NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 1

Name: Yiwen Cui

Question 1

A(1):

$$\begin{aligned}10011011_2 &= 1\times 2^7 + 0\times 2^6 + 0\times 2^5 + 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 \\ &= 128+0+0+16+18+0+2+1 = \boxed{155}\end{aligned}$$

A(2): $456_7 = 4\times 7^2 + 5\times 7^1 + 6\times 7^0 = 4\times 49 + 5\times 7 + 6\times 1 = \boxed{237}$

A(3): $38A_{16} = 3\times 16^2 + 8\times 16^1 + 10\times 16^0 = 768+128+10 = \boxed{906}$

A(4): $2214_5 = 2\times 5^3 + 2\times 5^2 + 1\times 5^1 + 4\times 5^0 = 250+50+5+4 = \boxed{309}$

B(1): $69_{10} = 1\times 2^6 + 0\times 2^5 + 0\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 = \boxed{1000101}$

B(2): $485_{10} = \boxed{111100101}$
$485\div 2 = 242/1$ $242\div 2 = 121/0$ $121\div 2 = 60/1$ $60\div 2 = 30/0$ $30\div 2 = 15/0$ $15\div 2 = 7/1$
$7\div 2 = 3/1$ $3\div 2 = 1/1$ $1\div 2 = 0/1$ From right to left fill in the remainder

B(3): $6D1A_{16} = 0110110100011010 = \boxed{110110100011010}$
$6_{16} = 0110$ $D_{16} = 1101$ $1_{16} = 0001$ $A_{16} = 1010$

C(1): $1101011_2 = 01101011_2 = \boxed{6B_{16}}$
$0110 = 6_{16}$ $1011 = B_{16}$

C(2): $895_{10} = \boxed{37F}$
$895\div 16 = 55/15, 15_{10} = F_{16}$ $55\div 16 = 3/7$ $3\div 16 = 0/3$ From right to left fill in the remainder

Question 2

1: $7566_8 + 4515_8 = \boxed{14303_8}$

$$
\begin{array}{cccccc}
 & & 7 & 5 & 6 & 6 \\
+ & & 4 & 5 & 1 & 5 \\
\hline
 & 1 & 4 & 3 & 0 & 3
\end{array}
$$

2: $10110011_2 + 1101_2 = \boxed{11000000_2}$

$$
\begin{array}{ccccccccc}
 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
+ & & & & & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

3: $7A66_{16} + 45C5_{16} = \boxed{C02B_{16}}$

$$
\begin{array}{ccccc}
 & 7 & \text{A} & 6 & 6 \\
+ & 4 & 5 & \text{C} & 5 \\
\hline
 & \text{C} & 0 & 2 & \text{B}
\end{array}
$$

4: $3022_5 - 2433_5 = \boxed{34_5}$

$$
\begin{array}{ccccc}
 & 3 & 0 & 2 & 2 \\
+ & 2 & 4 & 3 & 3 \\
\hline
 & 0 & 0 & 3 & 4
\end{array}
$$

Question 3

A1: 124_{10}= $01111100_{8-bit2'scomp}$

Positive integer's 2's complement representation is equal to itself. Therefore 124's 8-bit two's complement representation is 124 convert to binary representation. Adding one "0" at the very left to make it 8 bit

$124_{10} = 1\times2^6+1\times2^5+1\times2^4+1\times2^3+1\times2^2+0\times2^1+0\times2^0 = 01111100_{8-bit} = 01111100_{8-bit2'scomp}$

A2: -124_{10}= $10000100_{8-bit2'scomp}$

		0	1	1	1	1	1	0	0
+		1	0	0	0	0	1	0	0
	1	0	0	0	0	0	0	0	0

A3: 109_{10}= $01101101_{8-bit2'scomp}$

Positive integer's 2's complement representation is equal to itself. Therefore 124's 8-bit two's complement representation is 109 convert to binary representation. Adding one "0" at the very left to make it 8 bit

$124_{10} = 1\times2^6+1\times2^5+0\times2^4+1\times2^3+1\times2^2+0\times2^1+1\times2^0 = 01101101_{8-bit2'scomp}$

A4: -79_{10}= $10110001_{8-bit2'scomp}$

$79_{10} = 1\times2^6+0\times2^5+0\times2^4+1\times2^3+1\times2^2+1\times2^1+1\times2^0 = 1001111 = 01001111_{8-bit}$

		0	1	0	0	1	1	1	1
+		1	0	1	1	0	0	0	1
	1	0	0	0	0	0	0	0	0

B1: $00011110_{8-bit2'scomp}$ = 30

Since the first digit is 0, then it's a positive integer. Positive integer's 2's complement representation is equal to itself.

$00011110 = 1\times2^4+1\times2^3+1\times2^2+1\times2^1+0\times2^0 = 30$

B2: $11100110_{8-bit2'scomp}$ = -26

Since the first digit is 1, then it's a negative integer

		1	1	1	0	0	1	1	0
+		0	0	0	1	1	0	1	0
	1	0	0	0	0	0	0	0	0

$00011010 = 1\times2^4+1\times2^3+0\times2^2+1\times2^1+0\times2^0 = 26$

Since it's a negative integer, then it's -26

B3: $00101101_{8-bit2'scomp}$ = 45

Since the first digit is 0, then it's a positive integer. Positive integer's 2's complement representation is equal to itself.

$00011110 = 1\times2^5+0\times2^4+1\times2^3+1\times2^2+0\times2^1+1\times2^0 = 45$

B4: $10011110_{8-bit2'scomp} =$ -98
Since the first digit is 1, then it's a negative integer

$$\begin{array}{cccccccccc} & & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ + & & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$01100010 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 98$
Since it's a negative integer, then it's -98

Question 4

1: 1.2.4 b

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

1: 1.2.4 c

p	q	r	r$\vee(p \wedge \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2: 1.3.4 b

p	q	(p $\to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

2: 1.3.4 d

p	q	(p $\leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 5

1: 1.2.7 b $(B \wedge D \wedge M) \vee (B \wedge D \wedge \neg M) \vee (B \wedge \neg D \wedge M) \vee (\neg B \wedge D \wedge M)$

1: 1.2.7 c $B \vee (D \wedge M)$

2: 1.3.7 b $(s \vee y) \to p$

2: 1.3.7 c $p \to y$

2: 1.3.7 d $p \leftrightarrow (s \wedge y)$

2: 1.3.7 e $p \to (s \vee y)$

3: 1.3.9 c $c \to p$

3: 1.3.9 d $c \to p$

Question 6

1: 1.3.6 b If Joe is eligible for the honors program then he has to maintain a B average.

1: 1.3.6 c If Rajiv can go on the roller coaster, then he is at least four feet tall.

1: 1.3.6 d If Rajiv is at least for feet tall, then he can go on the roller.

2: 1.3.10 c $(p \vee r) \leftrightarrow (q \wedge r)$
False. $T \leftrightarrow F$ is False.

2: 1.3.10 d $(p \wedge r) \leftrightarrow (q \wedge r)$
Unknown. If r is True, then the expression is False. $(T \leftrightarrow F)$
If r is False, then the expression is True. $(F \leftrightarrow F)$

2: 1.3.10 e $p \rightarrow (r \vee q)$
Unknown. If r is True, then the expression is True. $(T \rightarrow T)$
If r is False, then the expression is False. $(T \rightarrow F)$

2: 1.3.10 f $(p \wedge q) \rightarrow r$
True. $p \wedge q$ is False. No matter r is True or False, the result is False. ($F \rightarrow F$ is True, $F \rightarrow T$ is True)

Question 7

1.4.5 b

$\neg j \to (l \lor \neg r)$
$(r \land \neg l) \to j$
Logically equivalent.

j	l	r	$\neg j \to (l \lor \neg r)$	$(r \land \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

1.4.5 c

$j \to \neg l$
$\neg j \to l$
Not logically equivalent.

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

1.4.5 d

$(r \lor \neg l) \to j$
$j \to (r \land \neg l)$
Not logically equivalent.

j	l	r	$(r \lor \neg l) \to j$	$j \to (r \land \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Question 8

1: 1.5.2 c

$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	conditional identities
$\neg p \lor (q \land r)$	distributive laws
$p \to (q \land r)$	conditional identities

1: 1.5.2 f

$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	de morgan's law
$\neg p \land (\neg\neg p \lor \neg q)$	de morgan's law
$\neg p \land (p \lor \neg q)$	double negation law
$(\neg p \land p) \lor (\neg p \land \neg q)$	distributive laws
$F \lor (\neg p \land \neg q)$	complement laws
$\neg p \land \neg q$	identity laws

1: 1.5.2 i

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	conditional identities
$(\neg p \lor \neg q) \lor r$	de morgan's laws
$(\neg p \lor r) \lor \neg q$	associative laws
$\neg(\neg p \lor r) \to \neg q$	conditional identities
$\neg\neg p \land \neg r \to \neg q$	de morgan's laws
$p \land \neg r \to \neg q$	double negation law

2: 1.5.3 c

$\neg r \lor (\neg r \to p)$	
$\neg r \lor (\neg\neg r \lor p)$	conditional identities
$\neg r \lor (r \lor p)$	double negation law
$(\neg r \lor r) \lor p$	associative laws
$T \lor p$	conditional identities
T	domination laws

2: 1.5.3 d

$\neg(p \to q) \to \neg q$	
$(p \to q) \lor \neg q$	conditional identities
$(\neg p \lor q) \lor \neg q$	conditional identities
$(\neg q \lor q) \lor \neg p$	associative laws
$T \lor \neg p$	conditional identities
T	domination laws

Question 9

1: 1.6.3 c $\exists x(x = x^2)$

1: 1.6.3 d $\forall x(x \leq x^2)$

2: 1.7.4 b $\forall x(\neg S(x) \wedge W(x))$

2: 1.7.4 c $\forall x(S(x) \rightarrow \neg W(x))$

2: 1.7.4 d $\exists x(S(x) \wedge W(x))$

Question 10

1: 1.7.9 c True

1: 1.7.9 d True

1: 1.7.9 e True

1: 1.7.9 f True

1: 1.7.9 g False. counter-example is c

1: 1.7.9 h True

1: 1.7.9 i True

2: 1.9.2 b True. When x =2 , Q(2,1),Q(2,2),Q(2,3) are all true

2: 1.9.2 c True. when x=1 that makes P(y,x) all true

2: 1.9.2 d False. There is no x and y make S(x,y) true

2: 1.9.2 e False. Not all x has a y that makes Q(x,y) true.

2: 1.9.2 f True. For every x, when y=1 makes P(x,y) all true.

2: 1.9.2 g False. P(1,2) is false

2: 1.9.2 h True. Q(2,1) is true

2: 1.9.2 i True. All $\neg S(x, y)$ are true

Question 11

1: 1.10.4 c $\exists x \exists y(x + y = xy)$

1: 1.10.4 d $\forall x \forall y(x > 0 \wedge y > 0 \rightarrow x/y > 0)$

1: 1.10.4 e $\forall x((0 < x < 1) \rightarrow (1/x > 1))$

1: 1.10.4 f $\neg \exists x \forall y(y \geq x)$

1: 1.10.4 g $\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$

2: 1.10.7 c $\exists x(N(x) \wedge D(x))$

2: 1.10.7 d $\exists x \forall y(x = Sam \rightarrow (P(x, y) \wedge D(y))$

2: 1.10.7 e $\exists x \forall y(N(x) \wedge P(x, y))$

2: 1.10.7 f $\exists x((N(x) \wedge D(x)) \wedge \forall y(N(y) \wedge D(y)) \rightarrow x = y)$

3: 1.10.10 c $\forall x \exists y(y \neq Math101 \rightarrow T(x, y))$

3: 1.10.10 d $\exists x \forall y(y \neq Math101 \rightarrow T(x, y))$

3: 1.10.10 e $\forall x \exists y \exists z(x \neq Sam \rightarrow (y \neq z \wedge T(x, y) \wedge T(x, z)))$

3: 1.10.10 f $\exists y \exists z(y \neq z \wedge T(Sam, y) \wedge T(Sam, z)) \wedge \forall w(T(Sam, w) \rightarrow (w = y \vee w = z))$

Question 12

1: 1.8.2 b

- $\forall x(D(x) \vee P(x) \vee (D(x) \wedge P(x))$
- Negation: $\neg\forall x(D(x) \vee P(x) \vee (D(x) \wedge P(x))$
- Applying De Morgan's law: $\exists x(\neg(D(x) \wedge \neg P(x) \wedge \neg(D(x) \wedge P(x)))$
- English: There is a patient who was not given placebo, and was not given medication and was not given both. (There is a patient who was not given anything)

1: 1.8.2 c

- $\exists x(D(x) \wedge M(x))$
- Negation: $\neg\exists x(D(x) \wedge M(x))$
- Applying De Morgan's law: $\forall x(\neg(D(x) \vee \neg M(x))$
- English: Every patient either did not take medication or did not have migraines.

1: 1.8.2 d

- $\forall x(P(x) \to M(x))$
- Negation: $\neg\forall x(P(x) \to M(x))$
- Applying De Morgan's law: $\exists x(P(x) \wedge \neg M(x))$
- English: There is a patient took the placebo and did not have migraines.

1: 1.8.2 e

- $\exists x(M(x) \wedge P(x))$
- Negation: $\neg\exists x(M(x) \wedge P(x))$
- Applying De Morgan's law: $\forall x(\neg P(x) \vee \neg M(x))$
- English: Every patient either did not have migraines or was not given the placebo.

2: 1.9.4 c

$\forall x\exists y(P(x, y) \wedge \neg Q(x, y))$

2: 1.9.4 d

$\forall x\exists y(P(x, y) \wedge \neg P(y, x)) \vee ((P(y, x) \wedge \neg P(x, y))$

2: 1.9.4 e

$\forall x\forall y\neg P(x, y) \vee \exists x\exists y\neg Q(x, y)$