Homework – Week 6

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Problem 1

- Current Stock Price: 151.03
- Strike Price: 165
- Current Date: 03/13/2022
- Options Expiration Date: 04/15/2022
- Risk-Free Rate: 4.25%
- Continuously Compounding Coupon: 0.53%

Tasks

1. Implement the closed-form Greeks for the Generalized Black-Scholes Model (GBSM).
2. Implement a finite difference derivative calculation.
3. Compare the values between the two methods for both a call and a put.
4. Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:
    - Pays a dividend on 04/11/2022 of 0.88.
5. Calculate the value of the call and the put. Calculate the Greeks of each.
6. Determine the sensitivity of the put and call to a change in the dividend amount.

```
American call option price without dividend: 0.65
American put option price without dividend: 14.18
American call option price with dividend: 0.60
American put option price with dividend: 14.69
```

The payment of dividends reduces the value of an American call option because investors holding a call option will tend to exercise the option early to capture the dividend before it is paid, which reduces the time value of the option. Conversely, the presence of a dividend slightly increases the value of a put option because the dividend payment causes the price of the underlying asset to fall, which increases the likelihood that the put option will be exercised.

```
Greeks:
Delta (call): 0.1261
Delta (put): -0.8732
Gamma: 0.0193
Vega: 11.1044
Theta (call): -9.4920
Theta (put): -3.3168
Rho (call): 2.3185
Rho (put): -18.3650
```

```
Greeks (call option):
Delta (FD): 0.1261
Gamma (FD): 0.0193
Vega (FD): 11.1082
Theta (FD): -9.4907
Rho (FD): 2.3188

Greeks (put option):
Delta (FD): -0.8732
Gamma (FD): 0.0193
Vega (FD): 11.1082
Theta (FD): -3.3156
Rho (FD): -18.3645
```

The results of the two methods are very close to each other, which indicates that the Greek letter values calculated by both methods are reliable under the current input conditions. The consistency of the results of these two methods verifies the accuracy of the calculations.

Problem 2

Using the options portfolios from Problem 3 last week (named `problem2.csv` in this week's repository) and assuming:

- American Options
- Current Date: 03/03/2023
- Current AAPL price: 165
- Risk-Free Rate: 4.25%
- Dividend Payment: 1.00 on 03/15/2023

Using `DailyPrices.csv`:

1. Fit a Normal distribution to AAPL returns, assuming a mean return of 0.
2. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price.
3. Calculate the Mean, Value at Risk (VaR), and Expected Shortfall (ES) of the simulated returns.
4. Calculate VaR and ES using the Delta-Normal method.
5. Present all VaR and ES values as dollar losses (not percentages).
6. Compare these results to last week's results.

```
implied volatility: 0.2303
```

```
Total Portfolio - Mean: $13.24
Total Portfolio - VaR: $8.45
Total Portfolio - ES: $7.28
Portfolio Straddle - Mean: $3.37, VaR: $0.98, ES: $0.39
Portfolio SynLong - Mean: $13.07, VaR: $10.68, ES: $10.09
Portfolio CallSpread - Mean: $5.41, VaR: $5.41, ES: $5.41
Portfolio PutSpread - Mean: $-3.01, VaR: $3.01, ES: $3.01
Portfolio Call  - Mean: $8.22, VaR: $5.83, ES: $5.24
Portfolio Put  - Mean: $-4.85, VaR: $4.85, ES: $4.85
Portfolio CoveredCall - Mean: $-5.97, VaR: $8.42, ES: $9.10
Portfolio ProtectedPut - Mean: $-3.01, VaR: $3.01, ES: $3.01
```

```
Hedged Portfolio - Mean: $-0.09
Hedged Portfolio - VaR: $21.98
Hedged Portfolio - ES: $27.10
Portfolio Straddle - Mean: $-0.04, VaR: $9.40, ES: $11.60
Portfolio SynLong - Mean: $-0.04, VaR: $10.84, ES: $13.37
Portfolio CallSpread - Mean: $-0.01, VaR: $2.41, ES: $2.97
Portfolio PutSpread - Mean: $0.00, VaR: $0.65, ES: $0.82
Portfolio Call  - Mean: $-0.04, VaR: $10.14, ES: $12.48
Portfolio Put  - Mean: $0.00, VaR: $0.72, ES: $0.91
Portfolio CoveredCall - Mean: $0.04, VaR: $9.16, ES: $11.50
Portfolio ProtectedPut - Mean: $0.00, VaR: $0.25, ES: $0.32
```

Return Comparison: Mean is close to 0 for most portfolios after hedging, indicating that the hedging operation effectively reduces the volatility of returns. Risk Comparison: The VaR and ES of the total portfolio increase after hedging, indicating that the hedging operation increases the potential risk in extreme market conditions. This may be due to the fact that hedging operations are less efficient in times of high market volatility. Strategy selection: CallSpread and PutSpread have significantly less risk after hedging, while Straddle and SynLong have higher extreme risk after hedging.

Problem 3

Use the Fama French 3-factor return time series (`F-F_Research_Data_Factors_daily.CSV`) as well as the Carhart Momentum time series (`F-F_Momentum_Factor_daily.CSV`) to fit a 4-factor model to the following stocks:

AAPL	META	UNH	MA
MSFT	NVDA	HD	PFE
AMZN	BRK-B	PG	XOM
TSLA	JPM	V	DIS
GOOGL	JNJ	BAC	CSCO

- Fama stores values as percentages, so you will need to divide by 100 (or multiply the stock returns by 100) to get like units.
- Based on the past 10 years of factor returns, find the expected **annual** return of each stock.
- Construct an annual covariance matrix for the 10 stocks.
- Assume the risk-free rate is 0.05. Find the super efficient portfolio.

```
The annualized expected return of each stock:
AAPL     0.247294
AMZN     0.240931
BAC      0.164900
BRK-B    0.138633
CSCO     0.147967
DIS      0.097690
GOOGL    0.195104
HD       0.211549
JNJ      0.134421
JPM      0.173053
MA       0.235449
META     0.219277
MSFT     0.272361
NVDA     0.488790
PFE      0.133443
PG       0.124346
TSLA     0.558743
UNH      0.275352
V        0.206561
XOM      0.100009
dtype: float64
```

```
Optimal portfolio weight:
[0.00000000e+00 0.00000000e+00 4.38098664e-17 6.51312431e-17
 9.71749803e-17 1.74895837e-16 4.74470112e-17 0.00000000e+00
 1.15568686e-16 0.00000000e+00 1.20657149e-17 0.00000000e+00
 7.18810742e-03 2.88208424e-01 2.15524647e-18 9.60536591e-18
 2.08992340e-01 4.95611128e-01 0.00000000e+00 0.00000000e+00]

The expected annualized return of the portfolio:  0.39607178349778605
The annualized volatility of the portfolio:  0.2760571665835868
The Sharpe ratio of the portfolio:  1.2536236163715013
```

The optimal portfolio focuses on a few high expected return stocks such as TSLA, NVDA, and MSFT, whose high returns with reasonable risk-adjusted returns give them significant weight in the optimal solution. The portfolio has an annualized expected return of 39.61%, a volatility of 27.61%, and a Sharpe

ratio of 1.25, suggesting that the portfolio achieves superior risk-adjusted returns while taking on a higher level of risk.