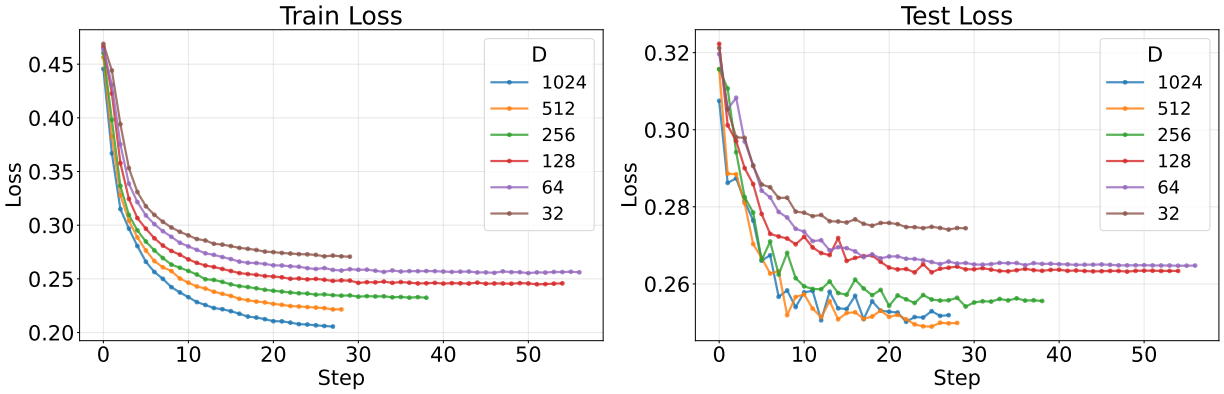
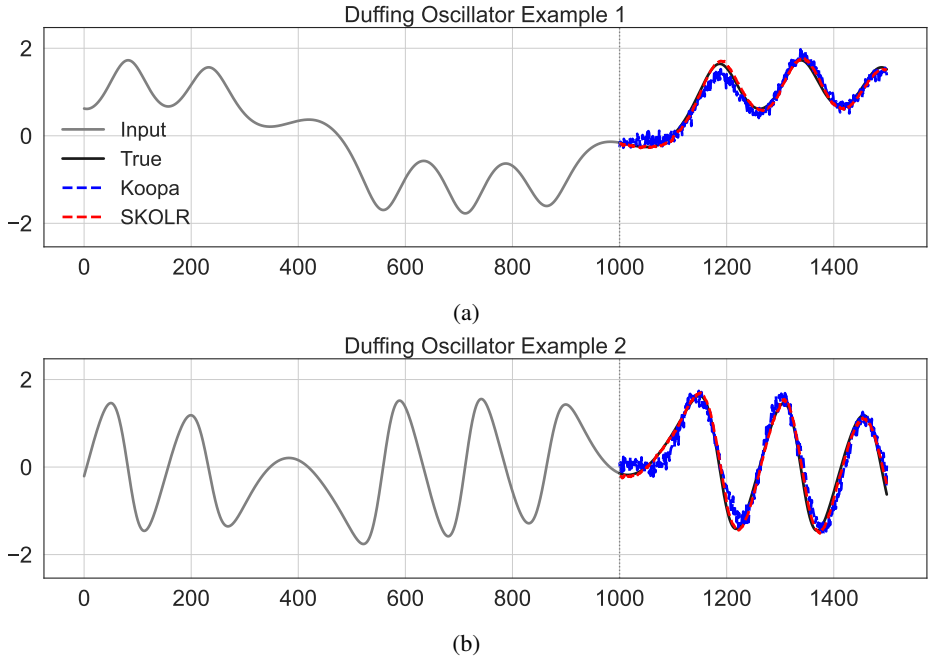


(a) Train Loss (b) Test Loss

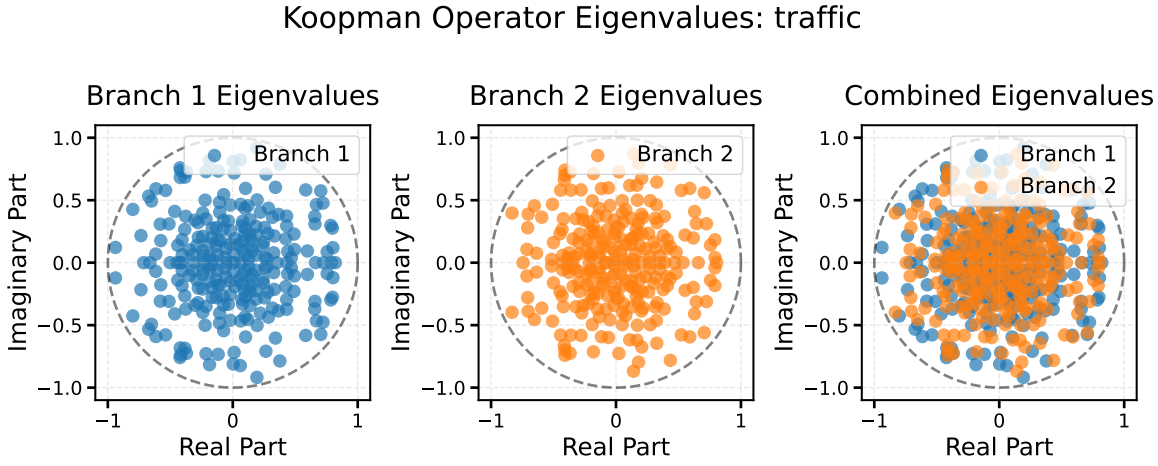
Figure 1: Train loss and test loss for different dynamic dimensions



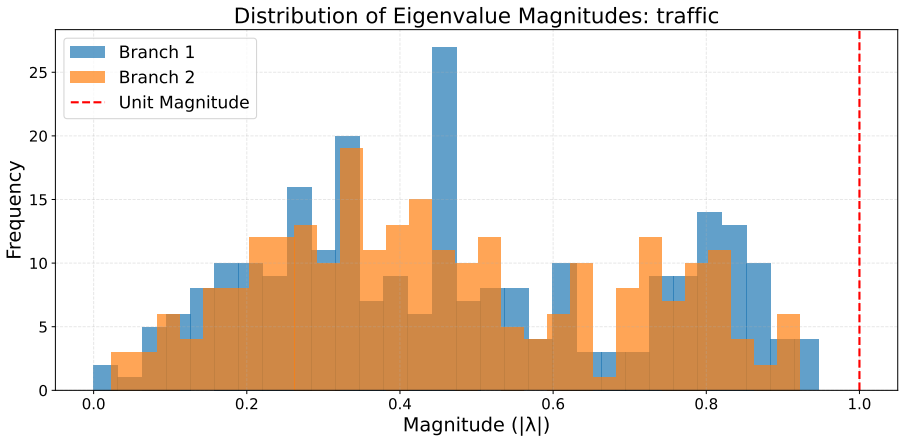
(a)

(b)

Figure 2: Comparison of prediction performance between SKOLR and KooPA on Duffing dynamical systems.



(a)



(b)

Figure 3: Koopman operator eigenvalue analysis for SKOLR on the Traffic dataset

Table 1: Scaling up forecast horizon: (T_tr, T_te) = (24, 48) for ILI and (T_tr, T_te) = (48, 144) for others. Koopa and SKOLR conducts vanilla rolling forecast and Koopa OA has operator adaptation.

	ETTh2 (ADF -4.135)		ILI (ADF -5.406)		ECL (ADF -8.483)		Traffic (ADF -15.046)		Weather (ADF -26.661)	
Metric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Koopa (T_tr)	0.226	0.300	1.621	0.800	0.130	0.234	0.415	0.274	0.126	0.168
Koopa(T_te)	0.437	0.429	2.836	1.065	0.199	0.298	0.709	0.437	0.237	0.276
Error(+ %)	93%	43%	75%	33%	53%	27%	71%	59%	88%	64%
Koopa OA(T_te)	0.372	0.404	2.427	0.907	0.182	0.271	0.699	0.426	0.225	0.264
Error(+ %)	65%	35%	50%	13%	40%	16%	68%	55%	79%	57%
SKOLR (T_tr)	0.238	0.306	1.556	0.760	0.137	0.229	0.400	0.258	0.131	0.170
SKOLR (T_te)	0.393	0.402	2.392	0.958	0.204	0.289	0.612	0.383	0.222	0.257
Error(+ %)	65%	31%	54%	26%	49%	26%	53%	48%	69%	51%

Table 2: Effect of hidden layer count on forecasting performance (MSE) with dimension 256 and 2 branches

Hidden Layer	ETTh1	ETTh2	ETTm1	ETTm2
1	0.333	0.238	0.280	0.134
2	0.337	0.253	0.281	0.136
3	0.339	0.239	0.283	0.135
4	0.344	0.254	0.294	0.137

Table 3: Effect of segment length on forecasting performance (MSE) with dimension 256 and 2 branches

Segment Length	Traffic	ETTm2	Weather	ECL
L/8	0.400	0.133	0.132	0.137
L/6	0.400	0.134	0.131	0.137
L/3	0.401	0.134	0.129	0.137
L/2	0.399	0.135	0.130	0.135

Table 4: Model Efficiency and Performance Comparison for Different Datasets with $T = 96$. Parameters (Params) are measured in millions (M), GPU memory (GPU) in MiB, computation time per epoch in seconds (s) on NVIDIA V100 GPU with batch size 32.

(a) Traffic Dataset

Model	**Params** (M)	**GPU**(MiB)	**Time** (s)	**MSE**
Autoformer	14.914	18.811	51.0	0.668
iTransformer	6.405	62.710	126.0	0.388
PatchTST	3.755	22.132	1042.0	0.413
MICN	236.151	32.310	84.0	0.511
TimesNet	30.170	111.998	6563.0	0.611
DLinear	0.009	12.861	7.7	0.485
Koopa	5.429	50.335	25.5	0.401
SKOLR	1.479	5.915	216.0	0.368

(b) Electricity Dataset

Model	**Params** (M)	**GPU**(MiB)	**Time** (s)	**MSE**
Autoformer	11.214	17.373	68.7	0.182
iTransformer	4.957	86.478	58.6	0.134
PatchTST	6.904	73.517	1231.0	0.143
MICN	6.635	32.668	18.0	0.165
TimesNet	15.037	33.435	11351.0	0.170
DLinear	0.019	76.016	6.8	0.153
Koopa	4.076	31.067	33.1	0.136
SKOLR	1.541	6.163	99.1	0.132

(c) ETTh1 Dataset

Model	**Params** (M)	**GPU**(MiB)	**Time** (s)	**MSE**
Autoformer	10.536	16.523	29.5	0.634
iTransformer	0.237	27.245	4.1	0.393
PatchTST	3.752	22.018	8.5	0.372
MICN	252.001	65.974	21.1	0.406
TimesNet	0.605	26.053	22.1	0.411
DLinear	0.140	26.440	0.6	0.379
Koopa	0.135	31.951	10.1	0.371
SKOLR	0.429	1.717	2.8	0.371

(d) ETTm2 Dataset

Model	**Params** (M)	**GPU**(MiB)	**Time** (s)	**MSE**
Autoformer	10.536	14.599	152.6	0.241
iTransformer	0.237	27.245	13.1	0.177
PatchTST	10.056	39.910	980.0	0.171
MICN	252.001	65.974	84.2	0.197
TimesNet	1.192	34.783	113.0	0.187
DLinear	18.291	9.312	1.9	0.172
Koopa	0.135	31.951	48.2	0.171
SKOLR	0.429	1.717	12.6	0.171

Table 5: Ablation study comparing SKOLR with versions without structure and without spectral encoder

Dataset	T	SKOLR		w/o Structure		w/o Spectral Encoder	
		MSE	MAE	MSE	MAE	MSE	MAE
ECL	48	**0.137**	**0.229**	0.148	0.238	0.149	0.238
	96	**0.132**	**0.225**	0.135	0.228	0.133	0.227
	144	0.143	0.236	0.146	0.241	**0.142**	**0.235**
	192	0.149	0.244	0.150	0.245	**0.148**	**0.243**
Traffic	48	0.400	0.258	**0.395**	**0.255**	0.397	0.257
	96	0.368	**0.248**	**0.367**	0.249	0.369	0.249
	144	0.375	0.255	0.375	0.255	0.375	0.255
	192	**0.377**	0.256	0.378	0.256	**0.377**	0.256
Weather	48	**0.131**	**0.170**	0.134	0.173	0.134	0.172
	96	**0.154**	**0.202**	0.157	0.203	0.158	**0.202**
	144	**0.172**	**0.220**	0.177	0.225	0.175	0.221
	192	**0.193**	**0.241**	0.195	0.242	0.197	0.244
ETTm1	48	**0.280**	**0.330**	0.284	0.334	0.282	0.332
	96	**0.287**	**0.340**	0.292	0.343	0.291	0.342
	144	**0.313**	**0.361**	0.325	0.365	0.319	**0.361**
	192	**0.328**	0.373	0.332	**0.372**	0.332	**0.372**
ETTm2	48	**0.134**	**0.228**	0.135	0.229	0.162	0.259
	96	0.171	0.255	0.174	0.259	**0.169**	**0.253**
	144	0.209	0.283	**0.206**	**0.280**	0.209	0.282
	192	0.241	0.304	0.241	0.305	**0.230**	**0.299**
ETTh1	48	**0.333**	**0.373**	0.338	0.377	0.336	0.374
	96	**0.371**	**0.398**	0.387	0.408	0.373	0.399
	144	**0.405**	**0.417**	0.414	0.423	0.410	0.420
	192	0.422	0.432	**0.409**	**0.421**	0.413	0.422
ETTh2	48	0.238	0.306	**0.233**	**0.304**	0.239	0.305
	96	**0.299**	**0.352**	0.301	0.350	0.303	0.350
	144	**0.335**	**0.377**	0.341	0.382	0.337	0.381
	192	**0.365**	**0.397**	0.370	0.398	0.370	0.401
ILI	24	1.556	0.760	1.795	0.842	**1.522**	**0.741**
	36	**1.462**	**0.728**	1.990	0.889	1.496	0.734
	48	**1.537**	**0.798**	1.875	0.909	1.571	0.810
	60	**2.187**	**0.995**	2.407	1.056	2.263	0.999

Table 6: Model performance across different datasets with mean ± standard deviation for MSE and MAE metrics.

Dataset	Models	MSE	MAE
ECL	48	0.137 ± 0.0003	0.229 ± 0.0003
	96	0.132 ± 0.0005	0.225 ± 0.0004
	144	0.143 ± 0.0001	0.236 ± 0.0001
	192	0.149 ± 0.0001	0.244 ± 0.0001
Traffic	48	0.400 ± 0.0003	0.258 ± 0.0040
	96	0.368 ± 0.0007	0.248 ± 0.0007
	144	0.375 ± 0.0003	0.255 ± 0.0002
	192	0.377 ± 0.0003	0.256 ± 0.0002
Weather	48	0.131 ± 0.0009	0.170 ± 0.0008
	96	0.154 ± 0.0015	0.202 ± 0.0015
	144	0.172 ± 0.0009	0.220 ± 0.0006
	192	0.193 ± 0.0004	0.241 ± 0.0005
ETTm1	48	0.280 ± 0.0013	0.330 ± 0.0015
	96	0.287 ± 0.0003	0.340 ± 0.0001
	144	0.313 ± 0.0020	0.361 ± 0.0023
	192	0.328 ± 0.0019	0.373 ± 0.0018
ETTm2	48	0.134 ± 0.0011	0.228 ± 0.0007
	96	0.171 ± 0.0015	0.255 ± 0.0013
	144	0.209 ± 0.0014	0.283 ± 0.0014
	192	0.241 ± 0.0013	0.304 ± 0.0015
ETTh1	48	0.333 ± 0.0009	0.373 ± 0.0007
	96	0.371 ± 0.0011	0.398 ± 0.0008
	144	0.405 ± 0.0019	0.417 ± 0.0020
	192	0.422 ± 0.0030	0.432 ± 0.0034
ETTh2	48	0.238 ± 0.0012	0.306 ± 0.0004
	96	0.299 ± 0.0034	0.352 ± 0.0042
	144	0.335 ± 0.0042	0.377 ± 0.0048
	192	0.365 ± 0.0033	0.397 ± 0.0040
ILI	24	1.556 ± 0.0213	0.760 ± 0.0159
	36	1.462 ± 0.0711	0.728 ± 0.0676
	48	1.537 ± 0.0038	0.798 ± 0.0030
	60	2.187 ± 0.0435	0.995 ± 0.0498

Table 7: Performance comparison of LRU (Orieto et al. (2023)) and SKOLR on non-linear dynamical systems (NLDS)

	SKOLR		**KooPA**		**LRU**	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE
Pendulum	**0.0001**	**0.0083**	0.0039	0.0470	0.0572	0.0242
Duffing	**0.0047**	**0.0518**	0.0365	0.1479	0.0573	0.5970
Lotka-Volterra	**0.0018**	**0.0354**	0.0178	0.1050	0.2058	0.3779
Lorenz '63	**0.9740**	**0.7941**	1.0937	0.8325	1.1905	0.8932

Table 8: Comparison of Models for short-term prediction. Best results and second best results are highlighted in **red** and blue respectively.

M4	Metric	SKOLR	KooPA	N-HiTS	N-BEATS	PatchTST	TimesNet	DLinear	MICN	KNF	FiLM	Autoformer
Year	sMAPE	**13.291**	13.352	13.371	13.466	13.517	13.394	13.866	14.532	13.986	14.012	14.786
	MASE	**2.996**	2.997	3.025	3.059	3.031	3.004	3.006	3.359	3.029	3.071	3.349
	OWA	**0.784**	0.786	0.790	0.797	0.795	0.787	0.802	0.867	0.804	0.815	0.874
Quarter	sMAPE	**9.986**	10.159	10.454	10.074	10.847	10.101	10.689	11.395	10.343	10.758	12.125
	MASE	**1.166**	1.189	1.219	1.163	1.315	1.183	1.294	1.379	1.202	1.306	1.483
	OWA	**0.878**	0.895	0.919	0.881	0.972	0.890	0.957	1.020	0.965	0.905	1.091
Month	sMAPE	**12.536**	12.730	12.794	12.801	14.584	12.866	13.372	13.829	12.894	13.377	15.530
	MASE	**0.921**	0.953	0.960	0.955	1.169	0.964	1.014	1.082	1.023	1.021	1.277
	OWA	**0.867**	0.901	0.895	0.893	1.055	0.894	0.940	0.988	0.985	0.944	1.139
Others	sMAPE	**4.652**	4.861	4.696	5.008	6.184	4.982	4.894	6.151	4.753	5.259	5.841
	MASE	3.233	**3.124**	3.130	3.443	4.818	3.323	3.358	4.263	3.138	3.608	4.308
	OWA	0.999	1.004	**0.988**	1.070	1.140	1.048	1.044	1.319	1.019	1.122	1.294
Average	sMAPE	**11.704**	11.863	11.960	11.910	13.022	11.930	12.418	13.023	12.126	12.489	14.057
	MASE	**1.572**	1.595	1.606	1.613	1.814	1.597	1.656	1.836	1.641	1.690	1.954
	OWA	**0.843**	0.858	0.861	0.862	0.954	0.867	0.891	0.960	0.874	0.902	1.029