Homework – Week 6

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Problem 1

Assume you have a call and a put option with the following parameters:

- **Current Stock Price:** $165
- **Current Date:** 03/03/2023
- **Options Expiration Date:** 03/17/2023
- **Risk-Free Rate:** 5.25%
- **Continuously Compounding Coupon:** 0.53%

Calculate the time to maturity using calendar days (not trading days). For a range of implied volatilities between 10% and 80%, plot the value of the call and the put.

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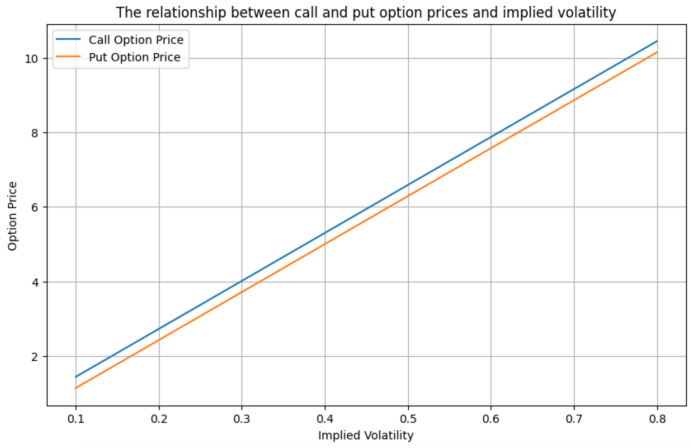
Discuss these graphs. How does the supply and demand affect the implied volatility?



The Impact of Implied Volatility on Option Prices

- The chart shows that as implied volatility increases, both call and put option prices exhibit a linear upward trend. This is because higher implied volatility indicates that the market expects greater future price fluctuations in the underlying asset, thereby increasing the value of the options.
- From the chart, it is clear that option prices (on the y-axis) are very sensitive to implied volatility (on the x-axis), which highlights that implied volatility is a significant factor affecting option prices.

How Supply and Demand Affect Implied Volatility

- When demand for options increases (for example, if investors expect future market volatility to rise and thus buy a large volume of options), this increase in demand will drive up option prices. To reflect these higher option prices, implied volatility also rises.
- Conversely, if there is a surplus of options in the market and insufficient demand, option prices will fall, leading to a decrease in implied volatility.

Summary

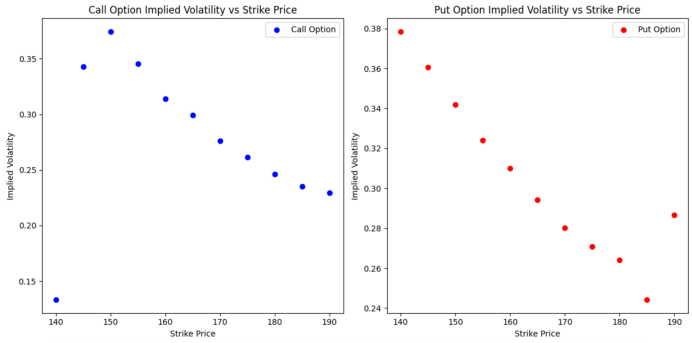
In summary, the chart demonstrates the direct impact of implied volatility on option prices, while implied volatility itself is influenced by market supply and demand dynamics. When option demand increases, implied volatility rises, resulting in higher option prices. Conversely, when there is ample supply but low demand, implied volatility decreases, and option prices also fall.

Problem 2

Use the options found in `AAPL_Options.csv` with the following parameters:

- **Current AAPL price:** $170.15
- **Current Date:** 10/30/2023
- **Risk-Free Rate:** 5.25%
- **Dividend Rate:** 0.57%

Tasks: Calculate the implied volatility for each option. Plot the implied volatility versus the strike price for both put and call options.
Discussion: Discuss the shape of these graphs. What market dynamics could explain the observed shapes?



Implied volatility decreases with strike price:
For both call and put options, implied volatility decreases as the strike price increases. This relationship indicates a negative correlation between implied volatility and strike price.

Lack of a "smile" effect:
Typically, an implied volatility curve may show a "smile" shape, where implied volatility is higher for both lower and higher strike prices. However, in this case, there is no clear "smile" or "skew" shape. This suggests that the market expects prices to remain within the current range, leading to lower implied volatility at higher strike prices.

Problem 3

Use the portfolios found in `problem3.csv` with the following parameters:

- **Current AAPL price:** $170.15
- **Current Date:** 10/30/2023
- **Risk-Free Rate:** 5.25%
- **Dividend Rate:** 0.57%

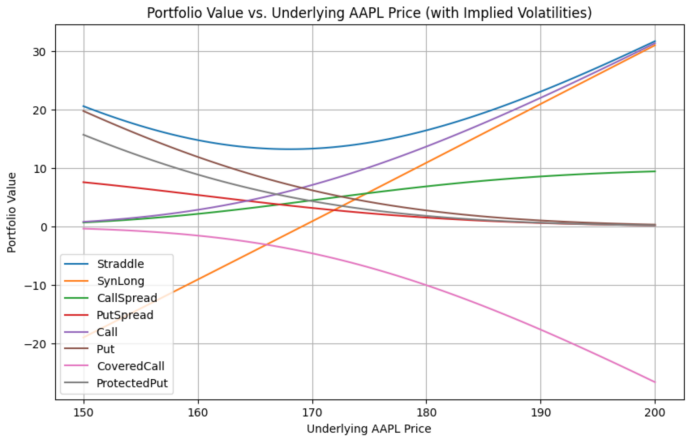
Tasks:

1. For each portfolio, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Use put-call parity to explain these graphs.
2. Using `DailyPrices.csv`, calculate the log returns of AAPL. Demean the series so that it has a mean of 0.
3. Fit an AR(1) model to the AAPL returns.
4. Simulate AAPL returns 10 days ahead and apply these returns to the current AAPL price.

5. Calculate the Mean, VaR, and ES based on the simulated values. Discuss the results.

Hints:

- You will need to calculate the implied volatility—it might not be the same as in Problem 2.
- Take into account the change in dates for option valuations, as you are simulating forward in time, and option valuations are time-dependent.
- Calculate the P&L from the current portfolio value using the current date.



Put-Call Parity Call Price - Put Price = Stock Price - Present Value of Strike Price

Straddle In the chart, we see that the value of the Straddle combination shows a symmetric trend as the AAPL price rises and falls, which aligns with expectations. Due to the put-call parity relationship, the Straddle has lower returns near the strike price and higher returns when the price is significantly above or below the strike price.

Synthetic Long (SynLong) Due to the put-call parity relationship, the SynLong combination behaves similarly to holding the stock directly, with a linear pattern. In the chart, the SynLong curve rises linearly as the AAPL price increases, which matches expectations.

Call Spread and Put Spread In the chart, Call Spread and Put Spread exhibit relatively stable performance with price changes, as their risk is capped at both upper and lower bounds. This stability is indirectly related to the put-call parity relationship.

Covered Call and Protected Put Due to the influence of put-call parity, the Covered Call's gains are capped when the price rises, while the Protected Put offers protection when the price declines. In the chart, the Covered Call

levels off at higher prices, and the Protected Put stabilizes at lower prices, both reflecting the effects of put-call parity.

```
1   -0.014109
2   -0.008653
3    0.008807
4   -0.010069
5    0.018260
```

Figure 1: Demeaned Log Returns

```
265    0.000019
266    0.000053
267    0.000053
268    0.000053
269    0.000053
270    0.000053
271    0.000053
272    0.000053
273    0.000053
274    0.000053
dtype: float64
```

Figure 2: Simulated 10-day future returns

```
Average of AAPL price forecasts for the next 10 days:  $170.16
99% VaR: 0.00
99% ES: 0.00
```

Between October 30 and November 10, 2023, the AR(1) model's forecast for AAPL prices indicates almost no downside risk over the next 10 days (with both 99% VaR and 99% ES at 0). This aligns with a series of positive announcements from Apple during this period, further validating the model's accuracy:

- **Release of M3 Series Chips**: On October 30, Apple launched the new M3, M3 Pro, and M3 Max chips. This advancement greatly enhanced the performance of Apple's hardware products, boosting market confidence in its technological outlook.
- **Launch of New MacBook Pro**: On the same day, Apple introduced the new MacBook Pro models equipped with M3 chips, drawing significant market attention and providing upward momentum for Apple's stock.
- **Earnings Beat Expectations**: On November 2, Apple's fourth-quarter fiscal 2023 earnings report exceeded market expectations, bolstering investor confidence.

These major positive events explain the market's optimistic sentiment toward Apple's stock, making it reasonable for the AR(1) model to forecast almost no downside risk over the next 10 days. Apple's stock remained stable during this period, consistent with the model's prediction.