\right)^{+} \\ +\frac{W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)-W_{\mathbf{s}}^{n+1}\left(a_i, z_{j-1}\right)}{\Delta z} \cdot \theta\left(\bar{z}_{\mathbf{s}}-z_j\right)^{-} \\ +\lambda_{\mathbf{e}}\left(W_{\mathbf{s}'}^{n+1}\left(a_i, z_j\right)-W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)\right) \end{array}\right) \tag{24}$$

By rearranging the terms, the equation becomes:

$$
\left(\frac{1}{\Delta}+\rho\right) W_{\mathbf{s}}^{n+1}(a_i,z_j) - \frac{1}{\Delta} W_{\mathbf{s}}^{n}(a_i,z_j) = u\left(c_{\mathbf{s}}^{n}(a_i,z_j)\right) + \left(\begin{array}{l}
\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}(a_i,z_j)+(r-\delta)a_i - c_{\mathbf{s}\,\mathbf{F}}^{n}(a_i,z_j)\right)^{+}}_{\left(\xi_{\mathbf{s}\,\mathbf{F}}^{n}(a_i,z_j)\right)^{+}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_{i+1},z_j) \\
+\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}(a_i,z_j)+(r-\delta)a_i - c_{\mathbf{s}\,\mathbf{B}}^{n}(a_i,z_j)\right)^{-}}_{\left(\xi_{\mathbf{s}\,\mathbf{B}}^{n}(a_i,z_j)\right)^{-}} - \underbrace{\left(y_{\mathbf{s}}(a_i,z_j)+(r-\delta)a_i - c_{\mathbf{s}\,\mathbf{F}}^{n}(a_i,z_j)\right)^{+}}_{\left(\xi_{\mathbf{s}\,\mathbf{F}}^{n}(a_i,z_j)\right)^{+}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_i,z_j) \\
-\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}(a_i,z_j)+(r-\delta)a_i - c_{\mathbf{s}\,\mathbf{B}}^{n}(a_i,z_j)\right)^{-}}_{\left(\xi_{\mathbf{s}\,\mathbf{B}}^{n}(a_i,z_j)\right)^{-}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_{i-1},z_j) \\
+\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{+}}_{(\mu_{s,j})^{+}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_i,z_{j+1}) \\
+\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{-}}_{(\mu_{s,j})^{-}} - \underbrace{\theta\left(\bar{z}_{\mathbf{e}}-z_j\right)^{+}}_{(\mu_{s,j})^{+}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_i,z_j) \\
-\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{-}}_{(\mu_{s,j})^{-}}\right)\cdot W_{\mathbf{s}}^{n+1}(a_i,z_{j-1}) \\
+\ \lambda_{\mathbf{e}}\left(W_{\mathbf{s}'}^{n+1}(a_i,z_j) - W_{\mathbf{s}}^{n+1}(a_i,z_j)\right)
\end{array}\right) \tag{25}
$$

For clarity and simplicity, in the following paragraphs, I use the notation " $W^{n+1}_{\mathbf{s}\,[i,j]} \equiv W^{n+1}_{\mathbf{s}}\left(a_i, z_j\right)$ " to rewrite the equations.The equation above therefore becomes:

$$\left(\frac{1}{\Delta}+\rho\right)W^{n+1}_{\mathbf{s}\,[i,j]}-\frac{1}{\Delta}W^{n}_{\mathbf{s}\,[i,j]}=u\left(c^{n}_{\mathbf{s}\,[i,j]}\right)+\ z_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i+1,j]}+y_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i,j]}+x_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i-1,j]}+\zeta_j\cdot W^{n+1}_{\mathbf{s}\,[i,j+1]}+\upsilon_j\cdot W^{n+1}_{\mathbf{s}\,[i,j]}+\chi_j\cdot W^{n+1}_{\mathbf{s}\,[i,j-1]}+\lambda_{\mathbf{s}}\cdot W^{n+1}_{\mathbf{s}'\,[i,j]}-\lambda_{\mathbf{s}}\cdot W^{n+1}_{\mathbf{s}\,[i,j]} \quad (26)$$

, where:

$$\begin{aligned}
z_{i,j} &= \frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{F}}[i,j]\right)^{+}}{\Delta a}\\
y_{i,j} &= \frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{B}}[i,j]\right)^{-}-\left(\xi^{n}_{\mathbf{s}\,\mathbf{F}}[i,j]\right)^{+}}{\Delta a}\\
x_{i,j} &= -\frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{B}}[i,j]\right)^{-}}{\Delta a}\\
\zeta_j &= \frac{\left(\mu_{\mathbf{s},j}\right)^{+}}{\Delta z}\\
\upsilon_j &= \frac{\left(\mu_{\mathbf{s},j}\right)^{-}-\left(\mu_{\mathbf{s},j}\right)^{+}}{\Delta z}\\
\chi_j &= -\frac{\left(\mu_{\mathbf{s},j}\right)^{-}}{\Delta z}
\end{aligned} \quad (27)$$

The system presented above consists of $\langle\ I\times J\times 2\ \rangle$ equations, which can be reformulated in a matrix-algebraic framework. In this setup, W^n and c^n are vectors of size $\langle\ I\times J\times 2\ \rangle$, while $\mathbf{A}$ and $\mathbf{B}$ are both $\langle\ (I\cdot J\cdot 2)\times(I\cdot J\cdot 2)\ \rangle$ matrices, where $\mathbf{B}$ is the combination of the transition matrix for the diffusion process, $\mathbf{C}$, and the transition matrix for the Poisson process, $\mathbf{\Lambda}$ respectively.

$$\left(\frac{1}{\Delta}+\rho\right)W^{n+1}-\frac{1}{\Delta}W^{n}=u\left(c^{n}\right)+\left(\tilde{\mathbf{A}}+\mathbf{B}\right)W^{n+1} \quad (28)$$

$$\mathbf{B}=\mathbf{C}+\mathbf{\Lambda}$$

Denoting $\mathbf{A} \equiv \tilde{\mathbf{A}} + \mathbf{B}$, the equation becomes:

$$\left(\frac{1}{\Delta} + \rho\right) W^{n+1} - \frac{1}{\Delta} W^n = u\left(c^n\right) + \mathbf{A} W^{n+1} \tag{29}$$

I manipulate the equation and derive the result as:

$$W^{n+1} = \left(\left(\frac{1}{\Delta} + \rho\right) \mathbf{I} - \mathbf{A}\right)^{-1} \left(u(c^n) + \frac{1}{\Delta} W^n\right) \tag{30}$$

C.2. Solving Consumer's Kolmogorov Forward Equation (Fokker-Planck Equation)

After solving the consumer's HJB equation, the solution of the consumer's KF equation is within arm's reach since we have obtained the matrix **A**. The consumer's KF equation is (at stationary equilibrium):

$$0 = -\frac{d}{da}\Big(\xi\left(a,z,s\right)\cdot g\left(a,z,s\right)\Big) - \frac{d}{dz}\Big(\theta\left(\bar{z}_e - z\right)\cdot g\left(a,z,s\right)\Big) + \underbrace{\lambda_u g\left(a,z,s'\right)}_{\text{those who find a job}} - \underbrace{\lambda_e g\left(a,z,s\right)}_{\text{those separated}} \tag{31}$$

For specific asset level a_i, productivity z_j, and employment status s, the density function, $g(a_i, z_j, s)$, is:

$$0 = -\frac{d}{da}\Big(\xi\left(a_i,z_j,s\right)\cdot g\left(a_i,z_j,s\right)\Big) - \frac{d}{dz}\Big(\theta\left(\bar{z}_e - z_j\right)\cdot g\left(a_i,z_j,s\right)\Big) + \lambda_u g\left(a_i,z_j,s'\right) - \lambda_e g\left(a_i,z_j,s\right) \tag{32}$$

Similar to the HJB equation, this system of equations can be solved by finite-difference. The first step is to discretize the system as:

$$0 = \left(\begin{array}{l} -\underbrace{\frac{\xi\left(a_{i+1},z_j,s\right)g\left(a_{i+1},z_j,s\right) - \xi\left(a_i,z_j,s\right)g\left(a_i,z_j,s\right)}{\Delta a}}_{\text{forward difference term}} - \underbrace{\frac{\xi\left(a_i,z_j,s\right)g\left(a_i,z_j,s\right) - \xi\left(a_{i-1},z_j,s\right)g\left(a_{i-1},z_j,s\right)}{\Delta a}}_{\text{backward difference term}} \\ -\underbrace{\frac{\theta\left(\bar{z}_s - z_{j+1}\right)g(a_i,z_{j+1},s) - \theta\left(\bar{z}_s - z_j\right)g(a_i,z_j,s)}{\Delta z}}_{\text{forward difference term}} - \underbrace{\frac{\theta\left(\bar{z}_s - z_j\right)g(a_i,z_j,s) - \theta\left(\bar{z}_s - z_{j-1}\right)g(a_i,z_{j-1},s)}{\Delta z}}_{\text{backward difference term}} \\ +\lambda_{s'} g\left(a_i,z_j,s'\right) - \lambda_s g\left(a_i,z_j,s\right) \end{array}\right) \tag{33}$$

By using the upwind scheme, the discretized system can be written as:

$$0 = \left(\begin{array}{l} - \underbrace{\frac{\overbrace{\left(\xi_{\mathbf{B}}\left(a_{i+1}, z_j, s\right)\right)^{-}}^{-x_{i+1,j}\cdot\Delta a} g\left(a_{i+1}, z_j, s\right) - \left(\xi_{\mathbf{B}}\left(a_i, z_j, s\right)\right)^{-} g\left(a_i, z_j, s\right)}{\Delta a}}_{\text{forward difference term}} - \underbrace{\frac{\left(\xi_{\mathbf{F}}\left(a_i, z_j, s\right)\right)^{+} g\left(a_i, z_j, s\right) - \overbrace{\left(\xi_{\mathbf{F}}\left(a_{i-1}, z_j, s\right)\right)^{+}}^{-z_{i-1,j}\cdot\Delta a} g\left(a_{i-1}, z_j, s\right)}{\Delta a}}_{\text{backward difference term}} \\ - \underbrace{\frac{\left(\theta\left(\bar{z}_s - z_{j+1}\right)\right)^{-} g(a_i, z_{j+1}, s) - \left(\theta\left(\bar{z}_s - z_j\right)\right)^{-} g(a_i, z_j, s)}{\Delta z}}_{\text{forward difference term}} - \underbrace{\frac{\left(\theta\left(\bar{z}_s - z_j\right)\right)^{+} g(a_i, z_j, s) - \left(\theta\left(\bar{z}_s - z_{j-1}\right)\right)^{+} g(a_i, z_{j-1}, s)}{\Delta z}}_{\text{backward difference term}} \\ + \underbrace{\lambda_{s'} g\left(a_i, z_j, s'\right) - \lambda_s g\left(a_i, z_j, s\right)}_{\text{Unlike the HJB, the subscripts (of the employment status) are different here.}} \end{array} \right) \tag{34}$$

By rearranging the terms, the system of equation can be rewritten by using the variables developed in previous subsection, $x_{i,j}$, $y_{i,j}$, $z_{i,j}$, $\chi_{i,j}$, $\upsilon_{i,j}$, and, $\zeta_{i,j}$. The resulting equation is:

$$x_{i+1,j}\cdot g\left(a_{i+1}, z_j, s\right) + y_{i,j}\cdot g\left(a_i, z_j, s\right) + z_{i-1.j}\cdot g\left(a_{i-1}, z_j, s\right) + \chi_{j+1}\cdot g\left(a_i, z_{j+1}, s\right) + \upsilon_j\cdot g\left(a_i, z_j, s\right) + \zeta_{j-1}\cdot g\left(a_i, z_{j-1}, s\right) + \lambda_{s'}\cdot g\left(a_i, z_j, s'\right) - \lambda_s\cdot g\left(a_i, z_j, s\right) \tag{35}$$

Or in a brief manner:

$$x_{i+1,j} \cdot g_{s\,[i+1,j]} + y_{i,j} \cdot g_{s\,[i,j]} + z_{i-1.j} \cdot g_{s\,[i-1,j]} + \chi_{j+1} \cdot g_{s\,[i,j+1]} + \upsilon_j \cdot g_{s\,[i,j]} + \zeta_{j-1} \cdot g_{s\,[i,j-1]} + \lambda_{s'} \cdot g_{s'\,[i,j]} - \lambda_s \cdot g_{s\,[i,j]} \tag{36}$$

The system can be expressed in matrix form as:

$$\mathbf{A}^T g = 0 \tag{37}$$

where g is a vector of dimension $(I \times J \times 2)$, and $\mathbb{A}$ represents the matrix from the final iteration (after convergence) of $\mathbf{A}$ in the previous subsection, which solves the consumer's HJB equation.

C.3. Solving Firm's HJB Equation

The representative firm's maximization problem (assuming free-entry condition, $V = 0$) with regard to asset level a and productivity level z is:

$$(r_t - \delta)\, J(a, z) = \max_k \left(zF(k) - r_t k - \omega(a, z) + \Big(J\,(a, z)\Big)'_a \xi_e\,(a, z) + \Big(J\,(a, z)\Big)'_z \theta\,(\bar{z}_e - z) - \lambda_e J\,(a, z) \right) \tag{38}$$

The free-entry condition ($V = 0$) implies that:

$$\phi = \lambda_f \int_{\underline{z}}^{\overline{z}} \int_{\underline{a}}^{\infty} J(a, z_j) \frac{g(a, z_j, u)}{u_t} \; da \;\; dz \tag{39}$$

The first-order condition is:

$$z_j F'(k) = r_t \Rightarrow (F')^{-1} \frac{r_t}{z_j} = k^* \tag{40}$$

$$(r_t - \delta)\, J(a_i, z_j) = z_j F\left(k^*\left(z_j, r_t\right)\right) - r_t k^*\left(z_j, r_t\right) - \omega(a_i, z_j) + \left(J\left(a_i, z_j\right)\right)'_a \xi_e\left(a_i, z_j\right) - \lambda_e J\left(a_i, z_j\right) \tag{41}$$

The problem can be written in a numerical way as:

$$\frac{J^{n+1} - J^n}{\Delta} + (\lambda_e + r - \delta)\, J^{n+1} = \Pi + \mathbb{A}_e J^{n+1} \Rightarrow \left(\left(\frac{1}{\Delta} + \lambda_e + r - \delta \right) I - \mathbb{A}_e \right) J^{n+1} = \frac{1}{\Delta} J^n + \Pi$$

, where $\mathbb{A}_\mathbf{e}$ denotes the $\mathbf{A_e}$ from the last converged iteration of the consumer's HJB in Appendix B.1.1. By iteration, there is a converged result of J^{n+1}

$$J^{n+1} = \left(\left(\frac{1}{\Delta} + \lambda_e + r - \delta \right) I - \mathbb{A}_e \right)^{-1} \left(\frac{1}{\Delta} J^n + \Pi \right) \tag{42}$$

D. Algorithm

1. Initial Setups

1.1. Set up Structural Parameters

1. General parameters

α production	δ depreciation	γ utility	ρ discount
.3	.021	2	.01

Table 7: Production and consumption parameters

2. Labor market parameters

β bargaining power	η LM tightness elasticity	ϕ vacancy posting cost
.72	.72	.395
χ matching efficiency	λ_e separation	h UI
1.7935	.1038	0

Table 8: Labor market parameters

1.2. Set up the Grids

1. Set up non-uniform asset grid a_i, $i = 1, 2, ..., I$. Here I choose $a_i \in [\ -1\ ,\ 2500\ ]$, $I = 2500$
2. Set up productivity grid z_j, $j = 1, 2, ..., J$. Here I choose $z_j \in [\ 0.65\ ,\ 1.0\ ]$, $J = 15$

1.3. Set up the Productivity Transition Matrix

Since the LoM doesn't change as the updating parameters (the parameters that change as the loop goes until reaching the equilibrium) change, the transition matrix of productivity can be set up here.

1.4. Set up the Solution Parameters

For the following numerical solution, I set (1) the tolerance level of the HJB equation as $1e - 10$, (2) the tolerance level of the free-entry condition and (3) the tolerance level of market clearing as $1e - 5$. I set the step size for the iteration of the HJB equation, $\Delta = 500$.

1.5. Set up the payroll Tax Rate τ

For the baseline, I set $\tau = .01$ (1%).

1.6. Initially Guess the Updating Parameters

The initial guess of these parameters are made by the following steps.

1. Make the initial guess on Θ. I guess $\Theta = 1$.
2. Obtain λ_u by having $\chi\Theta^\eta$.
3. Obtain the aggregate(average) productivity $\bar{z}_e$ by calculating the stationary distribution of productivity, z, condition on the employed consumers, which is $\mathbb{E}\left[z|s=e\right]$. The stationary equilibrium is calculated from the KFE:

$$0 = -\frac{d}{dz}\left(\left(\theta\left(z_e - z_j\right)\right) g(a_i, z_j, e)\right) + \lambda_u g(a_i, z_j, u) - \lambda_e g(a_i, z_j, e) \tag{43}$$

$$0 = -\frac{d}{dz}\left(\left(\theta\left(z_u - z_j\right)\right) g(a_i, z_j, u)\right) + \lambda_e g(a_i, z_j, e) - \lambda_u g(a_i, z_j, u) \tag{44}$$

4. Make a guess on the capital per worker, k, by taking the complete market value, $\left(\frac{\rho+\delta}{z\alpha}\right)^{\frac{1}{\alpha-1}}$, and times 1.01 to ensure nonexplosive asset accumulation.
5. Obtain the interest rate, r, since I have z, and k on my hand now.

1.7. Guess the wage schedule: $\omega(a_i, z_j)$

For guessing the initial wage schedule, I use the formula:

$$\tilde{\omega}(a_i, z_j) = \beta\left(z_j k^\alpha - rk\right)$$

1.8. Guess the Value Functions of the Consumers

I guess the initial value functions of each type of consumers, denoted as $W_{\mathbf{e}}(a, z)$ for the employed and $W_{\mathbf{u}}(a, z)$ for the unemployed, as 0.

1.9. Guess the Value Functions of the Representative Firm

1.10. Guess the Amount of the Lump-sum Unemployment Benefit

Given the values of labor market tightness Θ, payroll tax rate τ, exogenous separation intensity λ_e, the unemployment benefits can be written as:

$$h = \frac{\displaystyle\int_{\underline{z}}^{1} \int_{0}^{\infty} \tau\, \omega(a,z)\, da\, dz}{\left(\dfrac{\lambda_e}{\lambda_e + \lambda_u}\right)}$$

2. The Solution

The Outerloop

In each round of the outerloop updates the updating parameters so as to find the equilibrium outcome. Inside each outerloop, there are: (1) the innerloop for consumer optimization, (2) the solution to FPE (or KFE), (3) the innerloop for the firm's optimization, (4) wage bargaining, and (5) updating parameters.

2.1. Innerloop for Consumer Optimization

The detail of the solution is displayed in Appendix C.1.

2.2. Solution to the Fokker-Planck Equation

The detail of the solution is displayed in Appendix C.2.

2.3. Innerloop for the Firm's Optimization

The detail of the solution is displayed in Appendix C.3.

2.4. Wage Bargaining

2.5. Market Clearing

1. Given the newly computed $J(a,z)$ from Step D.2.3., calculate the vacancy-posting value, V, as in 10

2. Given the newly computed KFE in Step D.2.2., calculate the aggregate savings, and then divide it by $(1-u)$ (the proportion of population in the workplace) to have the k^{AS}

2.6. Check the Convergence

1. Check if the vacancy-posting value is sufficiently close to 0, by the criteria set in the step setting the solution parameter, Step D.1.4.
2. Check if the difference between the initial k and the k^{AS} computed in Step D.2.5.2. is sufficiently close to 0, by the criteria set in the step setting the solution parameter, Step D.1.4.

If the differences are small enough, break the loop; otherwise, let the algorithm continue.

2.7. Parameter Updates

This step updates the updating parameters in order to find the stationary recursive equilibrium of the economy. This step includes:

- Labor market parameter update: Updating Θ, then the value of job-finding intensity λ_u, unemployed population u, vacancy amounts v, will be updated.
- Capital per worker k: Updating k by $k^{NEW} = relax * k^{OLD} + (1 - relax) * k^{AS}$, which k^{AS} is obtained from the solved FPE (KFE).
- Aggregate productivity z: Updating z by using the FPE (KFE) with the newly obtained λ_u.
- The new price of capital r.
- The new wage schedule $\omega(a, z)$.
- The new unemployment benefits amount h from the new wage schedule and the solved FPE (KFE).

E. The Stationary Equilibrium

In this appendix, I define the economy's stationary recursive equilibrium, whose outcomes are presented in Subsection 4.1 for the quantitative analysis. The stationary equilibrium consists of:

1. A set of value functions:
$$\left\{ W_{\mathbf{e}}(a, z), W_{\mathbf{u}}(a, z), J(a, z), V \right\}$$
2. Individual consumption policies, $c(a, z, s)$, and saving policies, $\dot{a}(a_\ell, z_\ell, s_\ell) \equiv \xi(a_\ell, z_\ell, s_\ell)$, as a function of individual asset level a_ℓ, productivity z_ℓ, and employment status s_ℓ.

3. The prices of capital, r, and the price (wage schedule) of labor $\omega(a, z)$.

4. The amount of vacancies, v.

5. The capital stock per worker, k^d.

6. The labor market tightness, Θ, and the resulting intensities of job search and job filling, λ_u and λ_f respectively.

7. The amount of the lump-sum unemployment benefit, h, and the payroll tax rate, τ

8. The distribution of agents over asset level a, productivity level z, and employment status s, denoted as $g(a, z, s)$

such that:

1. Consumer optimization
   Given the equilibrium job-finding intensity λ_u, the (rental) price r, wage schedule $\omega(a_\ell, z_\ell)$, the consumption-savings choice, c and ξ, solves individual's maximization problem with asset level a_ℓ, productivity level z_ℓ, and employment status, s_ℓ, denoted as $W(a_\ell, z_\ell, \mathbf{e})$ for the employed and $W(a_\ell, z_\ell, \mathbf{u})$ for the unemployed.

2. Firm optimization
   Given the labor market tightness, Θ, the price of capital r and wage schedule $\omega(a_\ell, z_\ell)$, the equilibrium vacancy-filling intensity λ_f, and the stationary distribution of consumer $g(a, z, s)$, the capital per worker, k solves the firm's maximization problem of $J(a_\ell, z_\ell)$.

3. The consistency of stationary distribution
   Given the consumer's policies and labor market parameters, the distribution, $g(a, z, s)$ satisfies and .

4. The free-entry condition
   Given r, $\omega(a, z)$, Θ, and $g(a, z, s)$, the value of vacancy, V, satisfies 10

5. The asset market is cleared:

$$\int_{\underline{z}}^{1} \int_{0}^{\infty} \left\{ a \cdot (g(a, z, \mathbf{e}) + g(a, z, \mathbf{u})) \right\} \, da \, dz = k(1 - u)$$

6. The government budget is balanced as in equation 16