Modeling IX

Introduction

- Continue Model Selection Using Shrinkage Estimation Methods
- Tutorial
  - Download Rmd
  - Knit the Document
  - Extended from Last Tutorial
  - We are Beginning in Part 2 Chunk 4
- Do You Remember What We Learned? Let Me Remind You.

Irrelevant
Nonsense

Watch Me Whip Watch Me Lasso

Recall Info

- Classic Linear Model Estimation
  - Minimize Sum of Squared Error

$$SSE = \sum [y_i - (\beta_0 + \boldsymbol{x_i}'\boldsymbol{\beta})]^2$$

  - Optimization: Find $\widehat{\beta_0}$ and $\widehat{\boldsymbol{\beta}}$ that Make SSE as Small as Possible
  - $\widehat{\beta_0}$ and $\widehat{\boldsymbol{\beta}}$ are Easily Found Using Matrix Representation

- Regularized Estimation
  - Produces Biased Estimates
  - Shrinks Coefficients Toward 0
  - Favors Smaller Models
  - May Lead to a Better Model for Out-of-Sample Prediction

Recall Info

- Three Popular Methods
  - Download R Package

```
> library(glmnet)
```

  - Penalized SSE

$$PSSE = SSE + \lambda[(1-\alpha)\sum_{i=1}^{p}\beta_i^2 + \alpha\sum_{i=1}^{p}|\beta_i|]$$

  - Variations
    - Ridge (1970): $\lambda = 1 \,\&\, \alpha = 0$
    - Lasso (1996): $\lambda = 1 \,\&\, \alpha = 1$
    - Elastic Net (2005)

      $\lambda = 1 \;\&\; 0 < \alpha < 1$
  - Notice When
    - $\lambda = 0$ ➡ PSSE=SSE
    - As λ Gets Bigger, the Coefficients Approach 0

Next Steps

- Tuning Parameters
  - Use Cross-Validation to Choose Tuning Parameters λ & α
  - Constraints
    - $\lambda > 0$
    - $0 \leq \alpha \leq 1$
  - Best Approach:
    - Divide Data Into Train & Test
    - Loop Over a Vector of Alpha
    - Find Best Lambda for Each Alpha Considered Using CV in Train
    - For Each Alpha and Best Lambda, Predict on Test and Select Alpha and Lambda that Minimize MSE

Part 2: Shrinkage Estimation and More Meditation

- Run Chunk 4
  - Illustration of 10 Fold CV
  - Finding Best Combination of Alpha and Lambda

alpha	lambda	MSE
0.0	17.282127	176.3021
0.1	7.837234	146.4758
0.2	5.180181	139.9872
0.3	3.453454	133.7793
0.4	2.590091	130.7873
0.5	2.495819	132.6983
0.6	1.895081	129.1495
0.7	1.624355	128.1601
0.8	1.559887	129.2083
0.9	1.386566	128.5799
1.0	1.247909	128.0857

Best: $\alpha = 1$ & $\lambda = 1.25$

Part 2: Shrinkage Estimation and More Meditation

- Run Chunk 5
  - The Top 4 Models

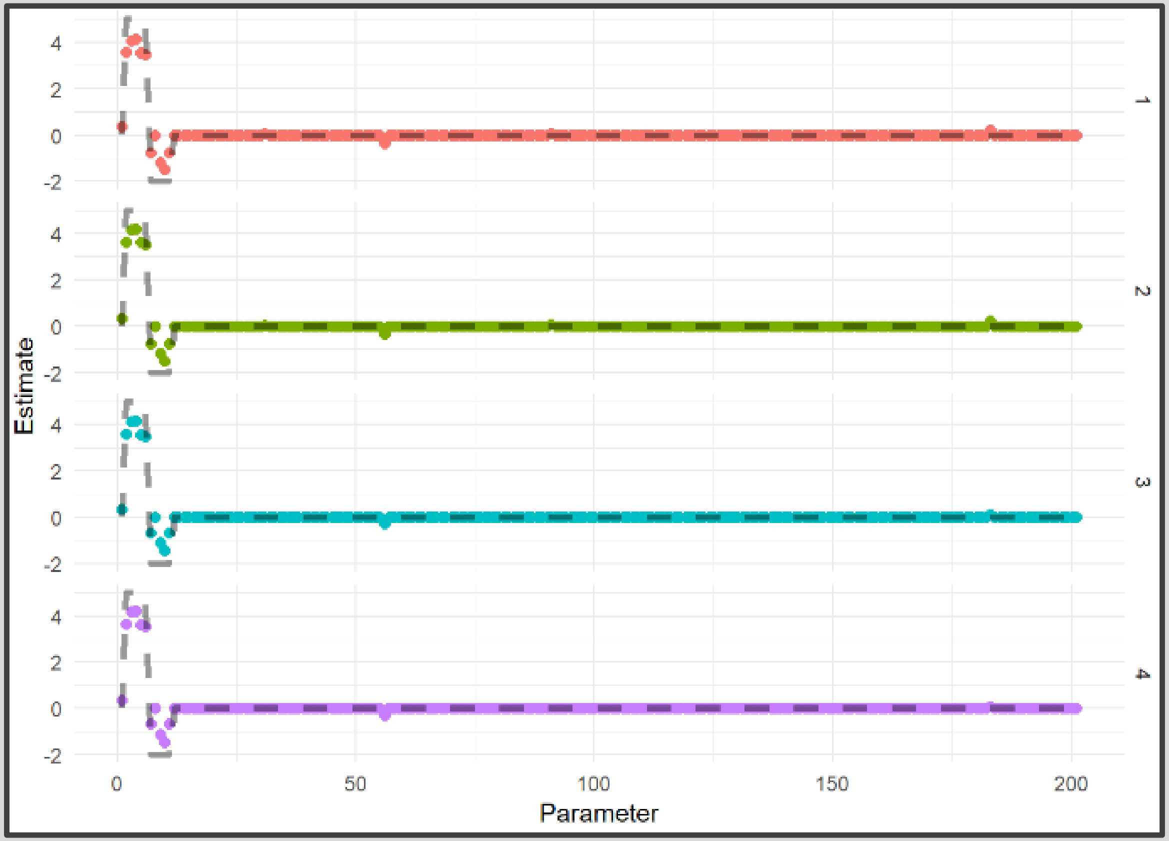
```
##    alpha   lambda      MSE
## 1   0.6 1.895081 129.1495
## 2   0.7 1.624355 128.1601
## 3   0.9 1.386566 128.5799
## 4   1.0 1.247909 128.0857
```

- Question: How Different Are These Models?

- For Each Alpha & Lambda,
  - Get Final Coefficients
  - Compare Across Models
  - Compare to True Values

Part 2: Shrinkage Estimation and More Meditation

- Chunk 5 (Continued)
  - Visualizing Top Four
    - Points Show Estimates
    - Dashed Line Shows Truth



Part 3: Less Meditation and More Application

- Built-In Data `> mpg`
  - n=234
  - Focus is on Modeling Hwy MPG
  - Subset Data to Include Only Wanted Covariates

year <int>	displ <dbl>	cyl <int>	drv <chr>	cty <int>	hwy <int>	fl <chr>	class <chr>
1999	1.8	4	f	18	29	p	compact
1999	1.8	4	f	21	29	p	compact
2008	2.0	4	f	20	31	p	compact
2008	2.0	4	f	21	30	p	compact
1999	2.8	6	f	16	26	p	compact
1999	2.8	6	f	18	26	p	compact

  - There are p=7 Covariates
  - Difficulty
    - Fitting all Combinations
    - Considering All 2-Way Interaction Terms

Part 3: Less Meditation and More Application

- Run Chunk 1
  - Creating Model Matrix
    - Up to 2-Way Interactions
    - Now, p=115
  - Model Selection is Difficult
  - Dividing Data into Train & Test is Not Advised (n=234)
- Run Chunk 2
  - Only a Few Options

alpha <dbl>	lambda <dbl>	CV.Error <dbl>
0.00	1.44063441	1.722966
0.25	0.55006214	1.620769
0.50	0.18956825	1.488094
0.75	0.10492193	1.456773
1.00	0.04942052	1.411025

Lowest Estimation of Prediction Error

Part 3: Less Meditation and More Application

- Chunk 2 (Continued)
  - Understanding cv.glmnet Object
    - $lambda = Contains Vector of Lambda Auto-Generated
    - $cvm = Cross Validated Estimate of Error for Each Lambda in $lambda
    - $lambda.min = The Lambda that Leads to Smallest CV Measure of Error
    - $lambda.1se = The Largest Value of Lambda Such That Error is Within 1 SD of the Error Using $lambda.min

Part 3: Less Meditation and More Application

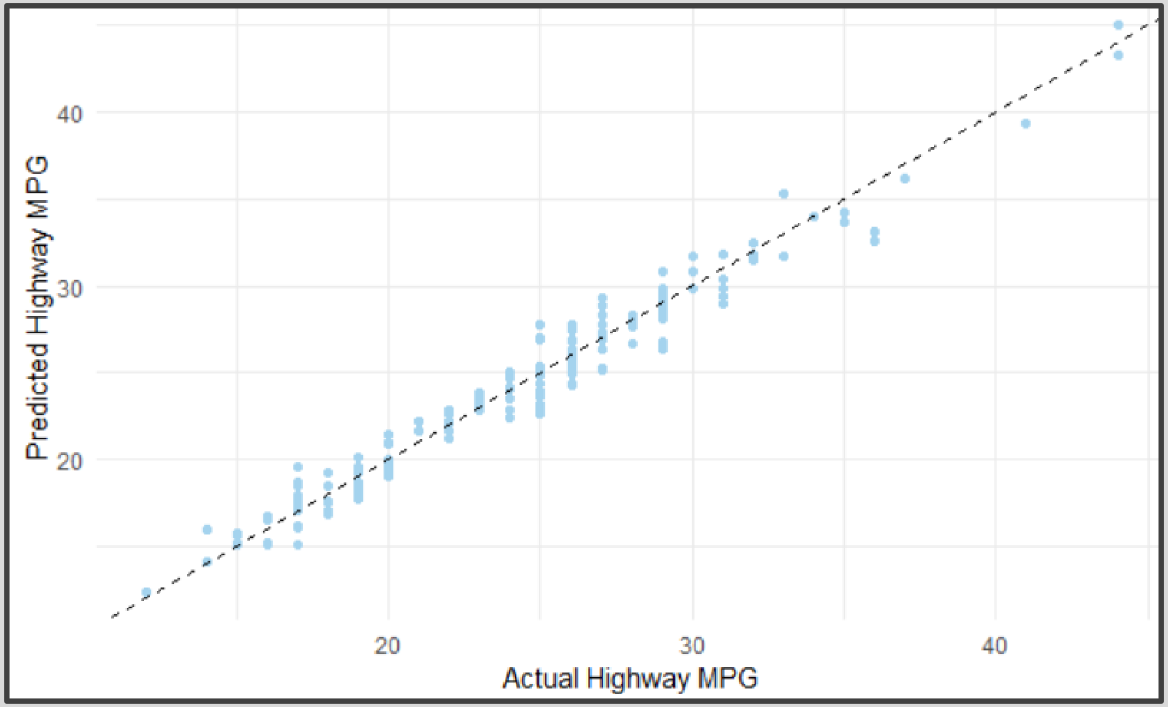
- Run Chunk 3
  - Next
    - Use Best Alpha and Lambda
    - Observe the Non-Zero Coefficients
    - Plot Predictions and Errors

  - Table of Non-Zero Coefficients
    - Before p=115
    - Now p=28

```
## # A tibble: 29 x 2
##    Parameter             Estimate
##    <chr>                    <dbl>
##  1 Int                  -123.
##  2 year                    0.0660
##  3 cty                     0.799
##  4 fle                    -1.37
##  5 flr                    -0.0629
##  6 classpickup            -0.104
##  7 classsuv               -1.37
##  8 year:cyl               -0.0000392
##  9 year:drvf               0.0000955
## 10 year:cty                0.0000565
## 11 year:classmidsize       0.0000259
## 12 year:classpickup       -0.000659
## 13 displ:drvr              0.127
## 14 displ:classmidsize      0.0317
## 15 displ:classsuv         -0.178
## 16 cyl:fle                -0.143
## 17 cyl:flr                -0.0973
## 18 cyl:classcompact        0.0462
## 19 cyl:classsuv           -0.0262
## 20 drvf:cty                0.0466
## 21 drvr:cty                0.0282
## 22 drvf:fld                2.54
## 23 drvr:classsubcompact   -0.0754
## 24 cty:classminivan       -0.0574
## 25 cty:classpickup        -0.106
## 26 flr:classmidsize        0.488
## 27 flp:classsubcompact    -1.42
## 28 fld:classsuv           -0.552
## 29 flp:classsuv           -0.431
```

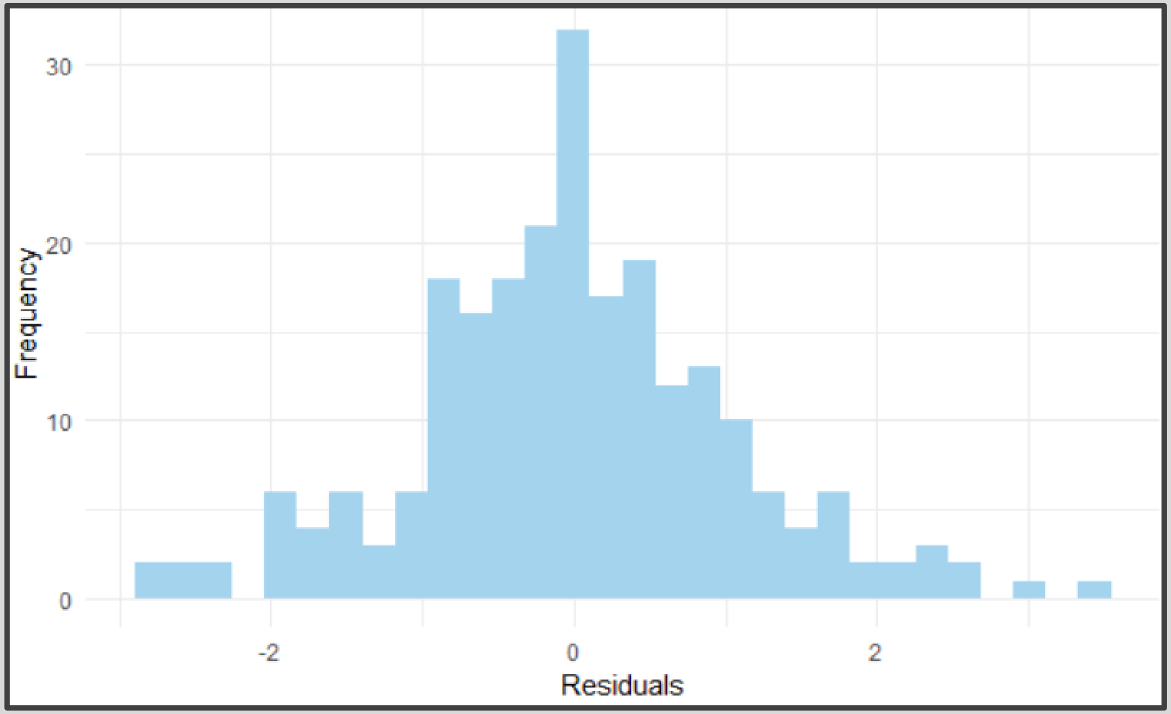
Part 3: Less Meditation and More Application

- Chunk 3 (Continued)
  - Comparing Predict and Actual



Part 3: Less Meditation and More Application

- Chunk 3 (Continued)
  - Distribution of Residuals



Part 3: Less Meditation and More Application

- Ecdat Data `> Participation`
  - Labor Market Participation of Married Women in Switzerland
  - Data From 1981
    - 872 Married Women
    - Variables
      - Participation (Binary)
      - Non-Labor Income (log transformed)
      - Age (Scaled by 10)
      - Education (Years)
      - # of Young Children
      - # of Older Children
      - Foreigner (Binary)

Part 3: Less Meditation and More Application

- Run Chunk 4
  - Observe the Data

	lfp	lnnlinc	age	educ	nyc	noc	foreign
1	no	10.78750	3.0	8	1	1	no
2	yes	10.52425	4.5	8	0	1	no
3	no	10.96858	4.6	9	0	0	no
4	no	11.10500	3.1	11	2	0	no
5	no	11.10847	4.4	12	0	2	no
6	yes	11.02825	4.2	12	0	1	no

  - We Would Like to Build a Model to Predict Labor Involvement
  - Method: Logistic Regression

Part 3: Less Meditation and More Application

- Run Chunk 5
  - Only a Few Options

alpha <dbl>	lambda <dbl>	CV.Error <dbl>
0.00	0.18805333	0.3405963
0.25	0.06542368	0.3291284
0.50	0.04745928	0.3348624
0.75	0.03472433	0.3348624
1.00	0.02604325	0.3348624

Lowest Estimation of Prediction Error

- Notice Using Binomial Family
- What is the Purpose of the Following? `type.measure="class"`

Part 3: Less Meditation and More Application

- Run Chunk 6
  - Only Considering Best Choices
  - Observe the Coefficients
    - Useful Variables?
    - Useless Variables?
  - Observe the Confusion Matrix
    - Misspecification Error Matches What We Saw
    - $0.329 = \frac{78+209}{393+78+209+192}$
  - Write Code That Counts
    - # of Labor Participants
    - # of Predicted Participants

Part 3: Less Meditation and More Application

- Results from Paper
  - Compares In-Sample Prediction of Four Competing Models
  - True # of Participants
  - Predicted # of Participants

	Swiss data (N = 873) number of participants
Actual	401
1. Method	
$\Sigma 1(P_{pb} > 0{\cdot}5)$	389
$\Sigma 1(P_{snp} > 0{\cdot}5)$	338
$\Sigma 1(P_{ks} > 0{\cdot}5)$	382
$\Sigma 1(xb_{sms} > 0)$	355

Closing

Disperse and Make Reasonable Decisions