

Homework 5 Q3-Q5

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Question 3

a: 4.1.3 b Not a well-defined function, when $x = 2$ or $x = -2$. Thus not a function.

a: 4.1.3 c This is a function. Range of f is $\{f(x) \geq 0\}$

b: 4.1.5 b $\{4, 9, 16, 25\}$

b: 4.1.5 d $\{0, 1, 2, 3, 4, 5\}$

b: 4.1.5 h $\{(1, 1), (2, 1), (3, 1), (1, 2), (2, 2), (3, 2), (1, 3), (2, 3), (3, 3)\}$

b: 4.1.5 i $\{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

b: 4.1.5 l $\{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$

Question 4

I a: 4.2.2 c

Not onto. For example, there is no integer x when $y=2$. One-to-one. If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

I a: 4.2.2 g

Not onto. For example, there is no pair (x,y) , $x \in Z$ and $y \in Z$ when $f(x,y)$ is $(1,5)$. One-to-one. If $(x_1,y_1) \neq (x_2,y_2)$, then $f(x_1,y_1) \neq f(x_2,y_2)$

I a: 4.2.2 k

Not onto. For example, there is no pair (x,y) , $x \in Z^+$ and $y \in Z^+$ when $f(x,y)=1$. One-to-one. If $(x_1,y_1) \neq (x_2,y_2)$, then $f(x_1,y_1) \neq f(x_2,y_2)$

I b: 4.2.4 b

Not Onto. There is no triple can make $f = 000$

Not One-to-one. For example, $f(001) = f(101) = 101$

I b: 4.2.4 c

Both Onto and One-to-one

I b: 4.2.4 d

One-to-one. But not Onto. Function f has a domain with 8 element and target with 16 elements. If it's Onto, domain should have at least 16 element.

I b: 4.2.4 g

Not One-to-one. $f(\{2,3\}) = f(\{1,2,3\}) = \{2,3\}$

Not Onto. There is no element in domain can make $f = \{1\}$

II a: $f : Z \rightarrow Z^+$,

$$f(x) = \begin{cases} 3x & x > 0 \\ 3|x| + 1 & x \leq 0 \end{cases}$$

II b: $f : Z \rightarrow Z^+$, $f(x) = |x| + 1$

II c: $f : Z \rightarrow Z^+$,

$$f(x) = \begin{cases} 2x + 1 & x \geq 0 \\ -2x & x < 0 \end{cases}$$

II d: $f : Z \rightarrow Z^+$, $f(x) = 1$

Question 5

- a: 4.3.2 c** $f^{-1}(x) = (x - 3)/2$
- a: 4.3.2 d** Since $|D| \neq |T|$, The function is not a bijection. There is no inverse function.
- a: 4.3.2 g** The output of f^{-1} is obtained by taking the input string and reversing the bits.
- a: 4.3.2 i** $f^{-1}(x, y) = (x - 5, y + 2)$
- b: 4.4.8 c** $f \circ h(x) = 2x^2 + 5$
- b: 4.4.8 d** $h \circ f(x) = 4x^2 + 12x + 10$
- c: 4.4.2 b** 121
- c: 4.4.2 c** 16
- c: 4.4.2 d** $h \circ f(x) = \lceil x^2/5 \rceil$
- d: 4.4.6 c** 111
- d: 4.4.6 d** $\{101, 111\}$
- d: 4.4.6 e** $\{001, 101, 011, 111\}$
- e: 4.4.4 c** No. We will show that if $g \circ f$ is one-to-one, then f must be one-to-one. If $g \circ f$ is one-to-one, which means if $f(x) \neq f(y)$ then $g(f(x)) \neq g(f(y))$. then $x \neq y$.
- e: 4.4.4 d** Yes. The diagram below illustrates an example: (see next page)

