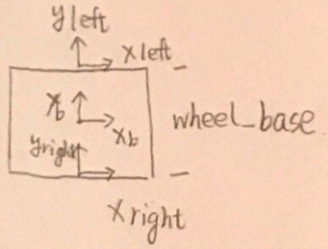
1)



$$V\_left = A\left(0, 0, -\frac{wheel\_base}{2}\right) V_b = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{wheel\_base}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W_b \\ V_{xb} \\ V_{yb} \end{bmatrix}$$

$$V\_left = \begin{bmatrix} W\_left \\ \dot{\varphi}\_left * wheel\_radius \\ V_y\_left \end{bmatrix}$$

$$\Rightarrow \dot{\varphi}\_left = \left(-\frac{wheel\_base}{2} W_b + V_{xb}\right) / wheel\_radius$$

the same: $\dot{\varphi}\_right = \left(\frac{wheel\_base}{2} W_b + V_{xb}\right) / wheel\_radius$.

2) The inverse of 1):

$$V_{xb} = \frac{(\dot{\varphi}\_left + \dot{\varphi}\_right) * wheel\_radius}{2}$$

$$W_b = \frac{(\dot{\varphi}\_right - \dot{\varphi}\_left) * wheel\_radius}{wheel\_base}$$

3) if $W_b = 0$: $V_b = \begin{bmatrix} 0 \\ V_{xb} \\ V_{yb} \end{bmatrix} \Rightarrow dq_b = \begin{bmatrix} 0 \\ V_{xb} \\ V_{yb} \end{bmatrix}$, denotes the integral of velocity in frame $\{b\}$ during the unit time.

$\Rightarrow dq = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} dq_b$, denotes the integral of velocity in the world frame.

θ is the angle or orientation of the diff_drive model.

if $W_b \neq 0$: $V_b = \begin{bmatrix} W_b \\ V_{xb} \\ V_{yb} \end{bmatrix}$ $\quad V_b(t) = \begin{bmatrix} W_b \\ V_{xb}\cos(W_b t) - V_{yb}\sin(W_b t) \\ V_{yb}\cos(W_b t) + V_{xb}\sin(W_b t) \end{bmatrix}$

$$\int_0^1 V_b(t)\,dt = \begin{bmatrix} W_b \\ (\sin(W_b)V_{xb} + V_{yb}(\cos(W_b) - 1))/W_b \\ (\sin(W_b)V_{yb} + V_{xb}(-\cos(W_b) + 1))/W_b \end{bmatrix}$$

$$\Rightarrow dq_b = \int_0^1 V_b(t)\,dt$$

$$\Rightarrow dq = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} dq_b$$