

# ÁLGEBRA

## Desigualdades

1. Si  $a < b$  y  $b < c$ , entonces  $a < c$
2. Si  $a < b$ , entonces  $a + c < b + c$
3. Si  $a < b$  y  $c > 0$ , entonces  $ac < bc$
4. Si  $a < b$  y  $c < 0$ , entonces  $ac > bc$
5. Si  $a < b$  y  $c < d$ , entonces  $a + c < b + d$
6. Si  $0 < a < b$  y  $0 < c < d$ , entonces  $ac < bd$
7. Si  $a < b$  y  $ab > 0$ , entonces  $\frac{1}{a} > \frac{1}{b}$

## Valor Absoluto

1.  $|a| < \sqrt{a^2}$
2.  $|ab| < |a| \cdot |b|$
3.  $\left| \frac{a}{b} \right| < \frac{|a|}{|b|}$
4.  $|x| < k$  ( $k > 0$ ) si, y solo si,  $-k < x < k$
5.  $|x| > k$  ( $k \geq 0$ ) si, y solo si,  $x < -k$  o  $x > k$

## Exponentes

1.  $b^m b^n = b^{m+n}$
2.  $\frac{b^m}{b^n} = b^{m-n}$
3.  $(b^m)^n = b^{mn}$
4.  $(ab)^m = a^m b^m$
5.  $\left( \frac{a}{b} \right)^m = \frac{a^m}{b^m}$
6.  $b^{-n} = \frac{1}{b^n}$

## Radicales

1.  $b^{\frac{1}{n}} = \sqrt[n]{b}$
2.  $b^{\frac{m}{n}} = \left( \sqrt[n]{b} \right)^m = \sqrt[n]{b^m}$
3.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
4.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
5.  $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
6.  $b^{-n} = \frac{1}{b^n}$

## Logaritmos

1.  $\log_b N = x$  quiere decir  $b^x = N$
2.  $\log_b MN = \log_b M + \log_b N$
3.  $\log_b \frac{M}{N} = \log_b M - \log_b N$
4.  $\log_b (N)^k = k \log_b N$
5.  $\log_b N = \frac{\text{Ln } N}{\text{Ln } b}$
6.  $\log_b b^x = x$
7.  $b^{\log_b x} = x$
8.  $\text{Ln } x = \log_e x$ ,  $\log x = \log_{10} x$

## Fórmulas de factorización y multiplicación

- |  |  |
|--|--|
| <ol style="list-style-type: none"> <li>1. <math>a^2 - b^2 = (a + b)(a - b)</math></li> <li>2. <math>a^3 - b^3 = (a - b)(a^2 + ab + b^2)</math></li> <li>3. <math>a^3 + b^3 = (a + b)(a^2 - ab + b^2)</math></li> </ol> | <ol style="list-style-type: none"> <li>4. <math>a^2 + 2ab + b^2 = (a + b)^2</math></li> <li>5. <math>a^2 - 2ab + b^2 = (a - b)^2</math></li> <li>6. <math>a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3</math></li> <li>7. <math>a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3</math></li> </ol> |
|--|--|

## Fórmula cuadrática

La ecuación cuadrática  $ax^2 + bx + c = 0$  tiene como soluciones:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

# TRIGONOMETRÍA

## Identidades Fundamentales

- $\operatorname{sen} x \csc x = 1$
- $\cos x \sec x = 1$
- $\tan x \cot x = 1$
- $\tan x = \frac{\operatorname{sen} x}{\cos x}$
- $\operatorname{sen}^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\cot x = \frac{\cos x}{\operatorname{sen} x}$

## Productos de Senos y Cosenos

- $\operatorname{sen} A \operatorname{sen} B = -\frac{1}{2} \cos(A+B) + \frac{1}{2} \cos(A-B)$
- $\operatorname{sen} A \cos B = \frac{1}{2} \operatorname{sen}(A+B) + \frac{1}{2} \operatorname{sen}(A-B)$
- $\cos A \operatorname{sen} B = \frac{1}{2} \operatorname{sen}(A+B) - \frac{1}{2} \operatorname{sen}(A-B)$
- $\cos A \cos B = \frac{1}{2} \cos(A+B) + \frac{1}{2} \cos(A-B)$

## Fórmulas para la suma

- $\operatorname{sen}(x+y) = \operatorname{sen} x \cos y + \operatorname{sen} y \cos x$
- $\cos(x+y) = \cos x \cos y - \operatorname{sen} x \operatorname{sen} y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

## Fórmulas para la resta

- $\operatorname{sen}(x-y) = \operatorname{sen} x \cos y - \operatorname{sen} y \cos x$
- $\cos(x-y) = \cos x \cos y + \operatorname{sen} x \operatorname{sen} y$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

## Fórmulas para el doble de un ángulo

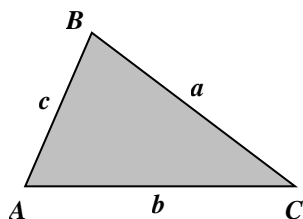
- $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$
- $\cos 2x = \cos^2 x - \operatorname{sen}^2 x$   
 $= 1 - 2 \operatorname{sen}^2 x$   
 $= 2 \cos^2 x - 1$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

## Fórmulas para la mitad de un ángulo

- $\operatorname{sen} \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$
- $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$
- $\tan \frac{x}{2} = \frac{1 - \cos x}{\operatorname{sen} x} = \frac{\operatorname{sen} x}{1 + \cos x}$

## Fórmulas para cuadrados

- $\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$
- $\cos^2 x = \frac{1 + \cos 2x}{2}$
- $\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$



## Ley de los cosenos

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

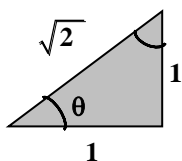
## Ley de los senos

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C}$$

Cuadrante	I	II	III	IV
Funciones				
$\operatorname{sen} \theta$ y $\csc \theta$	+	+	-	-
$\cos \theta$ y $\sec \theta$	+	-	-	+
$\tan \theta$ y $\cot \theta$	+	-	+	-

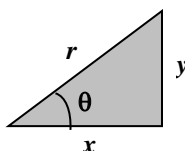
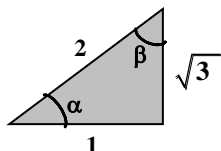
$$\pi \text{ Radianes} = 180 \text{ grados}$$

$$\theta = 45^\circ = \frac{\pi}{4}$$



$$\alpha = 60^\circ = \frac{\pi}{3}$$

$$\beta = 30^\circ = \frac{\pi}{6}$$



$$\begin{aligned} \operatorname{sen} \theta &= \frac{y}{r} ; \csc \theta = \frac{r}{y} \\ \cos \theta &= \frac{x}{r} ; \sec \theta = \frac{r}{x} \\ \tan \theta &= \frac{y}{x} ; \cot \theta = \frac{x}{y} \end{aligned}$$

$$\begin{aligned} \operatorname{sen}(-\theta) &= -\operatorname{sen} \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= +\cos \theta & \sec(-\theta) &= +\sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta \end{aligned}$$

Cuadrante	Angulo relacionado
I C	Directo
II C	Lo que falta para $\pi$ ( $180^\circ$ )
III C	Lo que se pasa de $\pi$ ( $180^\circ$ )
IV C	Lo que falta para $2\pi$ ( $360^\circ$ )

## Fórmulas de Integración

$$1. \int (du + dv) = \int du + \int dv$$

$$2. \int a du = a \int du$$

$$3. \int dx = x + C$$

$$4. \int u^n du = \frac{u^{n+1}}{n+1}, n \neq -1$$

$$5. \int \frac{du}{u} = \ln u + C$$

$$6. \int a^n du = \frac{a^n}{\ln a} + C$$

$$7. \int e^u du = e^u + C$$

$$8. \int \operatorname{sen} u du = -\cos u + C$$

$$9. \int \cos u du = \operatorname{sen} u + C$$

$$10. \int \sec^2 u du = \tan u + C$$

$$11. \int \csc^2 u du = -\cot u + C$$

$$12. \int \sec u \tan u du = \sec u + C$$

$$13. \int \csc u \cot u du = -\csc u + C$$

$$14. \int \tan u du = -\ln |\cos u| + C = \ln |\sec u| + C$$

$$15. \int \cot u du = \ln |\operatorname{sen} u| + C$$

$$16. \int \sec u du = \ln |\sec u + \tan u| + C$$

$$17. \int \csc u du = \ln |\csc u - \cot u| + C$$

$$18. \int \frac{du}{u^2 + a^2} = \frac{1}{a} \operatorname{arc} \tan \frac{u}{a} + C$$

$$19. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C; u^2 > a^2$$

$$20. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C; a^2 > u^2$$

$$21. \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{arc} \operatorname{sen} \frac{u}{a} + C$$

$$22. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$$

$$23. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{arc} \operatorname{sen} \frac{u}{a} + C$$

$$24. \int \sqrt{u^2 \pm a^2} du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$$

$$25. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arc} \sec \frac{u}{a} + C$$

### Variaciones Trigonométricas

$$26. \int \operatorname{sen}^n u \cos u du = \frac{\operatorname{sen}^{n+1} u}{n+1} + C$$

$$27. \int \cos^n u (-\operatorname{sen} u) du = \frac{\cos^{n+1} u}{n+1} + C$$

$$28. \int \tan^n u \sec^2 u du = \frac{\tan^{n+1} u}{n+1} + C$$

$$29. \int \cot^n u (-\csc^2 u) du = \frac{\cot^{n+1} u}{n+1} + C$$

$$30. \int \sec^n u \sec u \tan u du = \frac{\sec^{n+1} u}{n+1} + C$$

$$31. \int \csc^n u (-\csc u \cot u) du = \frac{\csc^{n+1} u}{n+1} + C$$

### Sustitución Trigonométrica

$$\sqrt{a^2 - u^2} \quad \text{hacer} \quad u = a \operatorname{sen} \theta$$

$$\sqrt{a^2 + u^2} \quad \text{hacer} \quad u = a \tan \theta$$

$$\sqrt{u^2 - a^2} \quad \text{hacer} \quad u = a \sec \theta$$

### Integración por Partes

$$\int u dv = uv - \int v du$$

## Fórmulas de Diferenciación

1. $\frac{d}{dx}(c) = 0$	15. $\frac{d}{dx}(\arccos v) = \frac{-\frac{d}{dx}(v)}{\sqrt{1-v^2}}$
2. $\frac{d}{dx}(x) = 1$	16. $\frac{d}{dx}(\arctan v) = \frac{\frac{d}{dx}(v)}{1+v^2}$
3. $\frac{d}{dx}(u+v) = \frac{d}{dx}(u) + \frac{d}{dx}(v)$	17. $\frac{d}{dx}(\operatorname{arccot} v) = \frac{-\frac{d}{dx}(v)}{1+v^2}$
4. $\frac{d}{dx}(cv) = c \frac{d}{dx}(v)$	18. $\frac{d}{dx}(\operatorname{arcsec} v) = \frac{\frac{d}{dx}(v)}{v\sqrt{v^2-1}}$
5. $\frac{d}{dx}(uv) = u \frac{d}{dx}(v) + v \frac{d}{dx}(u)$	19. $\frac{d}{dx}(\operatorname{arccsc} v) = \frac{-\frac{d}{dx}(v)}{v\sqrt{v^2-1}}$
6. $\frac{d}{dx}(v^n) = nv^{n-1} \frac{d}{dx}(v)$	20. $\frac{d}{dx}(\operatorname{Log}_a v) = \frac{\frac{d}{dx}(v)}{v} \operatorname{Log}_a e$
7. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{d}{dx}(u) - u \frac{d}{dx}(v)}{v^2}$	21. $\frac{d}{dx} \operatorname{Ln} v = \frac{\frac{d}{dx}(v)}{v}$
8. $\frac{d}{dx}(\operatorname{sen} v) = \cos v \frac{d}{dx}(v)$	22. $\frac{d}{dx}(a^v) = a^v \operatorname{Ln} a \frac{d}{dx}(v)$
9. $\frac{d}{dx}(\cos v) = -\operatorname{sen} v \frac{d}{dx}(v)$	23. $\frac{d}{dx}(e^v) = e^v \frac{d}{dx}(v)$
10. $\frac{d}{dx}(\tan v) = \sec^2 v \frac{d}{dx}(v)$	24. $\frac{d}{dx}(u^v) = vu^{v-1} \frac{d}{dx}(u) + u^v \operatorname{Ln} u \frac{d}{dx}(v)$
11. $\frac{d}{dx}(\cot v) = -\operatorname{csc}^2 v \frac{d}{dx}(v)$	
12. $\frac{d}{dx}(\sec v) = \sec v \tan v \frac{d}{dx}(v)$	
13. $\frac{d}{dx}(\csc v) = -\operatorname{csc} v \cot v \frac{d}{dx}(v)$	
14. $\frac{d}{dx}(\operatorname{arc sen} v) = \frac{\frac{d}{dx}(v)}{\sqrt{1-v^2}}$	