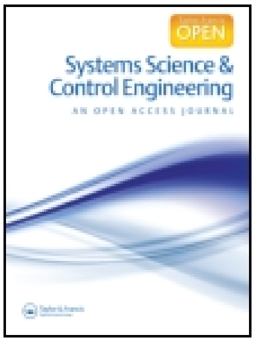

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Extended Kalman filter-based mobile robot localization with intermittent measurements

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In this paper, a theoretical study on extended Kalman filter (EKF)-based mobile robot localization with intermittent measurements is examined by analysing the measurement innovation characteristics. Even if measurement data are unavailable and existence of uncertainties during mobile robot observations, it is suggested that the mobile robot can effectively estimate its location in an environment. This paper presents the uncertainties bounds of estimation by analysing the measurement innovation to preserve good estimations although some measurements data are sometimes missing. Theoretical analysis of the EKF is proposed to demonstrate the conditions when the problem occurred. From the analysis of measurement innovation, Jacobian transformation has been found as one of the main factors that affects the estimation performance. Besides that, the initial state covariance, process and measurement noises must be kept smaller to achieve better estimation results. The simulation and experimental results obtained are showing consistent behaviour as proposed in this paper.

Keywords: estimation; extended Kalman filter (EKF); robot localization; intermittent measurements

1. Introduction

Mobile robot localization plays an important role in trying to realize the behaviour of an autonomous robot, where the robot must consistently identify its position while moving in a given map. The main issue of the mobile robot localization is the mobile robot must continuously affirm its location in order to successfully accomplish its given task. A simple illustration of the robot localization is shown in Figure 1.

Mobile robot localization has been an issue for more than over two decades. Since 1990s, a complex problem that combines the robot localization and mapping problems has appeared and is known as *simultaneous localization and mapping* (SLAM) (Ahmad & Namerikawa, 2010a, 2010b, 2011a, 2011b; Csorba & Durrant-Whyte, 1996; Huang & Dissayanake, 2007; Jun, Zidong, Huijun, & Lampros, 2012; Muraca, Pugliese, & Rocc, 2008). SLAM, which demonstrates the robot characteristics when observing an unknown environment, relies on efficient localization to improve its estimation, hence showing the importance of localization efficiency to accomplish its task. In realizing SLAM, localization problem must be encountered first as there are still many issues existing for further improvement.

The research on mobile robot localization includes both theoretical and practical approaches. Up to date, various kinds of techniques have been proposed by Tsai (1998). They developed a system that integrates a number of sensor types such as compass, gyroscope and four ultrasonic sensors to determine the exact location of mobile robot. They focussed on the reading of heading angle and claim that their system is unaffected by the magnetism effects. These findings are then proved by Huang and Dissayanake (2007) which explains that the heading angle plays an important role in localization. Analysis of different estimation techniques can also be found widely. Extended Kalman filter (EKF) is the popular method used for estimation purposes due to its simplicity and consistency compared with others (Tsai, 1998).

There are two types of mobile robot localization: local localization and global localization. Local localization, which is also known as position tracking, is the easiest case to be implemented as the mobile robot has prior information of its initial condition before observing an environment (Chen, Sun, Yang, & Chen, 2009; Martinelli & Siegwart, 2005; Se, Lowe, & Little, 2001). On the contrary, global localization is more difficult to achieve as the robot does not have any information regarding its initial condition (Se, Lowe, & Little, 2005; Stephen, Lowe, & Little, 2002). This study concentrates on EKF as the algorithm used for the intermittent measurements since it provides sufficient information for the estimation process.

The various researches on the intermittent measurements have been mainly addressing the packet drop characteristic issues in linear network system. Intermittent measurements' analysis was first investigated by Nahi (2002) where he analysed two different cases of data lost. He then derived an optimal estimator as a solution to the problem.

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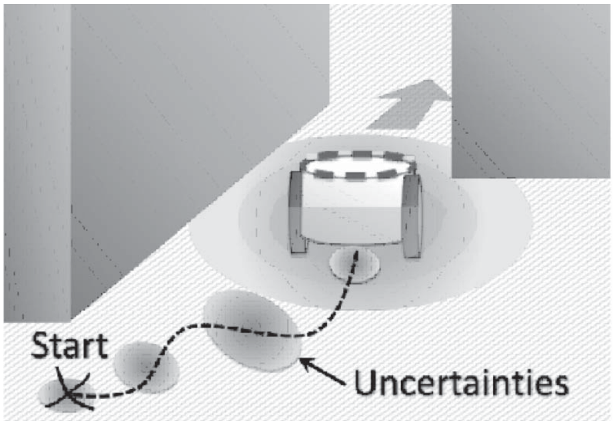

Figure 1. Mobile robot localization.

His study provided analytical descriptions of the system behaviour when some measurement data are suddenly lost. The studies were then continued by Lynch and Figueroa (1991) who conducted a research of Kalman filter that used sonar sensors to estimate a position of a mobile robot when measurement data are lost. They utilized the chain of information obtained during each robot observation to update the mobile robot estimation. Unfortunately, they did not explicitly describe how to determine the information when the measurement data are unavailable. As a result, this paper attempts to discover the system behaviour with a similar approach to Lynch et al. but with different approach of using measurement innovation as the main reference. The measurement innovation is examined to identify the cause of error that has led to the increasing uncertainties into the estimation.

In Kalman filtering, the main interest is to analyse the updated state covariance performance. This is due as the updated state covariance defines the amount of uncertainties existing in the system. In this perspective, Sinopoli et al. (2004) proposed that even if the problem happened, the state covariance should have its upper and lower bounds and converge to its critical value. Similar results have been suggested in different expression by Huang and Dissayanake (2007) who consistently proved that the state covariance is converging to a steady state covariance. Related to Sinopoli et al., Plarre and Bullo (2009) then configured that their results agreed with what Sinopoli et al. have proposed which determines that there is an exact critical value whenever observations are missing. They focussed on an analysis of a detectable system by investigating the system cones of the positive semidefinite (PsD) matrix. They claimed that the critical value is bounded to some exact value and have proposed some boundaries to the expected error. Yet, both studies were unable to determine explicitly about the error bounds.

In shaping up the performance of estimation in Kalman filtering, system parameters must also be taken into account. In this sense especially considering the measurement data lost, a number of papers have been published to characterize the estimation results. Mo and Sinopoli (2008) attempt to compute explicitly the expected probabilities when measurement data were unavailable under some limited conditions in a linear system. Based on their results, the transition and measurement matrices play an important role to determine the probability of data arrival. Those variables are used to establish constant information about the state error covariance (Censi, 2008). The upper bound analysis was also performed by Jun et al. (2012). By including the deterministic and stochastic nonlinearities in the system, the upper bound of state covariance was proposed. Interestingly, they claimed that the upper bound can be further reduced by controlling the Kalman gain. Kluge, Reif, and Brokate (2010) claimed that under some restrictions of small initial state and noise covariance the uncertainties can be bounded especially in a general nonlinear system. Even though some common assumptions were being ignored in their research, the errors remain bounded.

H_∞ filter was also one of the techniques available in analysing intermittent measurement problem. Some of its applications were investigated by Hwan Hur and Hyo-Sung (2013) and Zidong, Hongli, Bo, and Huijun (2013). They suggested that the state covariance never exceed the proposed upper bound when stochastic nonlinearities and multiple missing measurements occurred. In addition, they found that the state covariance has specific upper bound even though multiple data are missing. Another analysis on H_∞ filter by Zidong et al. (2013) described that the filter could also perform better than EKF in a wireless network system when inertial measurement unit and chirp-based spectrum ranging are applied. Their results also explain that the complexity of the system can be further reduced by using an eigenvector approach. Comparison with EKF determines that H_∞ filter could be an alternative solution for the mobile robot localization problem.

In contrast to the network system analysis in intermittent observations, there are very few case studies found in robotics research. This is probably due to the existence of nonlinearities which makes the analysis even more difficult to realize. Consider a mobile robot that is equipped with exteroceptive sensors. If some data suddenly become unavailable during its observations, then the mobile robot could be facing kidnapped robot problem. Owing to this condition, the performance could become more critical for a multi-robot system or multi-sensor network cases as it may cause system instability and inconsistency. Robot localization is also a partially observable problem (Vidal-Calleja, Andrade-Cetto, & Sanfeliu, 2004); and even though we can control the initial input, the output is still unpredictable. Hence, this problem is more difficult in a nonlinear system than in a linear system (Censi, 2008; Jun et al., 2012; Mo & Sinopoli, 2008; Plarre & Bullo, 2009; Sinopoli et al., 2004).

Early research on mobile robot localization for this issue was conducted by Payeur (2008). Using Jacobian approach to sketch an environment, he studied the propagation of uncertainties when measurement data are not available. This was done by merging all information through the occupancy grid to illustrate the system characteristics. Based on his approach, he found that Jacobian transformation and geometrical transformation have significant effects to the whole system. Later, Muraca et al. (2008) proposed a scanning strategy to estimate the mobile robot state using EKF in a sensor network framework. The proposed system only activates sensors that are really needed for specific times to avoid data lost during estimation. However, they did not perform any analysis regarding the state covariance behaviour. Then Ahmad and Namerikawa (2011a) proposed Fisher information matrix statistical bounds to examine the statistical bounds of SLAM in intermittent measurements. Their results showed that both the estimated upper and lower bounds never exceed the expected calculated upper and lower bounds. Inspired by these results, this study is conducted in order to provide adequate information when measurements data are unavailable in mobile robotics. Different to Hwan Hur and Hyo-Sung (2013), Jun et al. (2012) and Zidong et al. (2013), which define the state covariance upper and lower bounds in intermittent measurement, this paper investigates the reason why the state covariance has an upper bound and lower bound.

The search starts from the analysis of Kalman filter where we finally found that the measurement innovation can describe the estimation conditions whenever measurement data are lost during mobile robot observations. The contents also agree with what previous literatures have actually reported in Huang and Dissayanake (2007), Hwan Hur and Hyo-Sung (2013), Jun et al. (2012), Mo and Sinopoli (2008), Payeur (2008), Plarre and Bullo (2009) and Sinopoli et al. (2004). Using EKF measurement innovation and Jacobian transformation of measurement matrix, whenever measurement data are partially missing, the lower and upper limits can be determined via measurement innovation error. The information obtained from the measurement innovation can be used to determine the system performance for both the local and global localizations. Besides, the results also depict that the initial state covariance, the process and measurement noises must be kept smaller to realize good results.

This paper is organized as follows. Section 2 explains the problem statement and introduces robot dynamical system and EKF with some underlying theory about intermittent measurements. Section 3 discusses the convergence analysis for intermittent measurements, with some theoretical results. Then, Section 4 illustrates the experimental results. And finally, Section 5 concludes the paper.

2. Problem statement

In this section, the EKF-based mobile robot localization, when intermittent measurements occurred, is introduced. The case of a mobile robot localization that does not know its initial conditions with some measurements data missing is analysed. It is expected that the robot cannot detect any fault in the sensor or any lost measurement data during its observations. Before presenting further description of the system, the following assumption is made regarding the whole system.

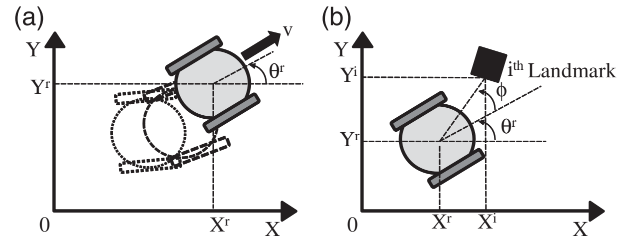

Figure 2. Process model (left) and measurement model (right) of robot localization.

ASSUMPTION 2.1 *Landmarks are stationary and robot cannot sense an occluded landmark.*

In this paper, the analysis utilizes EKF measurement innovation characteristics to identify the system conditions when measurements data are partially unavailable during observations. The study mainly examines the case when both measurement data from proprioceptive and exteroceptive sensors do not arrive at the same time.

2.1. *EKF-based localization*

Consider a nonlinear discrete-time dynamic system that defines both process and measurement models of a mobile robot, as shown in Figure 2. The process model that defines the robot kinematics can be expressed as follows:

$$\begin{aligned} X_{k+1} &= f(\theta_k, x_k^r, y_k^r, L_k, \omega_k, v_k, \delta\omega_k, \delta v_k) \\ &= f(X_k, \omega_k, v_k, \delta\omega_k, \delta v_k), \end{aligned} \tag{1}$$

where X_{k+1} is the augmented states ($\in \mathbb{R}^{3+2N}$) comprising of the robot states ($\in \mathbb{R}^3$) and the landmarks' states $L_k(\in \mathbb{R}^{2N}, N = 1, 2, \ldots)$ where N represents the landmark number. k defines the time integer. θ_k is the mobile robot pose angle, and x_k^r, y_k^r are the x, y mobile robot position in Cartesian coordinate. ω_k is the mobile robot turning rate/angular acceleration and v_k is its velocity while $\delta\omega_k, \delta v_k$ are the corresponding noise in ω_k, v_k, respectively. The process model for the landmarks is unchanged as the landmarks are assumed to be stationary at all times. The predicted state $\hat{X}_{k+1}^-$ is based on the system's previous information and can be expressed by

$$\hat{X}_{k+1}^- = f(\hat{X}_k^+, \omega_k, v_k, 0, 0), \tag{2}$$

where X_k^+ is the previous updated state covariance. Particularly, Equation (2) also defines that there are no process noise during the prediction state as both $\delta\omega_k = 0$ and $\delta v_k = 0$. The associated covariance P_{k+1}^- is given by

$$P_{k+1}^- = f_r P_k^+ f_r^{\mathrm{T}} + G_{\omega v} \Sigma_k G_{\omega v}^{\mathrm{T}}, \tag{3}$$

f_r is the Jacobian evaluated from the mobile robot motion in Equation (1) and Σ_k is the control noise covariance, where

$$f_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v_k T \sin\theta & 1 & 0 & 0 \\ v_k T \cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix}, \quad G_{\omega v} = \begin{bmatrix} g_{\omega v} \\ 0 \end{bmatrix}, \tag{4}$$

$g_{\omega v}$ is the Jacobian transformation for the robot process noise, I_n an identity matrix with an appropriate dimension and T the sampling rate. Note that Equation (4) also means that there is no process noise for the landmarks as it was assumed to be stationary at all times. The mobile robot then makes each measurement using its exteroceptive sensors. This action consequently results in a measurement model which can be expressed by

$$z_i = \begin{bmatrix} r_i \\ \phi_i \end{bmatrix} = \gamma_k \begin{bmatrix} \sqrt{(x_i - x_k^r)^2 + (y_i - y^r{}_k)^2} + \omega_{r_i} \\ \arctan \dfrac{y_i - y_k^r}{x_i - x_k^r} - \theta_k + \omega_{\theta_i} \end{bmatrix}, \tag{5}$$

where r_i and ϕ_i are the relative distance and angle between robot (x_k^r, y_k^r), and the ith landmark (x_i, y_i), respectively. This equation includes the associated noises of ω_{r_i} and ω_{θ_i} for both distance and angle measurements, respectively. The linearization of Equation (5) results in the following expression:

$$z_i = \gamma_k H_i X_k + \omega_{r_i\theta_i}, \tag{6}$$

$H_i X_k$ defines the nonlinear measurements done by the robot at time k. Notice that in Equations (5) and (6), γ_k is added to the system, making it different from the normal EKF measurement model, such as proposed by Dissanayake, Newman, Clark, Durrant-Whyte, and Csorba (2001). γ_k is the probability of measurement data arriving each time the measurement is made. γ_k, a scalar quantity and independent of the observation time k, takes either the value 1 or 0. When the measurement data are available at time k, $k = 1$; $k = 0$ when the measurement data are unavailable at time k. The probability of the random measurement data arrival in Equation (6) is equivalent to the results obtained by Mo and Sinopoli (2008).

$$\Pr\{\gamma_k = 1\} = p, \tag{7}$$

$$\Pr\{\gamma_k = 0\} = 1 - p, \tag{8}$$

$$E\gamma_k = E\gamma_k^2 = p. \tag{9}$$

After the measurement process, the robot then updates its current location. Under the probability of measurement data arrival (Sinopoli et al., 2004), the updated state covariance P_{k+1}^+ is given by

$$P_{k+1}^+ = P_{k+1}^- - \gamma_{k+1} K_{k+1} H_i P_{k+1}^-, \tag{10}$$

where $K_{k+1} = P_{k+1}^- H_i^{\mathrm{T}} (H_i P_{k+1}^- H_i^{\mathrm{T}} + R_{k+1})^{-1}$. Using the steps above, the corrected state update $\hat{X}_{k+1}^+$ yields the following expression:

$$\hat{X}_{k+1}^+ = \hat{X}_{k+1}^- + \gamma_{k+1} K_{k+1} (H_i X_k - H_i \hat{X}_{k+1}^-). \tag{11}$$

Assumption 2.2 *Above equation is similar to the normal Kalman filter algorithm except for the existence of γ_{k+1} where the updated state is relying on the previous priori state and the measurement innovation. Both the process model and measurement model noises are zero mean uncorrelated Gaussian noises and holds a PsD characteristic such that*

$$\mathbb{E}\left[\begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix} \begin{bmatrix} w_k & 0 \\ 0 & v_k \end{bmatrix}^{\mathrm{T}}\right] = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix},$$

where $Q_k \geq 0$ and $R_k > 0$ are the process and measurement noise covariances.

The following definition is used in this paper unless otherwise stated. This characteristic is a general assumption in EKF state estimation.

Definition 2.3 *Based on the above variable definitions for mobile robot and system, the Jacobian measurements matrix, when the robot observes its surroundings at point A, is written as*

$$H_A = \begin{bmatrix} 0 & -\dfrac{\mathrm{d}x_k^r}{r} & -\dfrac{\mathrm{d}y_k^r}{r} & \dfrac{\mathrm{d}x_k^r}{r} & \dfrac{\mathrm{d}y_k^r}{r} \\ -1 & \dfrac{\mathrm{d}y_k^r}{r^2} & -\dfrac{\mathrm{d}x_k^r}{r^2} & -\dfrac{\mathrm{d}y_k^r}{r^2} & \dfrac{\mathrm{d}x_k^r}{r^2} \end{bmatrix}$$
$$= [-e \;\; -A \;\; A], \tag{12}$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} \dfrac{\mathrm{d}x_A^r}{r_A} & \dfrac{\mathrm{d}y_A^r}{r_A} \\ -\dfrac{\mathrm{d}y_A^r}{r_A^2} & \dfrac{\mathrm{d}x_A^r}{r_A^2} \end{bmatrix}.$$

Let x_A^r, y_A^r be x_A, y_A to simplify the notations. The above equation elements are defined by

$$\mathrm{d}x_A = [x_i - x_A], \quad \mathrm{d}y_A = [y_i - y_A],$$
$$r_A = \sqrt{\mathrm{d}x_A^2 + \mathrm{d}y_A^2}.$$

3. Convergence analysis for intermittent measurements

In this paper, measurement innovation d_k is used to determine whether information during intermittent measurement is available or not. The idea originated from EKF algorithm, where it is found that the measurement innovation represents the robot's behaviour estimation when measurement data are unexpectedly missing. In other words, EKF do not conceal any information obtained during the estimation process. In the analysis, the Jacobian transformation described in *Definition 2.3* is used to aid the explanations. Assume that if the measurement data do not arrive at a specific time, then the updated states become as

$$\hat{X}_{k+1}^{+} = \hat{X}_{k+1}^{-} + \gamma_{k+1} K_{k+1} \underbrace{H_i(X_{k+1} - \hat{X}_{k+1}^{-})}_{\text{measurement innovation}} . \tag{13}$$

As expressed in Equation (13), note that the measurement innovation provides some bounded errors to the estimation such that if the measurement innovation is increased then the updated state has bigger error. According to EKF, if an observation data are lost at a specific time $k+1$, then the estimation at time $k+1$ would take the previous value at time k. This property is stated by Equation (13) which says that the state becomes $X_{k+1}^{+} = X_{k+1}^{-}$ if measurement data do not arrive at time $k+1$. However, it is found that this condition is never satisfied whenever intermittent measurement occurred.

The estimation error is found by subtracting the middle term of Equation (13) from the mobile robot kinematics in Equation (1). Hence, we have

$$\begin{aligned} e_{k+1} &= X_k - [\hat{X}_{k+1}^{-} + \gamma_{k+1} K_{k+1}(H_i X_k - H_i \hat{X}_{k+1}^{-})] \\ &= (I - \gamma_{k+1} K_{k+1} H_i)(X_{k+1} - \hat{X}_{k+1}^{-}), \end{aligned} \tag{14}$$

where I is an identity matrix with an appropriate dimension. To simplify the interpretation on what will happen to the measurement innovation, the process noise in Equation (14) is assumed to be so small that it can be neglected. After one-step estimation, we then have the following equation:

$$e_{k+2} = (I - \gamma_{k+2} K_{k+2} H_i)(X_{k+2} - \hat{X}_{k+2}^{-}).$$

Note the importance of the term $I - K_{k+2}H_i$ as it determines the system's stability, where this equation defines that the linearized system is asymptotically stable and bounded (Vidal-Calleja et al., 2004). It is evident that the measurement innovation is different every time the measurement is taken, and individually it describes the efficiency of each measurement. A bigger value of measurement innovation would result in bigger error. For $\gamma_{k+1} = 1$ or no missing observations, the performance is similar to the normal Kalman filter and guaranteed to converge for any initial condition, $P_0 > 0$. Equation (14) also indirectly demonstrates that the results agree with what has Jun et al. proposed where the Kalman gain is adjusted to minimize the state covariance. If the linearization error can be reduced, then the state covariance will become smaller. The investigation now proceeds to examine the convergence properties of mobile robot localization.

3.1. Localization convergence

The analysis begins by proposing the statements below. The initial state covariance P_0 is given by

$$P_0 = \begin{bmatrix} P_{vv} & 0 \\ 0 & P_{mm} \end{bmatrix},$$

where $P_{vv} \in \mathbb{R}^3$ and $P_{mm} \in \mathbb{R}^{2N}$ present both robot and landmarks initial covariance, respectively. Henceforth, the updated state covariance is represented as P_k to simplify the expression for better interpretation.

Proposition 3.1 *Let $P_0 = P_0^{\mathrm{T}} > 0$ and assume that Assumption 1 and Assumption 2 are satisfied. A stationary mobile robot state error covariance is monotonically reduced if n-times successive observations are made by the mobile robot.*

Proof If the measurement data are available during the update process, then the analysis is derived using the normal EKF algorithm. When the robot observes its surroundings at point A at time k, the information matrix ($\Omega_k^{-1} = P_k$) can be expressed as

$$\Omega_k = (P_0^{-1} + {H_A}^{\mathrm{T}} R_A^{-1} H_A).$$

Consider $P_0 = P_0^{\mathrm{T}} > 0$. Utilizing Equation (12) in a case of a robot observing a landmark at point A for n-times observations, we have the following equation:

$$\Omega_k^n = \left(P_0^{-1} + n \begin{bmatrix} -H_A^T \\ A^T \end{bmatrix} R_A^{-1} [-H_A \;\; A] \right). \tag{15}$$

Note that, P_{vv} and P_{mm} are the proposed initial mobile robot and landmarks' covariances, respectively. Therefore, if the expected $P_{vv} = P_{mm} >> 0$, then we arrive with the following equation:

$$\begin{aligned} \Omega_k^n &= \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} nH_A^{\mathrm{T}} R_A^{-1} H_A & -nH_A^{\mathrm{T}} R_A^{-1} A \\ -nA^{\mathrm{T}} R_A^{-1} H_A & nA^{\mathrm{T}} R_A^{-1} A \end{bmatrix}, \\ P_k^n &= (\Omega_k^n)^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \end{aligned} \tag{16}$$

where when the robot makes successive observations about its surroundings such that $n \to \infty$, then

$$\begin{aligned} P_{11} &= [nH_A^{\mathrm{T}} R_A^{-1} H_A + (nH_A^{\mathrm{T}} R_A^{-1} A \\ &\quad \times (nA^{\mathrm{T}} R_A^{-1} A)^{-1} nA^{\mathrm{T}} R_A^{-1} H_A)]^{-1} \to \infty, \\ P_{12} &= P_{11} nH_A^{\mathrm{T}} R_A^{-1} A (nA^{\mathrm{T}} R_A^{-1} A)^{-1} = P_{21}^{\mathrm{T}} \to \infty, \\ P_{22} &= (nA^{\mathrm{T}} R_A^{-1} A)^{-1} + [(nA^{\mathrm{T}} R_A^{-1} A)^{-1} nA^{\mathrm{T}} R_A^{-1} H_A P_{11} \\ &\quad \times nH_A^{\mathrm{T}} R_A^{-1} A (nA^{\mathrm{T}} R_A^{-1} A)^{-1}] \to \infty. \end{aligned}$$

Now it is evident that, in the case above, it is impossible to make the estimation, even if robot makes many observations of its surroundings. On the other hand, if $P_{vv} >> P_{mm} > 0$, then

$$\Omega_{22} = P_{mm}^{-1} + nA^{\mathrm{T}}R_A^{-1}A.$$

In this case Ω_{22} has more information than in the previous case. Consequently, the updated state error covariance is smaller hence resulting in better estimation. In fact, this characteristic determines the importance of the landmarks existence in order to achieve better results, as have been reported by previous researchers. Technically, if the robot made more observations at a particular point without loss of measurement data, then Ω_k presents bigger probabilities. In other words, P_k would become smaller and keep decreasing as more observations are made. After successive n-times observations, and if and only if the measurements of landmarks are available, the final state covariance shows that the updated state error covariance P_k^n gradually decreases as more information about its surroundings are received. ■

It has been demonstrated that the localization is possible for a single mobile robot observing a landmark, as shown by Equation (16), if the robot keeps observing its surroundings and if at least a number of landmarks are available. This result is also applicable for a group of robot navigating an environment. However, note that this situation is different in the case of two moving robot localizing itself relatively to each other (Chen et al., 2009). According to the EKF properties, the updated state error covariance is bigger than the true state covariance. This is well known as EKF is optimistic about its estimation. Nevertheless, *Proposition 1* has explained that the uncertainties reduce as more observations are made by robot.

3.2. Main results

As stated earlier in Equation (13), the measurement innovation d_k is one of the essential information in EKF algorithm especially when measurement data are partially lost during observations. To demonstrate, let us analyse the state error covariance which is expected to have the following criteria: $P_{X_{k+1}} = \mathbb{E}[(X_{k+1} - \hat{X}_{k+1}^-)^{\mathrm{T}}(X_{k+1} - \hat{X}_{k+1}^-)]$. Also, assume that the process noise is very small and therefore can be neglected; this assumption is so as to simplify the resulting equation. Consequently, the expectation of the cross-covariance between the process and measurement models $P_{xz_{k+1}}$ can be expressed in terms of $P_{X_{k+1}}H_i^{\mathrm{T}}$.

$$\begin{aligned} P_{z_{k+1}} &= \mathbb{E}[(z_i - \hat{z}_i)(z_i - \hat{z}_i)^{\mathrm{T}}] \\ &= \mathbb{E}[(H_i(X_{k+1} - \hat{X}_{k+1}^-))(H_i(X_{k+1} - \hat{X}_{k+1}^-))^{\mathrm{T}}] \\ &= H_i P_{X_{k+1}^-} H_i^{\mathrm{T}} + R_{k+1}. \end{aligned} \tag{17}$$

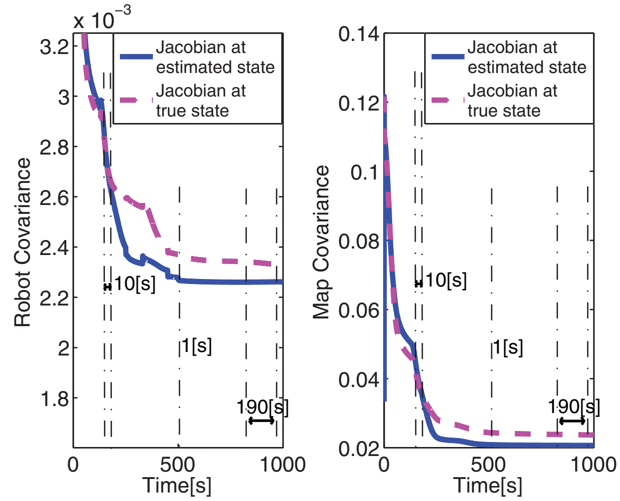

Figure 3. The updated state covariance comparison between the case of Jacobian being calculated at the true estimate and Jacobian being calculated at the estimated state.

To better visualize the situation, $P_{xz_{k+1}}$ that constitute Equation (17) can be expressed as

$$P_{xz_{k+1}} = \begin{bmatrix} P_{X_{k+1}^-} & P_{X_{k+1}^-}H_i^{\mathrm{T}} \\ H_i P_{X_{k+1}^-} & H_i P_{X_{k+1}^-} H_i^{\mathrm{T}} + R_{k+1} \end{bmatrix}. \tag{18}$$

As stated above, the measurement innovation brings significant information regarding the state covariance at each update process. Besides, the structure of Equation (18) is similar to the updated state error covariance as shown in *Proposition 1*. Equation (18) also defines that measurement innovation applied in Equation (13) is important and affects the updated state covariance. In fact, the condition of an updated state error covariance is accessible by Equation (18) such that if the measurement innovation increases, the updated state covariance also increases.

A simulation conducted to show the consistency of the system is shown in Figure 3, where the Jacobian calculated at the true state is compared with that calculated at the estimated state. The initial state error covariance is $P_0 = [P_{vv}\ P_{mm}]^{\mathrm{T}} = [1e^4\ 1e^{-2}]^{\mathrm{T}}$ and the measurement data are assumed to be unavailable for 10[s] at 100[s], 1[s] at 500[s] and 190[s] at 800[s] observations. The results from the simulation show that the updated true state covariance is bigger than the estimated state error covariance, and this is similar to the estimations obtained using EKF.

Now let us analyse the effect of the intermittent measurement to the robot localization. The expectation of state covariance at time $k+1$ is demonstrated by Sinopoli et al. (2004), Mo and Sinopoli (2008) and Kluge et al. (2010)

$$\begin{aligned} &\mathbb{E}[(z_{k+1} - \hat{z}_{k+1})(z_{k+1} - \hat{z}_{k+1})^{\mathrm{T}}] \\ &\quad = \mathbb{E}[d_{k+1}d_{k+1}^{\mathrm{T}}] \\ &\quad = H_i P_{k+1}^- H_i^{\mathrm{T}} + \gamma_{k+1}R_{k+1} + (1-\gamma_{k+1})\sigma^2 I. \end{aligned} \tag{19}$$

Equation (19) demonstrates that the probability of measurement data is given by $H_iP_{k+1}H_i^{\mathrm{T}} + \sigma^2 I$ when $\gamma = 0$, where $\sigma > 0$ is the random uncertainties and I is an identity matrix with an appropriate dimension. In contrast to the previous findings (Kluge et al., 2010; Mo & Sinopoli, 2008; Sinopoli et al., 2004), this behaviour of intermittent measurements can be represented by measurement innovation technique.

As previously mentioned, if at some specific time a measurement innovation is missing, then from Equation (13), the state covariance seems to acquire the previous state covariance data. In this respect, it has been shown that, when the measurements are missing, there exists some valuable information that forms statistical bounds for the estimation. The next lemma is proposed to illustrate the relevancy of EKF measurement innovation in evaluating the behaviour of intermittent measurements.

Lemma 3.2 *Consider a case of a stationary robot observing the landmarks. Suppose that $y_k = H_iX_k$ and $y_{k+1} = H_{i+1}X_{k+1}$. If H_i is a gradually increasing function, then the following situations are achieved:*

(1) $y_1 \geq y_0 \Rightarrow y_{k+1} \geq y_k$,
(2) $y_1 < y_0 \Rightarrow y_{k+1} < y_k$.

Proof The proof is analogous to *Lemma 2* as stated by Mo and Sinopoli (2008). By induction, this lemma can be proved. To prove number 1, consider the mobile robot is observing one landmarks and moving away from it. Hence, the measurement matrix is gradually increasing. Therefore, if the initial state of y is y_0 and the observation after one time update is y_1, then $y_1 \geq y_0$. The conditions follow as long as the mobile robot moves farther away from the landmarks. The case 2 can be proved oppositely when the mobile robot is moving towards the landmarks. ■

In a system, especially one with high nonlinearities, the above lemma is inappropriate and probably could not be achieved. This is due to the nonlinear robot movements and the information gained by mobile robot from its successive observations. By taking into account these nonlinearities, Theorem 3.3 is proposed to determine the properties of measurement innovation during intermittent measurements.

Theorem 3.3 *Assume that a robot is observing a landmark at point A. If the measurement data are intermittently unavailable for one time at $k > 1$, then the measurement innovation will exhibit the following equation:*

$$d_k = \gamma_{k-1}A_{k-1}(L_{m_{k-1}} - C_{k-1}), \tag{20}$$

where A_{k-1} is defined in Equation (12) and d_k is the measurement innovation. $L_{m_{k-1}}$ and C_{k-1} define the landmark and robot x, y locations, respectively.

Proof The measurement innovation for at $k+1$ as has been shown by Huang and Dissayanake (2007) can be expressed as

$$\begin{aligned} d_{k+1} &= z_k - \gamma_{k+1}H_i\hat{X}^-_{k+1} \\ &= \gamma_{k+1}H_iX_k + \omega_{r1\theta 1} - \gamma_{k+1}H_i\hat{X}^-_{k+1} \\ &= \gamma_{k+1}H_i(X_k - \hat{X}^-_{k+1}) + \omega_{r1\theta 1} \\ &= \gamma_{k+1}[-e_{k+1} \quad -A_{k+1} \quad A_{k+1}]\begin{bmatrix} \theta_k - \hat{\theta}^-_{k+1} \\ C_k - \hat{C}^-_{k+1} \\ L_{m_k} - \hat{L}^-_{m_{k+1}} \end{bmatrix} + \omega_{r1\theta 1} \\ &= \gamma_{k+1}[-e_{k+1}(\theta_k - \hat{\theta}^-_{k+1}) - A_{k+1}(C_k - \hat{C}^-_{k+1}) \\ &\quad + A_{k+1}(L_{m_k} - \hat{L}^-_{m_{k+1}})] + \omega_{r1\theta 1}, \end{aligned} \tag{21}$$

where θ_k, C_k, L_{m_k} are the robot angle, robot (x, y) position and landmarks' (x, y) locations, respectively. A_{k+1} is the Jacobian evaluation of measurement matrix between relative angle and position of robot and landmarks, and $\omega_{r1\theta 1}$ is the measurement noise. Reorganizing and neglecting the measurement error, the equation above becomes

$$\begin{aligned} &\gamma_{k+1}[e_{k+1}\theta_k + A_{k+1}R_k - A_{k+1}L_{m_k}] \\ &\quad = -d_{k+1} + \gamma_{k+1}[e_{k+1}\hat{\theta}^-_{k+1} + A_{k+1}\hat{R}^-_{k+1} - A_{k+1}\hat{L}^-_{m_{k+1}}]. \end{aligned} \tag{22}$$

To better understand the consequence of the measurement innovation, the analysis is carried out the next time the robot updates its measurement. This results in the Jacobian to be evaluated at a different time based on the estimated position. Each measurement innovation exhibits

$$\begin{aligned} &\gamma_{k+1}[e_{k+1}\theta_k + A_{k+1}R_k - A_{k+1}L_{m_k}] \\ &\quad = -d_{k+1} + \gamma_{k+1}[e_{k+1}\hat{\theta}^-_{k+1} + A_{k+1}\hat{C}^-_{k+1} - A_{k+1}\hat{L}^-_{m_{k+1}}], \\ &\gamma_{k+2}[e_{k+2}\theta_k + A_{k+2}R_k - A_{k+2}L_{m_k}] \\ &\quad = -d_{k+2} + \gamma_{k+2}[e_{k+2}\hat{\theta}^-_{k+2} + A_{k+2}\hat{C}^-_{k+2} - A_{k+2}\hat{L}^-_{m_{k+2}}]. \end{aligned}$$

Note that the equations above are calculated based on the estimated positions. For convenience, assume that $A_{k+2} = A_{k+1}$ and consider that the robot remains stationary at $k+1$ and $k+2$ when observing its surroundings. Therefore, after rearranging and subtracting the above equations, the following equations are obtained:

$$\begin{aligned} &\gamma_{k+1}A^{-1}_{k+1}e_{k+1}\theta_k - \gamma_{k+2}A^{-1}_{k+1}e_{k+1}\theta_k \\ &\quad = -A^{-1}_{k+1}d_{k+1} + A^{-1}_{k+1}d_{k+2} \\ &\qquad + \gamma_{k+1}A^{-1}_{k+1}e_{k+1}\hat{\theta}^-_{k+1} - \gamma_{k+2}A^{-1}_{k+1}e_{k+1}\hat{\theta}^-_{k+2} \\ &\qquad + \gamma_{k+1}A^{-1}_{k+1}\hat{C}^-_{k+1} - \gamma_{k+2}A^{-1}_{k+2}\hat{C}^-_{k+2} \\ &\qquad - \gamma_{k+1}A^{-1}_{k+1}\hat{L}^-_{m_{k+1}} + \gamma_{k+2}A^{-1}_{k+2}\hat{L}^-_{m_{k+2}}. \end{aligned}$$

The above equation determines the *measurement innovation error* of the system for observations from time $k+1$

to $k+2$. The above information is used to develop some knowledge about the system estimation.

$$\gamma_{k+1}A_{k+1}^{-1}e_{k+1}\theta_k = -A_{k+1}^{-1}d_{k+1} + A_{k+1}^{-1}d_{k+2} + \gamma_{k+1}A_{k+1}^{-1}e_{k+1}\hat{\theta}_{k+1}^{-} + \gamma_{k+1}\hat{C}_{k+1}^{-} - \gamma_{k+1}\hat{L}_{m_{k+1}}^{-}. \quad (23)$$

If the observation does not arrive at time $k+2$, then the following equations are obtained:

$$d_{k+2} = A_{k+1}[A_{k+1}^{-1}d_{k+1} + \gamma_{k+1}(A_{k+1}^{-1}e_{k+1}(\theta_k - \hat{\theta}_{k+1}^{-}) - A_{k+1}^{-1}\hat{C}_{k+1}^{-} + A_{k+1}^{-1}\hat{L}_{m_{k+1}}^{-})], \quad (24)$$

$$d_{k+2} = \gamma_{k+1}A_{k+1}(L_{m_k} - C_k). \quad (25)$$

Even though the measurement data are intermittently missing, the results above proved that estimation is still possible with a level of certainty, and it depends on the nonlinear transformation of Jacobian matrix A. ■

As shown above, the system now has sufficient information regarding the observation during intermittent measurements without demanding us to introduce additional tools for analysis (Jun et al., 2012) or the riccati equation (Kluge et al., 2010; Mo & Sinopoli, 2008; Plarre & Bullo, 2009; Sinopoli et al., 2004) as, in practice, some approaches are difficult to examine the state error covariance. Besides that, the proposed results are likely to be feasible to be utilized in a robotic system since the substantial recursive information can be used by the robot to infer its position. For some restricted conditions, such as when both $A_{k+1} = A_k$ and $d_{k+1} > 0, d_k > 0$, we have the following results.

Corollary 3.4 *Assume that $A_{k+1} = A_k$ and both $d_{k+1} > 0$ and $d_k > 0$. In the case of a stationary robot localization, when the measurement data do not arrive, the magnitude of the measurement innovation d_{k+1} is bigger than the d_k such that $d_{k+1} > d_k$.*

Proof The *measurement innovation error* when measurement data are lost can be expressed as

$$d_k = \gamma_k A_{k-1}(L_{m_{k-1}} - C_{k-1}) > 0,$$

and the measurement innovation, when the measurement data are available, can be expressed as

$$d_{k+1} = [-e_{k+1}(\theta_k - \hat{\theta}_{k+1}^{-}) - A_k(C_k - \hat{C}_{k+1}^{-}) + A_k(L_{m_k} - \hat{L}_{m_{k+1}}^{-})] > 0.$$

Comparing d_{k+1} and d_k,note that $d_{k+1} > d_k$. Therefore, if the measurement data are lost at k but available at $k+1$, then the measurement innovation at $k+1$ is bigger than the measurement innovation at k. ■

Now that a direct influence of the Jacobian transformation to a nonlinear system can be perceived. The Jacobian transformation actually provides information of the estimation properties during intermittent measurements. This also means that it contains a significant value of the estimation error at all times. In other words, when the robot lost its measurement data, the state estimation does not theoretically refer to Equation (11). It is not difficult to assess that the statistical bounds are available and those statistical bounds affect the robot movements when measurement data are lost. However, there is one variable that remains unknown, i.e. the relative angle between the robot and the landmarks. As a result, the estimation may become insufficient as this information is very important to define the robot movements in the given environment. Nevertheless, as Equation (20) defines the estimation error between the robot and any observed landmarks, it can be guaranteed to have correct information with a small error when no measurement data arrived at the system.

In addition, we found that *Theorem 3.3* is also applicable when measurement data are unavailable for a long period, especially when the robot angle is known with some certainty.

Lemma 3.5 *The measurement innovation is bounded even if there are multiple losses of measurement data after $k > 1$ observations. Moreover, if $A_k = A_{k+1}$, then the measurement innovation $d_{k+1} = A_k(L_{m_k} - C_k)$.*

Proof The proof can be similarly derived as in *Theorem 3.3*. Consider that the mobile robot lost its measurement at $k+2$, then Equation (25) is obtained. If the Jacobian transformation of $A_k = A_{k+1} = A_{k+2} \ldots = Ak + n$ where n is the number of observations, then the equation still exhibits (25). The only difference is the relative location between mobile robot and the landmarks it observed. If the mobile robot is stationary, then Equation (25) has the same bounds at all times. ■

Lemma 3.2 and *Theorem 3.3* above have sufficiently described that even if measurement data are missing after $k > 1$, the mobile robot is still able to estimate its location with a bounded certainty. Furthermore, the solution for the robot localization depends on the Jacobian transformation of A. If A is large, and increasing, when no measurement data are received, then the uncertainties also increase. Hence, there is still a possibility that this can lead to unbounded estimations for both the robot and landmarks' estimations. Regardless of this fact, the localization convergence properties are still the essential elements, as it determines whether the filter is efficient or not for the estimation purposes.

Now let us inspect the upper and lower bounds of the state error covariance in intermittent measurements. EKF algorithm illustrated that the state updates are given by Equation (11). This is the essential information required to represent the whole state covariance characteristics about the system. First, we define *measurement innovation error* as $S_k = d_k - d_{k-1}$ to evaluate the state covariance behaviour. This definition is made to show that S_k has a proportional relationship to the updated state error covariance P_k such that $S_k = H_k P_k H_k^{\mathrm{T}} + R_k$ as shown in Equations (17)–(19). In fact, it implicitly determines the bounds of measurement error which is very important for reliable information regarding the environment; if the measurement innovation error is decreasing then the state covariance is converging. Such information is very important in trying to understand the whole system characteristics.

The relationship between d_k and d_{k-1} is examined by further analysing *Theorem 3.3*.

Theorem 3.6 *Consider a case of a robot observing a landmark at point A. If measurement data are partially lost at $k > 1$, then the measurement innovation error S_k is shown by the following expression:*

$$S_k = e_{k-1}(\theta_{k-1} - \hat{\theta}_{k-1}^{-}) + A_{k-1}(\hat{C}_{k-1}^{-} - \hat{L}_{m_{k-1}}^{-}). \tag{26}$$

The uncertainties are decreasing if the equations below are satisfied.

$$\mathrm{d}x_A(\hat{L}_{m_x^i}^{-} - \hat{C}_{k_x}^{-}) - \mathrm{d}y_A(\hat{L}_{m_y^i} - \hat{C}_{k_y}^{-}) > 0, \tag{27}$$

$$\mathrm{d}x_A(\hat{L}_{m_y^i}^{-} - \hat{C}_{k_y}^{-}) - \mathrm{d}y_A(\hat{L}_{m_x^i} - \hat{C}_{k_x}^{-}) > r_A^2(\theta_{k-1} - \hat{\theta}_{k-2}^{-}). \tag{28}$$

This is in the case when a robot is observing at point A and $\mathrm{d}x_A$, $\mathrm{d}y_A$ and r_A are as shown in Equation (12). $\hat{L}_{m_x^i}^{-}$, $\hat{L}_{m_y^i}^{-}$ state the estimated landmark number in x, y location that are being observed by the robot which is situated at the estimated $\hat{C}_{k_x}^{-}, \hat{C}_{k_y}^{-}$ in x, y positions, respectively. Otherwise, the uncertainties are increasing.

Proof The result from *Theorem 3.3* is referred to prove that if measurement data are not available, then measurement innovation is equal to Equation (20). This property is predictable via $d_k - d_{k-1}$ as this equation is useful to define the robot confidence about its position.

$$\begin{aligned} d_k - d_{k-1} &= A_{k-1}(L_{m_{k-1}} - C_{k-1}) - [-e(\theta_{k-1} - \hat{\theta}_{k-2}^{-}) \\ &\quad - A_{k-1}(C_{k-1} - \hat{C}_{k-2}^{-}) + A_{k-2}(L_{m_{k-2}} - \hat{L}_{m_{k-2}}^{-})]. \end{aligned} \tag{29}$$

For convenience, assume that the Jacobian transformation of measurement matrix at $k-1$ is the same as the Jacobian transformation at time k such that $A_{k-1} = A_{k-2}$. Recognize that the robot is stationary at point A and the landmark is also static such that $L_{m_{k-1}} = L_{m_{k-2}}$ and $C_{k-1} = C_{k-2}$. Using these information, if the measurement innovation error is decreasing, then

$$\begin{aligned} S_k &= d_k - d_{k-1} \\ &= e(\theta_{k-1} - \hat{\theta}_{k-2}^{-}) - A_{k-1}(\hat{L}_{m_{k-2}}^{-} - \hat{C}_{k-2}^{-}) < 0. \end{aligned} \tag{30}$$

Calculating the equation further will result in

$$e(\theta_{k-1} - \hat{\theta}_{k-2}) < A_{k-1}(\hat{L}_{m_{k-2}}^{-} - \hat{C}_{k-2}^{-}). \tag{31}$$

Substituting the elements in Equation (12) into Equation (31) leads to

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} (\theta_{k-1} - \hat{\theta}_{k-2}^{-}) < \begin{bmatrix} \dfrac{\mathrm{d}x_A}{r_A} & \dfrac{\mathrm{d}y_A}{r_A} \\ -\dfrac{\mathrm{d}y_A}{r_A^2} & \dfrac{\mathrm{d}x_A}{r_A^2} \end{bmatrix} \begin{bmatrix} \hat{L}_{m_x^i}^{-} - \hat{C}_{k_x}^{-} \\ \hat{L}_{m_y^i}^{-} - \hat{C}_{k_y}^{-} \end{bmatrix}. \tag{32}$$

The relationships above provide two different conditions that must be satisfied to ensure that the measurement innovation error is decreasing as follows:

$$\mathrm{d}x_A(\hat{L}_{m_x^i}^{-} - \hat{C}_{k_x}^{-}) - \mathrm{d}y_A(\hat{L}_{m_y^i}^{-} - \hat{C}_{k_y}^{-}) > 0, \tag{33}$$

$$\mathrm{d}x_A(\hat{L}_{m_y^i}^{-} - \hat{C}_{k_y}^{-}) - \mathrm{d}y_A(\hat{L}_{m_x^i}^{-} - \hat{C}_{k_x}^{-}) > r_A^2(\theta_{k-1} - \hat{\theta}_{k-2}^{-}). \tag{34}$$

This is the case when a robot is observing at point A and $\mathrm{d}x_A$, $\mathrm{d}y_A$ and r_A are as shown in Equation (12). $\hat{L}_{m_x^i}^{-}, \hat{L}_{m_y^i}^{-}$ are the estimated landmark number in x, y locations and $\hat{C}_{k_x}^{-}, \hat{C}_{k_y}^{-}$ are mobile robot x, y positions, respectively. Equation (30), under the conditions of Equations (33) and (34), equivalently means that the error is decreasing and bounded by Equations (33) and (34). Thus, the analysis above states that Sk defines two different conditions such that if Equations (33) and (34) are not satisfied, then the measurement innovation error is increasing. ■

Theorem 3.6 suggested that through the measurement innovation, measurement data can be still estimated even though it is intermittently missing during observations. Equations (7) and (8) described that the measurement innovation can be one of the promising methods and has its own advantages compared with previous works (Censi, 2008; Jun et al., 2012; Mo & Sinopoli, 2008; Plarre & Bullo, 2009; Sinopoli et al., 2004) to investigate a system under intermittent measurements.

4. Experimental results

To evaluate the proposed theorems, experiments were conducted using an E-puck robot which uses a bluetooth device to transmit data to the processor. The measurement data are then feed into dSpace software to directly inspect the estimation results. In the test, the robot attempts to observe a small environment which consists of some landmarks while

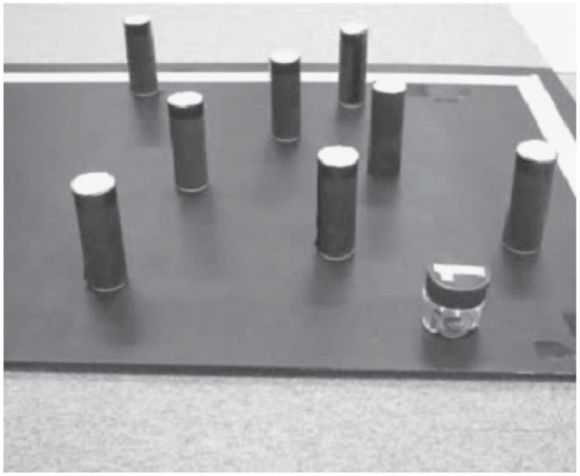

Figure 4. Experimental and environmental setup.

Table 1. Experimental parameters when measurements, data are missing at 50s, 80s and 120s.

Sampling time, T	0.1 (s)
Robot process noise, Q	$0.000001 * \mathrm{diag}(I_3)$
Observation noise, R	$0.001 * \mathrm{diag}(I_{3+2N})$
Robot initial covariance P_{vv}	$10,000 * \mathrm{diag}(I_3)$
Landmarks initial covariance P_{mm}	$0.00001 * \mathrm{diag}(I_{2N})$

moving in the given environment, as shown in Figure 4. A state is designed such that it demonstrates a situation where measurements data are missing at time 50[s], 80[s] and 120[s] for one-time observation. Even though this is impractical, this will help researcher to understand how actually the system behaves during intermittent measurement. Besides, the role of measurement innovations can be further noticed if this is considered. See Table 1 for experimental parameters that approximately describe the system and environment conditions. Every parameter included in Table 1 represents the selected parameters in appropriate dimensions.

As soon as the robot moves, the robot location and its measured landmarks' positions are concurrently illustrated. As shown in Figure 5, when the measurement data are missing for 1[s] each at $k = 50$[s], 80[s] and 120[s], the measurement innovation reveals that there still exist some small values defining the uncertainties characteristics at those specific times. This characteristic shows that when the measurement data are unavailable, the estimation does not refer to its previous estimation but hold its individual state. Therefore, these conditions agree and support the proposed results mentioned in the previous section. Moreover, this characteristic also corresponds to Equation (19) in the case where $y = 0$ and consistently pose the same meaning as reported by Mo and Sinopoli (2008). In addition, observe that when the measurement data are unavailable at 50[s], the estimation error becomes slightly less than the normal EKF estimation. Due to this, EKF with intermittent measurements is expected to hold a lower state error covariance than normal EKF. It is found that this is the starting point where the estimation of EKF with intermittent measurement is expected to exhibit better performance than normal EKF (refer to Equation (11) which defines that if measurement

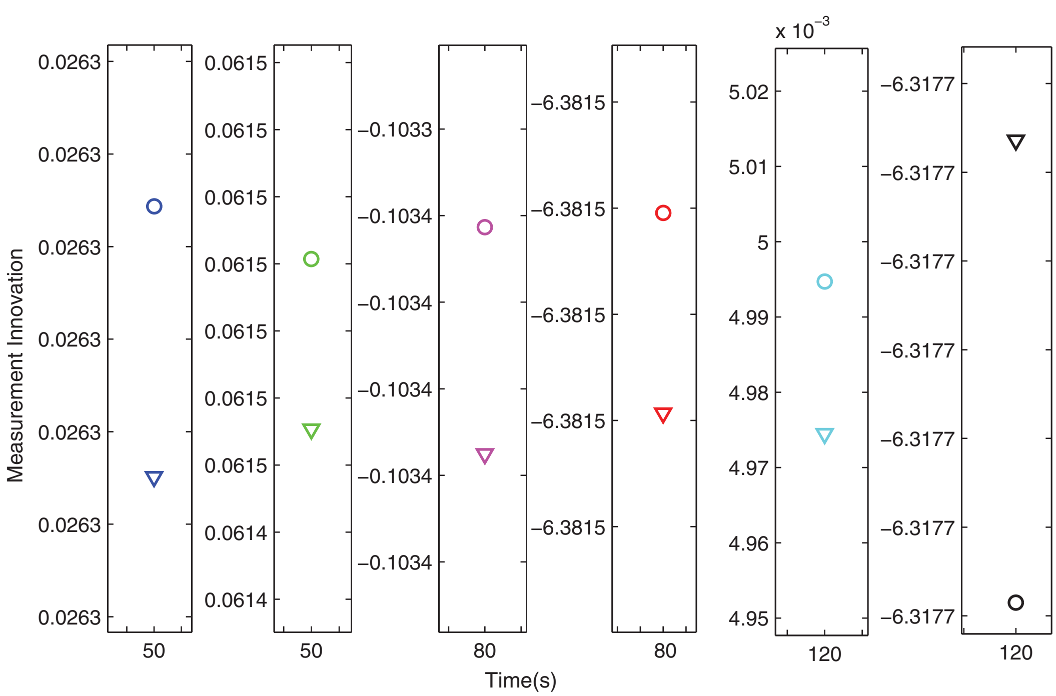

Figure 5. Some measurement innovation when measurement data are missing at 50[s], 80[s] and 120[s]. △ represents EKF estimation when measurements data are missing and ∘ illustrates normal EKF estimations.

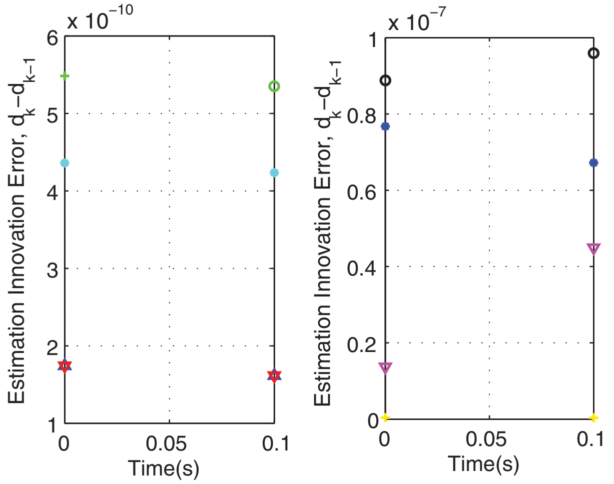

Figure 6. Measurement innovation error characteristics when measurements data are missing at 50[s].

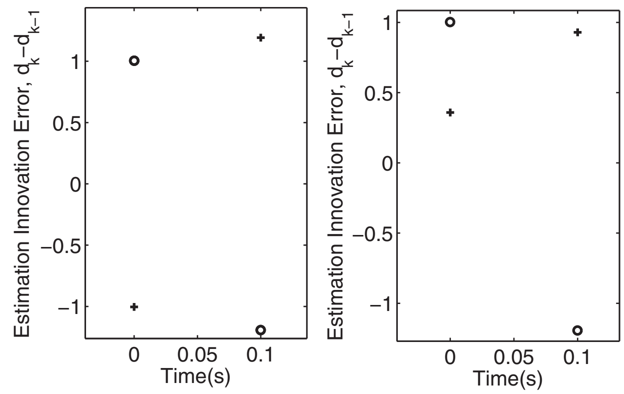

Figure 7. Total measurement innovation error S_k^{total} when measurements data are missing at 50[s] (left), and 120[s] (right) for normal EKF (○ mark) and EKF with intermittent measurements (+ mark).The total measurement innovation error for EKF with intermittent measurements increasing, while that for normal EKF decreasing.

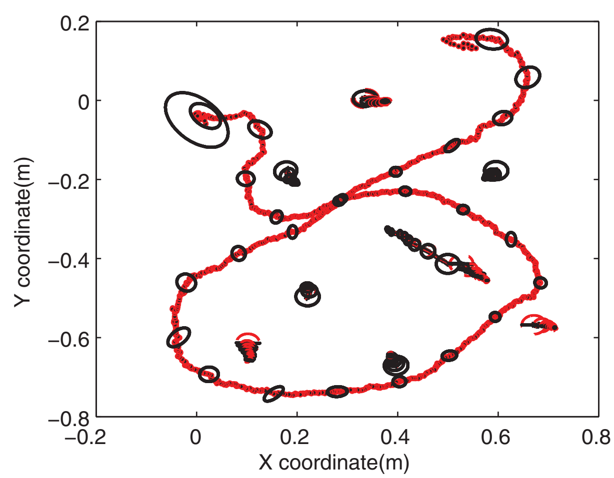

Figure 8. Comparison of robot movements between EKF (black) and normal EKF (red) when measurements data are missing at 50[s], 80[s] and 120[s] for 1 sampling time, together with its associated covariance (circles).

innovation is decreased, then the error becomes smaller). As the measurement data do not arrive at 50[s], it can be said that the measurement innovation at 50.1[s] is directly engaged to Equation (20). This equation explains that the estimated states have either larger or smaller states, such that its performance depends on the Jacobian transformation of measurement matrix A and the distances between the robot and the landmarks.

Next, Figure 6 illustrates the measurement innovation error characteristics for some states. In this figure, when the measurement data are missing, some of the measurement innovation errors are decreasing; as shown by Equations (33) and (34), otherwise the measurement innovation errors are increasing if Equations (33) and (34) are not satisfied. This information is then used to infer EKF measurement innovation covariance, and finally contribute to the successive information to update the state error covariance. Using Figure 7, the total amount of S_k^{total} was analysed to compare the performance between EKF with intermittent measurements and normal EKF. As a result, it is found that S_k^{total} is increasing for EKF with intermittent measurements when the measurement data are unavailable. This characteristic is opposite to that of the normal EKF estimation. Put it differently, this situation equivalently means that the state error covariance is increasing in the case of intermittent measurements. These transient characteristics are in accordance with the proposed *Theorem 2*.

In Figure 8, it can be seen that the estimation results for both robot and landmarks' positions are almost similar to the normal EKF estimation. This is due to the small measurement innovation error, and hence, there are only small differences between the two filtered estimations. Also, according to the properties as presented in Figure 6, the trace of state error covariance should increase and proportional to the total measurement innovation error, S_k^{total}. Now let us compare the performance of measurement innovation error to the updated state covariance. It can be seen that the uncertainties shown in Figure 9(a)–(c) demonstrate the same results as depicted in Figures 5–7; this is because the trace of the updated state error covariance with intermittent measurement is less than that of the normal EKF update for both the robot and landmarks covariance. Thus, these results show the relationship between the measurement innovation and the state error covariance, and that they have high dependency on each other. Indeed, these characteristics are in compliance with what has been proposed in this paper.

Considering a case when measurement data are missing for a longer time about five sampling time, the estimated results from the KF are found to be almost the same with that obtained from normal KF, as shown in Figure 10. As expected, the outcomes steadily exhibit that the estimation is still available with a level of confidence as shown in the analysis. Also, the mobile robot estimation error during

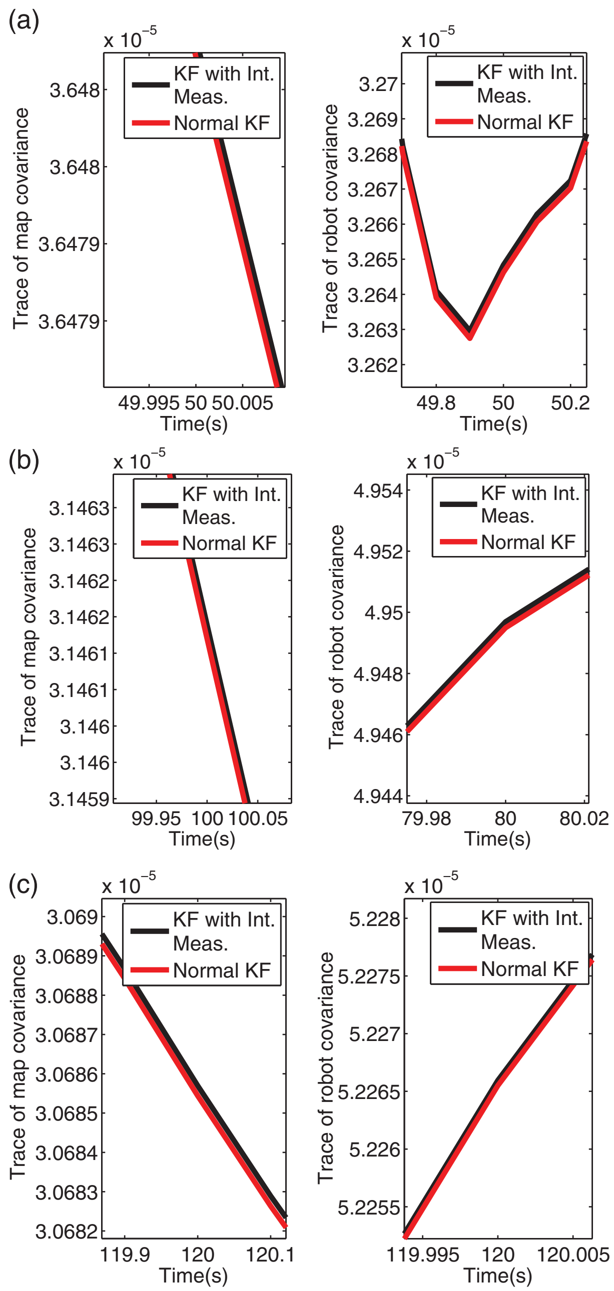

Figure 9. (a) Measurement lost at 50s, (b) measurement lost at 80s and (c) measurement lost at 120s.

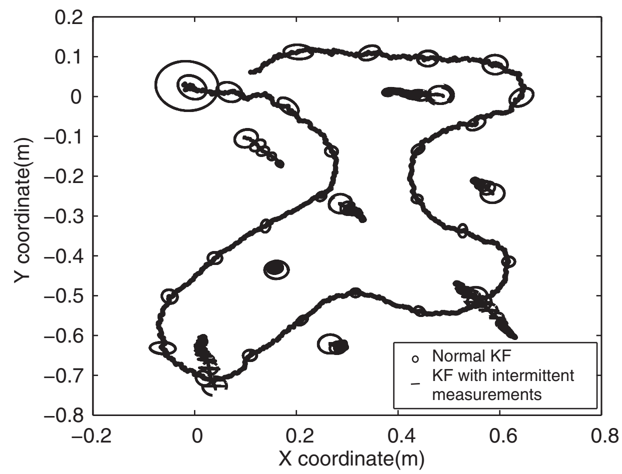

Figure 10. Robot movements when some measurements data are missing for five sampling time at 50[s], 80[s] and 120[s] with its associated covariance.

intermittent measurement, as shown in Figure 11, is consistent with the analysis; Figure 11 agrees with what has been presented in Figures 5–9. Note that after the measurement data loss at 50[s], 80[s] and 120[s], some of the estimation errors are gradually decreasing. Due to this behaviour, the estimation errors for that specified states become slightly smaller than that of the normal EKF. Consequently, the updated state error becomes smaller than normal EKF. In fact, this figure proved that even though some measurements are lost during observations, mobile robot is still able to infer its pose with bounded uncertainties, thus supporting the proposed *Theorem 2*. Hence, the experimental results do indeed agree with the analysis done in this study, and therefore, shows the importance of the measurement innovation information in Kalman filtering for recursively predicting and updating states whenever measurement data are missing.

In spite of the results presented above, the uncertainties can become unbounded if the initial state covariance, process noise and measurement noise are too big (Kluge et al., 2010). This is in accordance with the theory, as shown in Equation (13), if the measurement innovation d_k has a large measurement error then the estimation diverges. In the case where the initial state covariance and process noise are very large, the uncertainties characteristics also increase. The situation is observable in Equation (17); if P_0 is very large, then the measurement innovation error increases. The continuous measurements and updates under this condition then result in gradually increasing uncertainties especially if the robot is observing only one landmark; good estimation cannot be achieved by observing one landmark (Dissanayake et al., 2001). Under these circumstances, the estimation error is increasing and subsequently produces erroneous estimation. Therefore, a designer must carefully design the system by taking into account these parameters, not only of the robot's assigned workplace, especially so when the robot that has a very low sensing accuracy.

However, for a small initial state covariance with a small process and measurement noises, it is found that other than the robot and landmarks' distances, the Jacobian transformation for measurement process is a primary aspect that governed the estimation efficiency such that if it is too big, then the uncertainties bounds become bigger. In the experimental evaluations, the robot initial uncertainties are not too large and the noises are designed such that those parameters can accommodate the specified environment conditions. Through these settings, divergence of estimation was not perceived during the experimental evaluations.

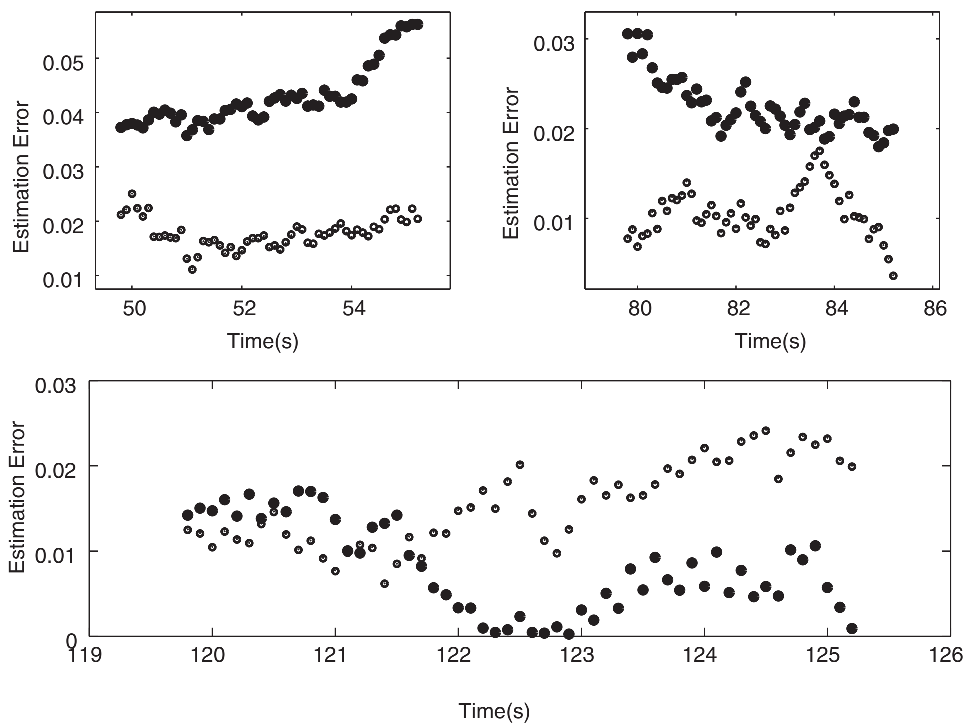

Figure 11. Estimation error when some measurement data are missing for five sampling time at 50[s], 80[s] and 120[s] for normal EKF (darker dot) and EKF with intermittent measurements (lighter dot).

5. Conclusion

This paper has demonstrated the information provided by measurement innovation technique in intermittent measurements when a mobile robot is observing its surroundings. Even though data are missing, the robot is still able to estimate its location, and its errors are statistically bounded if and only if the initial state covariance, process and measurement noises are sufficiently small such that robot has a fair confidence about its surroundings. The results are also coherent with the findings described in the literatures which defines Kalman gain must be reduced to minimize the state covariance. Instead of Kalman gain, our results focussed on minimizing the measurement innovation. Moreover, the linearization error must be reduced to achieve better estimation. It has been shown that measurement innovation is very helpful in deciding and pursuing the whole system uncertainties when measurement data are not entirely lost. Furthermore, experimental evaluation has shown the effectiveness of the proposed method.

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