NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 2 Q5-Q9

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Question 5

a-1: 1.12.2 b

1.	$p \to (q \wedge r)$	Hypothesis
2.	$\neg p \vee (q \wedge r)$	Conditional identities
3.	$(q \wedge r) \vee \neg p$	Commutative laws
4.	$\neg q$	Hypothesis
5.	$\neg q \vee \neg r$	Addition, 4.
6.	$\neg(q \wedge r)$	De Morgan's law
7.	$\neg p$	Disjunctive syllogism, 3,6.

a-1: 1.12.2 e

1.	$p \vee q$	Hypothesis
2.	$\neg p \vee r$	Hypothesis
3.	$q \vee r$	Resolution, 1,2.
4.	$\neg q$	Hypothesis
5.	r	Disjunctive syllogism, 3,4.

a-2: 1.12.3 c

1.	p $\vee q$	Hypothesis
2.	$\neg p \to q$	Conditional identities
3.	$\neg p$	Hypothesis
4.	q	Modus tollens

a-3: 1.12.5 c

- j: I will get a job
- c: I will buy a new car
- h: I will buy a new house

The form of the argument is

$$\begin{array}{l} (c \wedge h) \to j \\ \underline{\neg j \qquad\qquad} \\ \therefore \neg c \end{array}$$

The argument is not valid. when c = T, h = j = F, the hypothesis are both true and the conclusion$\neg c$ is false

a-3: 1.12.5 d

- j: I will get a job
- c: I will buy a new car

- h: I will buy a new house

The form of the argument is

$$\begin{array}{l} (c \wedge h) \to j \\ \neg j \\ h \\ \hline \\ \therefore \neg c \end{array}$$

The argument is valid.

1.	$(c \wedge h) \to j$	Hypothesis
2.	$\neg(c \wedge h) \vee j$	Conditional identities
3.	$j \vee \neg(c \wedge h)$	Commutative laws
4.	$\neg j$	Hypothesis
5.	$\neg(c \wedge h)$	Disjunctive syllogism, 3,4.
6.	$\neg c \vee \neg h$	De Morgan's laws
7.	$\neg h \vee \neg c$	Commutative laws
8.	h	Hypothesis
9.	$\neg c$	Disjunctive syllogism, 7,8.

b-1: 1.13.3 b

	P	Q
a	F	T
b	F	F

$\exists x(P(x) \vee Q(x))$ is true because when x=a, $P(a) \vee Q(a)$ is $F \vee T$ is true. when x=b, Q(b) =F, therefore $\exists x \neg Q(b)$ is true. However, whenever x=a or b, P(a)=false, P(b)=false. There are no element can make P(x)is true. Therefore, both hypotheses are true and the conclusion is false.

b-2: 1.13.5 d

- M(x): x missed class
- D(x): x got a detention

$\forall x(M(x) \to D(x))$
Penelope is a student in the class
$D(Penelope)$

$\therefore M(Penelope)$

Not Valid.
When the third hypothesis is true, D(Penelope) is true. the second hypothesis is true. The first hypothesis is true when M(Penelope) is false, $(\forall x(M(Penelope)) \to D(Penelope))$. However, the conclusion M(Penelope)is false. Therefore, all hypotheses are true and the conclusion is false.

b-2: 1.13.5 e

- M(x): x missed class
- A(x): x got A
- D(x): x got a detention

$\forall x((M(x) \lor D(x)) \rightarrow \neg A(x))$
Penelope is a student in the class
$A(Penelope)$

$\therefore \neg D(Penelope)$

Valid.

1.	$\forall x(M(x)) \lor D(x) \rightarrow \neg A(x)$	Hypothesis
2.	Penelope is a student in the class	Hypothesis
3.	$M(Penelope) \lor D(Penelope) \rightarrow \neg A(Penelope)$	Universal instantiation, 1,2.
4.	$A(Penelope)$	Hypothesis
5.	$\neg(M(Penelope) \lor D(Penelope))$	Modus tollens, 3,4.
6.	$\neg M(Penelope) \land \neg D(Penelope)$	De Morgan's laws
7.	$\neg D(Penelope) \land \neg M(Penelope)$	Commutative laws
8.	$\neg D(Penelope)$	Simplification, 7.

Question 6

2.2.1 d:
Proof.
Direct proof. Assume that a,b are two odd integer. We need to show a*b is also an integer. Since a is odd, a=2m+1. b is odd, b=2n+1.For some integer m,n. Plug the expression for a,b into a*b. $a * b = (2m + 1) * (2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$ Since m,n are integers, then (2mn + m + n)is also an integer. Since a*b=2c+1, where c = 2mn + m + n is an integer, then a*b is odd.

2.2.1 c:
Proof.
Direct proof. Assume that x is a real number and $x \leq 3$, then we need to show $12 - 7x + x^2 \geq 0$, which is $(x - 3)(x - 4) \geq 0$
Since $x \leq 3$, then $x - 3 \leq 0$ and definitely $x - 4 < 0$. In the meantime, since x is a real number, then x-3 and x-4 are also real number. The product of two negative numbers or 0 are equal or greater than 0. Therefore, $(x - 3)(x - 4) \geq 0$, $12 - 7x + x^2 \geq 0$)

Question 7

2.3.1 d:

Proof.
Proof by contrapositive. We assume that n is even and show that for every integer, $n^2 - 2n + 7$ is odd.
If n is even, then n = 2k for some integer k. Plugging in the expression 2k for n in $n^2 - 2n + 7$ gives $n^2 - 2n + 7 = (2k)^2 - 4k + 7 = 4k^2 - 4k + 7 = 2(2k^2 - 2k + 3) + 1$
Since k is an integer, $2k^2 - 2k + 3$ is also an integer. Since $n^2 - 2n + 7 = 2c + 1$, $n = 2k^2 - 2k + 3$, therefore, $n^2 - 2n + 7$ is odd.

2.3.1 f:

Proof.
Proof by contrapositive. We assume that 1/x is not irrational, then prove x is rational.
Since x is a non-zero real number, then 1/x is also a non-zero real number and $a \neq 0$. Every real number is either rational or irrational. Therefore 1/x is not irrational and is real number, 1/x must be rational.

$x = 1/(1/x) = 1/(a/b) = b/a$
x also equal to the ratio of two integers, b and a, since x is non-zero, then $b \neq 0$. Therefore x is rational.

2.3.1 g:

Proof.
Proof by contrapositive. We assume that for every pair of real number x and y, if $x > y$, then prove $x^3 + xy^2 > x^2y + y^3$.
Since x,y are real numbers, then $x^2 + y^2$ is a real number.
Since $x > y$, then $x^2 + y^2 \neq 0$. so $x^2 + y^2 > 0$
therefore $x(x^2 + y^2) > y(x^2 + y^2)$ which is $x^3 + xy^2 > x^2y + y^3$

2.3.1 l:

Proof.
Proof by contrapositive. We assume that for every pair of real number x and y, if $x \leq 10$ and $y \leq 10$, then prove $x + y \leq 20$.
Since x,y are real numbers, then x + y is a real number.
Since $x \leq 10$ and $y \leq 10$, then $x + y \leq 20$. Since the left side is the sum of two smaller numbers and the right side is sum of two larger numbers.

Question 8

2.4.1 c:

Proof.
Proof by contradiction. Suppose the average of three real number is greater than or equal to none of the numbers. In other word, all these three real numbers are less than the mean. assume these three numbers are a,b,c. Since a,b,c are real numbers, then $\overline{abc}$ = abc/3 is a real number. The sum of three numbers should be $a * b * c$.
If all three numbers are less than the mean, then $a < abc/3$, $b < abc/3$, $c < abc/3$, and the sum is $< abc$, which contradicts the fact that the sum is equal to abc.

2.4.1 e:

Proof.
Proof by contradiction. Suppose there is a smallest integer.
Assume there is a smallest negative integer called r. Since r is negative, then $r - 1$ also negative, $r - 1 < r$. $r - 1$ is a integer and smaller than r, which contradicts the assumption that r is the smallest integer.

Question 9

2.5.1 c:

Proof.
Considering two cases, x,y are both odd and x,y are both even.
Case 1: x,y are both odd, then x = 2m + 1 and y = 2n + 1, m and n are integer.
$x + y = (2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$ since m and n are integers, then m + n + 1 is also integer. Therefore, x + y is even.

Case 2: x,y are both even, then x = 2m and y = 2n, m and n are integer.
$x + y = 2m + 2n = 2(m + n)$ since m and n are integers, then m + n is also integer. Therefore, x + y is even.