# Modeling VI

# Introduction

- Now We Consider
  - Categorical Response Variables
  - Numerical/Categorical Explanatory Variables
- Focus is on Classification
- Bless Your Soul with Ch 4 in ISLR

# Introduction

- Basic Case: Binary Response
  - Variable Has Two Possible Outcomes
  - Typically, Yes or No Responses to a Question
  - Example
    - Y = Do You Enjoy Your Experience in the Presence of the Doctor?
    - Y = Did You Pass Your STOR 320 Class?
    - Y = Are You Comfortable Having Your Mind Blown?

# Scenario

- Question: Are Students Who Get Good Grades in STOR 320 Less Likely to Recommend This Class To an Enemy?
  - Y = Would You Recommend STOR 320 to an Enemy?
  - X = Grade in STOR 320
- Why is Linear Regression Inappropriate?

# Model Construction

- Bernouilli Random Variable

$$Y = \begin{cases} 1 & if\ Yes \\ 0 & if\ No \end{cases}$$

$$p = E(Y) = P(Y = 1)$$

- Sample $n$ Students

$$\mathrm{Y}' = \sum Y_i \sim Binomial(n, p)$$

$$\hat{p} = \frac{\sum y_i}{n}$$

Estimated Probability that a Student Would This Recommend Class to an Enemy Based on a Sample

- Analyze the Effect of $X$ on $p$

$$p = E(Y|X) \neq \beta_0 + \beta_1 X$$

## Model Construction

- Modeling the Mean
  - Logit Link Function

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

Odds of Recommending STOR 320

  - Understanding Odds
    - Odds of Recommending = 1
    - Odds of Recommending < 1
    - Odds of Recommending > 1

# Model Construction

- Solving for $\frac{p}{1-p}$

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

$$\frac{p}{1-p} = e^{\beta_0+\beta_1 X}$$

Odds of Recommending STOR 320 Given the Student's Grade

- Solving for $p$

$$p = e^{\beta_0+\beta_1 X} - pe^{\beta_0+\beta_1 X}$$

$$p(1 + e^{\beta_0+\beta_1 X}) = e^{\beta_0+\beta_1 X}$$

$$p = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}}$$

Probability a Student Will Recommend STOR 320 Given the Student's Grade

## Logistic Regression for Classification

- Recall: $Y = \begin{cases} 1 & if\ Yes \\ 0 & if\ No \end{cases}$
- After Getting Data, We Estimate
  - $\hat{\beta}_0$
  - $\hat{\beta}_1$
  - $\hat{p} = \frac{e^{\hat{\beta}_0+\hat{\beta}_1 X}}{1+e^{\hat{\beta}_0+\hat{\beta}_1 X}}$

Estimated Probability a Student Recommends Course Given the Student's Grade

- Two Scenarios
  - $\hat{p} < 0.5 \Rightarrow \hat{Y} = 0$
  - $\hat{p} > 0.5 \Rightarrow \hat{Y} = 1$

## Evaluating the Logistic Regression Model

- Two Methods
  - Leave Out Data Intentionally
  - Use Cross-Validation

- Positives and Negatives
  - True Positive = Predicted a Recommendation and the Student Recommended
  - False Positive=Predicted a Recommendation and the Student Didn’t Recommend
  - False Negative = Predicted a Student Wouldn’t Recommend and They Did Recommend
  - True Negative = Predicted a Student Wouldn’t Recommend and They Didn’t Recommend

# Evaluating the Logistic Regression Model

- Confusion Matrix

| | Predicted | |
|---|---|---|
| **Actual** | *Will Recommend* | *Won't Recommend* |
| *Recommends* | $n_{11}$ | $n_{12}$ |
| *Doesn't Recommend* | $n_{21}$ | $n_{22}$ |

- Sensitivity:
  $n_{11}/(n_{11}+n_{12})$
- Specificity:
  $n_{22}/(n_{21}+n_{22})$
- False Positive Rate:
  $n_{12}/(n_{11}+n_{12})$
- False Negative Rate:
  $n_{21}/(n_{21}+n_{22})$

## I Won't Go Down With This Ship

- Titanic Survival Data

```
> library(titanic)
```

- Response Variable

$$Y = \begin{cases} 1 & if \;\; Survived \\ 0 & if \;\; Did\ Not\ Survive \end{cases}$$

- Explanatory Variables
  - Passenger Class
  - Sex
  - Age
  - Siblings/Spouses Aboard
  - Parents/Children Aboard
  - Passenger Fare
  - Port of Embarkation

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- Titanic Survival Data (Continued)
  - Selecting Variables of Interest

```
> TRAIN=titanic_train[,c(2,3,5,6,7,8,10,12)]
> TEST=titanic_test[,c(2,4,5,6,7,9,11)])
```

  - Glimpse of Data

```
glimpse(TRAIN)
```

```
## Observations: 891
## Variables: 8
## $ Survived <int> 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1,...
## $ Pclass   <int> 3, 1, 3, 1, 3, 3, 1, 3, 3, 2, 3, 1, 3, 3, 3, 2, 3, 2,...
## $ Sex      <chr> "male", "female", "female", "female", "male", "male",...
## $ Age      <dbl> 22, 38, 26, 35, 35, NA, 54, 2, 27, 14, 4, 58, 20, 39,...
## $ SibSp    <int> 1, 1, 0, 1, 0, 0, 0, 3, 0, 1, 1, 0, 0, 1, 0, 0, 4, 0,...
## $ Parch    <int> 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 1, 0, 0, 5, 0, 0, 1, 0,...
## $ Fare     <dbl> 7.2500, 71.2833, 7.9250, 53.1000, 8.0500, 8.4583, 51....
## $ Embarked <chr> "S", "C", "S", "S", "S", "Q", "S", "S", "S", "C", "S"...
```

```
glimpse(TEST)
```

Problem?

```
## Observations: 418
## Variables: 7
## $ Pclass   <int> 3, 3, 2, 3, 3, 3, 3, 2, 3, 3, 3, 1, 1, 2, 1, 2, 2, 3,...
## $ Sex      <chr> "male", "female", "male", "male", "female", "male", "...
## $ Age      <dbl> 34.5, 47.0, 62.0, 27.0, 22.0, 14.0, 30.0, 26.0, 18.0,...
## $ SibSp    <int> 0, 1, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 0, 0,...
## $ Parch    <int> 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,...
## $ Fare     <dbl> 7.8292, 7.0000, 9.6875, 8.6625, 12.2875, 9.2250, 7.62...
## $ Embarked <chr> "Q", "S", "Q", "S", "S", "S", "Q", "S", "C", "S", "S"...
```

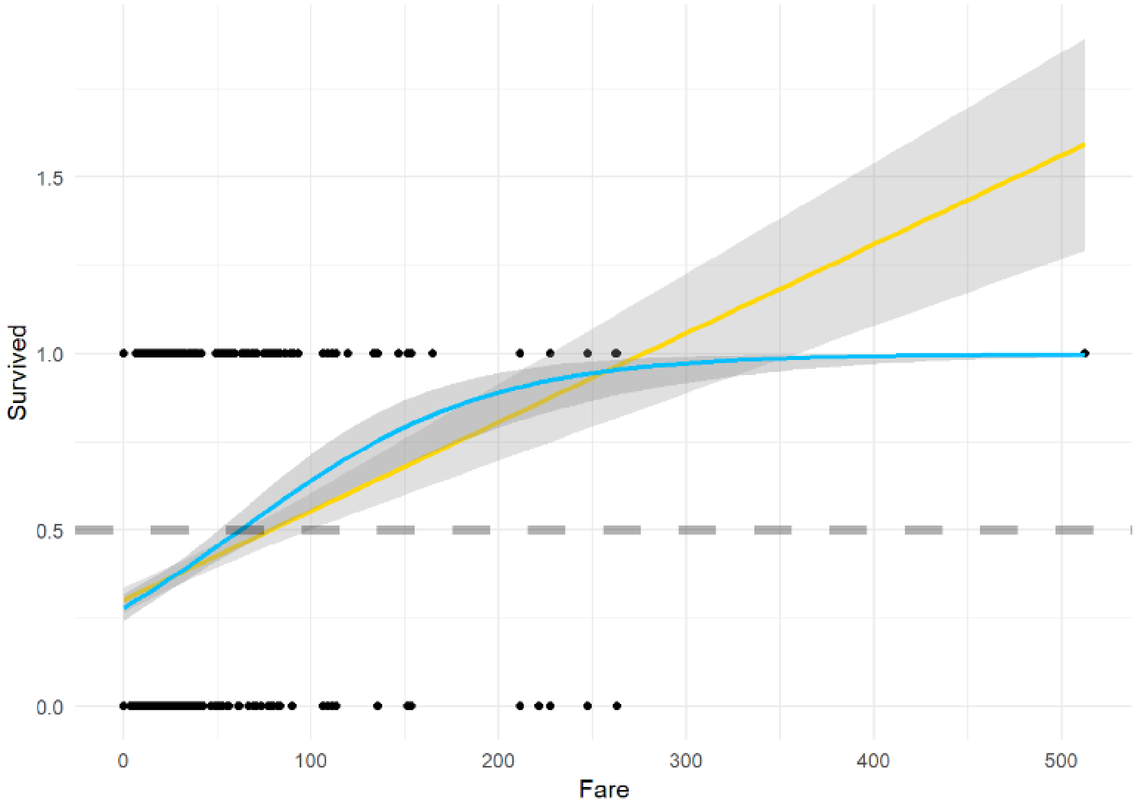
Pause For Lyrics

Every night in my dreams
I see you, I feel you
That is how I know you go on

# I Won't Go Down With This Ship

- Visualizing the Data

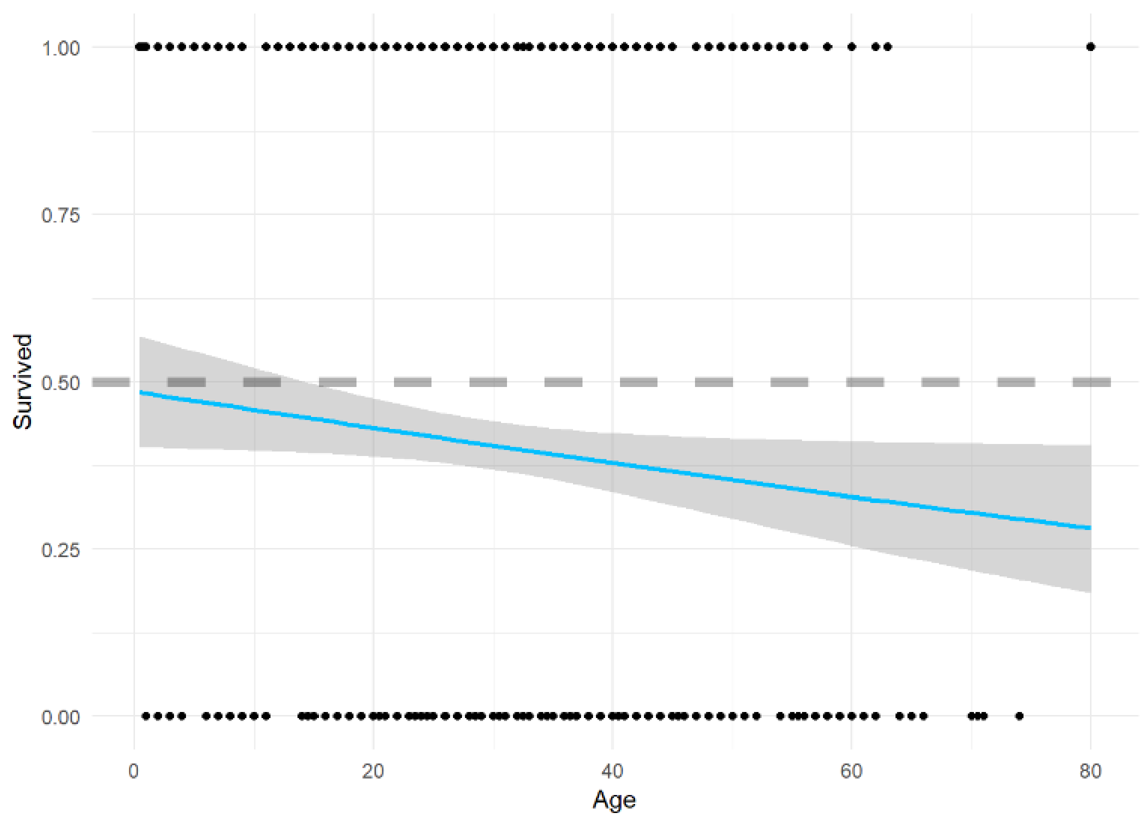
```
ggplot(TRAIN) + geom_point(aes(x=Fare,y=Survived)) + theme_minimal() +
  geom_smooth(aes(x=Fare,y=Survived),method="lm",alpha=0.3,color="gold") +
  geom_smooth(aes(x=Fare,y=Survived),method="glm",
              method.args=list(family="binomial"),color="deepskyblue1") +
  geom_hline(yintercept=0.5,linetype="dashed",size=2,alpha=0.3)
```



# I Won’t Go Down With This Ship

- Visualizing the Data (Continued)

```r
ggplot(TRAIN) + geom_point(aes(x=Age,y=Survived)) + theme_minimal() +
  geom_smooth(aes(x=Age,y=Survived),method="glm",
              method.args=list(family="binomial"),color="deepskyblue1") +
  geom_hline(yintercept=0.5,linetype="dashed",size=2,alpha=0.3)
```



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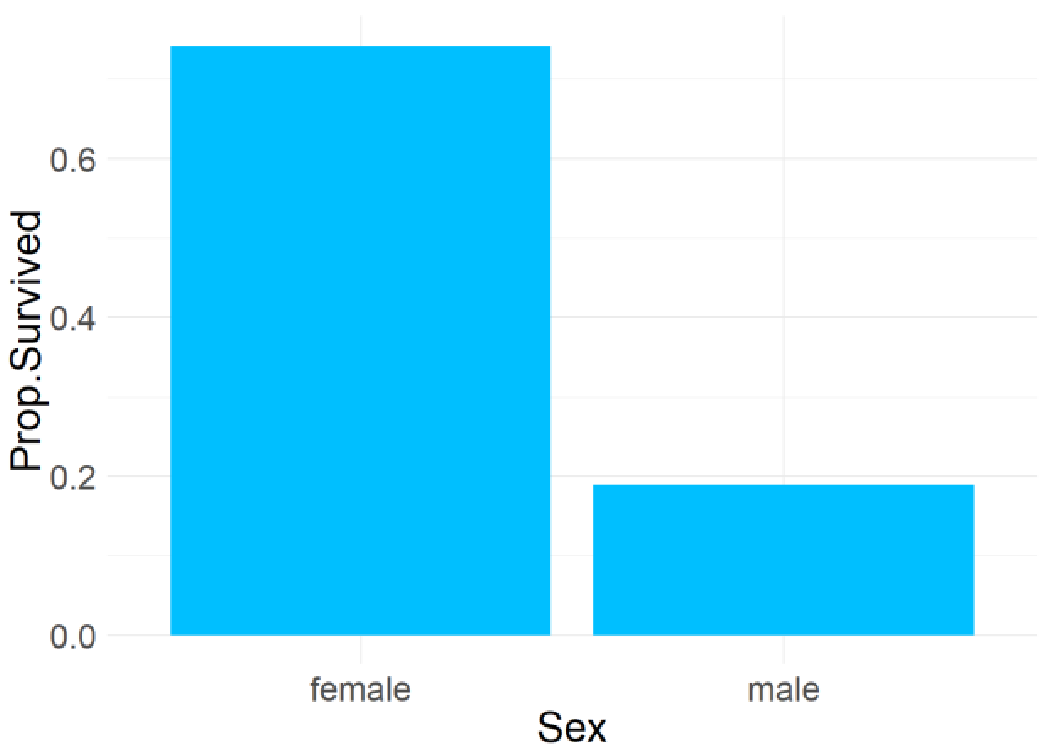
- Visualizing the Data (Continued)

```
TRAIN %>%
  mutate(Sex=factor(Sex)) %>%
  group_by(Sex) %>%
  summarize(Prop.Survived=mean(Survived)) %>%
  ggplot() +
  geom_bar(aes(x=Sex,y=Prop.Survived),
           stat="Identity",fill="deepskyblue1") +
  theme_minimal() +
  theme(text=element_text(size=20))
```



Pause For Lyrics

Far across the distance
And spaces between us
You have come to show you go on

I Won't Go Down With This Ship

- Logistic Regression Models
    - Split Training Set Up

```
> set.seed(216)
> sample.in=sample(1:dim(TRAIN)[1],
               size=floor(0.8*dim(TRAIN)[1]))
> TRAIN.IN=TRAIN[sample.in,
               c("Survived","Fare","Sex","Age")]
> TRAIN.OUT=TRAIN[-sample.in,
                 c("Survived","Fare","Sex","Age")]
```

- Modeling the Probability of Survival Given the Ticket Fare, the Sex of the Passenger, and the Age of the Passenger

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- Logistic Regression Models (Cont.)
    - Including 3-Way Interaction

```
logmod1=glm(Survived~.^3,family="binomial",data=TRAIN.IN)
tidy(logmod1)[,c("term","estimate","p.value")]
```

```
## # A tibble: 8 x 3
##   term              estimate p.value
##   <chr>                <dbl>   <dbl>
## 1 (Intercept)       0.959    0.0719
## 2 Fare             -0.0132   0.357
## 3 Sexmale          -1.54     0.0182
## 4 Age              -0.0362   0.0745
## 5 Fare:Sexmale      0.0180   0.255
## 6 Fare:Age          0.00177  0.00684
## 7 Sexmale:Age      -0.000359 0.988
## 8 Fare:Sexmale:Age -0.00168  0.0140
```

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- Logistic Regression Models (Cont.)
    - Only 2-Way Interactions

```
logmod2=glm(Survived~.*.,family="binomial",data=TRAIN.IN)
tidy(logmod2)[,c("term","estimate","p.value")]
```

```
## # A tibble: 7 x 3
##   term            estimate p.value
##   <chr>              <dbl>   <dbl>
## 1 (Intercept)    0.0835   0.846
## 2 Fare           0.0202   0.0459
## 3 Sexmale       -0.472    0.355
## 4 Age            0.00244  0.858
## 5 Fare:Sexmale -0.0204   0.0225
## 6 Fare:Age       0.000255 0.188
## 7 Sexmale:Age   -0.0456   0.00482
```

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- Logistic Regression Models (Cont.)
  - No Way Interactions

```
logmod3=glm(Survived~.,family="binomial",data=TRAIN.IN)
tidy(logmod3)[,c("term","estimate","p.value")]
```

```
## # A tibble: 4 x 3
##   term        estimate  p.value
##   <chr>          <dbl>    <dbl>
## 1 (Intercept)   1.03   1.42e- 4
## 2 Fare          0.0117 2.23e- 5
## 3 Sexmale      -2.32   6.58e-28
## 4 Age          -0.0157 2.87e- 2
```

Pause For Lyrics

Near, far, wherever you are
I believe that the heart does go on

I Won’t Go Down With This Ship

- Getting Predictions

```
TRAIN.OUT2 = TRAIN.OUT %>%
             mutate(p1=predict(logmod1,newdata=TRAIN.OUT,type="response"),
                    p2=predict(logmod2,newdata=TRAIN.OUT,type="response"),
                    p3=predict(logmod3,newdata=TRAIN.OUT,type="response")) %>%
             select(Survived,p1,p2,p3) %>%
             mutate(S1=ifelse(p1<0.5,0,1),
                    S2=ifelse(p2<0.5,0,1),
                    S3=ifelse(p3<0.5,0,1))
head(TRAIN.OUT2,15)
```

```
##    Survived        p1        p2        p3 S1 S2 S3
## 1         1 0.9690919 0.9092749 0.7802745  1  1  1
## 2         1 0.7754082 0.7600334 0.6058744  1  1  1
## 3         1 0.2080353 0.2054202 0.2124202  0  0  0
## 4         0 0.6660041 0.6390900 0.7598035  1  1  1
## 5         0        NA        NA        NA NA NA NA
## 6         1        NA        NA        NA NA NA NA
## 7         0 0.5144529 0.6150895 0.6255526  1  1  1
## 8         0        NA        NA        NA NA NA NA
## 9         0 0.3504463 0.3477779 0.2826244  0  0  0
## 10        0 0.2084528 0.2141609 0.1755685  0  0  0
## 11        0 0.3588175 0.3684181 0.2646063  0  0  0
## 12        0 0.2278485 0.2365545 0.1841222  0  0  0
## 13        0 0.1588185 0.1560858 0.1590190  0  0  0
## 14        1 0.2135621 0.2103355 0.2445736  0  0  0
## 15        1        NA        NA        NA NA NA NA
```

Why?

# I Won't Go Down With This Ship

- Getting Predictions

```
TRAIN.OUT3=na.omit(TRAIN.OUT2)
head(TRAIN.OUT3,20)
```

```
##     Survived        p1        p2        p3 S1 S2 S3
## 1          1 0.9690919 0.9092749 0.7802745  1  1  1
## 2          1 0.7754082 0.7600334 0.6058744  1  1  1
## 3          1 0.2080353 0.2054202 0.2124202  0  0  0
## 4          0 0.6660041 0.6390900 0.7598035  1  1  1
## 7          0 0.5144529 0.6150895 0.6255526  1  1  1
## 9          0 0.3504463 0.3477779 0.2826244  0  0  0
## 10         0 0.2084528 0.2141609 0.1755685  0  0  0
```

```
mean(TRAIN.OUT3$S1==TRAIN.OUT3$S2)
```

```
## [1] 0.993007
```

```
mean(TRAIN.OUT3$S2==TRAIN.OUT3$S3)
```

```
## [1] 1
```

What Do You Notice About the Predictions?

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- Getting Predictions

```
TRAIN.OUT4=TRAIN.OUT3 %>% select(-p2,-S2)
head(TRAIN.OUT4,8)
```

```
##    Survived        p1        p3 S1 S3
## 1         1 0.9690919 0.7802745  1  1
## 2         1 0.7754082 0.6058744  1  1
## 3         1 0.2080353 0.2124202  0  0
## 4         0 0.6660041 0.7598035  1  1
## 7         0 0.5144529 0.6255526  1  1
## 9         0 0.3504463 0.2826244  0  0
## 10        0 0.2084528 0.1755685  0  0
## 11        0 0.3588175 0.2646063  0  0
```

Where Do You See Error?

Pause For Lyrics

Once more you open the door
And you're here in my heart
And my heart will go on and on

I Won’t Go Down With This Ship

- Evaluating Results
  - Helpful Modifications

```
TRAIN.OUT5 = TRAIN.OUT4 %>%
             select(-p1,-p3) %>%
             mutate(Survived=factor(Survived),S1=factor(S1),S3=factor(S3)) %>%
             mutate(Survived=fct_recode(Survived,"Survived"="1","Died"="0"),
                    S1=fct_recode(S1,"Will Survive"="1","Will Die"="0"),
                    S3=fct_recode(S3,"Will Survive"="1","Will Die"="0")) %>%
             mutate(Survived=factor(Survived,levels=c("Survived","Died")),
                    S1=factor(S1,levels=c("Will Survive","Will Die")),
                    S3=factor(S3,levels=c("Will Survive","Will Die")))
head(TRAIN.OUT5)
```

```
##   Survived           S1           S3
## 1 Survived Will Survive Will Survive
## 2 Survived Will Survive Will Survive
## 3 Survived     Will Die     Will Die
## 4     Died Will Survive Will Survive
## 5     Died Will Survive Will Survive
## 6     Died     Will Die     Will Die
```

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- Evaluating Results (Continued)
  - Confusion Matrix
    - Including 3-Way Interactions

```
RESULTS1=table(TRAIN.OUT5$Survived,TRAIN.OUT5$S1) %>%
          prop.table()
print(RESULTS1)
```

```
##
##              Will Survive   Will Die
##   Survived    0.32867133 0.13986014
##   Died        0.07692308 0.45454545
```

- No Way Interactions

```
RESULTS3=table(TRAIN.OUT5$Survived,TRAIN.OUT5$S3) %>%
          prop.table()
print(RESULTS3)
```

```
##
##              Will Survive   Will Die
##   Survived    0.33566434 0.13286713
##   Died        0.07692308 0.45454545
```

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- Evaluating Results (Continued)
  - Error Statistics
    - Code

```
ERROR.RESULTS = tibble(
    Model=c("3 Way","No Way"),
    Sensitivity=c(RESULTS1[1,1]/sum(RESULTS1[1,]),RESULTS3[1,1]/sum(RESULTS3[1,])),
    Specificity=c(RESULTS1[2,2]/sum(RESULTS1[2,]),RESULTS3[2,2]/sum(RESULTS3[2,])),
    FPR=c(RESULTS1[1,2]/sum(RESULTS1[1,]),RESULTS3[1,2]/sum(RESULTS3[1,])),
    FNR=c(RESULTS1[2,1]/sum(RESULTS1[2,]),RESULTS3[2,1]/sum(RESULTS3[2,]))
)
print(ERROR.RESULTS)
```

    - Results

| Model | Sensitivity | Specificity | FPR | FNR |
|---|---|---|---|---|
| <chr> | <dbl> | <dbl> | <dbl> | <dbl> |
| 3 Way | 0.701 | 0.855 | 0.299 | 0.145 |
| No Way | 0.716 | 0.855 | 0.284 | 0.145 |

Closing

# Disperse and Make Reasonable Decisions