

Functional Dependencies

(a)

Suppose $X \rightarrow Y$ doesn't hold, then it means there exists $t \in R$ and $s \in R$, such that $\pi_X(t) = \pi_X(s)$ but $\pi_Y(t) \neq \pi_Y(s)$. In this case, denote $K = R - \{t, s\}$ as the rest of tuples in R . We know the number of tuples in $\pi_X(K)$ is less or equal to the number of tuples in K . However, for $\{t, s\}$, we know the number of tuples in $\pi_X(\{t, s\})$ is 1 and the total number of tuples in $\{t, s\}$ is 2. Thus, the number of tuples in $\pi_X\{R\}$ less or equal to the number of tuples in $\pi_X(K)$ plus the number of tuples in $\pi_X(\{t, s\})$ which is 1, and is strictly less than the number of tuples in R , contradiction. Thus, $X \rightarrow Y$ must be true.

(b)

" \Rightarrow " Suppose there doesn't exist a FD $X \rightarrow Y$ such that $F - \{X \rightarrow Y\} \models X \rightarrow Y$, but F is redundant. Then there exists a F' such that $F' \subset F$ but $(F')^+ = (F)^+$. Without loss of generality, we assume $X_0 \rightarrow Y_0 \in F$ but $X_0 \rightarrow Y_0 \notin F'$. Since $(F')^+ = (F)^+$ and $X_0 \rightarrow Y_0 \in (F)^+ = (F')^+$, thus we know $X_0 \rightarrow Y_0 \in (F')^+$. Thus we know $F - \{X_0 \rightarrow Y_0\} \subset F' \models X_0 \rightarrow Y_0$, which is a contradiction. Thus, if F is redundant, there must exist a $X \rightarrow Y$, such that $F - \{X \rightarrow Y\} \models X \rightarrow Y$.

" \Leftarrow " Suppose there exists a FD $X \rightarrow Y$ such that $F - \{X \rightarrow Y\} \models X \rightarrow Y$, then let $F' = F - \{X \rightarrow Y\}$. Then we know $(F')^+ = (F - \{X \rightarrow Y\})^+ = (F - \{X \rightarrow Y\} \cup \{X \rightarrow Y\})^+ = (F)^+$ and $F' \subset F$, thus we know F is redundant.

(c)

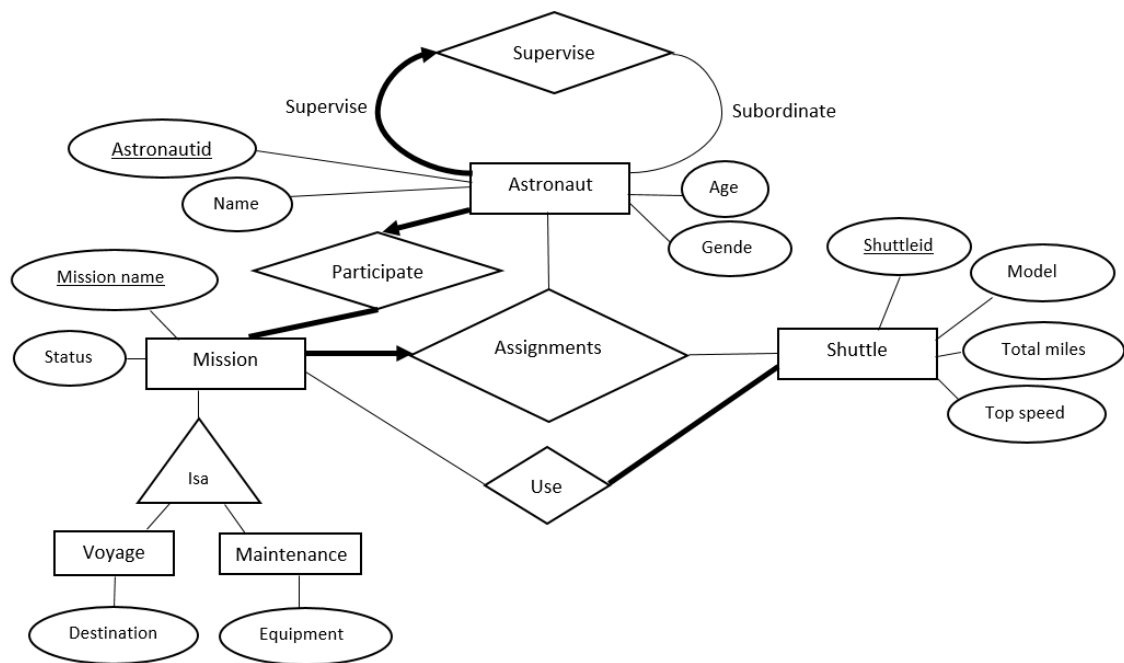
" \Rightarrow " Suppose R with FDs F is in BCNF, then for all $X \rightarrow A$ in F^+ , we know either $A \in X$ or X contains a key for R . Thus for all $X \rightarrow A$ in F^+ , we know at least one of the $A \in X$, X contains a key for R and A is part of some key for R holds true. Thus the condition for 3NF holds and therefore R with FDs F is in 3NF.

" \Leftarrow " Suppose R with FDs F is in 3NF and R has only one key K . Suppose R with FDs is not a BCNF, then there exists a FD $X \rightarrow A$ such that A doesn't contain K and A is not part of X . However, in this case, we know $X \cup (K - A) \rightarrow R$ (since $X \cup (K - A) \rightarrow A \cup (K - A) = K$, thus $X \cup (K - A)$ is a superkey) and $X \cup (K - A)$ is a superkey, which means $K \subset X \cup (K - A)$. $K \subset X \cup (K - A) \implies A \subset X$,

which is a contradiction. Thus, we know R with FDs F is in 3NF and R has only one key K is also in BCNF.

ER Diagram

Assumption: From “One astronaut per mission supervises all the other astronauts for this mission” in the instruction, we infer that each mission should have at least one astronaut.



The relation “Use” constrains that every shuttle should be assigned to at least one mission. And the relation “Participate” constrains that each astronaut can only participate in one mission. The relation “Supervise” constrains that only one astronaut per mission supervises all the other astronauts for this mission. The relation “Assignments” contains astronauts, shuttle and mission and it constrains that each assignment only have one mission.

Normal Forms

(a)

Advantage of decomposing tables: It could remove the redundancy of the tables.
 Disadvantage of decomposing tables: The decomposing of a table may cause a lossy problem. And some queries require join after the decomposition.

(b)

The keys of R are: DFG

Knowing that $D \rightarrow BC$, we have $D \rightarrow BCD$. We also know that $B \rightarrow AC$ and thus we have $D \rightarrow ABCD$. Combine F and we have $DF \rightarrow ABCDF$. Knowing that $AF \rightarrow E$, we have $DF \rightarrow ABCDEF$. Combine G and we have $DFG \rightarrow ABCDEFG$.

(c)

$D \rightarrow BC$, $AF \rightarrow E$ and $B \rightarrow AC$ violate the BCNF rule.

According to $AF \rightarrow E$, first we decompose R into R1=AEF and R2=ABCDFG. The FD of R1=AEF is $AF \rightarrow E$, so R1 is BCNF. The FDs of R2=ABCDFG are $B \rightarrow AC$ and $D \rightarrow BC$ and R2 is not BCNF.

Thus we decompose R2=ABCDFG into R21=ABC and R22= BDFG. The FD of R21=ABC is $B \rightarrow AC$ and it is BCNF. And R22=BDFG contains the key DFG of R, so R22 is BCNF.

Thus, the BCNF decomposition of R is AEF, ABC, BDFG.