Homework – Week 4

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Problem 1

Calculate and compare the expected value and standard deviation of the price at time P_t, given each of the 3 types of price returns, assuming $r_t \sim N(0, \sigma^2 = 0.01)$ and the price at time $t-1$ P_{t-1}. Simulate each return equation using $r_t \sim N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

Generate a set of returns drawn from a standard normal distribution (n = 1000 simulations). Each return indicates the percentage change in price over a given time period (e.g., the return generated on one occasion might be 0.05, indicating a 5% increase in price.)

```
[ 0.01330705  0.0639265   0.10938794  0.07306423  0.09001904 -0.01579259
  0.00152958  0.09248533  0.02209078  0.09478699]
```

Figure 1: Random Returns

Calculate returns using classical Brownian motion, arithmetic return systems and log return (geometric Brownian motion).

```
      Returns  Classical Brownian Motion  Arithmetic Return  Log Return (Geometric Brownian)
0    0.017125                 100.017125         101.712454                       101.727200
1    0.007900                 100.007900         100.790000                       100.793129
2   -0.089105                  99.910895          91.089465                        91.474920
3   -0.055860                  99.944140          94.413968                        94.567121
4    0.156631                 100.156631         115.663067                       116.956358
..        ...                        ...                ...                              ...
995  0.072312                 100.072312         107.231249                       107.499121
996 -0.102147                  99.897853          89.785266                        90.289651
997  0.037399                 100.037399         103.739900                       103.810714
998 -0.003299                  99.996701          99.670058                        99.670602
999 -0.077782                  99.922218          92.221792                        92.516601

[1000 rows x 4 columns]
```

Figure 2: Calculate Returns with the 3 types of Price Returns

Calculate the mean and standard deviation of each of the three calculation methods

Mean & Standard Deviation of returns:

	Model	Mean	Standard Deviation
0	Classical Brownian Motion	99.999740	0.099464
1	Arithmetic Return	99.974048	9.946425
2	Log Return (Geometric Brownian)	100.469299	10.002250

Figure 3: Mean & standard deviation

Classical Brownian Motion

- **Theoretical Expectation**: Since the rate of return $r_t \sim N(0, 0.01)$, the price P_t should fluctuate around the initial price P_{t-1} (assumed to be 100). The expected value should be close to the initial price 100 and the standard deviation should be close to the standard deviation of the yield 0.1.
- **Simulation results**:
  - Mean: 99.999740, almost 100.
  - Standard deviation: 0.099464, close to 0.1.
- **Conclusion**: The results are in line with theoretical expectations. The classical Brownian motion model simply adds normally distributed stochastic returns, keeping price fluctuations small and in line with the expected mean and standard deviation.

Arithmetic Return System

- **Theoretical Expectation**: In the Arithmetic Return System, the price is calculated by $P_t = P_{t-1}(1+r_t)$. Due to the multiplicative relationship, the price should fluctuate more than in a classical Brownian Motion, especially if the return fluctuates more, and the standard deviation will be higher.
- **Simulation results**:
  - Mean: 99.974048, close to 100.
  - Standard deviation: 9.946425, significantly larger than the standard deviation of the classical Brownian motion.
- **Conclusion**: The results are in line with theoretical expectations. Although the mean is close to 100, the standard deviation is much larger than in the classical Brownian motion, i.e., the arithmetic return model exhibits greater volatility because the volatility is amplified by the fact that the price changes are scaled.

Log Return/Geometric Brownian Motion

- **Theoretical Expectation**: The log return system $P_t = P_{t-1}e^{r_t}$ calculates price changes via an exponential function, and the compound growth characteristic leads to greater price volatility.
- **Simulation results**:
  - Mean: 100.469299, slightly higher than the initial price.
  - Standard deviation: 10.002250, with the highest volatility.
- **Conclusion**: The results are in line with theoretical expectations. Due to the growth of the index, the price rises quickly when there is a positive return, and the mean is slightly higher than 100. The standard deviation is also larger, which reflects the cumulative effect of compounded returns, and therefore the highest volatility.

In conclusion, by comparing the means and standard deviations of the three models, all results are in line with theoretical expectations.

Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation. Use DailyPrices.csv. Calculate the arithmetic returns for all prices. You own 1 share of META. Remove the mean from the series so that the mean(META)=0. Calculate VaR.

Calculate the arithmetic returns for META, then remove the mean of the returns so that Mean(META_Returns) = 0 to see the adjusted returns

```
Adjusted META_Returns Mean: 2.2293635032633663e-19
            META_Returns  META_Adjusted
Date
2023-09-06     -0.003265      -0.005754
2023-09-07     -0.001671      -0.004161
2023-09-08     -0.002612      -0.005101
2023-09-11      0.032462       0.029972
2023-09-12     -0.019183      -0.021673
```

2.1 Using a normal distribution.
Calculate the mean and variance of the returns and calculate the VaR according to the formula var = mean + z_score * std_dev.

```
VaR (Normal Distribution): -0.03817295454890712
```

2.2 Using a normal distribution with an Exponentially Weighted variance (= 0. 94)
Calculate volatility using EWMA and then calculate VaR based on EWMA

```
VaR (EWMA): -0.030991424222976006
```

2.3 Using a MLE fitted T distribution.
The t-distribution was fitted using MLE and the VaR of the t-distribution was calculated

```
VaR (T Distribution): -0.03242585905836266
```

2.4 Using a fitted AR(1) model.
Train the AR(1) model to predict future returns and calculate the standard deviation, then calculate the VAR based on AR(1) model

```
VaR (AR(1) Model: -0.03837900413254759
```

2.5 Using a Historic Simulation.
The quartiles were calculated from the historical data, and then the VaR based on the historical simulation was calculated.

```
VaR (Historical Simulation): -0.02884348903974016
```

Compare the 5 values.

```
VaR Comparison:
1. Normal Distribution: -0.03817295454890712
2. EWMA: -0.030991424222976006
3. T Distribution: -0.03242585905836266
4. AR(1) Model: -0.03837900413254759
5. Historical Simulation: -0.02884348903974016
```

The normal distribution assumes that the distribution of returns is symmetric and there are no fat tails, so the estimate of VaR is usually small (-0.0381).
Historical volatility is computed by EWMA by assigning a higher weight to recent returns ($\lambda = 0.94$). By giving more weight to recent data, the EWMA is more sensitive to the most recent market fluctuations and therefore the value of VaR is smaller than the normal distribution (-3.09%).
The T-distribution is used to handle return distributions with thick tails. The T-distribution captures the risk of extreme events better than the normal distribution. As a result, the value of VaR is between normal distribution and EWMA (-3.24%).
The AR(1) model captures autocorrelation in the time series and predicts future returns based on past data. If the market has strong autocorrelation, the AR(1) model may significantly affect the forecasts. However, here the AR(1) model gives VaR values that are very similar to the normal distribution, suggesting that autocorrelation in market returns may be weak or that the volatility of returns is not significantly self-correlated.
The historical simulation method is calculated directly using past historical return data without assuming the form of the return distribution. The VaR estimated here based on historical data is -2.88%, which is one of the smallest VaR values among the five methods, probably due to the recent smoother market performance.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.
This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.97, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $

3.1 Discuss your methods and your results.

Filter the stocks that are common to Portfolio.csv and DailyPrices.csv and the covariance matrix, compute the daily returns, compute the covariance matrix using EWMA (lambda = 0.97), and then compute the VaR of the portfolio based on the covariance matrix and convert the VaR to dollar values.

```
VaR for Portfolio A (in $): 18435.70
VaR for Portfolio B (in $): 11599.78
VaR for Portfolio C (in $): 18080.01
```

Finally, each combination is traversed and the VaR is summarized, and then the total VaR of all combinations is calculated.

```
Total EWMR VaR for all portfolios (in $): 48115.49
```

3.2.1 Choose a different model for returns and calculate VaR again.

I chose Historic Simulation to calculate the VaR for each group and the sum of the VaR for all groups

```
Historical VaR for Portfolio A: 0.0122
Historical VaR for Portfolio B: 0.0148
Historical VaR for Portfolio C: 0.0129
```

```
Total Historical VaR for all portfolios (in $): 41905.81
```

3.2.2 Why did you choose that model?

Reasons for choosing the historical simulation:
The historical simulation method does not rely on normal distribution assumptions; it uses historical data directly and is able to capture extreme volatility in the market.
Historical simulation is more reflective of potential risks when markets experience extreme events.

Compare the two:
EWMA VaR is better suited for smoothing risk assessment in the current market environment, and it is more sensitive to capturing recent changes in volatility.
Historical VaR is better at capturing historical extremes, and therefore has a higher VaR in some cases, especially in high volatility markets.

3.3 How did the model change affect the results?

Higher EWMA VaR value:
- EWMA model gives a higher VaR value of $48,115.49 in the portfolio. Since EWMA gives a higher weight to recent market volatility, this suggests that based on the greater volatility of the current market, EWMA considers the portfolio to have greater short-term risk.

Lower Historical Simulation VaR:
- Historical Simulation has a lower VaR of $41,905.81 than the EWMA VaR. Historical Simulation does not over-amplify recent market volatility, but rather is based on a long period of time, a result that suggests that past market volatility may not have been as dramatic as current volatility when viewed in the context of historical data.

Overall, the EWMA model puts more emphasis on the current market environment and reflects risks in the short term. The Historical Simulation Approach focuses more on historical data and is suitable for assessing risk over the long term, especially for capturing tail risk (i.e., the likelihood of extreme events). As a result, the historical simulation approach provides a more stable perspective when we consider portfolio performance over the long term.