"The Impact of Unemployment Benefits in an Economy with Search Frictions and Unemployment-Induced Productivity Loss"

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ABSTRACT

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This paper examines the impact of unemployment benefits on an economy characterized by search frictions in the labor market and unemployment-induced productivity loss on *ex ante* homogeneous workers facing idiosyncratic unemployment risks by building an extension on Krusell et al. (2010). The analysis is framed as a continuous-time dynamic programming problem, with the solution implemented using the finite-difference method outlined in Achdou et al. (2022). The results indicate that increasing unemployment benefits consistently improves welfare compared to the baseline scenario with a 1% payroll tax. However, this welfare improvement is accompanied by a significant rise in the variances of individual asset holdings and consumption levels. Furthermore, the analysis reveals that in the extension incorporating unemployment-induced productivity loss, firm's profits and filled job values are higher compared to the original Krusell et al. (2010) framework.

Contents

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1 Introduction

Unemployment benefits (UB hereafter) helps household mitigate excessive precautionary savings and smooth consumption in face of income and unemployment risks. It is especially crucial for those experiencing liquidity constraints. Moreover, in models that incorporate *ex-ante* skill heterogeneity as in Setty and Yedid-Levi (2021), UB also serves as a tool of redistribution to eliminate consumption inequality. However, when it comes to aggregate welfare, their overall effect remains ambiguous. Some studies suggest that UB, depending on the specific structures of different program, may have limited or even negative impacts on aggregate welfare (Young, 2004; Krusell et al., 2010; Popp, 2017).

Over the past few decades, research has highlighted the trade-offs governments face when implementing or expanding UB, particularly in the labor market. Such programs can distort both the supply and demand sides of the market. In traditional search models with endogenous search effort, UB reduce incentives to seek employment by increasing reservation wages and consequently the cost of job search, this leads to more unemployment spells and higher unemployment rates (Pissarides, 2000). In addition to this moral hazard effect, UB also (and mainly) channels its impact via providing liquidity for those at the borrowing constraint and make it less urgent to search for a job (Chetty, 2008). On the demand side, recent studies indicate that higher UB raise equilibrium wages, which diminishes firm profits and discourages firms from creating new vacancies (Krusell et al., 2010), this also leads to prolonged unemployment. Empirically, the supply side effect of UB has been studied by Nakajima (2012) and the demand side one has been explored latter in Hagedorn et al. (2013), both studies have demonstrated that extended UB duration during the Great Recession contributed to increased unemployment via different mechanisms.

This paper examines the impact of (an increase in) UB in an Krusell et al. (2010) (KMS hereafter) styled economy with *ex ante* homogeneous but *ex post* heterogeneous agents facing unemployment risks and unemployment-induced productivity loss (UIPL hereafter), which is estimated to reach up to 30% after one year of unemployment (Keane and Wolpin, 1997). As in the original KMS framework, this study has a particular focus on the demand-side effects of UB on the labor market, especially on firm's vacancy creation. In addition to examining the impact of UB, this paper investigates the rationale for incorporating UIPL and compares the differences between the extension built in this paper and the original KMS models.

Introducing UIPL into the labor market framework yields two critical dynamics. At the individual level, UIPL lowers future productivity upon re-employment. This reduction, combined with a weakened outside option, places workers at a disadvantage during wage bargaining sessions compared to the original KMS model without UIPL. From the firm's perspective, productivity depreciation among workers decreases aggregate productivity and profits, disincentivizing vacancy creation. This feedback loop exacerbates the negative effects of prolonged unemployment on labor market outcomes.

To address these questions, this paper conducts two stepwise analyses. First, to understand the impact UB has on an economy, an extension of the KMS model is developed by incorporating UIPL. The government is assumed to implement a lump-sum UB program financed through payroll taxes. The baseline scenario (especially for welfare analysis) considers a lump-sum UB funded by a 1% payroll tax on employed individuals. The paper then investigates equilibrium outcomes and welfare changes (comparing to the baseline) under various payroll tax rates and UB, characterized by payroll tax rates ranging from 0% to 2.5% and their corresponding lump-sum UBs. Welfare changes (from the baseline) are evaluated using the criterion proposed by Aiyagari (1994).

In the second analysis, this paper explores the rationale for incorporating UIPL into the KMS models by examining how the incorporation affects the equilibrium properties of such models. Specifically, it compares equilibrium outcomes between the extension, as introduced in the first analysis, and the original KMS model, where individuals do not experience UIPL. For simplicity, the comparison is performed in two scenarios: scenario without tax and scenario with 2% payroll tax and UB, allowing for a clear evaluation of the implications of UIPL on equilibrium dynamics.

To solve these models numerically, this paper forms the problem in a continuous-time dynamic programming framework and follows the traditional finite-difference method presented in Achdou et al. (2022) and Bardóczy (2017). The calibration strategy closely follows Bardóczy (2017), which serves as the continuous-time counterpart to Krusell et al. (2010), and draws on Keane and Wolpin (1997) and Ljungqvist and Sargent (1998) to parameterize productivity loss.

The quantitative results from the first analysis indicate that, relative to the baseline scenario, increasing the payroll tax rate—and consequently, the lump-sum UB—can enhance overall welfare as well as increase the proportion of population who are benefited from the policy change, from approximately 65% to 75%. This outcome is primarily driven by the redistributive effects inherent in this type of UB program.

In the second analysis, the comparison between the extension and the original KMS model highlights significant differences. A notable feature of the extension is that employed individuals within a moderate asset range (i.e., not at extremely high levels of asset holdings) exhibit dissaving behavior—a pattern absent in the original KMS model, where employed individuals consistently save within this asset interval.[1] Furthermore, while UIPL reduces the attractiveness of the outside option for unemployed workers, its impact on the difference between the value functions for the employed and the un-

[1] I restrict the analysis to the "moderate" asset level because in both case, with and without UIPL, agents does dissave, but only when at a very high asset level. However, in equilibrium, only few consumers are at this extreme high asset level.

employed, given same other state variables (asset and productivity), is relatively small. Specifically, in the original model, the value function of the unemployed is, at most, 0.089% lower than that of the employed at the same asset and productivity levels. In the model incorporating UIPL, the value of this drop increases marginally to about 0.096%. [2] Despite the fact that UIPL seems to marginally increase the scale of the drop of value function between being employed and being unemployed, the model with UIPL predicts lower wages, while for the firm, both the filled job value [3] and the expected job value (from recruiting, condition on individuals from the pool of unemployed) are higher, given same state variables (asset and productivity). These factors lead to greater equilibrium labor market tightness in the extension, compared to the original KMS model, as firms in the extension with UIPL are more incentivized to create vacancies.

The structure of this paper is as follows: Section 2 introduces a model that extends the KMS framework by incorporating UIPL. Section 3 describes the calibration procedure used to generate the equilibrium results for the subsequent analyses. Section 4 is divided into two parts. The first part examines the equilibrium outcomes and welfare changes in models with UIPL, ranging from the baseline model to alternative scenarios with different payroll tax rates, from 0 to 2.5%. The second part compares the equilibrium outcomes of the extension built in this paper with those of the original model, which excludes UIPL. Finally, Section 5 discusses the study's limitations and concludes with a summary of the key findings and their broader implications.

2 Model

Similar to the structure of Krusell et al. (2010) and its continuous-time version in Bardóczy (2017), the model comprises three key agents: (1) heterogeneous consumers, (2) a representative firm and (3) a benevolent government. The labor market is search-frictional as in the traditional Diamond (1982); Mortensen (1982); Pissarides (1985) (DMP hereafter) models with random search and matching. Employed consumers lose their jobs with an exogenous separation intensity, λ_e. The unemployed consumer and the representative firm meet each other with a certain endogenous job-finding intensity λ_u and job-filling intensity λ_f, and agree on the wage via Nash bargaining. All agents act in infinite-horizon continuous-time settings.

[2] The difference is calculated by: $\frac{W_{\mathbf{u}}-W_{\mathbf{e}}}{W_{\mathbf{e}}}$, expressing the percentage drop in value function when changing from being employed to being unemployed.

[3] Actually, it is the filled job value of individuals with the highest productivity, so as to be comparable with the original KMS model.

2.1 Labor Market

There is search friction in the labor market in which the firm creates vacancies to hire workers. Both search and job posting are random and therefore not directed. For simplicity, although workers might be different in terms of their own productivity, vacancy is non-discriminating, and vacancy-creating costs are assumed to be the same for every vacancy. Matches between workers and vacant positions are random. The labor market can be characterized by (1) the elasticity of the matching function, η, (2) the bargaining power of workers, β, (3) the exogenous given separation intensity, λ_e, (4) the cost of vacancy creation, ϕ, and (5) the matching function $M(u, v)$, where u denotes the unemployed population and v denotes the amounts of opened vacancies. Having these parameters and the matching function, the equilibrium labor market tightness, Θ, the job-finding intensity, λ_e, and the job-filling intensity, λ_f, can be determined. The market can be featured by the equations below:

$$\begin{aligned} \Theta &= \tfrac{v}{u} \\ \lambda_u &= \chi \Theta^{1-\eta} \\ u &= \tfrac{\lambda_e}{\lambda_e + \lambda_u} \end{aligned}$$

In the stationary equilibrium, the unemployment rate doesn't change, meaning that: $\dot{u} = 0$. This implies:

$$\lambda_e \, (1 - u) = \lambda_u \, u$$

2.2 Asset Market

In each period, consumers can only save in a non-state-contingent risk-free asset, denoted by a_t. This asset provides a return of r per unit while simultaneously depreciating at a rate δ. The resulting return on the asset a_t can be expressed as:

$$(1 + r - \delta) a_t,$$

where r is endogenously determined by the firm's production, and δ is an exogenously specified parameter.

2.3 Heterogeneity

Consumers are *ex ante* homogeneous but *ex post* heterogeneous in both their productivity, denoted as z_t, and their employment status, denoted as s_t. The employment status of each consumer follows a two-state Poisson process, with $\lambda_\mathbf{e}$ being the exogenous separation intensity of the employed and $\lambda_\mathbf{u}$ being the job-finding intensity of the unemployed. Unlike $\lambda_\mathbf{e}$, the value of $\lambda_\mathbf{u}$ depends on the equilibrium values of the parameters of the labor market: labor market tightness Θ and the intensity of exogenous separation $\lambda_{\mathbf{e}}$.

An employed worker has a theoretical long-term employed productivity z_e and the unemployed has a theoretical long-term jobless productivity z_u, where $z_e > z_u$ by assumption. Once a consumer becomes unemployed, her productivity gradually and deterministically drops from z_e to z_u. On the other hand, once she becomes employed again, her productivity increases gradually and deterministically until it reaches z_e. The process can be written as follows:

$$\dot{z}_t = \begin{cases} \theta_e(z_{\mathbf{e}} - z_t) & \text{if employed,} \\ \theta_u(z_{\mathbf{u}} - z_t) & \text{if unemployed,} \end{cases}$$

, where $\theta_e < \theta_u$ therefore productivity recovery takes more time than productivity decline.

Given the features above, in this model, the *ex ante* homogeneous consumers are *ex post* heterogeneous in three dimensions: the individual asset level (a), individual productivity (z), and employment status (s)

2.4 Consumers

Employed Consumers

The employed consumer (each is indexed ℓ, where $\ell = 1, 2, 3, ..., L$) with asset level a_ℓ and productivity z_ℓ receives pre-tax labor income, $\omega(a_\ell, z_\ell)$, which is determined by bilateral Nash bargaining (specified in Subsection 2.6) and pay the labor income tax τ (specified in Subsection 2.7) to the government to finance the UB program for the unemployed. Let $W_{\mathbf{e}}(a_\ell, z_\ell)$ be the value function of the employed consumer ℓ, its HJB equation [4] can be written as:

$$\rho W_{\mathbf{e}}(a_\ell, z_\ell) = \max_c \left\{ \begin{array}{c} u(c) + \partial_a W_{\mathbf{e}}(a_\ell, z_\ell)\Big((1-\tau)\,\omega(a_\ell, z_\ell) + (r-\delta)\,a_\ell - c\Big) \\ +\partial_z W_{\mathbf{e}}(a_\ell, z_\ell)\Big(\theta\,(z_{\mathbf{e}} - z_\ell)\Big) + \lambda_{\mathbf{e}}\Big(W_{\mathbf{u}}(a_\ell, z_\ell) - W_{\mathbf{e}}(a_\ell, z_\ell)\Big) \end{array} \right\} \tag{1}$$

The state variable a and z evolve as:

$$\begin{aligned} \dot{a}_\ell &= (1-\tau)\,\omega(a_\ell, z_\ell) + (r-\delta)\,a_\ell - c \\ \dot{z}_\ell &= \theta\,(z_{\mathbf{e}} - z_\ell) \end{aligned}$$

The first-order necessary condition for this problem is an optimal consumption choice, $\mathbf{c}^*_\ell$, given the asset level a_ℓ and the productivity z_ℓ:

[4] For the details of the derivations of this subsection, please see Appendix B.1.1.

$$u'\left(\mathbf{c}^*_{\mathbf{e},\ell}\left(a_\ell, z_\ell\right)\right) \equiv u'\left(c^*_{\mathbf{e}}(a_\ell, z_\ell)\right) = \partial_a W_{\mathbf{e}}(a_\ell, z_\ell)$$

Using the first-order condition above, the maximization problem can be reformulated in terms of the optimal consumption, $\mathbf{c}^*_{\mathbf{e},\ell}$:

$$\begin{aligned}\rho W_{\mathbf{e}}(a_\ell, z_\ell) = u\left(\mathbf{c}^*_{\mathbf{e},\ell}\right) + \partial_a W_{\mathbf{e}}(a_\ell, z_\ell)\left((1-\tau)\,\omega(a_\ell, z_\ell) + r a_\ell - \mathbf{c}^*_\ell\right) \\ +\partial_z W_{\mathbf{e}}(a_\ell, z_\ell)\left(\theta\left(z_{\mathbf{e}} - z_\ell\right)\right) + \lambda_{\mathbf{e}}\left(W_{\mathbf{u}}(a_\ell, z_\ell) - W_{\mathbf{e}}(a_\ell, z_\ell)\right)\end{aligned} \tag{2}$$

In the following paragraph, I use $\xi(a_\ell, z_\ell, s_\ell)$ to denote the optimal savings policy of individual ℓ with asset level a_ℓ, productivity level z_ℓ, and employment status s_ℓ.

Unemployed Consumers

The unemployed consumer (indexed ℓ) with asset level a_ℓ and productivity z_ℓ enjoys home production f and receives a lump-sum unemployment benefits h, whose value depends on the government UB program (specified in Subsection 2.7). Let $W_{\mathbf{u}}(a_\ell, z_\ell)$ be the value function of the unemployed consumer ℓ, its HJB equation [5] can be written as:

$$\rho W_{\mathbf{u}}(a_\ell, z_\ell) = \max_c \left\{ \begin{array}{c} u(c) + \partial_a W_{\mathbf{u}}\left(a_\ell, z_\ell\right)\left(f + h + (r-\delta)\,a_\ell - c\right) \\ +\partial_z W_{\mathbf{u}}\left(a_\ell, z_\ell\right)\left(\theta\left(z_{\mathbf{u}} - z_\ell\right)\right) + \lambda_{\mathbf{u}}\left(W_{\mathbf{e}}\left(a_\ell, z_\ell\right) - W_{\mathbf{u}}\left(a_\ell, z_\ell\right)\right) \end{array} \right\} \tag{3}$$

The state variable a and z evolve as:

$$\begin{aligned}\dot{a}_\ell &= f + h + (r-\delta)\,a_\ell - c \\ \dot{z}_\ell &= \theta\left(z_{\mathbf{u}} - z_\ell\right)\end{aligned}$$

As in the case of employed consumers, using the first-order condition, the maximization problem can be reformulated in terms of optimal consumption $\mathbf{c}^*_{\mathbf{u},\ell}$:

$$\begin{aligned}\rho W_{\mathbf{u}}(a_\ell, z_\ell) = u\left(\mathbf{c}^*_{\mathbf{u},\ell}\right) + \partial_a W_{\mathbf{u}}(a_\ell, z_\ell)\left(f + h + r a_\ell - \mathbf{c}^*_\ell\right) \\ +\partial_z W_{\mathbf{u}}(a_\ell, z_\ell)\left(\theta\left(z_{\mathbf{u}} - z_\ell\right)\right) + \lambda_{\mathbf{u}}\left(W_{\mathbf{e}}(a_\ell, z_\ell) - W_{\mathbf{u}}(a_\ell, z_\ell)\right)\end{aligned} \tag{4}$$

In addition, consumptions should be non-negative for all agents and satisfy the borrowing constraint. This yields the boundary condition (at the borrowing constraint $\underline{a}$) that for all possible levels of productivity z, I have:

[5] For the details of the derivations of this subsection, please see Appendix B.1.2.

$$W'_{\mathbf{e}}(\underline{a}, z) \geq u'\left((1-\tau)\,\omega(\underline{a}, z) + r\,\underline{a}\right) \tag{5}$$

$$W'_{\mathbf{u}}(\underline{a}, z) \geq u'\left(f + h + r\,\underline{a}\right) \tag{6}$$

General Expression

For simplicity, the two above HJB equations (for the employed and the unemployed, respectively) can be generalized as:

$$\begin{aligned} \rho W_{\mathbf{s}}(a, z) = u(\mathbf{c}^*_{\mathbf{i,j}}) + \partial_a W_{\mathbf{s}}(a, z)\left(y_{\mathbf{s}}(a, z) + ra - \mathbf{c}^*_{\mathbf{i,j}}\right) + \partial_z W_{\mathbf{s}}(a, z)\theta\,(z_{\mathbf{s}} - z) \\ + \lambda_{\mathbf{s}}\left(W_{\mathbf{s}'}(a, z) - W_{\mathbf{s}}(a, z)\right) \end{aligned} \tag{7}$$

, where $\mathbf{s}$ takes either $\mathbf{e}$ or $\mathbf{u}$, and $\mathbf{s}'$ takes the other value than $\mathbf{s}$, and:

$$y_{\mathbf{s}}(a, z) = \begin{cases} \omega(a, z) & \text{, if } \mathbf{s} = \mathbf{e}, \\ f + h & \text{, if } \mathbf{s} = \mathbf{u}. \end{cases}$$

Stationary Distribution

In the stationary equilibrium, the distribution of consumers across asset levels a, productivity z, and employment status s is characterized by the Hamilton-Jacobi-Bellman (HJB) equations outlined above, along with the Kolmogorov Forward equation (also known as the Fokker-Planck equation) featuring the invariant distribution, which takes the following form, one for the employed consumers and one for the unemployed consumers respectively:

$$0 = -\frac{d}{da}\left(\xi_{\mathbf{e}}(a, z) g_{\mathbf{e}}(a, z)\right) - \frac{d}{dz}\left(\left(\theta\,(z_{\mathbf{e}} - z)\right) g_{\mathbf{e}}(a, z)\right) + \lambda_{\mathbf{u}} g_{\mathbf{u}}(a, z) - \lambda_{\mathbf{e}} g_{\mathbf{e}}(a, z) \tag{8}$$

$$0 = -\frac{d}{da}\left(\xi_{\mathbf{u}}(a, z) g_{\mathbf{u}}(a, z)\right) - \frac{d}{dz}\left(\left(\theta\,(z_{\mathbf{u}} - z)\right) g_{\mathbf{u}}(a, z)\right) + \lambda_{\mathbf{e}} g_{\mathbf{e}}(a, z) - \lambda_{\mathbf{u}} g_{\mathbf{u}}(a, z) \tag{9}$$

2.5 The Firm

There is one representative firm that hires workers and rents capital to maximize its profits while facing a search-frictional labor market with random search and matching. The firm posts each vacancy with fixed cost ϕ, and waits until it randomly encounters the interviewee, (with intensity λ_f. Both parties are supposed to always accept the match regardless of the level of individual productivity, and wage scales are determined by Nash bargaining (detailed in Subsection 2.6) by both sides. In addition, wages are determined period-by-period. Therefore, for each employee with asset level a and productivity z, the firm has the values of each filled job denoted as $J(a,z)$, implying both asset level and productivity affects individual wage scale. The value for the firm to post a vacancy for recruitment, denoted as V, can be written as:

$$V = -\phi + \lambda_f \mathbb{E}\left[J(a,z)|\mathbf{u}\right] \tag{10}$$

The free entry condition requires the value of opening a position, V, to be 0 in equilibrium. By adding this fact, the value of opening up a vacancy implies:

$$\phi = \lambda_f \int_{\underline{z}}^{\overline{z}} \int_0^{\infty} J(a,z)\, \frac{g(a,z,u)}{u}\, da\, dz \tag{11}$$

By assuming the free-entry condition, $V = 0$, is satisfied, the value a filled job creates can be written as:

$$J\left(a,z\right) = \max_k \left\{ \left(\underbrace{zF\left(k\right) - rk - \omega\left(a,z\right)}_{\pi(a,z)} \right) \Delta + \left(1 - \rho_F \Delta\right)\left(1 - \lambda_{\mathbf{e}} \Delta\right) J\left(a_{t+\Delta}, z_{t+\Delta}\right) \right\}$$

where $\rho_F = r_t - \delta$. After taking $\Delta \to 0$, the problem simplifies into:

$$\left(\lambda_{\mathbf{e}} + r - \delta\right) J\left(a,z\right) = \max_k \left\{ \begin{array}{c} \underbrace{zF\left(k_t\right) - rk - \omega\left(a,z\right)}_{\pi(a,z)} \\ +\partial_a J\left(a,z\right) \xi_{\mathbf{e}}(a,z) + \partial_z J(a,z) \left(\theta\left(z_{\mathbf{e}} - z\right)\right) \end{array} \right\} \tag{12}$$

2.6 Wage Determination

Similar to other DMP models, the worker's wage levels are determined by bargaining between the worker and the firm. I use Nash bargaining as in the traditional literature. The maximization problem of wage bargaining, given the employee's asset level a_i and productivity z_j (here I use the subscripts to emphasize the fact that the wage varies not only between different asset levels a but also between different productivity levels z), can be written as:

$$\omega(a_i, z_j) = \arg\max_{w} \left\{ \Big(W_{\mathbf{e}}\,(a_i, z_j) - W_{\mathbf{u}}\,(a_i, z_j) \Big)^{\gamma} \Big(J\,(a_i, z_j) - V \Big)^{1-\gamma} \right\} \tag{13}$$

It is noteworthy that the F.O.C. of this maximization problem, if unbounded, might generate negative profit for firms in the practical computation process. Therefore, the strategy is to add a constraint that profits should be non-negative:

$$0 \leq \pi(a_i, z_j) = z_j f(k) - r\ k - \omega(a_i, z_j) \tag{14}$$

2.7 The Government

The benevolent government collect payroll taxes from the labor income of the employed, with τ denoting the proportion of income that is taxed, and provide lump-sum UB, denoted as h, to those who are unemployed. Suppose that the government can only observe individual's employment status and therefore individual asset level and productivity are unobservable by the government, the amount of UB is therefore identical among those who are unemployed.

$$h(a_\ell, z_\ell) \equiv h \tag{15}$$

Further assuming there is no cost for arranging the UB program, the government balances its budget by having the tax revenue (LHS) equal to the expenditure for the UB program (RHS):

$$\int_{z_u}^{z_e} \int_{\underline{a}}^{\infty} \Big\{ \tau(a, z)\ \omega(a, z)\ g_{\mathbf{e}}(a, z) \Big\}\ da\ dz = \int_{z_u}^{z_e} \int_{\underline{a}}^{\infty} \Big\{ h\ g_{\mathbf{u}}(a, z) \Big\}\ da\ dz \tag{16}$$

I define the stationary recursive equilibrium in Appendix E.

3 Calibration

Similar to the calibration in Bardóczy (2017), in which the model inherits parameters from Krusell et al. (2010) but translates them into their continuous-time counterpart. To model productivity gains and declines associated with employment and unemployment respectively, I draw on the parameterization of Ljungqvist and Sargent (1998) and fix the fraction (of the speed of each mean-reverting process), $\frac{\theta_e}{\theta_u}$ to be $\frac{1}{2}$. For the exact number of these parameters, I refer to Keane and Wolpin (1997) and set that for a worker, being unemployed for one-year duration causes 30% skill depreciation (productivity loss). By converting this rate into a continuous-time format, I have $z_e = 1$, $\theta_e = .25$ for the employed and $z_u = .65$, $\theta_u = .5$ for the unemployed. The detail of the parameterization is presented in Table 1 below.

Table 1: Calibration Parameters

Description	Parameter	Value
Production parameters		
Capital share	α	0.3
Depreciation	δ	0.021
Productivity of a long-run employed	z_e	1
Productivity of a long-run unemployed	z_u	0.65
Speed of productivity acquirement	θ_e	0.25
Speed of productivity loss	θ_u	0.50
Home production (when unemployed)	f	0.0001
Consumption parameters		
Substitution elasticity	γ	1
Discount factor	ρ	0.01
Labor market parameters		
Worker's bargaining power	β	0.72
Labor market tightness elasticity	η	0.72
Matching efficiency	χ	1.7935
Separation intensity	λ_e	0.1038
Vacancy flow cost	ϕ	0.395

4 Results

In this section, I first discuss and compare the equilibrium outcomes and welfare changes between the baseline model (with a 1% payroll tax and UB) and other scenarios with different payroll tax rates and UB in Subsection 4.1. Subsequently, I examine and compare the differences between the extension (incorporating UIPL) built in this paper and the original KMS model in Subsection 4.2. To make efficient use of space, some of the supporting figures are provided in Appendix A.

4.1 The Impacts of Unemployment Benefit

The equilibrium outcomes [6] are presented in Table 2, in which results of the baseline model are highlighted by lines above and below. The remainder of this section will be divided into several parts to study the impacts of the increase in UB on the economy, with the distribution of agents by asset level presented in Subsection 4.1.1, the discussion about consumption-savings behavior in Subsection 4.1.2, the cross-sectional moments in Subsection 4.1.3, the filled job value in Subsection 4.1.4, and the welfare changes in Subsection 4.1.5.

[6] For the numerical solution which produce these results, please see Appendix C.

Table 2: Equilibrium Outcomes with Different Payroll Taxes

Tax (%)	h	UR (%)	K	r (%)	Θ	$\mathbb{E}[a_\ell]$	$\mathbb{V}ar[a_\ell]$	$\mathbb{E}[J(a,z)\|\mathbf{u}]$
0.00	0.00	4.68	25.15	3.09	1.7991	23.97	121	0.3362
0.50	0.18	4.69	25.12	3.09	1.7795	23.94	156	0.3336
0.75	0.27	4.70	25.11	3.10	1.7704	23.93	168	0.3324
1.00	0.36	4.71	25.11	3.10	1.7615	23.93	180	0.3312
1.25	0.45	4.71	25.10	3.10	1.7528	23.92	191	0.3300
1.50	0.54	4.72	25.09	3.10	1.7442	23.91	202	0.3288
2.00	0.72	4.73	25.08	3.10	1.7275	23.90	227	0.3265
2.50	0.89	4.74	25.07	3.10	1.7113	23.88	255	0.3243

4.1.1 Distribution of Agents across Asset Level

The equilibrium distributions of consumers by asset level are shown in Figure 1 respectively for the scenario without UB (0% payroll tax), the baseline model, and the scenario with 2% payroll tax. The graphs show that as the benefits increases, the distribution tends to be more flattened and right-skewed. The distance between the first and third quartiles (Q1 and Q3 in the graph) also becomes wider as the benefit increases.

4.1.2 Consumption and Savings Behavior

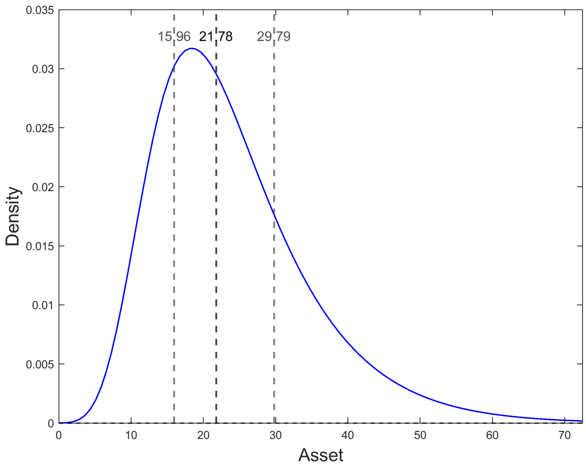
In this subsection, I compare the consumption and savings behavior of consumers when employed and unemployed under three scenarios: (1) scenario without tax and UB, (2) the baseline model (scenario with UB financed through a 1% payroll tax), (3) scenario with UB financed through a 2.5% payroll tax. To simplify the analysis, I focus on individuals with two specific productivity levels: $z = 1$, representing the productivity of a long-term employed individual, and $z = 0.7$, approximately corresponding to the productivity of an individual being jobless for one year.

Consumption

This subsection examines the consumption difference (in percentage) between the employed and the unemployed, holding the same asset and productivity levels (specifically, $z = 1$ and $z = -.7$ for the analysis here). This consumption difference is calculated and expressed as:

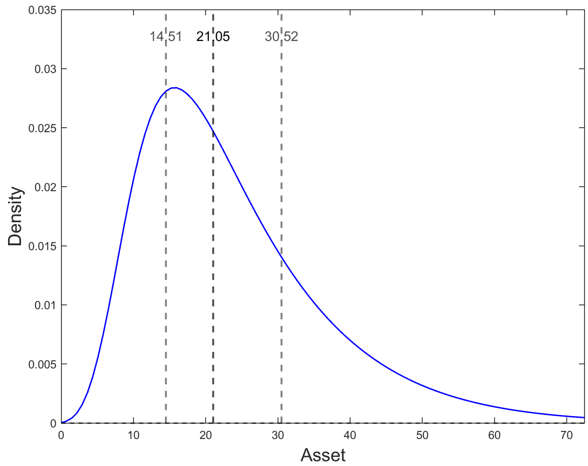
$$\text{Consumption Difference (\%)} \equiv \frac{c_{\mathbf{u}}(a, z) - c_{\mathbf{e}}(a, z)}{c_{\mathbf{e}}(a, z)}$$

The graphical results are presented in Figure 6 in Appendix A.1., with each sub-figure depicting the outcomes under different scenarios (0%, 1% as the baseline, and 2.5% payroll tax). The results indicate that, across all three scenarios, individuals with higher productivity ($z = 1$) who are near the borrowing constraint ($a \to 0$) would experience a larger



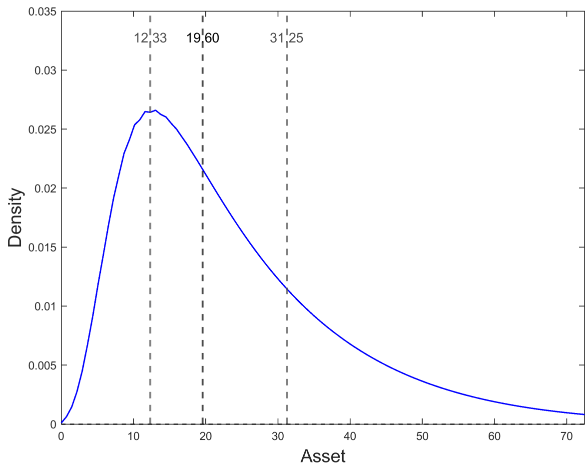
(a) Scenario **without** Tax and UB

Q1: 15.96, Q2: 21.78, Q3: 29.79



(b) **Baseline** with **1%** Payroll Tax

Q1: 14.51, Q2: 21.05, Q3: 30.52



(c) Scenario with **2.5%** Payroll Tax

Q1: 12.33, Q2: 19.60, Q3: 31.25

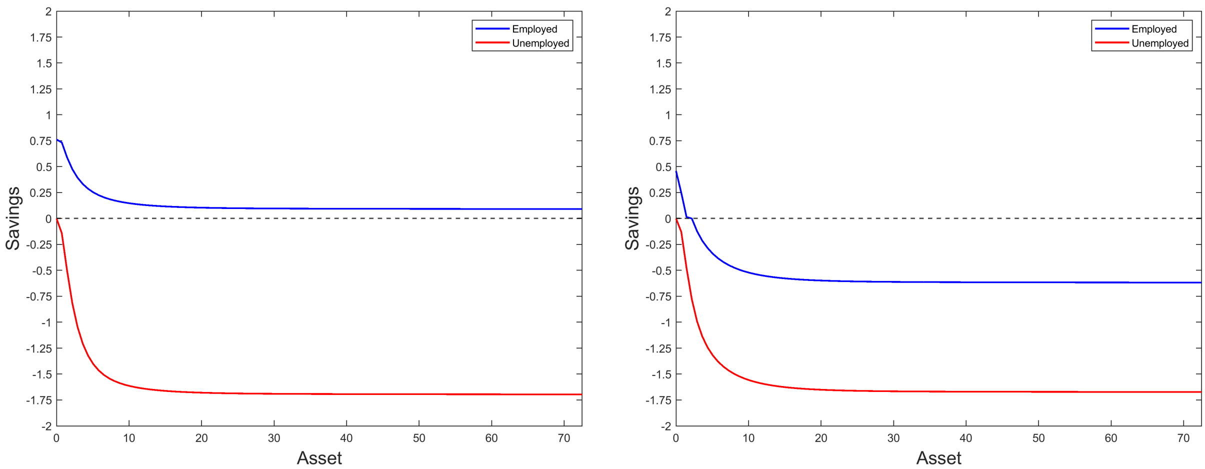
Figure 1: Distribution of Consumers by Asset Level under Different Payroll Tax and UB

consumption difference (in terms of percentage) between being employed and being unemployed compared to those with lower productivity and near the borrowing constraint. This outcome is intuitive: borrowing-constrained individuals have fewer means to smooth consumption when they become unemployed. Consequently, the gap is more pronounced in individuals with higher productivity. In addition, the analysis reveals the increase in UB reduces this consumption gap associated with employment status. This effect is particularly significant for individuals close to the borrowing constraint, emphasizing the role of UB as a liquidity-providing tool.

Savings

This subsection discusses individual savings behavior when employed (blue curve) and unemployed (red curve) given the same asset (a) and productivity (z) levels. The results are shown in Figures 2, 3, and 4 below. It shows that, across all three scenarios, the savings of the unemployed with **high** productivity ($z = 1$), denoted as $\xi(a, 1, \mathbf{u})$, do not differ a lot from that of the unemployed with **low** productivity ($z = .7$), denoted as $\xi(a, 0.7, \mathbf{u})$. This contrasts with the case of employed individuals, whose savings behavior varies significantly across different productivity levels. The analysis further reveals that the increase in UB affects more the behavior of unemployed individuals. From Figures 2 to 4, it is evident that as UB increases, the unemployed individuals tend to dissave less, as reflected by the less negative savings rates under higher UB levels. On the other hand, although less affected by the increase in UB, the employed's savings behavior differs with their individual productivity level, that individuals with high productivity tend to save more, while individual with lower productivity dissave even when being employed. This can be attributed to their expectations of productivity and wage growth in future periods if they remain employed. In summary, the increase in UB reduces individual consumer's gap of savings when being employed and being unemployed, given same asset and productivity level, primarily by mitigating the dissaving scale of the unemployed consumers.

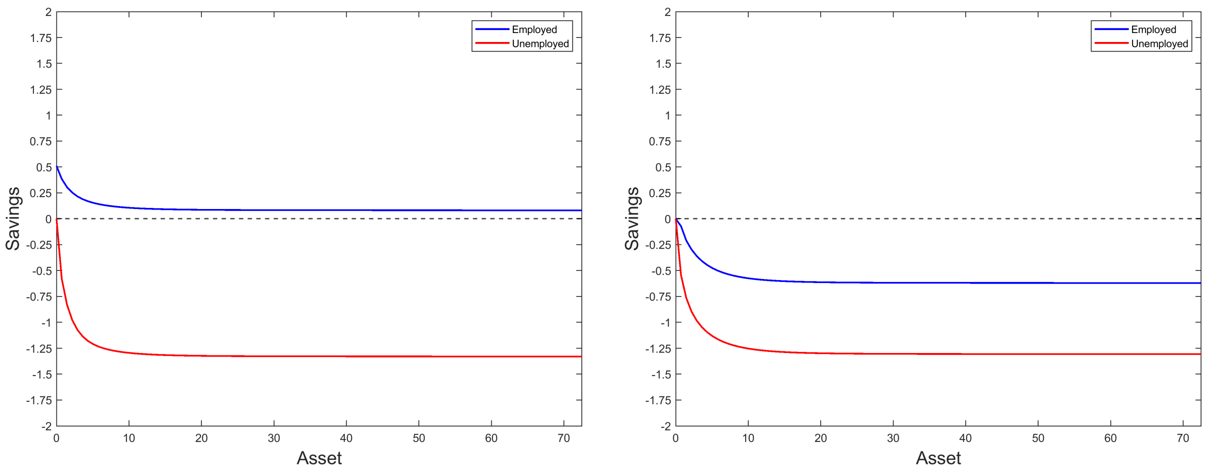
Figure 5 below further highlights the differences in savings for an employed individual with highest productivity ($z = 1$, implying being employed for a long time) under various taxation programs. [7] The blue curve represents the difference between the scenario with a 2.5% payroll tax and the baseline, while the red curve compares the scenario with a 2.5% payroll tax to the scenario without payroll tax and UB. The result shows that, by increasing the amount of UB, the change in savings amount of these individuals mostly happens in the area close to the borrowing constraint, 0.



(a) Consumer with Highest Productivity ($z = 1$) (b) Consumer with Low Productivity ($z = .7$)

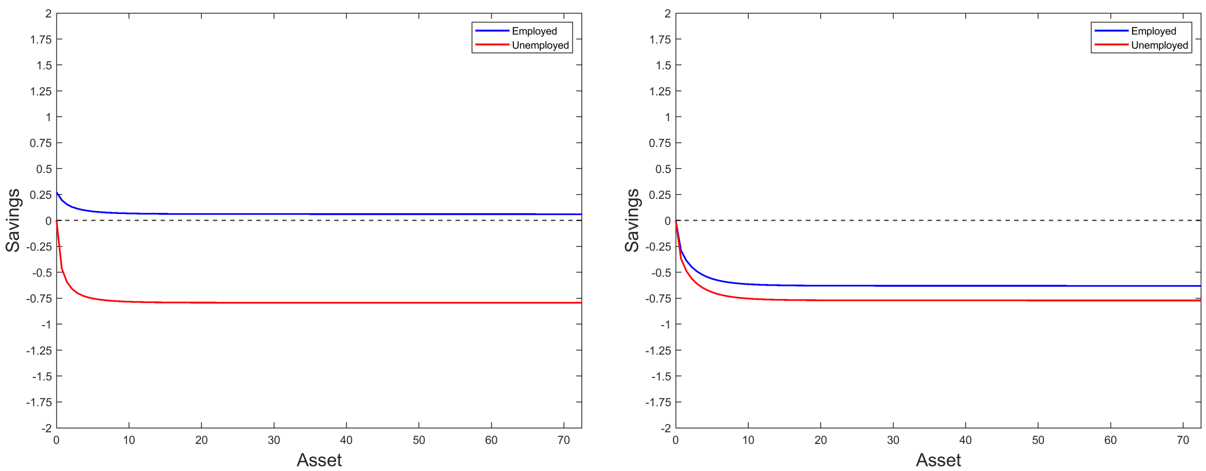
Figure 2: Difference in Savings Behavior between the Employed and Unemployed: Scenario **without** Tax and UB

[7] For better understanding, the calculation can be expressed as, for example: $\xi(a, z, 1)| \quad \tau = 2.5\% - \xi(a, z, 1)| \quad \tau = 0$, for comparison between the scenario with $\tau = 0$ and the scenario with $\tau = 2.5\%$



(a) Consumer with Highest Productivity ($z = 1$) (b) Consumer with Low Productivity ($z = .7$)

Figure 3: Difference in Savings Behavior between the Employed and Unemployed: **Baseline** with **1%** Payroll Tax



(a) Consumer with Highest Productivity ($z = 1$) (b) Consumer with Low Productivity ($z = .7$)

Figure 4: Difference in Savings Behavior between the Employed and Unemployed: Scenario with **2.5%** Payroll Tax

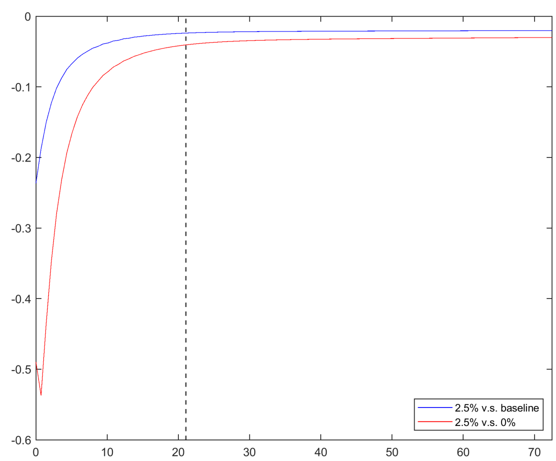


Figure 5: Comparing the Savings of the Employed with $z = 1$ under Different Tax Rates

4.1.3 Cross-Sectional Variation in Individual's Asset and Consumption Levels

From Table 3, it is evident that the variance of asset level increases with the increase in UB, by 42% from the baseline to the scenario with 2.5% payroll tax, or by 108% from the scenario without UB to the scenario with 2.5% payroll tax. However, the variance of individual consumption only increases around 30% from baseline to 2.5% payroll or 66% from the non-tax scenario to 2.5% payroll.

Table 3: Detailed Metrics of Consumer Assets and Consumption

Tax	$\mathbb{E}[a_\ell]$	$\mathbb{V}ar[a]$	$\mathbb{E}[w_\ell]$	$\mathbb{E}[c_\ell]$	$\mathbb{V}ar[c_\ell]$
0.00%	23.97	121	1.7759	1.93	0.0162
0.50%	23.94	156	1.7756	1.93	0.0190
0.75%	23.93	168	1.7755	1.93	0.0200
1.00%	23.93	180	1.7755	1.93	0.0209
1.50%	23.91	202	1.7754	1.93	0.0227
2.00%	23.90	227	1.7753	1.93	0.0253
2.50%	23.88	255	1.7753	1.93	0.0271

4.1.4 Filled Job Value

The graphical results of filled job values under different taxation scenarios are presented in Figure 7 in Appendix A.1. Notably, for individuals with the same productivity, those near the borrowing constraint ($a_\ell = 0$) receive lower wages, whereas wages for others exhibit minimal variation. The graph highlights that an increase in UB increases the wages of individuals near the borrowing constraint, thereby reducing the firm's profits. On the other hand, in the case without UB programs, the firm can exploit a huge proportion of profit via the Nash bargaining session. However, these mechanisms have only a negligible impact on firm profits within the model. Although UB significantly influences individual job seekers near the borrowing constraint through the wage channel, their overall impact on the firm's profits remains relatively limited.

4.1.5 Welfare Changes

To evaluate the welfare effects of the UB program, I adopt the welfare criterion introduced in Aiyagari (1994), which is later used by Krusell et al. (2010); Popp (2017); Setty and Yedid-Levi (2021). They use the metric Ω to assess whether a certain individual, given its state variables (a, z), gains from the implementation of the UB program. The metric

Ω is expressed as:[8]

$$\mathbb{E}\left[\int_0^\infty \left\{ e^{-\rho t} \log \left((1+\Omega)\, c_t \right) \right\} dt \right] = \mathbb{E}\left[\int_0^\infty \left\{ e^{-\rho t} \log \left(\quad \tilde{c}_t \quad \right) \right\} dt \right] \tag{17}$$

Moreover, to evaluate welfare changes across different levels of UBs, I analyze six distinct groups of consumers:

1. **Approximate Median-Asset Consumers**: Consumers whose asset levels fall between the 40th and 60th percentiles, regardless of employment status, denoted by $\mathbb{E}[\Omega^{\mathcal{M}}]$.

2. **Exact Median Consumers**: Consumers who hold exactly the median asset level (50th percentile), regardless of the employment status, denoted by $\mathbb{E}[\Omega^m]$.

3. **Median-Asset Median-Productivity Employed Consumers**: Employed consumers with asset levels at the 50th percentile among the employed and the median (and also the highest) productivity, denoted by $\mathbb{E}[\Omega^m | s = \mathbf{e}, z = z_{m\mathbf{e}}]$.

4. **Median-Asset Median-Productivity Unemployed Consumers**: Unemployed consumers with asset levels at the 50th percentile among the unemployed and the highest productivity, defined as $z = z_{\mathbf{e}}$, denoted by $\mathbb{E}[\Omega^m | s = \mathbf{u}, z = z_{m\mathbf{u}}]$.

5. **All Employed Consumers**: Denoted by $\mathbb{E}[\Omega | \mathbf{e}]$.

6. **All Unemployed Consumers**: Denoted by $\mathbb{E}[\Omega | \mathbf{u}]$.

The results, presented in Table 4, shows that the welfare improvements following the increase in UB are minimal. However, eliminating all UB coverage is not optimal for all individuals since 0% individuals benefited from that contraction. It is interesting that for the employed individuals, a reduction in the scale of UB may lead to welfare improvements, although total elimination of UB still remains suboptimal for this group of individuals. The results further suggest that a modest increase (that is, 1.25%) in payroll tax rate (and therefore the UB) can make most individuals (71%) better off.

[8] I actually have referred to Nuño and Moll (2018) for checking the continous-time version of this criterion.

Table 4: Welfare changes comparing to the baseline model

unit: percent (%)

Tax	$\mathbb{E}[\Omega]$	$\mathbb{E}[\Omega^{\mathcal{M}}]$	$\mathbb{E}[\Omega^m]$	$\mathbb{E}[\Omega\|\mathbf{e}]$	$\mathbb{E}[\Omega\|\mathbf{u}]$	$\mathbb{E}[\Omega^m\|\mathbf{e}, z_{m\mathbf{e}}]$	$\mathbb{E}[\Omega^m\|\mathbf{u}, z_{m\mathbf{u}}]$	Benefited
0	-0.0215	-0.0053	-0.0049	-0.0168	-0.1166	-0.0003	-0.0916	0
0.5	-0.0075	-0.0023	-0.0022	-0.0053	-0.0527	0.0001	-0.0453	10
0.75	-0.0034	-0.0011	-0.0010	-0.0023	-0.0259	0.0001	-0.0226	16
1.25	0.0029	0.0008	0.0007	0.0018	0.0250	-0.0004	0.0222	71
1.5	0.0053	0.0013	0.0012	0.0031	0.0495	-0.0010	0.0442	65
2.0	0.0091	0.0020	0.0019	0.0048	0.0969	-0.0026	0.0877	58
2.5	0.0118	0.0024	0.0023	0.0053	0.1428	-0.0045	0.1308	53

4.2 The Difference between the Extension with Unemployment-Induced Productivity Loss and the Original Krusell et al. (2010) Model

This subsection compares the extension (with UIPL) with the original KMS model. It should be noted that, in the extension, productivity evolves with equilibrium outcomes, whereas in the original model, productivity remains constant. To facilitate the comparison, the original KMS model is simulated under two settings: one with aggregate productivity fixed at $z = 1$ and the other with aggregate productivity set to be the average productivity of the employed under the equilibrium outcomes of the extension. Table 5 and 6 compare the equilibrium outcomes of the extension built in this paper and the original KMS model. Some of the graphical results are shown in Appendix A.2.

Table 5: Equilibrium Outcomes of the Original KMS Model and the Extension: Scenarios without Tax and UB

Scenario	h	UR (%)	K	r (%)	Θ	$\mathbb{E}[a_\ell]$	$\mathbb{V}ar[a_\ell]$	$\mathbb{E}[J(a_\ell, z_\ell)\|\mathbf{u}]$
Original, z = E.O.	0.00	4.77	25.12	3.10	1.6788	23.92	112.68	0.3199
Original, z = 1	0.00	4.74	25.65	3.10	1.7150	24.44	119.05	0.3248
Extension	0.00	4.68	25.15	3.09	1.7991	23.97	120.99	0.3362

Table 6: Equilibrium Outcomes of the Original KMS Model and the Extension: Scenarios with 2% Payroll Tax and UB

Scenario	h	UR (%)	K	r (%)	Θ	$\mathbb{E}[a_\ell]$	$\mathbb{V}ar[a_\ell]$	$\mathbb{E}[J(a_\ell, z_\ell)\|\mathbf{u}]$
Original, z = E.O.	0.70	4.82	25.06	3.10	1.6073	23.85	232.06	0.3100
Original, z = 1	0.72	4.80	25.60	3.10	1.6422	24.38	244.70	0.3148
Extension	0.72	4.73	25.08	3.10	1.7275	23.90	226.93	0.3265

4.2.1 Difference in the Drops in Value upon Unemployment

The first notable difference is that the value function gap between employment and unemployment is larger in the extension with UIPL compared to the original model.

4.2.2 Difference in Savings Behavior

Moreover, as shown in Figure 10 and 11 in Appendix A.2., in the extension with UIPL, workers with medium productivity in the scenario with a 2% payroll tax rate (and its corresponding UB) tend not to save.[9] This behavior arises from their expectation of future income growth as they remain employed for longer durations. This is different from the scenario in the original model, as most of the employed consumers save. [10]

4.2.3 Difference in the Labor Market

Another notable difference between the extension and the original KMS model is in the labor market. The results indicate that, compared to the original model, the extension with UIPL yields lower equilibrium wages for individuals with the highest productivity (z = 1), in Figure 12 and 13 in Appendix A.2. Consequently, for the firm, both the filled job values and the expected job values (condition on the unemployed) are higher (see Figures 14 and 15 in Appendix A.2., and it makes the firm more willing to create vacancies. As a result, labor market tightness (Θ) is higher in models incorporating UIPL, as shown in Table 5 and 6.

5 Conclusion

5.1 Limitation

The model focuses solely on the equilibrium outcomes of each UB program, rather than examining the transition path associated with the extension or contraction of UB. However, understanding this transition path is crucial for evaluating the effects of UB programs, as emphasized in much of the literature. The government's role in mitigating business cycle fluctuations is particularly important, and historical evidence suggests that UB extensions during economic downturns have been the subject of debate.

Furthermore, the model assumes that the maximum productivity depreciation for a currently long-term employed individual is approximately 65%. However, it remains unclear how productivity would decline over longer periods of unemployment, introduc-

[9]It is not graphically shown here but in the extension with UIPL, under the scenario with 2.5% payroll tax and UB, workers with productivity levels below 0.8 do not save, regardless of their asset levels.

[10]Except those having extremely high asset level, but the density of these individuals is very low

ing ambiguity into the results. This limitation warrants further investigation to better understand the dynamics of productivity loss over extended unemployment durations.

Last, for computational simplicity, this study assumes a deterministic dynamics of productivity for both cases when employed and unemployed respectively. In reality, however, the dynamics should be more noised like in Ljungqvist and Sargent (1998). This simplification is due to computational concern to diminish the task load of the program, but at the meantime lose the capability to thoroughly reflect individual's expectation and aggregate labor force in the real world.

5.2 Conclusive Remarks

This study investigates the effects of unemployment benefits on the economy, focusing on unemployment-induced productivity loss and labor market search frictions. The analysis is conducted in two parts. The first evaluates the impact of unemployment benefits in an extension built in this paper that incorporates UIPL into the KMS framework. The second compares the equilibrium outcomes of this extension with the original KMS model, in which there is no UIPL.

The first analysis indicates that the increase in UB has a negligible effect on firms' profits, neither through the wage bargaining mechanism nor the aggregate productivity channel. Regarding the wage bargaining channel, the increase in UB reduces the expected job value (from recruiting new employees from the unemployed pool) only by 3.5% (from the scenario without taxation and UB to the scenario with 2.5% payroll tax and UB) and by 2.0% (from the baseline to the scenario with 2.5% payroll tax and UB). The reason is that, although the increase in UB does significantly raise the reservation wage for those at the borrowing constraint, very few individuals are borrowing constrained in the equilibrium conditions. Consequently, the wage schedules of most individuals remain almost unaffected by the increase in UB. Regarding the aggregate productivity channel, the increase in UB does not significantly affect the firm's aggregate productivity given that the expectation of productivity on those who is finding a job (the unemployed) only drops .06% (from the scenario without tax and UB to the scenario with 2.5% payroll tax and UB) and .03% (from the baseline to the scenario with 2.5% payroll tax and UB).

However, the increase in UB apparently changes the consumption-saving behavior of individuals. Given same state variables (asset and productivity level), it reduces individual's consumption difference (in%) between being employed and being unemployed[11]. Moreover, it increases the economy's variances of asset level and consumption level. Nevertheless, an increase in UB does significantly improve aggregate welfare, evaluated with the welfare criterion of Aiyagari (1994). One of the reason that unemployment benefits is

[11] As in the discussion about consumption in Subsection 4.1.2, this difference (in percentage) is denoted as $\frac{\mathbf{c_u}(a,z)-\mathbf{c_e}(a,z)}{\mathbf{c_e}(a,z)}$

welfare improving is that the criterion used in this analysis is naturally in favor of redistribution from the rich to the poor (Mukoyama, 2010) [12]. Apart from the feature of the welfare criterion itself which prefers redistribution, the welfare improvement is brought about by the design of the simple payroll-tax-lump-sum-payment UB program in which the government is assumed to be unable to distinguish individuals with their asset level and productivity, therefore contributes to a certain level of redistribution.

The result of the second analysis, which compares the extension incorporating UIPL into the KMS model, shows that the equilibrium wage are lower even for individual with highest productivity ($z = 1$), comparing to those in the original KMS model. This might be caused by the worse outisde option of the workers for wage bargaining. Becuase of this feature, the firm in the extension incorporating UIPL has higher profit and filled job value (denoted as $J(a, z)$ in this paper) given same state variables (asset and productivity level) of the worker.The expected job value from recruiting the unemployed is also higher in the extension, and therefore the firm is more willing to open vacancies. This makes the labor market tighter in the extension comparing to the original model.

However, two concerns about the results are noteworthy when considering an extension of UB program. First, the optimistic perspective on the increase in unemplyoment benefits on the model with unemployment-induced productivity loss (UIPL) in this paper also relies on several key assumptions, such as (1) the omission of leisure in individual's optimization problem, which results in inelastic labor supply (therefore, both reduced income and increased benefits doesn't affect individual labor supply), (2) the inelastic job search effort (therefore unemployment benefits doesn't discourage individual search effort and then increase unemployment spell), and (3) exogenous separation rate. In addition, as highlighted by Ljungqvist and Sargent (1998), incorporating business cycle fluctuations into the model reveals potential drawbacks for a generous government that provides unemployment benefits without careful and precise planning.

[12]Since $\frac{\partial \Omega}{\partial \tilde{W}(a,z)}$ decreases in $W(\tilde{a}, z)$ and the latter increases in a (Mukoyama, 2010)

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Bardóczy, B. (2017). Labor-market matching with precautionary savings. *Note URL https://benjaminmoll. com/wp-content/uploads/2020/06/kms. pdf.*

Chetty, R. (2008). Moral hazard versus liquidity and optimal unemployment insurance. *Journal of political Economy*, 116(2):173–234.

Diamond, P. A. (1982). Aggregate demand management in search equilibrium. *Journal of political Economy*, 90(5):881–894.

Hagedorn, M., Karahan, F., Manovskii, I., and Mitman, K. (2013). Unemployment benefits and unemployment in the great recession: the role of macro effects. Technical report, National Bureau of Economic Research.

Keane, M. P. and Wolpin, K. I. (1997). The career decisions of young men. *Journal of political Economy*, 105(3):473–522.

Krusell, P., Mukoyama, T., and Şahin, A. (2010). Labour-market matching with precautionary savings and aggregate fluctuations. *The Review of Economic Studies*, 77(4):1477–1507.

Ljungqvist, L. and Sargent, T. J. (1998). The european unemployment dilemma. *Journal of political Economy*, 106(3):514–550.

Mortensen, D. T. (1982). Property rights and efficiency in mating, racing, and related games. *The American Economic Review*, 72(5):968–979.

Mukoyama, T. (2010). Welfare effects of unanticipated policy changes with complete asset markets. *Economics Letters*, 109(2):134–138.

Nakajima, M. (2012). A quantitative analysis of unemployment benefit extensions. *Journal of Monetary Economics*, 59(7):686–702.

Nuño, G. and Moll, B. (2018). Social optima in economies with heterogeneous agents. *Review of Economic Dynamics*, 28:150–180.

Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment, vacancies, and real wages. *The American Economic Review*, 75(4):676–690.

Pissarides, C. A. (2000). *Equilibrium unemployment theory*. MIT press.

Popp, A. (2017). Unemployment insurance in a three-state model of the labor market. *Journal of Monetary Economics*, 90:142–157.

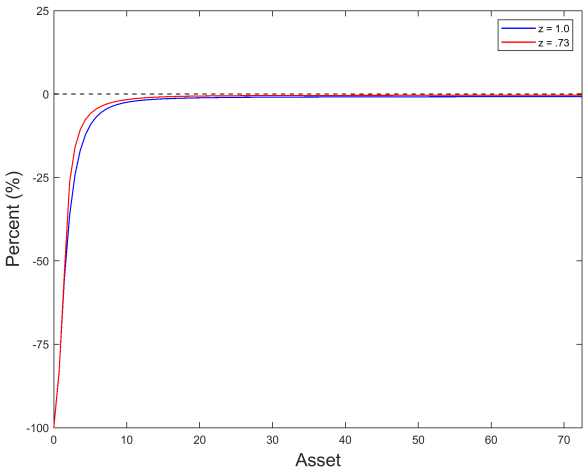
Setty, O. and Yedid-Levi, Y. (2021). On the provision of unemployment insurance when workers are ex-ante heterogeneous. *Journal of the European Economic Association*, 19(1):664–706.

Young, E. R. (2004). Unemployment insurance and capital accumulation. *Journal of Monetary Economics*, 51(8):1683–1710.

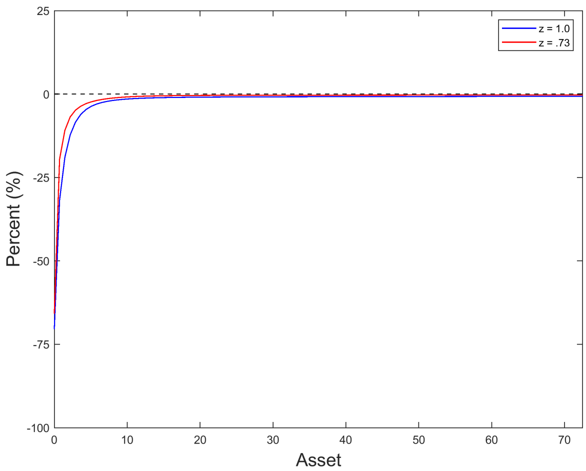
A. Figures

A.1. Figures related to Subsection 4.1

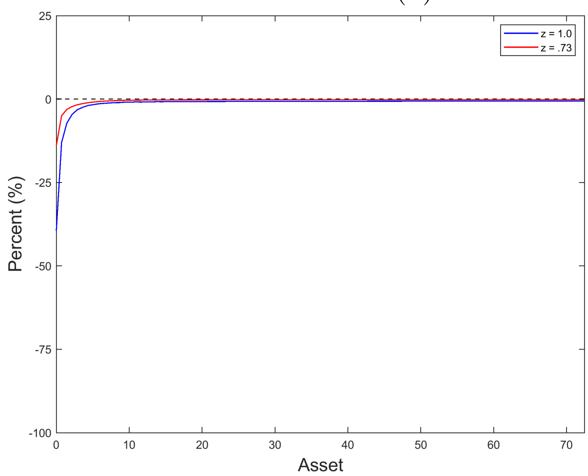
A.1.1. Difference in Consumption



(a) Scenario **without** tax



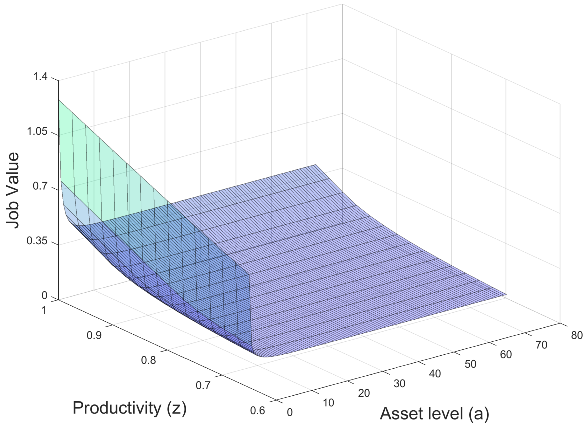
(b) Baseline with **1%** payroll tax



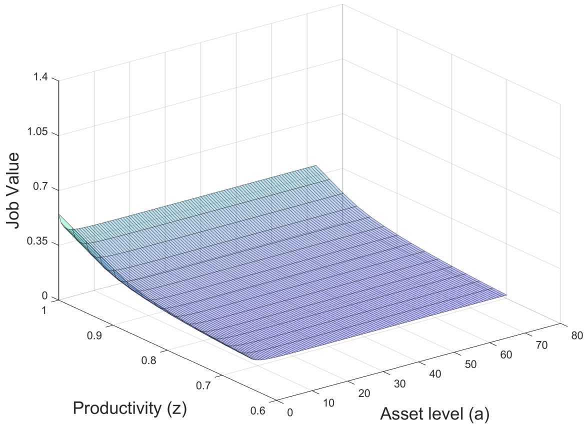
(c) Scenario with **2.5%** payroll tax

Figure 6: Distribution of consumers by asset level under different UB programs

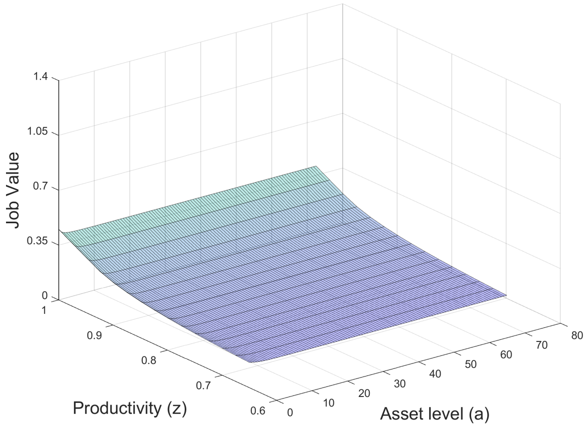
A.1.2. Filled Job Value



(a) Scenario without tax



(b) Baseline model with 1% payroll tax

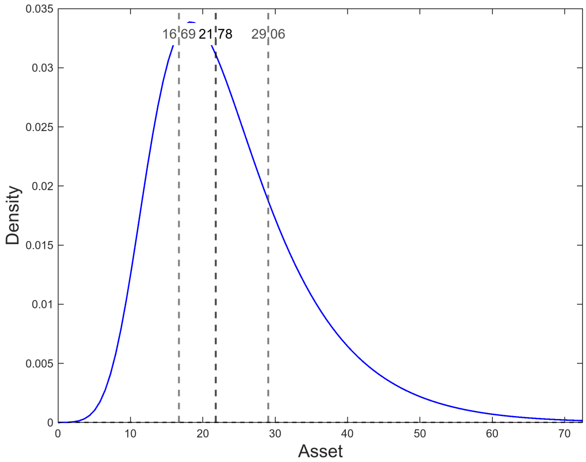


(c) Scenario with 2.5% payroll tax

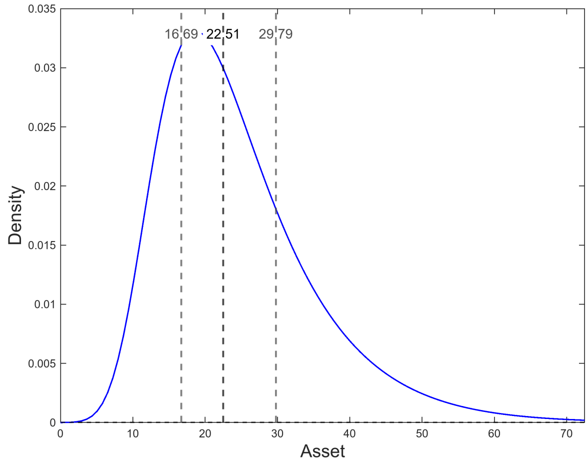
Figure 7: Filled job values under different UB programs: 0%, 1%, and 2.5% payroll taxation

A.2. Figures related to Subsection 4.2

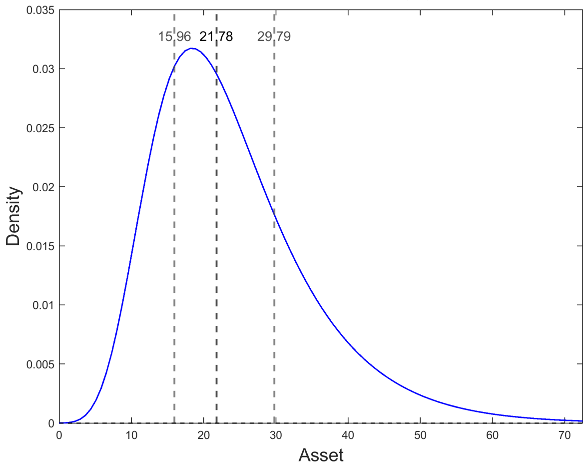
A.2.1. Distribution of Consumers by Asset Level



(a) Scenario **without** UIPL, z = E.O.

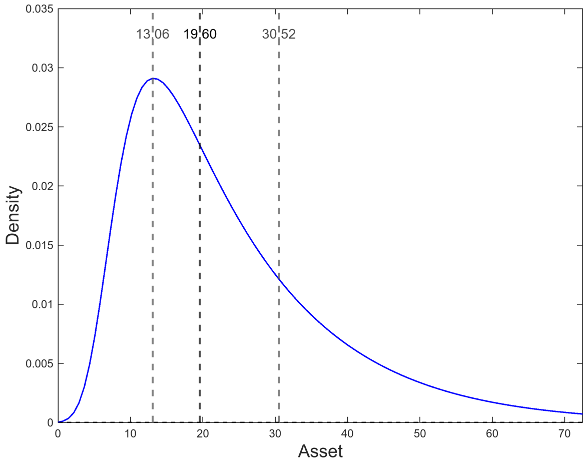


(b) Scenario **without** UIPL, z = 1

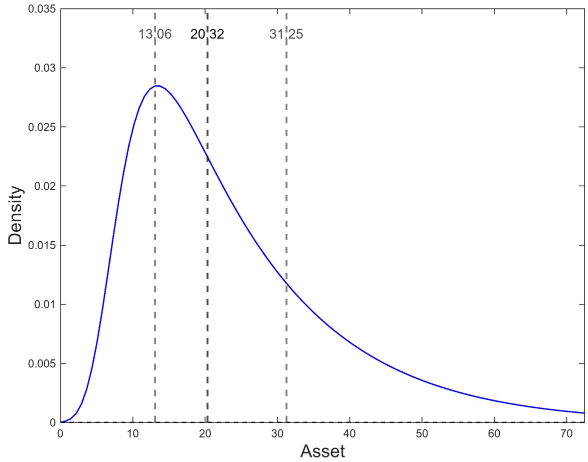


(c) Scenario **with** UIPL

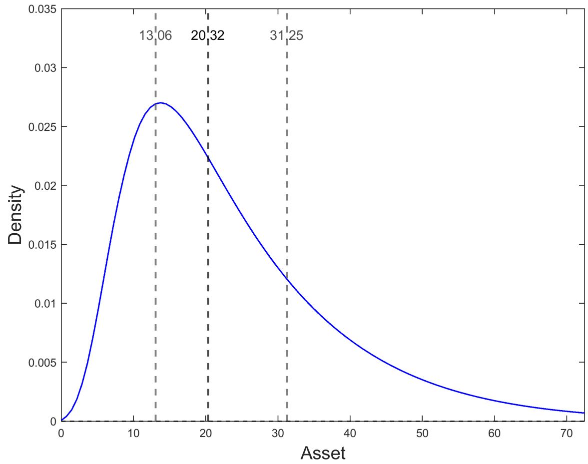
Figure 8: Distribution of consumers by Asset Level: Scenario without Tax and UB



(a) Scenario **without** UIPL, z = E.O.



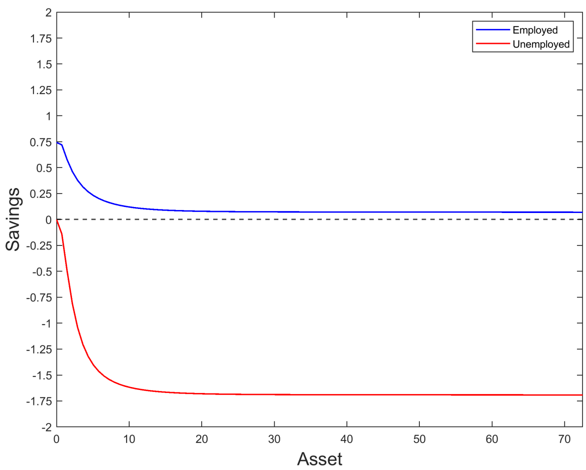
(b) Scenario **without** UIPL, z = 1



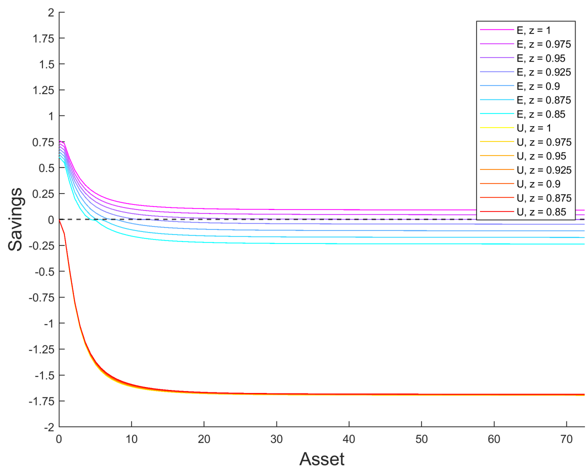
(c) Scenario **with** UIPL

Figure 9: Distribution of Consumers by Asset Level: Scenario with 2% Payroll Tax

A.2.2. Savings Behavior

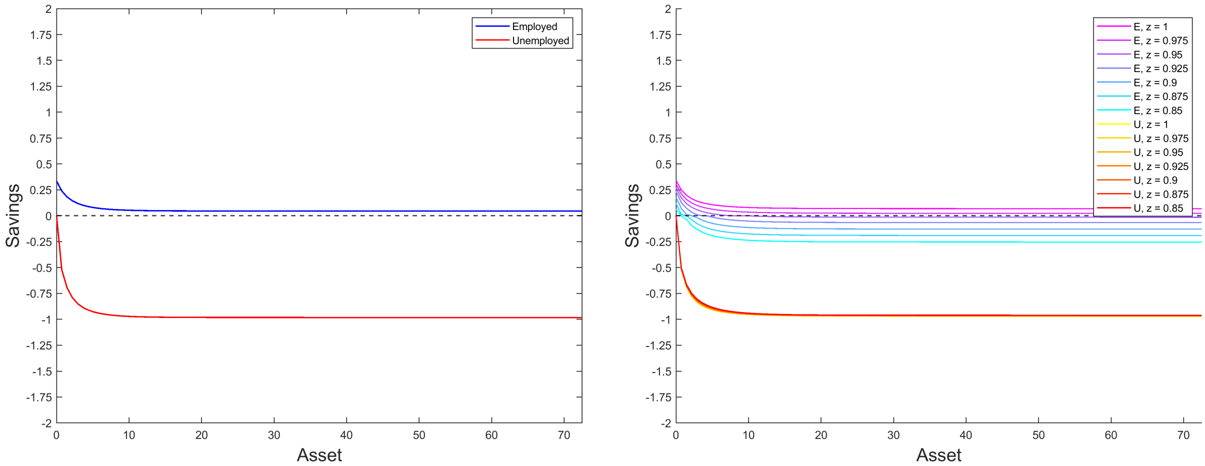


(a) Scenario **without** UIPL



(b) Scenario **with** UIPL

Figure 10: Savings Behavior: Scenario without Tax and UB

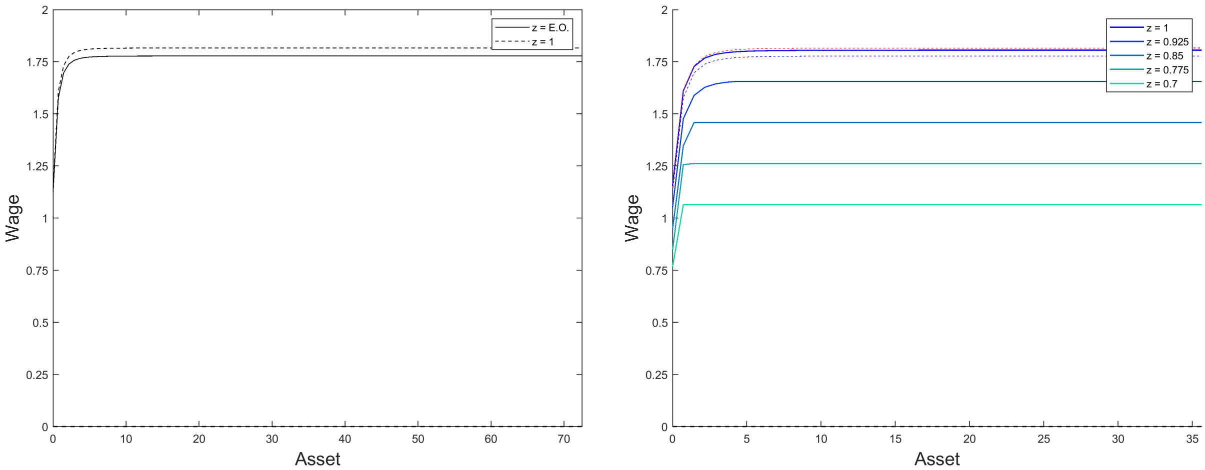


(a) Scenario **without** UIPL

(b) Scenario **with** UIPL

Figure 11: Savings Behavior: Scenario with 2% Payroll Tax

A.2.3. Wage Schedule

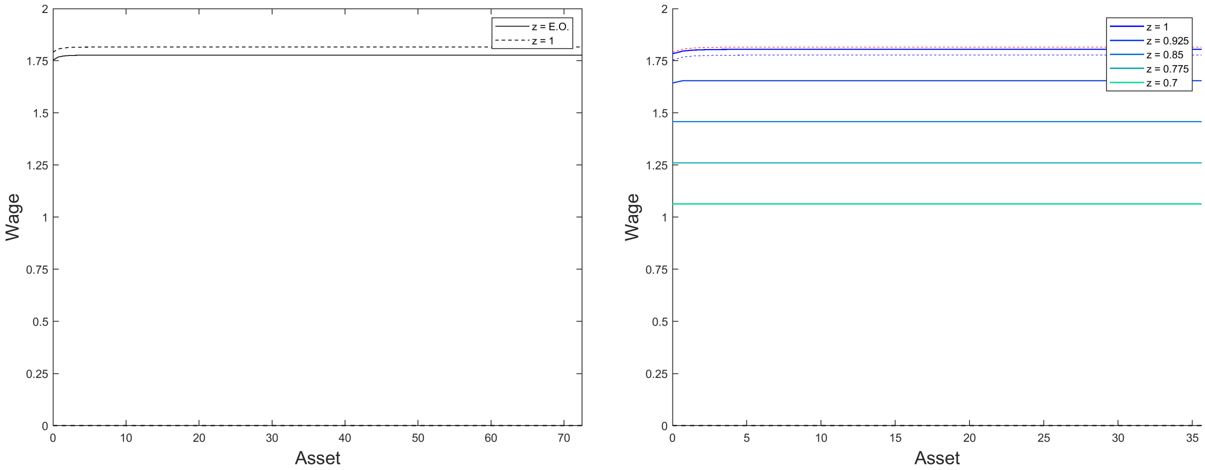


(a) Scenario **without** UIPL

(b) Scenario **with** UIPL

The red and blue dashed lines just represent the wage schedules with z = E.O. and z = 1 in the left scenario without UIPL

Figure 12: Wage Schedule: Scenario without Tax and UB

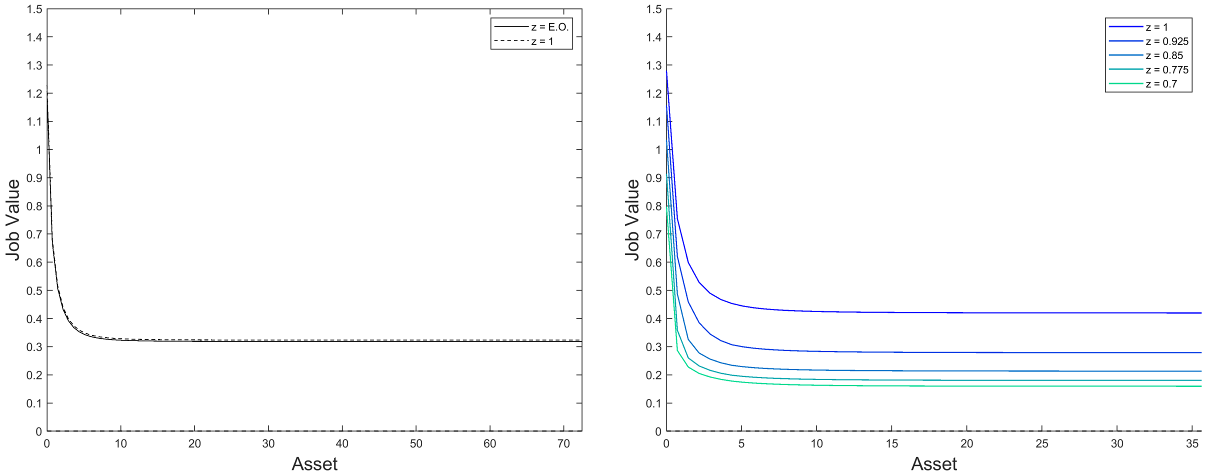


(a) Scenario **without** UIPL (b) Scenario **with** UIPL

The red and blue dashed lines just represent the wage schedules with z = E.O. and z = 1 in the left scenario without UIPL

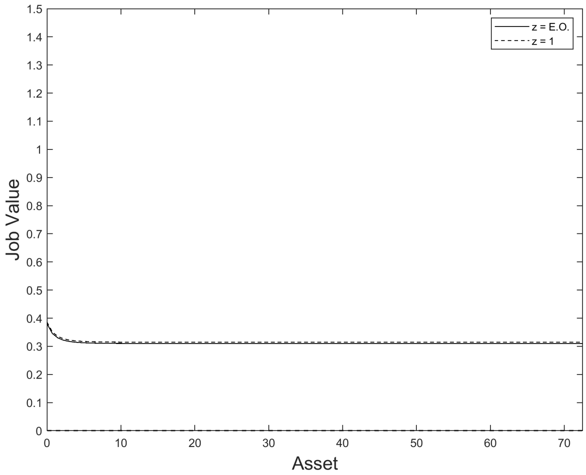
Figure 13: Wage Schedule: Scenario with 2% Payroll Tax

A.2.4. Filled Job Value

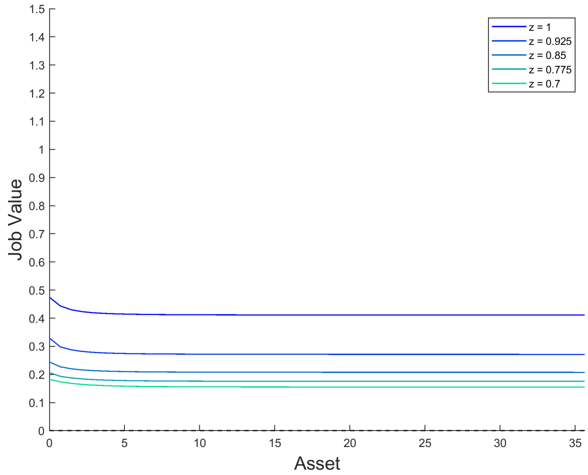


(a) Scenario **without** UIPL (b) Scenario **with** UIPL

Figure 14: Filled Job Value: Scenario without Tax and UB



(a) Scenario **without** UIPL



(b) Scenario **with** UIPL

Figure 15: Filled Job Value: Scenario with 2% Payroll Tax

B. Mathematical Derivations

Notations:

1. The 1° partial derivative uses the notation: $\frac{\partial f(x_1,x_2)}{\partial x_1} \equiv \Big(f(x_1,x_2)\Big)'_{x_1}$.
2. The parentheses, $(\cdot)$, are used exclusively for functions, i.e., $f(x)$; the brackets, $(\cdot)$ are used for calculations.

B.1. Consumer's HJB Equations

B.1.1. Employed Consumer

The employed consumer solves the optimization problem:

$$W_{\mathbf{e}}(a_t,z_t) = \max_c \left\{ u(c)\Delta + (1-\rho\Delta)\Big((1-\lambda_{\mathbf{e}}\Delta)\, W_{\mathbf{e}}(a_{t+\Delta},z_{t+\Delta}) + \lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta},z_{t+\Delta})\Big)\right\}$$

Perform a first-order Taylor expansion of $W_{\mathbf{e}}(a_{t+\Delta},z_{t+\Delta})$ around $W_{\mathbf{e}}(a_t,z_t)$:

$$W_{\mathbf{e}}(a_t,z_t) = \max_c \left\{ u(c)\Delta + (1-\rho\Delta) \left(\begin{array}{l} (1-\lambda_{\mathbf{e}}\Delta)\left(W_{\mathbf{e}}(a_t,z_t) + \Big(W_{\mathbf{e}}(a_t,z_t)\Big)'_a \Big(\omega(a_t,z_t) + (r-\delta)\,a_t - c\Big)\,\Delta + \Big(W_{\mathbf{e}}(a_t,z_t)\Big)'_z \Big(\theta\,(z_{\mathbf{e}}-z_t)\,\Delta\Big)\right) \\ +\ \lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta},z_{t+\Delta}) \end{array} \right) \right\}$$

Subtract $(1-\rho\Delta)\, W_{\mathbf{e}}(a_t,z_t)$ from both sides:

$$\rho \Delta W_{\mathbf{e}}(a_t, z_t) = \max_c \left\{ u(c)\Delta + (1-\rho\Delta) \left(\begin{array}{c} +(1-\lambda_{\mathbf{e}}\Delta) \left(\left(W_{\mathbf{e}}(a_t, z_t) \right)'_a \left(\omega(a_t, z_t) + (r-\delta)\, a_t - c \right) \Delta + \left(W_{\mathbf{e}}(a_t, z_t) \right)'_z \left(\theta\,(z_{\mathbf{e}} - z_t) \right) \Delta \right) \\ -\lambda_{\mathbf{e}}\Delta W_{\mathbf{e}}(a_t, z_t) + \lambda_{\mathbf{e}}\Delta W_{\mathbf{u}}(a_{t+\Delta}, z_{t+\Delta}) \end{array} \right) \right\}$$

Divide by Δ and take $\Delta \to 0$:

$$\rho W_{\mathbf{e}}(a_t, z_t) = \max_c \left\{ u(c) + \left(W_{\mathbf{e}}(a_t, z_t) \right)'_a \left(\omega(a_t, z_t) + (r-\delta)\, a_t - c \right) + \left(W_{\mathbf{e}}(a_t, z_t) \right)'_z \left(\theta\,(z_{\mathbf{e}} - z_t) \right) + \lambda_{\mathbf{e}} \left(W_{\mathbf{u}}(a_t, z_t) - W_{\mathbf{e}}(a_t, z_t) \right) \right\} \tag{18}$$

B.1.2. Unemployed Consumer

From the same logic of 19 , the HJB for the unemployed consumer can be written as:

$$\rho W_{\mathbf{u}}(a_t, z_t) = \max_c \left\{ u(c) + \left(W_{\mathbf{u}}(a_t, z_t) \right)'_a \left(h(z_t) + (r-\delta)\, a_t - c \right) + \left(W_{\mathbf{u}}(a_t, z_t) \right)'_z \left(\theta\,(z_{\mathbf{u}} - z_t) \right) + \lambda_{\mathbf{u}} \left(W_{\mathbf{e}}(a_t, z_t) - W_{\mathbf{u}}(a_t, z_t) \right) \right\} \tag{19}$$

B.2. Firm's HJB Equation

The representative firm solve the maximization problem:

$$J(a_t, z_t) = \max_k \left\{ \underbrace{\Big(z_t F(k_t) - r_t \cdot k_t - \omega(a_t, z_t) \Big) \Delta}_{\pi_t} + \Big(1 - (r_t - \delta)\Delta \Big) \cdot \underbrace{(1 - \lambda_e \Delta) \cdot \Big(J(a_{t+\Delta}, z_{t+\Delta}) \Big)}_{\substack{\text{the expected value of this position,} \\ \text{taking separation into consideration}}} \right\} \tag{20}$$

Comparing to the consumer's problem, this is relatively easier to solve. The derivation is done by taking $\Delta \to 0$ and rearranging the terms.

C. Numerical Solutions

C.1. Solving Consumer's HJB Equation

To solve the consumers' optimization problem, I follow the semi-implicit method as outlined in the appendices of Achdou et al. (2022) and in Bardóczy (2017).
Rewrite the problems into numerical formulation for the following finite difference iteration:

$$\frac{W_{\mathbf{s}}^{n+1}(a,z)-W_{\mathbf{s}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{s}}^{n+1}(a,z)=u(c_{\mathbf{s}}^{n})+\left(W_{\mathbf{s}}^{n+1}(a,z)\right)_a^{\prime}\left(y_{\mathbf{s}}(a,z)+(r-\delta)\,a-c_{\mathbf{s}}^{n}\right)+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\,(z_{\mathbf{s}}-z)\right)+\lambda_{\mathbf{s}}\left(W_{\mathbf{s}'}^{n+1}(a,z)-W_{\mathbf{s}}^{n+1}(a,z)\right) \tag{21}$$

where $\quad s\in\{e,u\}\quad$ and $\quad y_{\mathbf{s}}(a,z)=\begin{cases}\omega(a,z) & \text{, if } \mathbf{s}=\mathbf{e},\\ f+h(z) & \text{, if } \mathbf{s}=\mathbf{u}.\end{cases}$

To have a clear expression, the HJB equations for the employed and the unemployed are as follows:

1. The employed:

$$\frac{W_{\mathbf{e}}^{n+1}(a,z)-W_{\mathbf{e}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{e}}^{n+1}(a,z)=u(c_{\mathbf{e}}^{n})+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_a^{\prime}\left(\omega(a,z)+(r-\delta)\,a-c_{\mathbf{e}}^{n}\right)+\left(W_{\mathbf{e}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\,(z_{\mathbf{e}}-z)\right)+\lambda_{\mathbf{e}}\left(W_{\mathbf{u}}^{n+1}(a,z)-W_{\mathbf{e}}^{n+1}(a,z)\right) \tag{22}$$

where $c_{\mathbf{e}}^{n}=\left(W_{\mathbf{e}}^{n}\,(a,z)\right)_a^{\prime}$

2. The unemployed:

$$\frac{W_{\mathbf{u}}^{n+1}(a,z)-W_{\mathbf{u}}^{n}(a,z)}{\Delta}+\rho W_{\mathbf{u}}^{n+1}(a,z)=u(c_{\mathbf{u}}^{n})+\left(W_{\mathbf{u}}^{n+1}(a,z)\right)_a^{\prime}\left(f+h(z)+(r-\delta)\,a-c_{\mathbf{u}}^{n}\right)+\left(W_{\mathbf{u}}^{n+1}(a,z)\right)_z^{\prime}\left(\theta\,(z_{\mathbf{u}}-z)\right)+\lambda_{\mathbf{u}}\left(W_{\mathbf{e}}^{n+1}(a,z)-W_{\mathbf{u}}^{n+1}(a,z)\right) \tag{23}$$

where $c_{\mathbf{u}}^{n}=\left(W_{\mathbf{u}}^{n}\,(a,z)\right)_a^{\prime}$

Numerically, given asset level, productivity and employment status, (a_i, z_j, s), the problem can be reformulated as:

$$\left(\frac{1}{\Delta}+\rho\right) W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right) - \frac{1}{\Delta} W_{\mathbf{s}}^{n}\left(a_i, z_j\right) = u\left(c_{\mathbf{s}}^{n}\left(a_i, z_j\right)\right) + \left(\begin{array}{l} \quad \dfrac{W_{\mathbf{s}}^{n+1}\left(a_{i+1}, z_j\right) - W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)}{\Delta a} \cdot \left(\underbrace{y_{\mathbf{s}}\left(a_i, z_j\right)\left(a_i, z_j\right) + \left(r-\delta\right) a_i - c_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)}_{\xi_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)}\right)^{+} \\ + \dfrac{W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right) - W_{\mathbf{s}}^{n+1}\left(a_{i-1}, z_j\right)}{\Delta a} \cdot \left(\underbrace{y_{\mathbf{s}}\left(a_i, z_j\right)\left(a_i, z_j\right) + \left(r-\delta\right) a_i - c_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)}_{\xi_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)}\right)^{-} \\ + \dfrac{W_{\mathbf{s}}^{n+1}\left(a_i, z_{j+1}\right) - W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)}{\Delta z} \cdot \theta\left(\underbrace{\bar{z}_{\mathbf{s}} - z_j}_{\mu_{\mathbf{s},j}}\right)^{+} \\ + \dfrac{W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right) - W_{\mathbf{s}}^{n+1}\left(a_i, z_{j-1}\right)}{\Delta z} \cdot \theta\left(\bar{z}_{\mathbf{s}} - z_j\right)^{-} \\ + \lambda_{\mathbf{e}}\left(W_{\mathbf{s}'}^{n+1}\left(a_i, z_j\right) - W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)\right) \end{array}\right) \tag{24}$$

By rearranging the terms, the equation becomes:

$$
\left(\frac{1}{\Delta}+\rho\right) W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)-\frac{1}{\Delta} W_{\mathbf{s}}^{n}\left(a_i, z_j\right)=u\left(c_{\mathbf{s}}^{n}\left(a_i, z_j\right)\right)+\left(\begin{array}{l}
\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)\right)^{+}}_{\left(\xi_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)\right)^{+}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_{i+1}, z_j\right) \\
+\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)\right)^{-}}_{\left(\xi_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)\right)^{-}}-\underbrace{\left(y_{\mathbf{s}}\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)\right)^{+}}_{\left(\xi_{\mathbf{s}\,\mathbf{F}}^{n}\left(a_i, z_j\right)\right)^{+}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right) \\
-\frac{1}{\Delta a}\left(\underbrace{\left(y_{\mathbf{s}}\left(a_i, z_j\right)+(r-\delta) a_i-c_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)\right)^{-}}_{\left(\xi_{\mathbf{s}\,\mathbf{B}}^{n}\left(a_i, z_j\right)\right)^{-}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_{i-1}, z_j\right) \\
+\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{+}}_{\left(\mu_{s,j}\right)^{+}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_i, z_{j+1}\right) \\
+\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{-}}_{\left(\mu_{s,j}\right)^{-}}-\underbrace{\theta\left(\bar{z}_{\mathbf{e}}-z_j\right)^{+}}_{\left(\mu_{s,j}\right)^{+}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right) \\
-\frac{1}{\Delta z}\left(\underbrace{\theta\left(z_{\mathbf{e}}-z_j\right)^{-}}_{\left(\mu_{s,j}\right)^{-}}\right) \cdot W_{\mathbf{s}}^{n+1}\left(a_i, z_{j-1}\right) \\
+\quad \lambda_{\mathbf{e}}\left(W_{\mathbf{s}'}^{n+1}\left(a_i, z_j\right)-W_{\mathbf{s}}^{n+1}\left(a_i, z_j\right)\right)
\end{array}\right) \tag{25}
$$

For clarity and simplicity, in the following paragraphs, I use the notation " $W^{n+1}_{\mathbf{s}\,[i,j]} \equiv W^{n+1}_{\mathbf{s}}(a_i, z_j)$ " to rewrite the equations.The equation above therefore becomes:

$$\left(\frac{1}{\Delta}+\rho\right) W^{n+1}_{\mathbf{s}\,[i,j]} - \frac{1}{\Delta} W^{n}_{\mathbf{s}\,[i,j]} = u\left(c^{n}_{\mathbf{s}\,[i,j]}\right) + \; z_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i+1,j]} + y_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i,j]} + x_{i,j}\cdot W^{n+1}_{\mathbf{s}\,[i-1,j]} + \zeta_j\cdot W^{n+1}_{\mathbf{s}\,[i,j+1]} + \upsilon_j\cdot W^{n+1}_{\mathbf{s}\,[i,j]} + \chi_j\cdot W^{n+1}_{\mathbf{s}\,[i,j-1]} + \lambda_{\mathbf{s}}\cdot W^{n+1}_{\mathbf{s}'\,[i,j]} - \lambda_{\mathbf{s}}\cdot W^{n+1}_{\mathbf{s}\,[i,j]} \tag{26}$$

, where:

$$\begin{aligned}
z_{i,j} &= \frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{F}}[i,j]\right)^{+}}{\Delta a} \\
y_{i,j} &= \frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{B}}[i,j]\right)^{-} - \left(\xi^{n}_{\mathbf{s}\,\mathbf{F}}[i,j]\right)^{+}}{\Delta a} \\
x_{i,j} &= -\frac{\left(\xi^{n}_{\mathbf{s}\,\mathbf{B}}[i,j]\right)^{-}}{\Delta a} \\
\zeta_j &= \frac{\left(\mu_{\mathbf{s},j}\right)^{+}}{\Delta z} \\
\upsilon_j &= \frac{\left(\mu_{\mathbf{s},j}\right)^{-} - \left(\mu_{\mathbf{s},j}\right)^{+}}{\Delta z} \\
\chi_j &= -\frac{\left(\mu_{\mathbf{s},j}\right)^{-}}{\Delta z}
\end{aligned} \tag{27}$$

The system presented above consists of $\langle\, I \times J \times 2 \,\rangle$ equations, which can be reformulated in a matrix-algebraic framework. In this setup, W^n and c^n are vectors of size $\langle\, I \times J \times 2 \,\rangle$, while $\mathbf{A}$ and $\mathbf{B}$ are both $\langle\, (I\cdot J\cdot 2)\times(I\cdot J\cdot 2) \,\rangle$ matrices, where $\mathbf{B}$ is the combination of the transition matrix for the diffusion process, $\mathbf{C}$, and the transition matrix for the Poisson process, $\mathbf{\Lambda}$ respectively.

$$\left(\frac{1}{\Delta}+\rho\right) W^{n+1} - \frac{1}{\Delta} W^{n} = u\left(c^{n}\right) + \left(\tilde{\mathbf{A}} + \mathbf{B}\right) W^{n+1} \tag{28}$$

$$\mathbf{B} = \mathbf{C} + \mathbf{\Lambda}$$

Denoting $\mathbf{A} \equiv \tilde{\mathbf{A}} + \mathbf{B}$, the equation becomes:

$$\left(\frac{1}{\Delta} + \rho\right) W^{n+1} - \frac{1}{\Delta} W^{n} = u\left(c^{n}\right) + \mathbf{A} W^{n+1} \tag{29}$$

I manipulate the equation and derive the result as:

$$W^{n+1} = \left(\left(\frac{1}{\Delta} + \rho\right) \mathbf{I} - \mathbf{A}\right)^{-1} \left(u(c^{n}) + \frac{1}{\Delta} W^{n}\right) \tag{30}$$

C.2. Solving Consumer's Kolmogorov Forward Equation (Fokker-Planck Equation)

After solving the consumer's HJB equation, the solution of the consumer's KF equation is within arm's reach since we have obtained the matrix $\mathbf{A}$. The consumer's KF equation is (at stationary equilibrium):

$$0 = -\frac{d}{da}\Big(\xi\left(a,z,s\right)\cdot g\left(a,z,s\right)\Big) - \frac{d}{dz}\Big(\theta\left(\bar{z}_e - z\right)\cdot g\left(a,z,s\right)\Big) + \underbrace{\lambda_u g\left(a,z,s'\right)}_{\text{those who find a job}} - \underbrace{\lambda_e g\left(a,z,s\right)}_{\text{those separated}} \tag{31}$$

For specific asset level a_i, productivity z_j, and employment status s, the density function, $g(a_i, z_j, s)$, is:

$$0 = -\frac{d}{da}\Big(\xi\left(a_i,z_j,s\right)\cdot g\left(a_i,z_j,s\right)\Big) - \frac{d}{dz}\Big(\theta\left(\bar{z}_e - z_j\right)\cdot g\left(a_i,z_j,s\right)\Big) + \lambda_u g\left(a_i,z_j,s'\right) - \lambda_e g\left(a_i,z_j,s\right) \tag{32}$$

Similar to the HJB equation, this system of equations can be solved by finite-difference. The first step is to discretize the system as:

$$0 = \left(\begin{array}{l} -\underbrace{\frac{\xi\left(a_{i+1},z_j,s\right)g\left(a_{i+1},z_j,s\right) - \xi\left(a_i,z_j,s\right)g\left(a_i,z_j,s\right)}{\Delta a}}_{\text{forward difference term}} - \underbrace{\frac{\xi\left(a_i,z_j,s\right)g\left(a_i,z_j,s\right) - \xi\left(a_{i-1},z_j,s\right)g\left(a_{i-1},z_j,s\right)}{\Delta a}}_{\text{backward difference term}} \\ -\underbrace{\frac{\theta\left(\bar{z}_s - z_{j+1}\right)g(a_i,z_{j+1},s) - \theta\left(\bar{z}_s - z_j\right)g(a_i,z_j,s)}{\Delta z}}_{\text{forward difference term}} - \underbrace{\frac{\theta\left(\bar{z}_s - z_j\right)g(a_i,z_j,s) - \theta\left(\bar{z}_s - z_{j-1}\right)g(a_i,z_{j-1},s)}{\Delta z}}_{\text{backward difference term}} \\ +\lambda_{s'} g\left(a_i,z_j,s'\right) - \lambda_s g\left(a_i,z_j,s\right) \end{array}\right) \tag{33}$$

By using the upwind scheme, the discretized system can be written as:

$$
0 = \left(\begin{array}{l}
-\underbrace{\frac{\overbrace{\left(\xi_{\mathbf{B}}\left(a_{i+1}, z_j, s\right)\right)^{-}}^{-x_{i+1,j}\cdot\Delta a} g\left(a_{i+1}, z_j, s\right) - \left(\xi_{\mathbf{B}}\left(a_i, z_j, s\right)\right)^{-} g\left(a_i, z_j, s\right)}{\Delta a}}_{\text{forward difference term}} - \underbrace{\frac{\left(\xi_{\mathbf{F}}\left(a_i, z_j, s\right)\right)^{+} g\left(a_i, z_j, s\right) - \overbrace{\left(\xi_{\mathbf{F}}\left(a_{i-1}, z_j, s\right)\right)^{+}}^{-z_{i-1,j}\cdot\Delta a} g\left(a_{i-1}, z_j, s\right)}{\Delta a}}_{\text{backward difference term}} \\
-\underbrace{\frac{\left(\theta\left(\bar{z}_s - z_{j+1}\right)\right)^{-} g(a_i, z_{j+1}, s) - \left(\theta\left(\bar{z}_s - z_j\right)\right)^{-} g(a_i, z_j, s)}{\Delta z}}_{\text{forward difference term}} - \underbrace{\frac{\left(\theta\left(\bar{z}_s - z_j\right)\right)^{+} g(a_i, z_j, s) - \left(\theta\left(\bar{z}_s - z_{j-1}\right)\right)^{+} g(a_i, z_{j-1}, s)}{\Delta z}}_{\text{backward difference term}} \\
+ \underbrace{\lambda_{s'} g\left(a_i, z_j, s'\right) - \lambda_s g\left(a_i, z_j, s\right)}_{\text{Unlike the HJB, the subscripts (of the employment status) are different here.}}
\end{array}\right) \tag{34}
$$

By rearranging the terms, the system of equation can be rewritten by using the variables developed in previous subsection, $x_{i,j}$, $y_{i,j}$, $z_{i,j}$, $\chi_{i,j}$, $\upsilon_{i,j}$, and, $\zeta_{i,j}$. The resulting equation is:

$$
x_{i+1,j}\cdot g\left(a_{i+1}, z_j, s\right) + y_{i,j}\cdot g\left(a_i, z_j, s\right) + z_{i-1.j}\cdot g\left(a_{i-1}, z_j, s\right) + \chi_{j+1}\cdot g\left(a_i, z_{j+1}, s\right) + \upsilon_j\cdot g\left(a_i, z_j, s\right) + \zeta_{j-1}\cdot g\left(a_i, z_{j-1}, s\right) + \lambda_{s'}\cdot g\left(a_i, z_j, s'\right) - \lambda_s\cdot g\left(a_i, z_j, s\right) \tag{35}
$$

Or in a brief manner:

$$
x_{i+1,j}\cdot g_{s\;[i+1,j]} + y_{i,j}\cdot g_{s\;[i,j]} + z_{i-1.j}\cdot g_{s\;[i-1,j]} + \chi_{j+1}\cdot g_{s\;[i,j+1]} + \upsilon_j\cdot g_{s\;[i,j]} + \zeta_{j-1}\cdot g_{s\;[i,j-1]} + \lambda_{s'}\cdot g_{s'\;[i,j]} - \lambda_s\cdot g_{s\;[i,j]} \tag{36}
$$

The system can be expressed in matrix form as:

$$
\mathbf{A}^T g = 0 \tag{37}
$$

where g is a vector of dimension $(I \times J \times 2)$, and $\mathbb{A}$ represents the matrix from the final iteration (after convergence) of $\mathbf{A}$ in the previous subsection, which solves the consumer's HJB equation.

C.3. Solving Firm's HJB Equation

The representative firm's maximization problem (assuming free-entry condition, $V = 0$) with regard to asset level a and productivity level z is:

$$(r_t - \delta)\, J(a,z) = \max_k \left(zF(k) - r_t k - \omega(a,z) + \Big(J\,(a,z)\Big)'_a \xi_e\,(a,z) + \Big(J\,(a,z)\Big)'_z \theta\,(\bar{z}_e - z) - \lambda_e J\,(a,z) \right) \tag{38}$$

The free-entry condition ($V = 0$) implies that:

$$\phi = \lambda_f \int_{\underline{z}}^{\overline{z}} \int_{\underline{a}}^{\infty} J(a,z_j) \frac{g(a,z_j,u)}{u_t} \; da \;\; dz \tag{39}$$

The first-order condition is:

$$z_j F'(k) = r_t \Rightarrow (F')^{-1} \frac{r_t}{z_j} = k^* \tag{40}$$

$$(r_t - \delta)\, J(a_i,z_j) = z_j F\left(k^*\left(z_j, r_t\right)\right) - r_t k^*\left(z_j, r_t\right) - \omega(a_i,z_j) + \left(J\left(a_i,z_j\right)\right)'_a \xi_e\left(a_i,z_j\right) - \lambda_e J\left(a_i,z_j\right) \tag{41}$$

The problem can be written in a numerical way as:

$$\frac{J^{n+1} - J^n}{\Delta} + \left(\lambda_e + r - \delta\right) J^{n+1} = \Pi + \mathbb{A}_e J^{n+1} \Rightarrow \left(\left(\frac{1}{\Delta} + \lambda_e + r - \delta\right) I - \mathbb{A}_e \right) J^{n+1} = \frac{1}{\Delta} J^n + \Pi$$

, where $\mathbb{A}_{\mathbf{e}}$ denotes the $\mathbf{A_e}$ from the last converged iteration of the consumer's HJB in Appendix B.1.1. By iteration, there is a converged result of J^{n+1}

$$J^{n+1} = \left(\left(\frac{1}{\Delta} + \lambda_e + r - \delta\right) I - \mathbb{A}_e \right)^{-1} \left(\frac{1}{\Delta} J^n + \Pi\right) \tag{42}$$

D. Algorithm

1. Initial Setups

1.1. Set up Structural Parameters

1. General parameters

α production	δ depreciation	γ utility	ρ discount
.3	.021	2	.01

Table 7: Production and consumption parameters

2. Labor market parameters

β bargaining power	η LM tightness elasticity	ϕ vacancy posting cost
.72	.72	.395
χ matching efficiency	λ_e separation	h UI
1.7935	.1038	0

Table 8: Labor market parameters

1.2. Set up the Grids

1. Set up non-uniform asset grid a_i, $i = 1, 2, ..., I$. Here I choose $a_i \in [\ -1\ ,\ 2500\ ]$, $I = 2500$
2. Set up productivity grid z_j, $j = 1, 2, ..., J$. Here I choose $z_j \in [\ 0.65\ ,\ 1.0\ ]$, $J = 15$

1.3. Set up the Productivity Transition Matrix

Since the LoM doesn't change as the updating parameters (the parameters that change as the loop goes until reaching the equilibrium) change, the transition matrix of productivity can be set up here.

1.4. Set up the Solution Parameters

For the following numerical solution, I set (1) the tolerance level of the HJB equation as $1e - 10$, (2) the tolerance level of the free-entry condition and (3) the tolerance level of market clearing as $1e - 5$. I set the step size for the iteration of the HJB equation, $\Delta = 500$.

1.5. Set up the payroll Tax Rate τ

For the baseline, I set $\tau = .01$ (1%).

1.6. Initially Guess the Updating Parameters

The initial guess of these parameters are made by the following steps.

1. Make the initial guess on Θ. I guess $\Theta = 1$.
2. Obtain λ_u by having $\chi\Theta^{\eta}$.
3. Obtain the aggregate(average) productivity $\bar{z}_e$ by calculating the stationary distribution of productivity, z, condition on the employed consumers, which is $\mathbb{E}[z|s=e]$. The stationary equilibrium is calculated from the KFE:

$$0 = -\frac{d}{dz}\left(\left(\theta\left(z_e - z_j\right)\right) g(a_i, z_j, e)\right) + \lambda_u g(a_i, z_j, u) - \lambda_e g(a_i, z_j, e) \tag{43}$$

$$0 = -\frac{d}{dz}\left(\left(\theta\left(z_u - z_j\right)\right) g(a_i, z_j, u)\right) + \lambda_e g(a_i, z_j, e) - \lambda_u g(a_i, z_j, u) \tag{44}$$

4. Make a guess on the capital per worker, k, by taking the complete market value, $\left(\frac{\rho+\delta}{z\alpha}\right)^{\frac{1}{\alpha-1}}$, and times 1.01 to ensure nonexplosive asset accumulation.
5. Obtain the interest rate, r, since I have z, and k on my hand now.

1.7. Guess the wage schedule: $\omega(a_i, z_j)$

For guessing the initial wage schedule, I use the formula:

$$\tilde{\omega}(a_i, z_j) = \beta\left(z_j k^{\alpha} - rk\right)$$

1.8. Guess the Value Functions of the Consumers

I guess the initial value functions of each type of consumers, denoted as $W_{\mathbf{e}}(a, z)$ for the employed and $W_{\mathbf{u}}(a, z)$ for the unemployed, as 0.

1.9. Guess the Value Functions of the Representative Firm

1.10. Guess the Amount of the Lump-sum Unemployment Benefit

Given the values of labor market tightness Θ, payroll tax rate τ, exogenous separation intensity λ_e, the unemployment benefits can be written as:

$$h = \frac{\displaystyle\int_{\underline{z}}^{1} \int_{0}^{\infty} \tau\, \omega(a, z)\, da\, dz}{\left(\dfrac{\lambda_e}{\lambda_e + \lambda_u}\right)}$$

2. The Solution

The Outerloop

In each round of the outerloop updates the updating parameters so as to find the equilibrium outcome. Inside each outerloop, there are: (1) the innerloop for consumer optimization, (2) the solution to FPE (or KFE), (3) the innerloop for the firm's optimization, (4) wage bargaining, and (5) updating parameters.

2.1. Innerloop for Consumer Optimization

The detail of the solution is displayed in Appendix C.1.

2.2. Solution to the Fokker-Planck Equation

The detail of the solution is displayed in Appendix C.2.

2.3. Innerloop for the Firm's Optimization

The detail of the solution is displayed in Appendix C.3.

2.4. Wage Bargaining

2.5. Market Clearing

1. Given the newly computed $J(a, z)$ from Step D.2.3., calculate the vacancy-posting value, V, as in 10
2. Given the newly computed KFE in Step D.2.2., calculate the aggregate savings, and then divide it by $(1 - u)$ (the proportion of population in the workplace) to have the k^{AS}

2.6. Check the Convergence

1. Check if the vacancy-posting value is sufficiently close to 0, by the criteria set in the step setting the solution parameter, Step D.1.4.

2. Check if the difference between the initial k and the k^{AS} computed in Step D.2.5.2. is sufficiently close to 0, by the criteria set in the step setting the solution parameter, Step D.1.4.

If the differences are small enough, break the loop; otherwise, let the algorithm continue.

2.7. Parameter Updates

This step updates the updating parameters in order to find the stationary recursive equilibrium of the economy. This step includes:

- Labor market parameter update: Updating Θ, then the value of job-finding intensity λ_u, unemployed population u, vacancy amounts v, will be updated.
- Capital per worker k: Updating k by $k^{NEW} = relax * k^{OLD} + (1 - relax) * k^{AS}$, which k^{AS} is obtained from the solved FPE (KFE).
- Aggregate productivity z: Updating z by using the FPE (KFE) with the newly obtained λ_u.
- The new price of capital r.
- The new wage schedule $\omega(a, z)$.
- The new unemployment benefits amount h from the new wage schedule and the solved FPE (KFE).

E. The Stationary Equilibrium

In this appendix, I define the economy's stationary recursive equilibrium, whose outcomes are presented in Subsection 4.1 for the quantitative analysis. The stationary equilibrium consists of:

1. A set of value functions:

$$\left\{ W_{\mathbf{e}}(a, z), W_{\mathbf{u}}(a, z), J(a, z), V \right\}$$

2. Individual consumption policies, $c(a, z, s)$, and saving policies, $\dot{a}(a_\ell, z_\ell, s_\ell) \equiv \xi(a_\ell, z_\ell, s_\ell)$, as a function of individual asset level a_ℓ, productivity z_ℓ, and employment status s_ℓ.

3. The prices of capital, r, and the price (wage schedule) of labor $\omega(a, z)$.
4. The amount of vacancies, v.
5. The capital stock per worker, k^d.
6. The labor market tightness, Θ, and the resulting intensities of job search and job filling, λ_u and λ_f respectively.
7. The amount of the lump-sum unemployment benefit, h, and the payroll tax rate, τ
8. The distribution of agents over asset level a, productivity level z, and employment status s, denoted as $g(a, z, s)$

such that:

1. Consumer optimization
   Given the equilibrium job-finding intensity λ_u, the (rental) price r, wage schedule $\omega(a_\ell, z_\ell)$, the consumption-savings choice, c and ξ, solves individual's maximization problem with asset level a_ℓ, productivity level z_ℓ, and employment status, s_ℓ, denoted as $W(a_\ell, z_\ell, \mathbf{e})$ for the employed and $W(a_\ell, z_\ell, \mathbf{u})$ for the unemployed.
2. Firm optimization
   Given the labor market tightness, Θ, the price of capital r and wage schedule $\omega(a_\ell, z_\ell)$, the equilibrium vacancy-filling intensity λ_f, and the stationary distribution of consumer $g(a, z, s)$, the capital per worker, k solves the firm's maximization problem of $J(a_\ell, z_\ell)$.
3. The consistency of stationary distribution
   Given the consumer's policies and labor market parameters, the distribution, $g(a, z, s)$ satisfies and .
4. The free-entry condition
   Given r, $\omega(a, z)$, Θ, and $g(a, z, s)$, the value of vacancy, V, satisfies 10
5. The asset market is cleared:
$$\int_{\underline{z}}^{1} \int_{0}^{\infty} \left\{ a \cdot (g(a, z, \mathbf{e}) + g(a, z, \mathbf{u})) \right\} \, da \, dz = k(1 - u)$$
6. The government budget is balanced as in equation 16