Homework – Week 5

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Problem 2

Use the data in `problem1.csv`.

Calculate VaR and ES:

1. **Using a normal distribution with an exponentially weighted variance (lambda=0.97)**
2. **Using a MLE fitted T distribution**
3. **Using a Historic Simulation**

Compare the difference between VaR and ES under different probabilistic distributions. Explain the differences.

Under the assumption of normal distribution, the VaR is 9.10% and the ES is 11.41%, which is the highest VaR value among the three methods. This is due to the fact that the normal distribution assumes a concentrated data distribution, which leads to an overestimation of volatility.The EWMA method emphasizes recent data fluctuations and is sensitive to changes in market volatility, but is prone to underperform when thick tails and extreme events occur.

```
VaR (Normal Distribution with EWMA, 95%): 0.0910
ES (Normal Distribution with EWMA, 95%): 0.1141
```

Figure 1: normal distribution with an exponentially weighted variance

The T-distribution assumes a VaR of 7.65% and an ES of 11.44%, with the VaR lower than the normal distribution and the ES close. This is because the T-distribution is better adapted to the thick-tailed phenomenon and provides more reasonable estimates when extremes occur.The T-distribution has a greater probability mass in the tails than the normal distribution, meaning it is more sensitive to extreme events and is suitable for capturing extreme market volatility.

```
VaR (T Distribution, 95%): 0.0765
ES (T Distribution, 95%): 0.1144
```

Figure 2: MLE fitted T distribution

The Historical Simulation Method has a VaR of 7.59%, which is similar to the T-distribution, while the ES is 11.68%, which is slightly higher than the other methods. The historical simulation method relies directly on real data from the past and therefore reflects the extreme loss scenarios of the actual history. This method does not assume data distributions and estimates losses directly from historical data, thus giving more accurate estimates on past extreme events.

```
VaR (Historical Simulation, 95%): 0.0759
ES (Historical Simulation, 95%): 0.1168
```

Figure 3: Historic Simulation

Explanation of Differences:

- **Sensitivity to extreme events:** Normal distribution assumes higher VaR and ES, but may underestimate extreme events; T-distribution is better at capturing extreme risks through thick-tailed characteristics, while historical simulation method is directly based on historical data, which can truly reflect the losses of past extreme events.
- **Conservatism in risk assessment:** Normal distribution is more conservative in estimation and is suitable for use in a stable market; T-distribution and historical simulation are more reflective of true extreme losses in a more volatile market.
- **Applicability of distributional assumptions:** Normal distribution is suitable for smooth markets, while the T-distribution is suitable for thick-tailed data, and the historical simulation method is most realistic and reliable when there is sufficient data.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Calculate arithmetic returns. Assume the expected return on all stocks is 0. This file contains the stock holdings of 3 portfolios. You own each of these portfolios.

Fit Generalized T models to stocks in portfolios A and B, and fit a normal distribution to stocks in portfolio C.

Calculate the VaR and ES of each portfolio as well as your total VaR and ES. You will need to use a copula. Compare the results from this to your VaR from Problem 3 from Week 4.

Portfolio A has low VaR and ES values, indicating that the portfolio has a low risk of loss at the 95% confidence level. This may be due to the low volatility of Portfolio A or the fact that its constituents do not have high thick tail risk.

```
Total VaR for Portfolio B (95%): 0.0093
Total ES for Portfolio B (95%): 0.0010
```

Figure 4: Portfolio A

Portfolio B has the lowest VaR and ES of the three portfolios, especially the ES value of 0.0010, which suggests that the components of Portfolio B are the least volatile or have a very low probability of extreme risk events.

```
Total VaR for Portfolio C (95%): 0.0135
Total ES for Portfolio C (95%): 0.0171
```

Figure 5: Portfolio B

Portfolio C has a higher VaR and ES than Portfolios A and B. In particular, the ES value of 0.0171 is much higher than that of Portfolios A and B. This may be due to the fact that Portfolio C uses a normal distribution assumption, which underestimates the risk of extreme events relative to the generalized T-distribution.

```
Total Portfolio VaR (95%): 1.1785
Total Portfolio ES (95%): 1.5792
```

Figure 6: Portfolio C

The VaR and ES of the total portfolio are an order of magnitude higher than those of the individual portfolios, suggesting that correlations between different portfolios magnify overall losses under extreme events when multiple portfolios are held. The VaR and ES of individual portfolios are only their individual risks, but when there are correlations between portfolios, the total VaR and ES will be much greater than the simple sum of the risks of the respective portfolios. This suggests that risk accumulation emphasizes the importance of considering correlations and extreme loss distributions in multi-portfolio strategies.

Figure 7: Total Portfolio