

final project

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Motivation

Breast cancer prognosis is essential for guiding treatment and improving patient survival. This study uses logistic regression to identify key factors that influence whether patients survive or not (status: Alive or Dead). The goal is to build a clear and understandable model to predict survival and see how well it works across different racial groups. It also focuses on fairness by examining performance differences between groups and finding ways to improve generalizability of the model.

Data Cleaning and Preprocessing

```
# Import data and clean column names
data <- read.csv("./data/Project_2_data.csv") %>%
  clean_names()

# Select relevant covariates (variables 1-14) and outcome variable
model_data <- data %>%
  dplyr::select(-survival_months)

# Convert categorical variables to factors and relabel `grade`
model_data <- model_data %>%
  mutate(
    race = factor(race),
    marital_status = factor(marital_status),
    t_stage = factor(t_stage),
    n_stage = factor(n_stage),
    x6th_stage = factor(x6th_stage),
    differentiate = factor(differentiate),
    a_stage = factor(a_stage),
    estrogen_status = factor(estrogen_status),
    progesterone_status = factor(progesterone_status),
    status = factor(status, levels = c("Alive", "Dead")),
    grade = case_when(
      grade == "1" ~ "1",
      grade == "2" ~ "2",
      grade == "3" ~ "3",
      grade == "anaplastic; Grade IV" ~ "4",
      TRUE ~ NA_character_
    ) %>% factor(levels = c("1", "2", "3", "4"))
  )
```

```
# Summarize structure of the cleaned dataset
summary(model_data)
```

```
##      age      race      marital_status t_stage  n_stage  x6th_stage
## Min.   :30.00  Black: 291  Divorced : 486  T1:1603  N1:2732  IIA :1305
## 1st Qu.:47.00  Other: 320  Married  :2643  T2:1786  N2: 820  IIB :1130
## Median :54.00  White:3413  Separated: 45  T3: 533  N3: 472  IIIA:1050
## Mean   :53.97                Single   : 615  T4: 102                IIIB: 67
## 3rd Qu.:61.00                Widowed  : 235                IIIC: 472
## Max.   :69.00
##
##      differentiate grade      a_stage      tumor_size
## Moderately differentiated:2351  1: 543  Distant : 92  Min.   : 1.00
## Poorly differentiated      :1111  2:2351  Regional:3932  1st Qu.: 16.00
## Undifferentiated           : 19  3:1111                Median : 25.00
## Well differentiated        : 543  4: 19                Mean   : 30.47
##                                     3rd Qu.: 38.00
##                                     Max.   :140.00
## estrogen_status progesterone_status regional_node_examined
## Negative: 269  Negative: 698  Min.   : 1.00
## Positive:3755  Positive:3326  1st Qu.: 9.00
##                                     Median :14.00
##                                     Mean   :14.36
##                                     3rd Qu.:19.00
##                                     Max.   :61.00
## reginol_node_positive status
## Min.   : 1.000  Alive:3408
## 1st Qu.: 1.000  Dead : 616
## Median : 2.000
## Mean   : 4.158
## 3rd Qu.: 5.000
## Max.   :46.000
```

Exploratory Data Analysis

Summary statistics

```
# Summary statistics for continuous and categorical variables
skim(model_data)
```

Table 1: Data summary

Name	model_data
Number of rows	4024
Number of columns	15
Column type frequency:	
factor	11
numeric	4
Group variables	None

Variable type: factor

skim_variable	n_missing	complete_rate	ordered	n_unique	top_counts
race	0	1	FALSE	3	Whi: 3413, Oth: 320, Bla: 291
marital_status	0	1	FALSE	5	Mar: 2643, Sin: 615, Div: 486, Wid: 235
t_stage	0	1	FALSE	4	T2: 1786, T1: 1603, T3: 533, T4: 102
n_stage	0	1	FALSE	3	N1: 2732, N2: 820, N3: 472
x6th_stage	0	1	FALSE	5	IIA: 1305, IIB: 1130, III: 1050, III: 472
differentiate	0	1	FALSE	4	Mod: 2351, Poo: 1111, Wel: 543, Und: 19
grade	0	1	FALSE	4	2: 2351, 3: 1111, 1: 543, 4: 19
a_stage	0	1	FALSE	2	Reg: 3932, Dis: 92
estrogen_status	0	1	FALSE	2	Pos: 3755, Neg: 269
progesterone_status	0	1	FALSE	2	Pos: 3326, Neg: 698
status	0	1	FALSE	2	Ali: 3408, Dea: 616

Variable type: numeric

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
age	0	1	53.97	8.96	30	47	54	61	69	
tumor_size	0	1	30.47	21.12	1	16	25	38	140	
regional_node_examined	0	1	14.36	8.10	1	9	14	19	61	
reginol_node_positive	0	1	4.16	5.11	1	1	2	5	46	

```
# Key statistics grouped by survival status
summary_by_status <- model_data %>%
  group_by(status) %>%
  summarise(
    mean_age = mean(age, na.rm = TRUE),
    sd_age = sd(age, na.rm = TRUE),
    mean_tumor_size = mean(tumor_size, na.rm = TRUE),
    sd_tumor_size = sd(tumor_size, na.rm = TRUE),
    prop_white = mean(race == "White", na.rm = TRUE),
    prop_black_other = mean(race != "White", na.rm = TRUE),
    n_obs = n()
  )
print(summary_by_status)
```

```
## # A tibble: 2 x 8
##   status mean_age sd_age mean_tumor_size sd_tumor_size prop_white
##   <fct>    <dbl> <dbl>         <dbl>         <dbl>         <dbl>
## 1 Alive     53.8   8.81           29.3           20.3           0.852
## 2 Dead     55.2   9.70           37.1           24.1           0.828
## # i 2 more variables: prop_black_other <dbl>, n_obs <int>
```

Distribution Visualization

Categorical Variables Our modified dataset contains **10 categorical variables**:

- **race**: Patient's race (Black, White, Other).

- **marital_status**: Patient's marital status (Divorced, Married, Separated, Single, Widowed).
- **t_stage**: Tumor stage (T1, T2, T3, T4). ("T" refers to the size of the primary tumor) - **n_stage**: Lymph node stage (N1, N2, N3). (extent of cancer spread to nearby lymph nodes) - **x6th_stage**: Adjusted AJCC 6th stage (IIA, IIB, IIIA, IIIB, IIIC).
- **differentiate**: Tumor differentiation (Well, Moderately, Poorly, Undifferentiated).
- **grade**: Grade of the tumor (1-4). - **a_stage**: Tumor spread stage (Regional, Distant).
- **estrogen_status**: Estrogen receptor status (Positive, Negative).
- **progesterone_status**: Progesterone receptor status (Positive, Negative).

To examine the association of these variables with the binary outcome **status** (Alive/Dead), I used **Cramér's V**, which quantifies the strength of the association between two categorical variables based on the Chi-Square statistic. Cramér's V ranges from **0** (no association) to **1** (perfect association). This method helps identify the predictors most strongly associated with survival status, enabling us to prioritize variables for modeling.

```
# Define categorical variables to analyze
variables <- c("race", "marital_status", "t_stage", "n_stage", "x6th_stage",
              "differentiate", "grade", "a_stage", "estrogen_status", "progesterone_status")

# Initialize a vector to store Cramér's V results
results <- numeric(length(variables))

# Calculate Cramér's V for each variable
for (i in seq_along(variables)) {
  var <- variables[i]

  # Select outcome and predictor variable, omitting missing values
  df_temp <- model_data %>%
    dplyr::select(status, all_of(var)) %>%
    na.omit()

  # Convert both columns to factors
  x <- droplevels(as.factor(df_temp$status))
  y <- droplevels(as.factor(df_temp[[var]]))

  # Create contingency table and calculate Cramér's V
  table_var <- table(x, y)
  results[i] <- cramersV(table_var)
}
```

```
## Warning in stats::chisq.test(...): Chi-squared approximation may be incorrect
## Warning in stats::chisq.test(...): Chi-squared approximation may be incorrect
```

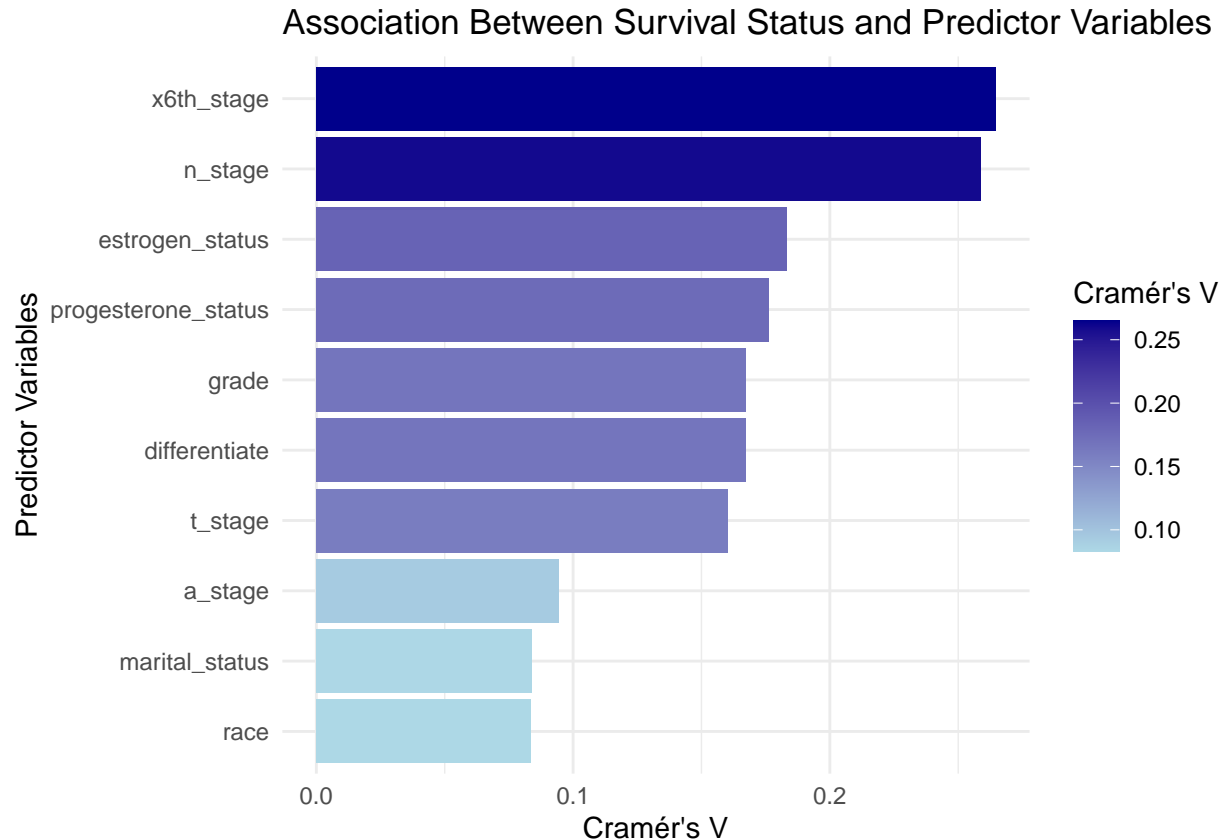
```
# Create a dataframe with results
association_df <- data.frame(Variable = variables, CramersV = results)

# Plot Cramér's V values
ggplot(association_df, aes(x = reorder(Variable, CramersV), y = CramersV, fill = CramersV)) +
  geom_bar(stat = "identity") +
  coord_flip() +
  scale_fill_gradient(low = "lightblue", high = "darkblue") +
  labs(
    title = "Association Between Survival Status and Predictor Variables",
    x = "Predictor Variables",
```

```

y = "Cramér's V",
fill = "Cramér's V"
) +
theme_minimal()

```

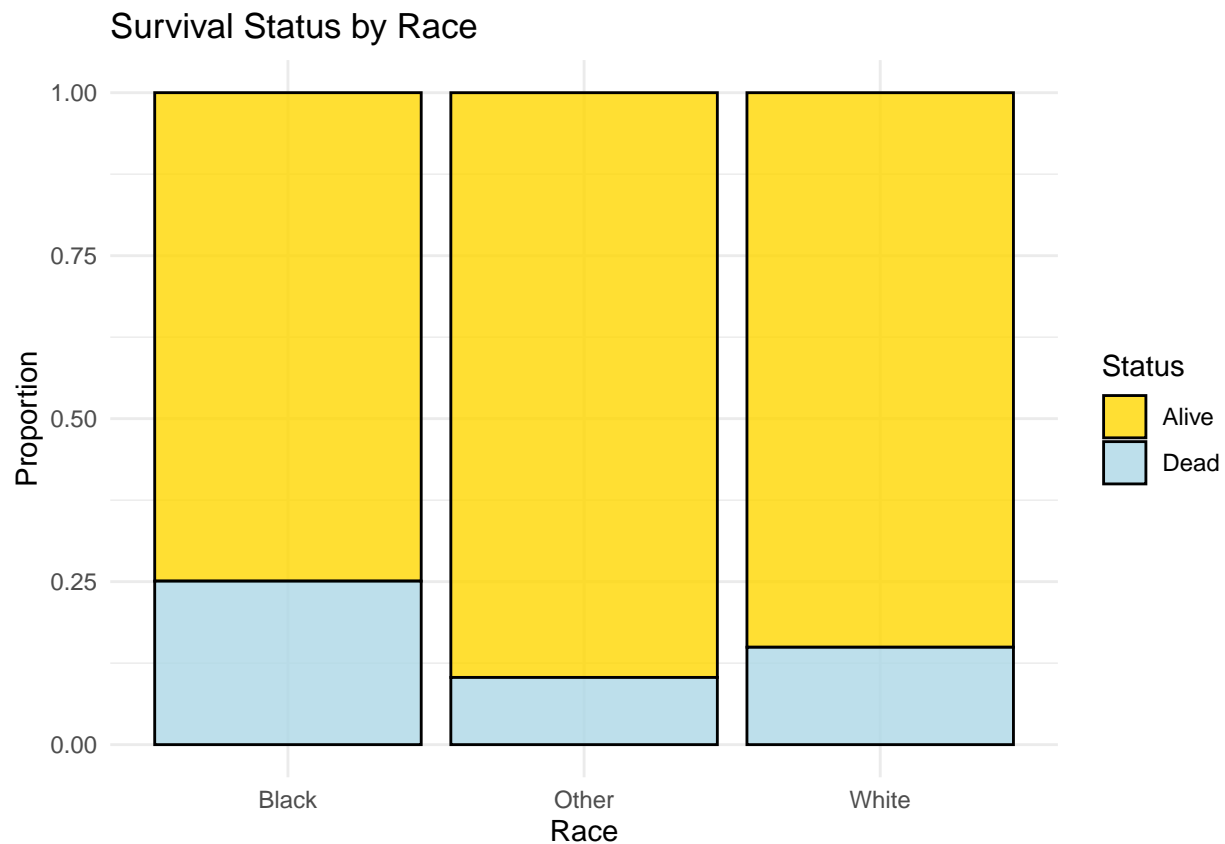


The plot shows the strength of association (Cramér's V) between **survival status** and each categorical predictor:

1. **x6th_stage** and **n_stage** have the **highest Cramér's V values** (around 0.25), indicating they are the most informative predictors of survival status in the dataset. These variables reflect tumor stage and lymph node involvement, which are critical factors in breast cancer prognosis.
2. **Estrogen_status**, **progesterone_status**, and **grade** exhibit **moderate associations** (Cramér's V around 0.15–0.2). These variables provide meaningful information about hormone status and differentiation, making them important contributors to survival prediction.
3. **Differentiate** and **t_stage** show moderate but slightly lower associations compared to the top variables, suggesting their relevance to survival status.
4. **a_stage**, **marital_status**, and **race** have **lower Cramér's V values** (less than 0.1), indicating weaker associations with survival status. Although these variables contribute limited information individually, they may still be useful in interaction terms or when combined with other predictors.

By focusing on variables with higher Cramér's V values, we can build more efficient and predictive models for survival status analysis.

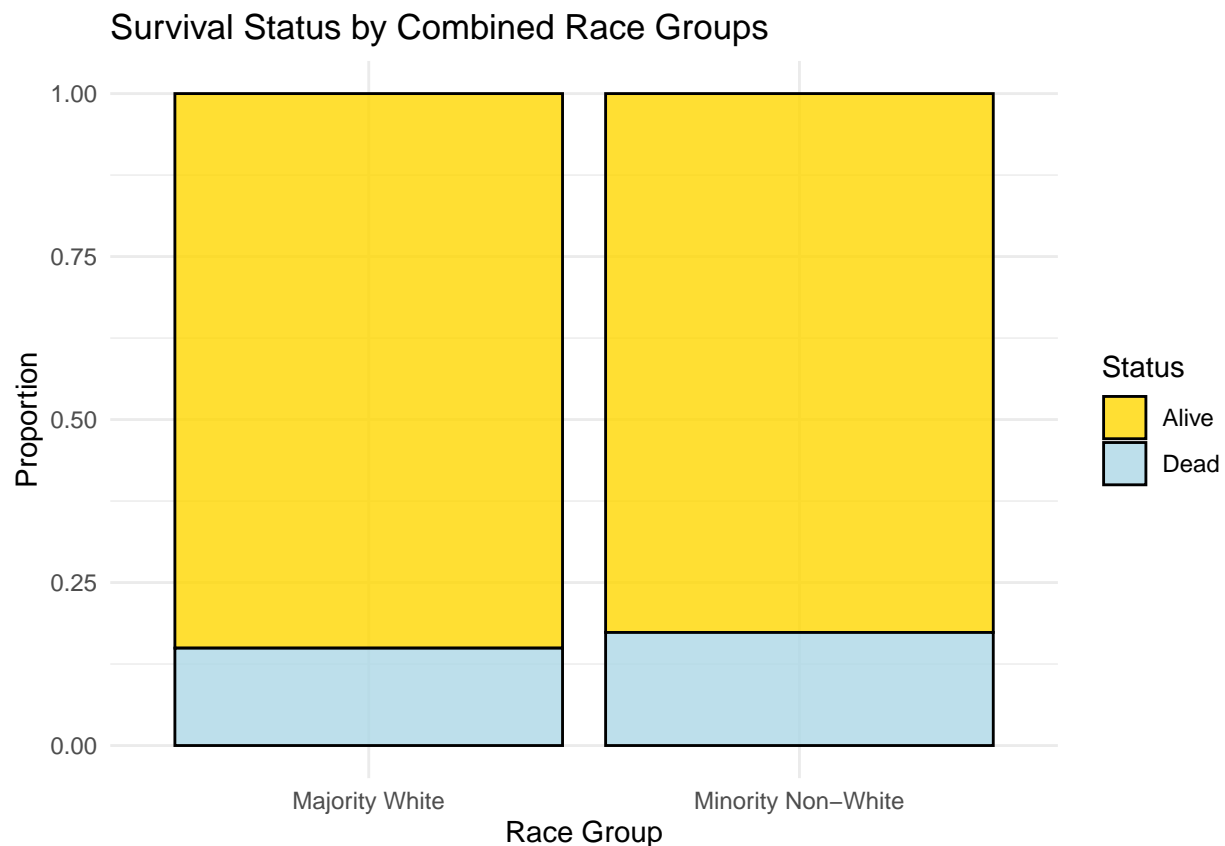
```
# Proportional Bar Plot for Survival Status by Race
ggplot(model_data, aes(x = race, fill = status)) +
  geom_bar(position = "fill", alpha = 0.8, color = "black") +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Survival Status by Race",
    x = "Race",
    y = "Proportion",
    fill = "Status"
  )
)
```



```
# Combine "Black" and "Other" into a single group "Minority Non-White"
model_data_race_combined <- model_data %>%
  mutate(
    race_combined = case_when(
      race == "White" ~ "Majority White",
      race %in% c("Black", "Other") ~ "Minority Non-White"
    ),
    race_combined = factor(race_combined, levels = c("Majority White", "Minority Non-White"))
  )

# Proportional Bar Plot for Combined Race Groups
ggplot(model_data_race_combined, aes(x = race_combined, fill = status)) +
  geom_bar(position = "fill", alpha = 0.8, color = "black") +
```

```
scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
theme_minimal() +
labs(
  title = "Survival Status by Combined Race Groups",
  x = "Race Group",
  y = "Proportion",
  fill = "Status"
)
```



Both plots confirm a disparity in survival outcomes across racial groups. Non-White patients, particularly Black patients, show a higher likelihood of death. Effect of Combining Groups: Combining Black and Other into Minority Non-White simplifies the comparison and reduce the racial disparities between Majority White and Minority Non-White groups.

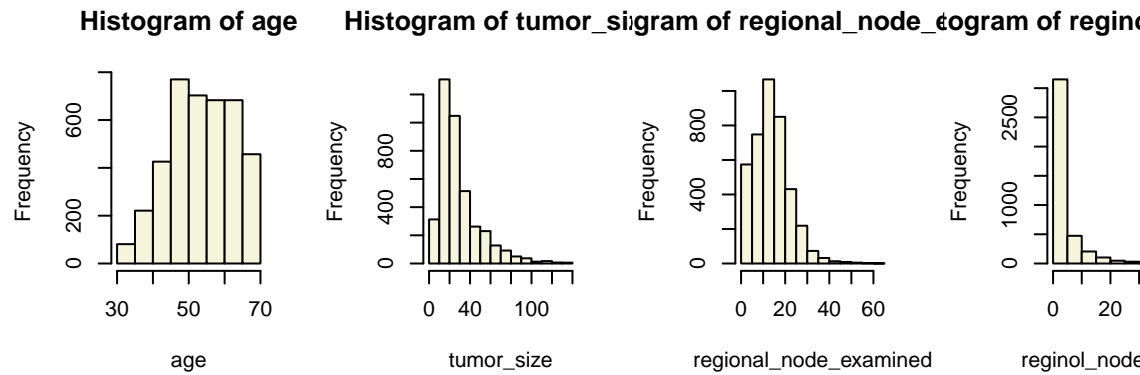
```
continuous_vars <- model_data %>%
  dplyr::select(age, tumor_size, regional_node_examined, reginol_node_positive) %>%
  na.omit()

df1 <- as.data.frame(continuous_vars)
par(mfrow = c(2, 4))
for (col in names(df1)) {
  hist(df1[[col]],
    main = paste("Histogram of", col),
```

```

    xlab = col,
    col = "beige",
    border = "black")
}
par(mfrow = c(1, 1))

```



Continuous Variables

Histograms were created for each continuous variable to visually examine their distributions and identify potential skewness. The results revealed that, except for **age**, the other three variables exhibited significant right skewness, indicating the need for transformation if we want to perform linear regression. Though we are not doing linear regression, transformation has still been performed here, we see that after performing log-transformation, there is a improvement on the skewness of **tumor_size**, **regional_node_examined** and **reginol_node_positive**.

```

df_log <- df1 %>%
  dplyr::select(-tumor_size, -regional_node_examined, -reginol_node_positive) %>%
  mutate(
    tumor_size_log = log(df1$tumor_size + 1),
    rn_examined_log = log(df1$regional_node_examined + 1),
    rn_positive_log = log(df1$reginol_node_positive + 1)
  )

par(mfrow = c(2, 4))
for (col in names(df_log)) {
  hist(
    df_log[[col]],
    main = paste("Histogram of", col),

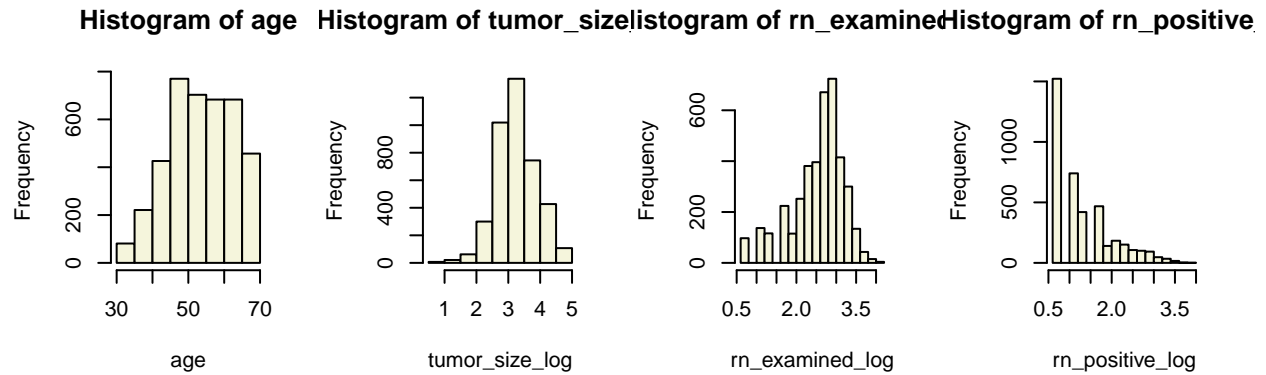
```



```

    xlab = col,
    col = "beige",
    border = "black"
  )
}
par(mfrow = c(1, 1))

```

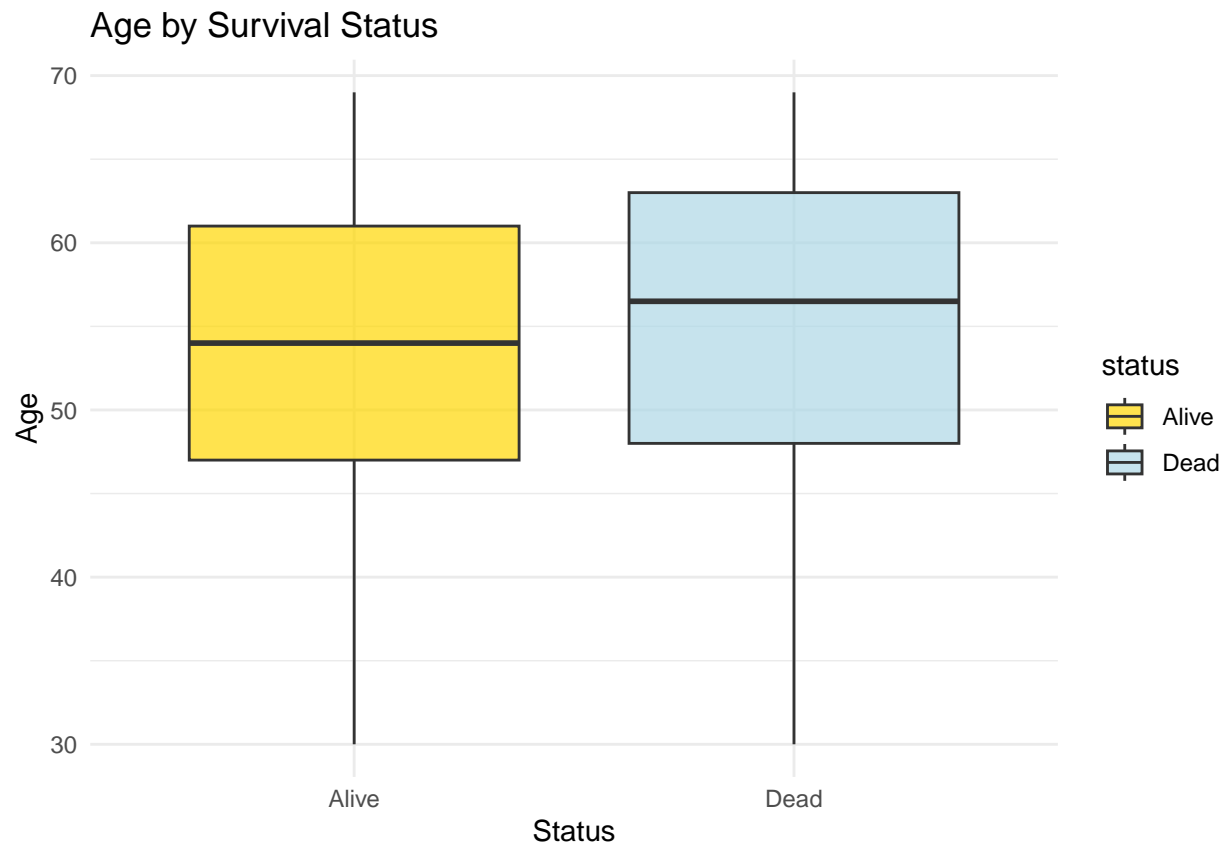


Boxplots were then generated for each continuous variable stratified by survival status to visually assess their relationships with the binary outcome.

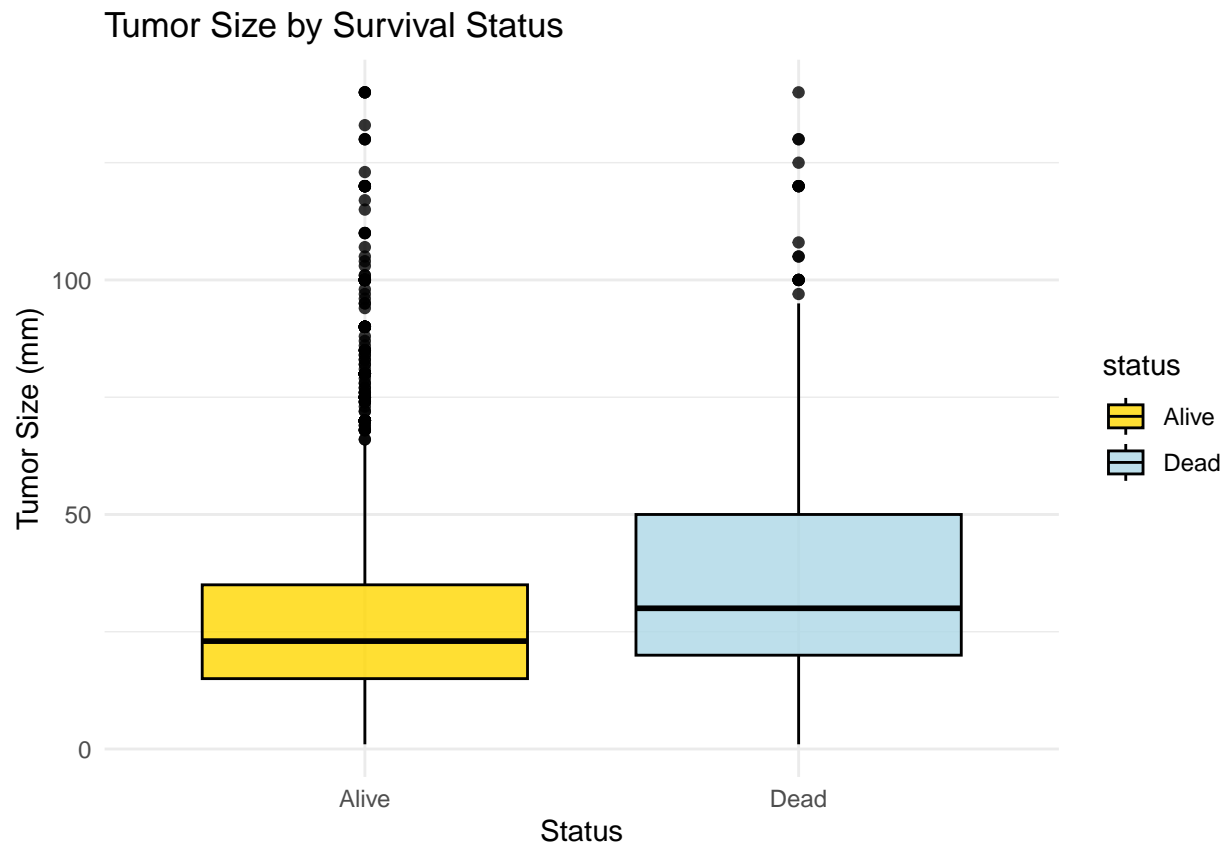
```

# Age by survival status
ggplot(model_data, aes(x = status, y = age, fill = status)) +
  geom_boxplot(alpha = 0.7) +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Age by Survival Status",
    x = "Status",
    y = "Age"
  )

```

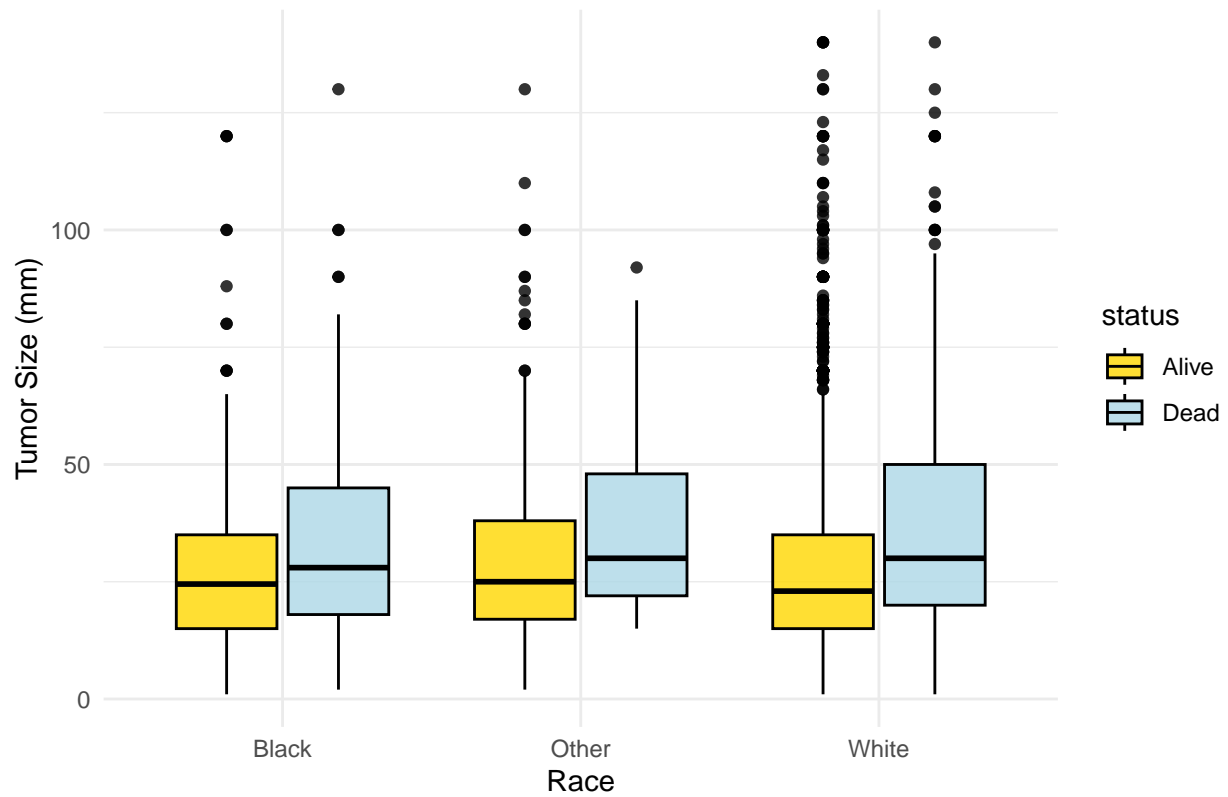


```
# Tumor size by survival status
ggplot(model_data, aes(x = status, y = tumor_size, fill = status)) +
  geom_boxplot(alpha = 0.8, color = "black") +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Tumor Size by Survival Status",
    x = "Status",
    y = "Tumor Size (mm)"
  )
```

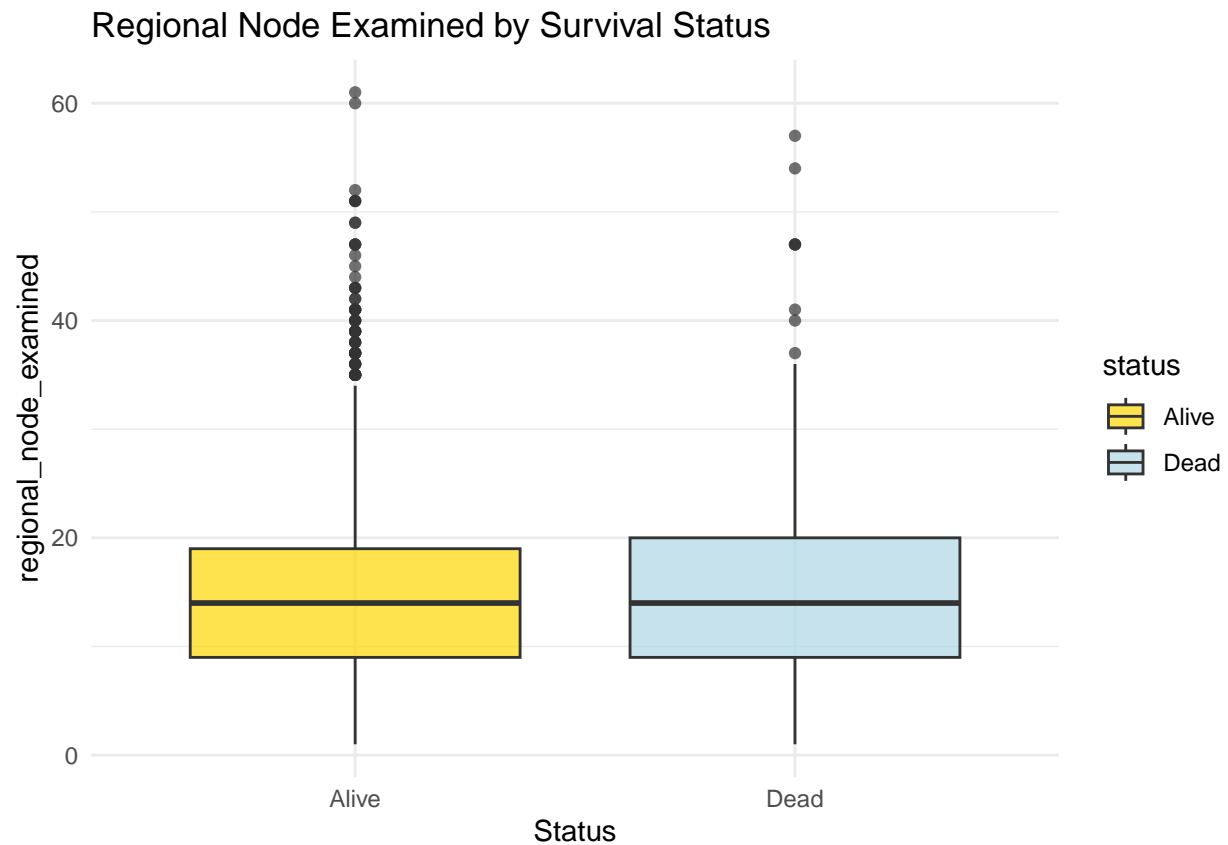


```
# Tumor size by race and survival status
ggplot(model_data, aes(x = race, y = tumor_size, fill = status)) +
  geom_boxplot(alpha = 0.8, color = "black") +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Tumor Size by Race and Survival Status",
    x = "Race",
    y = "Tumor Size (mm)"
  )
)
```

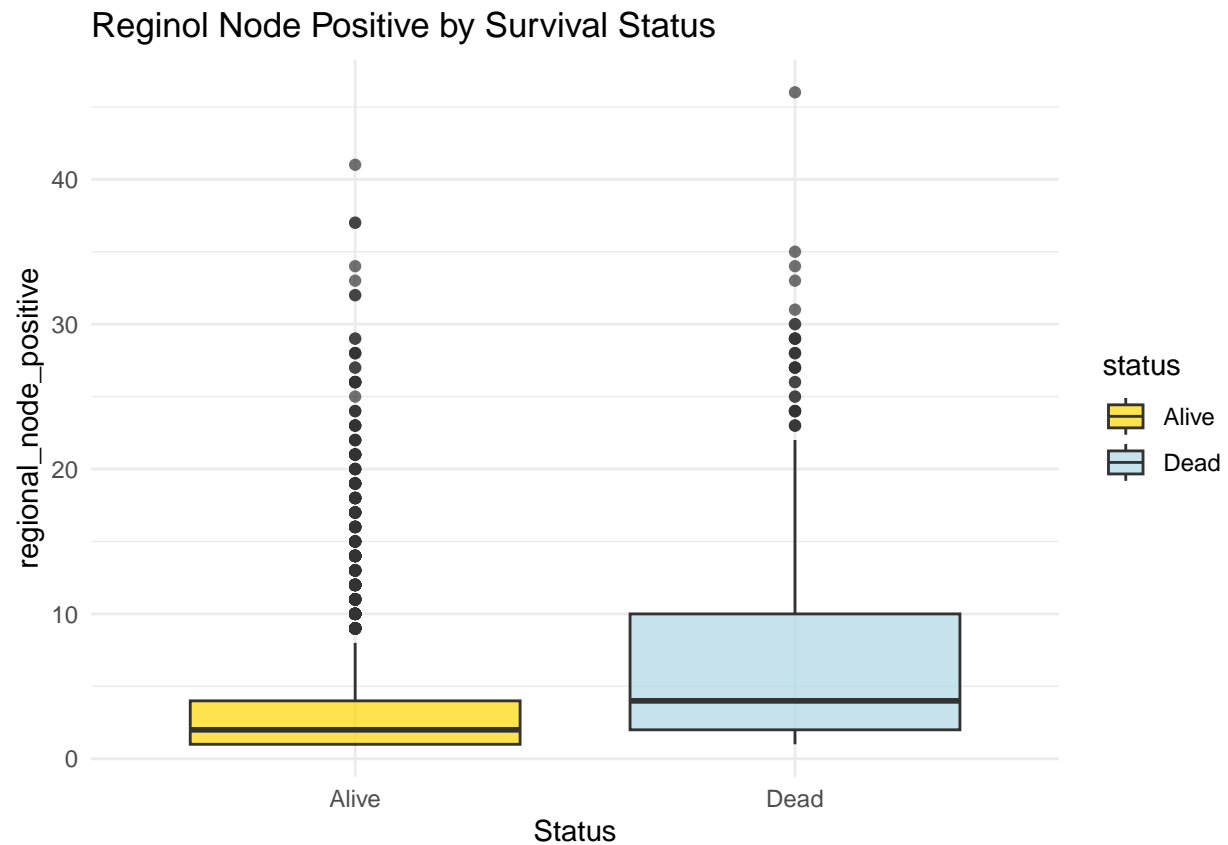
Tumor Size by Race and Survival Status



```
# Regional Node Examined by survival status
ggplot(model_data, aes(x = status, y = regional_node_examined, fill = status)) +
  geom_boxplot(alpha = 0.7) +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Regional Node Examined by Survival Status",
    x = "Status",
    y = "regional_node_examined"
  )
```



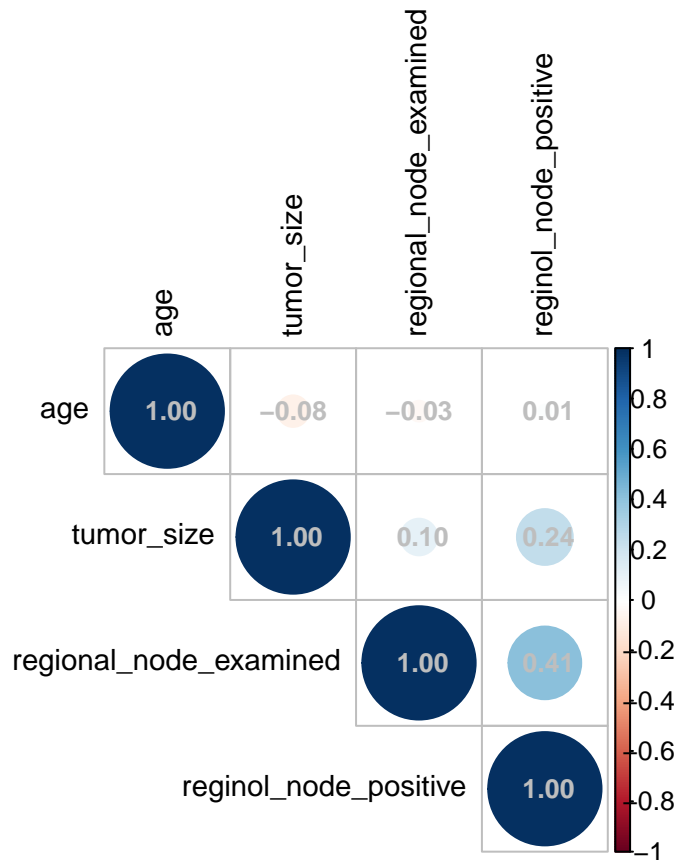
```
# Reginol Node Positive by survival status
ggplot(model_data, aes(x = status, y = reginol_node_positive, fill = status)) +
  geom_boxplot(alpha = 0.7) +
  scale_fill_manual(values = c("Alive" = "gold", "Dead" = "lightblue")) +
  theme_minimal() +
  labs(
    title = "Reginol Node Positive by Survival Status",
    x = "Status",
    y = "regional_node_positive"
  )
```



Pairwise Relationships and Interactions

```
correlation_matrix <- cor(continuous_vars, use = "pairwise.complete.obs")

# Pairwise relationships (correlation matrix for continuous variables)
corrplot(correlation_matrix, method = "circle",
  type = "upper", tl.col = "black", addCoef.col = "grey",
  number.cex = 0.8, tl.cex = 0.9)
```



A correlation matrix was generated to examine the relationships between the variables. Small circles (near-zero correlations) are observed for age and all other variables; tumor_size and regional_node_examined suggesting weak or no relationships. regional_node_positive is moderately associated with both tumor size (0.24) and regional nodes examined (0.41), which might influence modeling decisions. There are no strong correlations (close to ± 1), suggesting no immediate multicollinearity issues among these variables.

Modeling - Logistic Regression

Since the outcome is Binary (e.g., Alive/Dead), We decide to proceed with logistic regression instead of linear regression. Logistic regression outputs odds ratios, which are more interpretable for binary classification problems. For example, it tells you how much the odds of death increase with a unit increase in a predictor.

Checking Assumptions and Transformations

```
unique(model_data$status)
```

Binary or Dichotomous Response Variable

```
## [1] Alive Dead
## Levels: Alive Dead
```

The response variable (status) has exactly two categories: Alive and Dead.

```

# Fit an initial logistic regression model
model_data_1 <- model_data %>%
  mutate(
    race = relevel(race, ref = "White"), # Set "White" as reference
    grade = relevel(grade, ref = "1"), # Set "Grade 1" as reference
    x6th_stage = relevel(x6th_stage, ref = "IIA") # Set "IIA" as reference
  )

alias(glm(status ~ ., data = model_data_1, family = binomial))

```

No Multicollinearity

```

## Model :
## status ~ age + race + marital_status + t_stage + n_stage + x6th_stage +
##      differentiate + grade + a_stage + tumor_size + estrogen_status +
##      progesterone_status + regional_node_examined + reginol_node_positive
##
## Complete :
##      (Intercept) age raceBlack raceOther marital_statusMarried
## x6th_stageIIIC  0      0 0      0      0
## grade2          1      0 0      0      0
## grade3          0      0 0      0      0
## grade4          0      0 0      0      0
##      marital_statusSeparated marital_statusSingle
## x6th_stageIIIC  0      0
## grade2          0      0
## grade3          0      0
## grade4          0      0
##      marital_statusWidowed t_stageT2 t_stageT3 t_stageT4 n_stageN2
## x6th_stageIIIC  0      0      0      0      0
## grade2          0      0      0      0      0
## grade3          0      0      0      0      0
## grade4          0      0      0      0      0
##      n_stageN3 x6th_stageIIB x6th_stageIIIA x6th_stageIIIB
## x6th_stageIIIC  1      0      0      0
## grade2          0      0      0      0
## grade3          0      0      0      0
## grade4          0      0      0      0
##      differentiatePoorly differentiated differentiateUndifferentiated
## x6th_stageIIIC  0      0
## grade2          -1     -1
## grade3          1      0
## grade4          0      1
##      differentiateWell differentiated a_stageRegional tumor_size
## x6th_stageIIIC  0      0      0
## grade2          -1     0      0
## grade3          0      0      0
## grade4          0      0      0
##      estrogen_statusPositive progesterone_statusPositive
## x6th_stageIIIC  0      0
## grade2          0      0
## grade3          0      0

```



```
## grade4          0          0
##                regional_node_examined regional_node_positive
## x6th_stageIIIC  0          0
## grade2          0          0
## grade3          0          0
## grade4          0          0
```

From the `alias()` output: `grade2`, `grade3`, and `grade4` are collinear with other predictors. `x6th_stageIIIC` is collinear with `n_stage` or other levels of `x6th_stage`. `differentiate` levels also overlap in prediction with `grade`.

Given that both `x6th_stage` and `n_stage` are strong predictors, with `x6th_stage` showing a slightly higher association, `n_stage` can be dropped as its information is already captured by `x6th_stage`. Similarly, `t_stage`, which is related to tumor size, can also be removed to avoid redundancy. The variable `differentiate` can be excluded as well, as it overlaps with `grade`, with both variables reflecting tumor differentiation. In the context of breast cancer, “regional node positive” indicates the presence of cancer cells in nearby lymph nodes, while “regional node examined” refers to the surgical removal and analysis of these nodes to determine cancer spread. From the previous correlation matrix, `regional_node_positive` shows a moderate association with both `tumor_size` (0.24) and `regional_node_examined` (0.41), suggesting some redundancy. To simplify the model and minimize multicollinearity, we choose to drop `regional_node_positive`.

```
model_data_2 <- model_data_1 %>%
  dplyr::select(-differentiate, -n_stage, -t_stage, -regional_node_positive)

alias(glm(status ~ ., data = model_data_2, family = binomial))
```

```
## Model :
## status ~ age + race + marital_status + x6th_stage + grade + a_stage +
##      tumor_size + estrogen_status + progesterone_status + regional_node_examined
```

```
# Fit an initial logistic regression model
model_vif <- glm(status ~ ., data = model_data_2, family = binomial)

# Check VIF for predictors
vif_values <- vif(model_vif)
print(vif_values)
```

```
##                GVIF Df GVIF^(1/(2*Df))
## age                1.106908 1      1.052097
## race               1.058083 2      1.014215
## marital_status     1.127489 4      1.015112
## x6th_stage         1.967732 4      1.088293
## grade              1.118473 3      1.018836
## a_stage            1.210950 1      1.100432
## tumor_size         1.365304 1      1.168462
## estrogen_status    1.484914 1      1.218570
## progesterone_status 1.434692 1      1.197786
## regional_node_examined 1.225729 1      1.107127
```

```
# Identify predictors with VIF > 5 (indicating multicollinearity)
vif_values[vif_values > 5]
```

```
## numeric(0)
```

The results show that all VIF values are below 5, which indicates that multicollinearity is no longer a concern in your logistic regression model.

```
continuous_vars_log_odds <- model_data_2 %>%
  dplyr::select(age, tumor_size, regional_node_examined, status) %>% # Include 'status'
  na.omit()

# Log-transform tumor size and regional nodes examined
df_log_odds <- continuous_vars_log_odds %>%
  mutate(
    age_log = log(age + 1),
    tumor_size_log = log(tumor_size + 1),
    rn_examined_log = log(regional_node_examined + 1)
  )

linearity_test <- glm(status ~ age_log + tumor_size_log + rn_examined_log,
  data = df_log_odds,
  family = binomial)

df_log_odds$logit <- predict(linearity_test, type = "link")

plot1 <- ggplot(df_log_odds, aes(x = age_log, y = logit)) +
  geom_point(alpha = 0.4) +
  geom_smooth(method = "loess", color = "purple") +
  labs(title = "Logit vs Age (Log-Transformed)",
    x = "Log(Age + 1)", y = "Logit (Log Odds)") +
  theme_minimal()

plot2 <- ggplot(df_log_odds, aes(x = tumor_size_log, y = logit)) +
  geom_point(alpha = 0.4) +
  geom_smooth(method = "loess", color = "purple") +
  labs(title = "Logit vs Tumor Size (Log-Transformed)", x = "Log(Tumor Size + 1)", y = "Logit (Log Odds)") +
  theme_minimal()

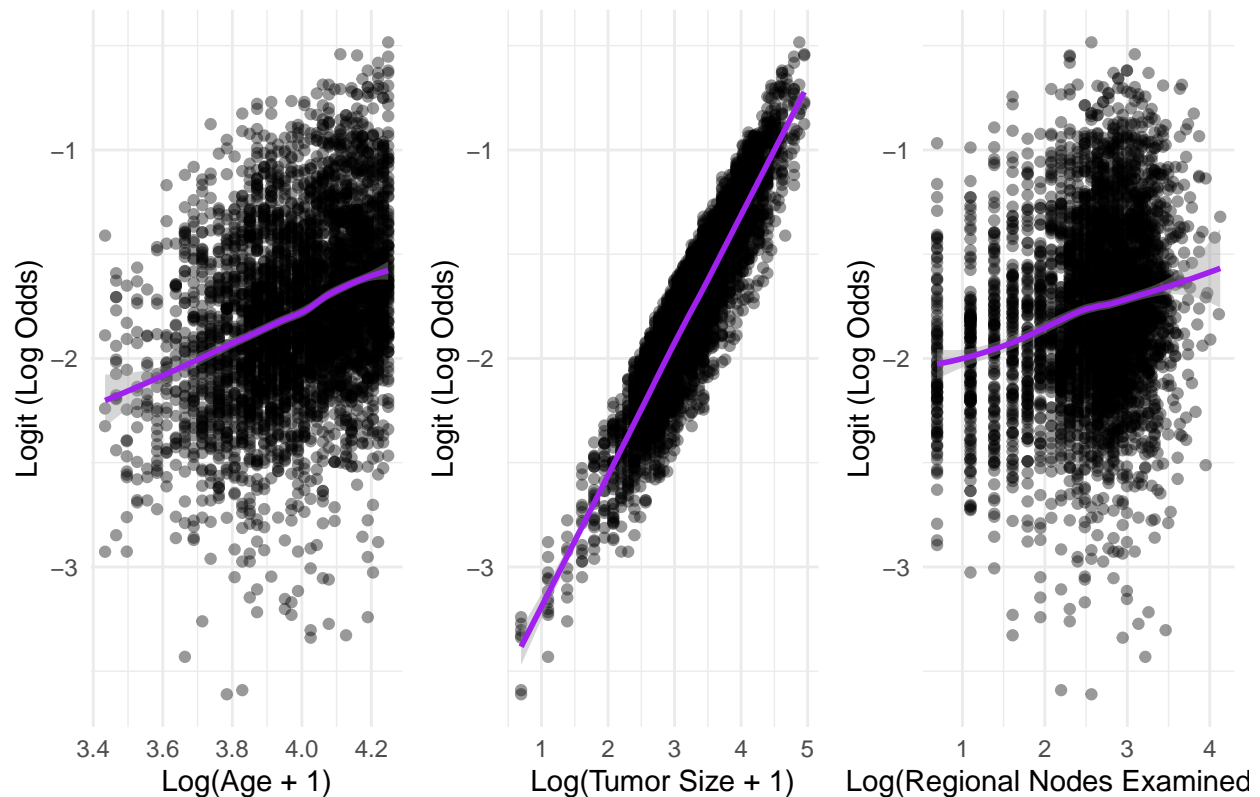
plot3 <- ggplot(df_log_odds, aes(x = rn_examined_log, y = logit)) +
  geom_point(alpha = 0.4) +
  geom_smooth(method = "loess", color = "purple") +
  labs(title = "Logit vs Regional Nodes Examined (Log-Transformed)",
    x = "Log(Regional Nodes Examined + 1)", y = "Logit (Log Odds)") +
  theme_minimal()

grid.arrange(plot1, plot2, plot3, ncol = 3)
```

Linear Relationship to Log Odds

```
## 'geom_smooth()' using formula = 'y ~ x'
## 'geom_smooth()' using formula = 'y ~ x'
## 'geom_smooth()' using formula = 'y ~ x'
```

Logit vs Age (Log-Transformed) vs Tumor Size (Log-Transformed) vs Regional Nodes Examined



```
# Update
model_data_3 <- model_data_2 %>%
  mutate(
    tumor_size_log = log(tumor_size + 1),
    rn_examined_log = log(regional_node_examined + 1),
    age_log = log(age + 1)
  ) %>%
  dplyr::select(-tumor_size, -regional_node_examined, -age)
```

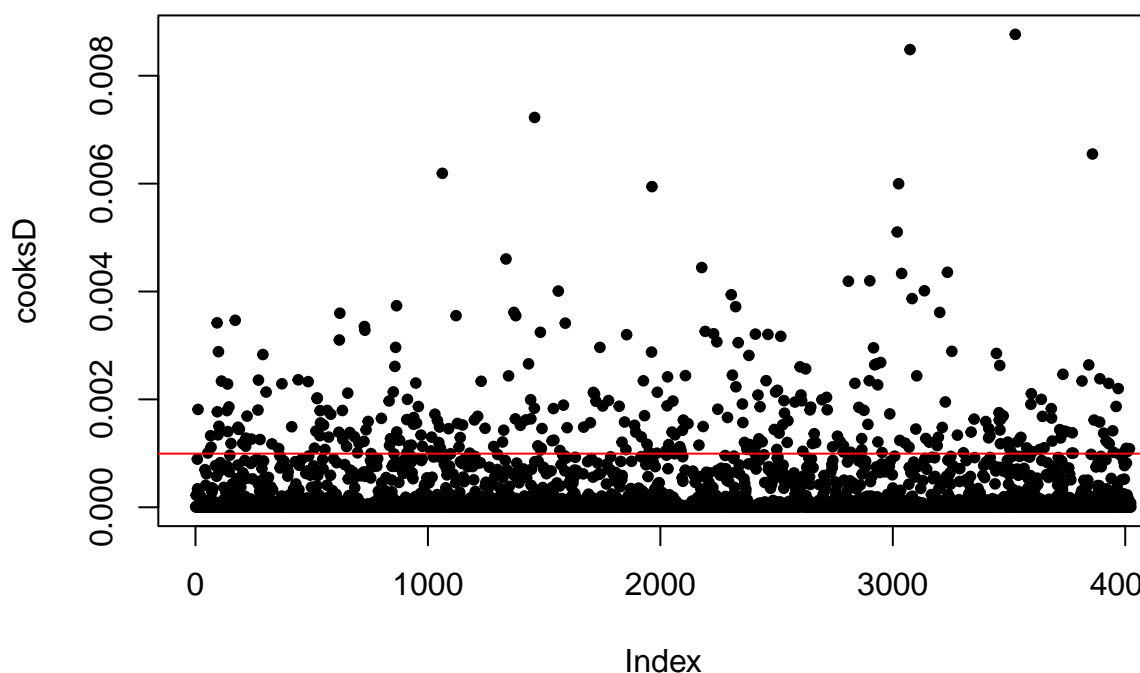
From the plot, we see that there's a linear relationship between continuous predictors (log transformed) and the log odds of the outcome.

```
# Fit logistic regression model with log-transformed predictors
full_model <- glm(status ~ .,
  data = model_data_3, family = binomial)

# Calculate Cook's Distance
cooksD <- cooks.distance(model_vif)

# Plot Cook's Distance
plot(cooksD, pch = 20, main = "Cook's Distance for Outlier Detection")
abline(h = 4 / nrow(model_data_3), col = "red")
```

Cook's Distance for Outlier Detection



No Extreme Outliers

```
# Identify influential observations
influential_obs <- which(cooksD > 4 / nrow(model_data_3))

# Build Model
model_no_outliers <- glm(status ~ .,
                        data = model_data_3[-influential_obs, ], family = binomial)

model_robust <- glmrob(status ~ .,
                      data = model_data_3, family = binomial, method = "Mqle")

# Compare summary statistics
summary(full_model) # Original model
```

```
##
## Call:
## glm(formula = status ~ ., family = binomial, data = model_data_3)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -5.89244    1.30699  -4.508 6.53e-06 ***
## raceBlack      0.48857    0.16068   3.041 0.002361 **
## raceOther     -0.42724    0.20104  -2.125 0.033573 *
## marital_statusMarried -0.22122    0.14008  -1.579 0.114283
## marital_statusSeparated  0.68972    0.38114   1.810 0.070355 .
## marital_statusSingle  -0.03701    0.17287  -0.214 0.830499
## marital_statusWidowed  0.03200    0.21828   0.147 0.883459
```

```
## x6th_stageIIB          0.43732    0.15999    2.733 0.006267 **
## x6th_stageIIIA         0.87662    0.16766    5.229 1.71e-07 ***
## x6th_stageIIIB         1.40443    0.33436    4.200 2.66e-05 ***
## x6th_stageIIIC         1.91444    0.18715   10.229 < 2e-16 ***
## grade2                 0.53100    0.18314    2.899 0.003739 **
## grade3                 0.90600    0.19171    4.726 2.29e-06 ***
## grade4                 1.85041    0.54216    3.413 0.000642 ***
## a_stageRegional       -0.10496    0.25449   -0.412 0.680023
## estrogen_statusPositive -0.72590    0.17578   -4.130 3.63e-05 ***
## progesterone_statusPositive -0.56562    0.12677   -4.462 8.13e-06 ***
## tumor_size_log         0.11412    0.09327    1.224 0.221128
## rn_examined_log       -0.29256    0.08015   -3.650 0.000262 ***
## age_log                1.13021    0.29507    3.830 0.000128 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 3444.7  on 4023  degrees of freedom
## Residual deviance: 2999.8  on 4004  degrees of freedom
## AIC: 3039.8
##
## Number of Fisher Scoring iterations: 5
```

```
summary(model_no_outliers) # Model without influential points
```

```
##
## Call:
## glm(formula = status ~ ., family = binomial, data = model_data_3[-influential_obs,
##    ])
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -30.59062   438.72668  -0.070  0.94441
## raceBlack      -0.03825    0.27480  -0.139  0.88930
## raceOther     -2.87714    0.72537  -3.966 7.30e-05 ***
## marital_statusMarried    0.52865    0.21259    2.487  0.01289 *
## marital_statusSeparated    1.07921    0.76198    1.416  0.15668
## marital_statusSingle     0.20186    0.26742    0.755  0.45033
## marital_statusWidowed   -0.59893    0.39427   -1.519  0.12874
## x6th_stageIIB     0.88704    0.27030    3.282  0.00103 **
## x6th_stageIIIA     1.79248    0.27114    6.611 3.82e-11 ***
## x6th_stageIIIB     1.12274    0.72170    1.556  0.11978
## x6th_stageIIIC     3.26591    0.29062   11.238 < 2e-16 ***
## grade2          16.65859   438.72233    0.038  0.96971
## grade3          17.20845   438.72234    0.039  0.96871
## grade4          17.64000   438.72590    0.040  0.96793
## a_stageRegional    0.50122    0.40802    1.228  0.21930
## estrogen_statusPositive -1.28606    0.23200   -5.543 2.97e-08 ***
## progesterone_statusPositive -0.48613    0.17594   -2.763  0.00573 **
## tumor_size_log     0.17561    0.12402    1.416  0.15679
## rn_examined_log   -0.51872    0.11898   -4.360 1.30e-05 ***
## age_log           2.86076    0.44464    6.434 1.24e-10 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 2228.9  on 3696  degrees of freedom
## Residual deviance: 1615.2  on 3677  degrees of freedom
## AIC: 1655.2
##
## Number of Fisher Scoring iterations: 18
```

```
summary(model_robust)
```

```
##
## Call:  glmrob(formula = status ~ ., family = binomial, data = model_data_3,      method = "Mqle")
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -5.02731    1.37993  -3.643 0.000269 ***
## raceBlack         0.42645    0.16787   2.540 0.011076 *
## raceOther        -0.57455    0.22817  -2.518 0.011802 *
## marital_statusMarried -0.15607    0.14924  -1.046 0.295641
## marital_statusSeparated  0.70131    0.39251   1.787 0.073986 .
## marital_statusSingle   0.02045    0.18299   0.112 0.911025
## marital_statusWidowed   0.02631    0.23244   0.113 0.909863
## x6th_stageIIB         0.39706    0.17156   2.314 0.020641 *
## x6th_stageIIIA        0.83151    0.17919   4.640 3.48e-06 ***
## x6th_stageIIIB        1.31418    0.34430   3.817 0.000135 ***
## x6th_stageIIIC        1.86721    0.19649   9.503 < 2e-16 ***
## grade2              0.40796    0.19353   2.108 0.035035 *
## grade3              0.78014    0.20139   3.874 0.000107 ***
## grade4              1.69667    0.54726   3.100 0.001933 **
## a_stageRegional      -0.12449    0.25587  -0.487 0.626597
## estrogen_statusPositive -0.74561    0.17889  -4.168 3.07e-05 ***
## progesterone_statusPositive -0.55819    0.13157  -4.243 2.21e-05 ***
## tumor_size_log       0.15034    0.09790   1.536 0.124607
## rn_examined_log     -0.30303    0.08523  -3.555 0.000378 ***
## age_log             0.92791    0.31225   2.972 0.002962 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Robustness weights w.r * w.x:
## 3546 weights are ~= 1. The remaining 478 ones are summarized as
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.2123 0.4537 0.5997 0.6159 0.7762 0.9983
##
## Number of observations: 4024
## Fitted by method 'Mqle' (in 5 iterations)
##
## (Dispersion parameter for binomial family taken to be 1)
##
## No deviance values available
## Algorithmic parameters:
##   acc   tcc
## 0.0001 1.3450
```

```
## maxit
##      50
## test.acc
##      "coef"
```

In large datasets, Cook's Distance*can flag numerous observations as influential, often due to large sample size. These flagged points may represent true variability in the population rather than errors. The code results indicate that removing influential points harms the model, likely by disrupting essential data structure. Coefficients for variables like `grade` and intercept became unstable and unreliable, with extreme standard errors.

In contrast, the robust logistic regression (`model_robust`) provided stable and reliable estimates while effectively mitigating the impact of influential observations. Key predictors, such as `age_log`, `rn_examined_log`, `grade`, `x6th_stage`, and `race`, retained coefficients similar to the standard logistic regression. The robustness weights further show that about 12% of the data (478 out of 4024 observations) had reduced influence, while the majority (~88%, or 3546 observations) remained largely unaffected with weights close to 1. This approach accounts for influential points by down-weighting their impact, ensuring the model is less sensitive to extreme values without the need to remove data.

While both original and robust logistic regression are viable options for future analysis, we will proceed with the original logistic regression due to our limited familiarity with robust logistic regression techniques

Independence of Errors Since there are no group IDs, the independence assumption is satisfied.

Model Evaluation and Selection

We use original Logistic Regression as the primary models, considering its earlier performance.

```
# Forward selection
forward_model <- step(glm(status ~ ., data = model_data_3, family = binomial),
  scope = list(lower = ~1, upper = full_model),
  direction = "forward", trace = 0)

# Backward elimination
backward_model <- step(glm(status ~ .,
  data = model_data_3, family = binomial),
  direction = "backward", trace = 0)

# Stepwise selection
stepwise_model <- step(glm(status ~ ., data = model_data_3, family = binomial),
  scope = list(lower = ~1, upper = full_model),
  direction = "both", trace = 0)

summary(full_model)
```

```
##
## Call:
## glm(formula = status ~ ., family = binomial, data = model_data_3)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -5.89244    1.30699  -4.508 6.53e-06 ***
```

```
## raceBlack          0.48857    0.16068    3.041 0.002361 **
## raceOther         -0.42724    0.20104   -2.125 0.033573 *
## marital_statusMarried -0.22122    0.14008   -1.579 0.114283 .
## marital_statusSeparated 0.68972    0.38114    1.810 0.070355 .
## marital_statusSingle  -0.03701    0.17287   -0.214 0.830499
## marital_statusWidowed  0.03200    0.21828    0.147 0.883459
## x6th_stageIIB        0.43732    0.15999    2.733 0.006267 **
## x6th_stageIIIA       0.87662    0.16766    5.229 1.71e-07 ***
## x6th_stageIIIB       1.40443    0.33436    4.200 2.66e-05 ***
## x6th_stageIIIC       1.91444    0.18715   10.229 < 2e-16 ***
## grade2              0.53100    0.18314    2.899 0.003739 **
## grade3              0.90600    0.19171    4.726 2.29e-06 ***
## grade4              1.85041    0.54216    3.413 0.000642 ***
## a_stageRegional     -0.10496    0.25449   -0.412 0.680023
## estrogen_statusPositive -0.72590    0.17578   -4.130 3.63e-05 ***
## progesterone_statusPositive -0.56562    0.12677   -4.462 8.13e-06 ***
## tumor_size_log      0.11412    0.09327    1.224 0.221128
## rn_examined_log    -0.29256    0.08015   -3.650 0.000262 ***
## age_log             1.13021    0.29507    3.830 0.000128 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 3444.7  on 4023  degrees of freedom
## Residual deviance: 2999.8  on 4004  degrees of freedom
## AIC: 3039.8
##
## Number of Fisher Scoring iterations: 5
```

```
summary(forward_model)
```

```
##
## Call:
## glm(formula = status ~ race + marital_status + x6th_stage + grade +
##      a_stage + estrogen_status + progesterone_status + tumor_size_log +
##      rn_examined_log + age_log, family = binomial, data = model_data_3)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -5.89244    1.30699  -4.508 6.53e-06 ***
## raceBlack      0.48857    0.16068   -2.125 0.033573 *
## raceOther     -0.42724    0.20104   -2.125 0.033573 *
## marital_statusMarried -0.22122    0.14008   -1.579 0.114283 .
## marital_statusSeparated 0.68972    0.38114    1.810 0.070355 .
## marital_statusSingle  -0.03701    0.17287   -0.214 0.830499
## marital_statusWidowed  0.03200    0.21828    0.147 0.883459
## x6th_stageIIB    0.43732    0.15999    2.733 0.006267 **
## x6th_stageIIIA   0.87662    0.16766    5.229 1.71e-07 ***
## x6th_stageIIIB   1.40443    0.33436    4.200 2.66e-05 ***
## x6th_stageIIIC   1.91444    0.18715   10.229 < 2e-16 ***
## grade2          0.53100    0.18314    2.899 0.003739 **
## grade3          0.90600    0.19171    4.726 2.29e-06 ***
## grade4          1.85041    0.54216    3.413 0.000642 ***
```



```
## a_stageRegional          -0.10496    0.25449   -0.412 0.680023
## estrogen_statusPositive  -0.72590    0.17578   -4.130 3.63e-05 ***
## progesterone_statusPositive -0.56562    0.12677   -4.462 8.13e-06 ***
## tumor_size_log           0.11412    0.09327    1.224 0.221128
## rn_examined_log         -0.29256    0.08015   -3.650 0.000262 ***
## age_log                  1.13021    0.29507    3.830 0.000128 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 3444.7  on 4023  degrees of freedom
## Residual deviance: 2999.8  on 4004  degrees of freedom
## AIC: 3039.8
##
## Number of Fisher Scoring iterations: 5
```

```
summary(backward_model)
```

```
##
## Call:
## glm(formula = status ~ race + marital_status + x6th_stage + grade +
##       estrogen_status + progesterone_status + rn_examined_log +
##       age_log, family = binomial, data = model_data_3)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -5.54025    1.23601  -4.482 7.38e-06 ***
## raceBlack         0.47961    0.16043   2.990 0.002794 **
## raceOther        -0.42884    0.20087  -2.135 0.032767 *
## marital_statusMarried -0.22574    0.13985  -1.614 0.106502
## marital_statusSeparated  0.68440    0.38070   1.798 0.072218 .
## marital_statusSingle  -0.03553    0.17263  -0.206 0.836915
## marital_statusWidowed   0.03218    0.21818   0.148 0.882727
## x6th_stageIIB        0.52325    0.14394   3.635 0.000278 ***
## x6th_stageIIIA        0.98706    0.14117   6.992 2.71e-12 ***
## x6th_stageIIIB        1.57287    0.30208   5.207 1.92e-07 ***
## x6th_stageIIIC        2.03890    0.16069  12.688 < 2e-16 ***
## grade2              0.53257    0.18299   2.910 0.003610 **
## grade3              0.91053    0.19152   4.754 1.99e-06 ***
## grade4              1.87333    0.54185   3.457 0.000546 ***
## estrogen_statusPositive -0.72329    0.17548  -4.122 3.76e-05 ***
## progesterone_statusPositive -0.56810    0.12665  -4.486 7.27e-06 ***
## rn_examined_log     -0.29822    0.08001  -3.727 0.000193 ***
## age_log             1.09581    0.29380   3.730 0.000192 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 3444.7  on 4023  degrees of freedom
## Residual deviance: 3001.5  on 4006  degrees of freedom
## AIC: 3037.5
##
```

```
## Number of Fisher Scoring iterations: 5
```

```
summary(stepwise_model)
```

```
##
## Call:
## glm(formula = status ~ race + marital_status + x6th_stage + grade +
##     estrogen_status + progesterone_status + rn_examined_log +
##     age_log, family = binomial, data = model_data_3)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -5.54025     1.23601  -4.482 7.38e-06 ***
## raceBlack         0.47961     0.16043   2.990 0.002794 **
## raceOther        -0.42884     0.20087  -2.135 0.032767 *
## marital_statusMarried -0.22574     0.13985  -1.614 0.106502
## marital_statusSeparated  0.68440     0.38070   1.798 0.072218 .
## marital_statusSingle  -0.03553     0.17263  -0.206 0.836915
## marital_statusWidowed   0.03218     0.21818   0.148 0.882727
## x6th_stageIIB         0.52325     0.14394   3.635 0.000278 ***
## x6th_stageIIIA        0.98706     0.14117   6.992 2.71e-12 ***
## x6th_stageIIIB        1.57287     0.30208   5.207 1.92e-07 ***
## x6th_stageIIIC        2.03890     0.16069  12.688 < 2e-16 ***
## grade2             0.53257     0.18299   2.910 0.003610 **
## grade3             0.91053     0.19152   4.754 1.99e-06 ***
## grade4             1.87333     0.54185   3.457 0.000546 ***
## estrogen_statusPositive -0.72329     0.17548  -4.122 3.76e-05 ***
## progesterone_statusPositive -0.56810     0.12665  -4.486 7.27e-06 ***
## rn_examined_log      -0.29822     0.08001  -3.727 0.000193 ***
## age_log            1.09581     0.29380   3.730 0.000192 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 3444.7  on 4023  degrees of freedom
## Residual deviance: 3001.5  on 4006  degrees of freedom
## AIC: 3037.5
##
## Number of Fisher Scoring iterations: 5
```

```
best_subset <- regsubsets(`status` ~ ., data = model_data_3, nvmax = ncol(model_data_3) - 1)
best_summary <- summary(best_subset)
```

```
subset_table <- data.frame(
  Num_Predictors = 1:length(best_summary$adjr2),
  Adj_R2 = best_summary$adjr2,
  Cp = best_summary$c_p,
  BIC = best_summary$bic
)
print(subset_table)
```

```
##    Num_Predictors    Adj_R2      Cp      BIC
```

```
## 1          1 0.05417048 298.36391 -208.5087
## 2          2 0.08087514 177.39536 -316.4581
## 3          3 0.08796390 146.00298 -340.3144
## 4          4 0.09450591 117.12118 -361.9835
## 5          5 0.09895252  97.81162 -374.4942
## 6          6 0.10268662  81.76122 -383.9066
## 7          7 0.10609541  67.20233 -391.9243
## 8          8 0.10904786  54.73108 -397.9391
## 9          9 0.11128153  45.54176 -400.7425
## 10         10 0.11336393  37.04659 -402.8850
```

```
best_adjr2_model <- which.max(best_summary$adjr2)
adjr2_predictor <- names(coef(best_subset, best_adjr2_model))[-1]
best_cp_model <- which.min(best_summary$cp)
cp_predictor <- names(coef(best_subset, best_cp_model))[-1]
best_bic_model <- which.min(best_summary$bic)
bic_predictor <- names(coef(best_subset, best_bic_model))[-1]

cat("Best Model by Adjusted R2: ", best_adjr2_model, "variables,", adjr2_predictor, "\n")
```

```
## Best Model by Adjusted R2:  10 variables, raceBlack x6th_stageIIIA x6th_stageIIIB x6th_stageIIIC grade3 grade4
```

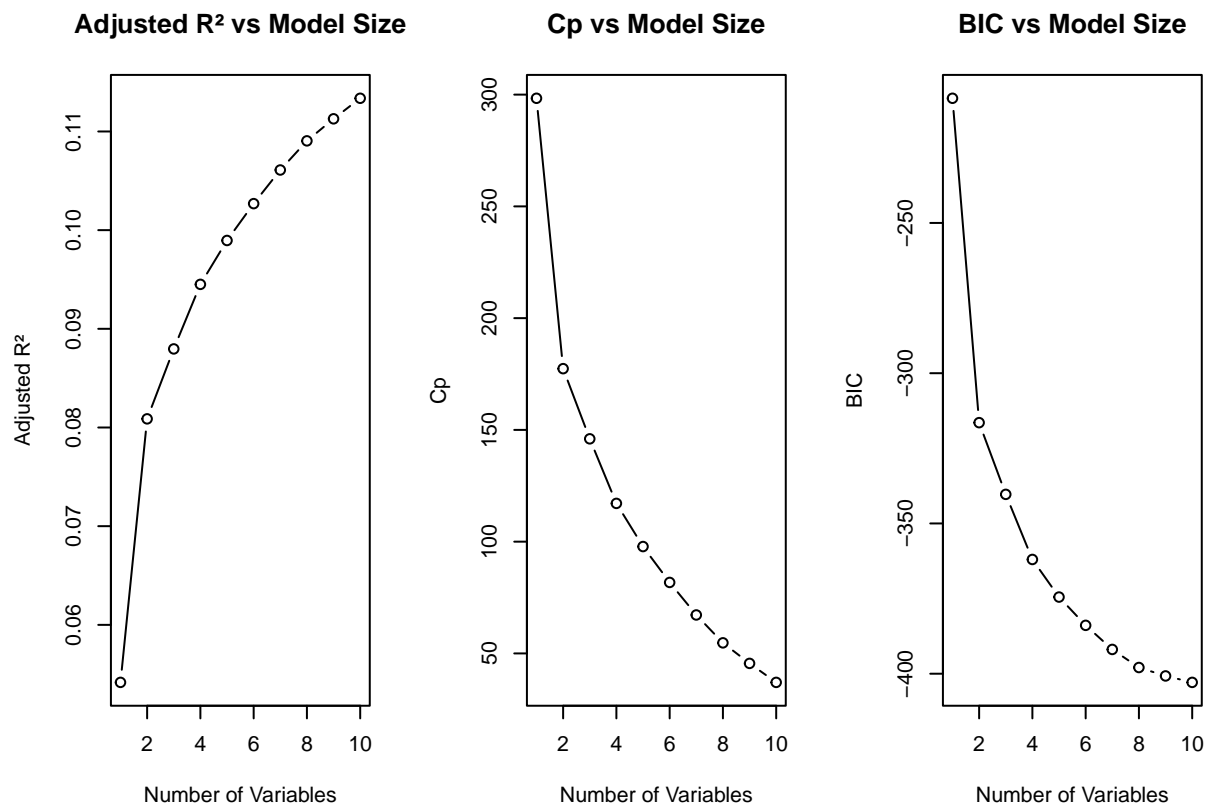
```
cat("Best Model by Cp: ", best_cp_model, "variables,", cp_predictor, "\n")
```

```
## Best Model by Cp:  10 variables, raceBlack x6th_stageIIIA x6th_stageIIIB x6th_stageIIIC grade3 grade4
```

```
cat("Best Model by BIC: ", best_bic_model, "variables,", bic_predictor, "\n")
```

```
## Best Model by BIC:  10 variables, raceBlack x6th_stageIIIA x6th_stageIIIB x6th_stageIIIC grade3 grade4
```

```
par(mfrow = c(1, 3))
plot(best_summary$adjr2,
     type = "b",
     main = "Adjusted R2 vs Model Size",
     xlab = "Number of Variables",
     ylab = "Adjusted R2")
plot(best_summary$cp,
     type = "b",
     main = "Cp vs Model Size",
     xlab = "Number of Variables",
     ylab = "Cp")
plot(best_summary$bic,
     type = "b",
     main = "BIC vs Model Size",
     xlab = "Number of Variables",
     ylab = "BIC")
```



```
par(mfrow = c(1, 1))

# Define predictors
predictors <- c("race", "marital_status", "x6th_stage", "grade",
               "estrogen_status", "progesterone_status", "rn_examined_log", "age_log")

# Create a dataframe to store results
interaction_results <- data.frame(
  Predictor1 = character(),
  Predictor2 = character(),
  P_Value = numeric(),
  stringsAsFactors = FALSE
)

# Loop through each pair of predictors
for (i in seq_along(predictors)) {
  for (j in seq_along(predictors)) {
    if (i < j) {
      predictor1 <- predictors[i]
      predictor2 <- predictors[j]

      # Fit interaction model
      formula <- as.formula(paste("status ~", paste(predictors, collapse = " + "),
                                "+", predictor1, "*", predictor2))
      interaction_model <- glm(formula, family = binomial, data = model_data_3)
```

```

# Extract interaction term
interaction_term <- paste(predictor1, predictor2, sep = ":")
coef_names <- names(coef(interaction_model))

if (interaction_term %in% coef_names) {
  # Extract p-value for the interaction term
  p_value <- coef(summary(interaction_model))[interaction_term, "Pr(>|z|)"]

  # Append to results if p-value < 0.05
  if (p_value < 0.05) {
    interaction_results <- rbind(interaction_results,
                                data.frame(Predictor1 = predictor1,
                                           Predictor2 = predictor2,
                                           P_Value = p_value,
                                           stringsAsFactors = FALSE))
  }
}
}
}

# Sort results by p-value
interaction_results <- interaction_results[order(interaction_results$P_Value), ]
print(interaction_results)

```

```

## [1] Predictor1 Predictor2 P_Value
## <0 rows> (or 0-length row.names)

```

Since resulting dataframe is empty, it suggests that none of the interaction terms tested in our dataset have p-values less than 0.05. This outcome aligns with our earlier exploratory analyses (Cramér's V, correlation matrix, and model diagnostics), which indicated that interaction effects are likely weak or non-existent in your data.

Final Model Since Best subset selection evaluates all possible combinations of predictors (including all levels of factor variables) to find the model with the best fit according to criteria like adjusted R², Cp and BIC, each level of a factor variable (e.g., x6th_stageIIIA) can be treated as an independent binary variable (dummy variable) in this process. Thus we will not use the best subset selection, as we need consider the factor as a whole for interpretability.

The final model was chosen through forward, backward, and stepwise selection methods using AIC as the selection criterion. The model balances interpretability and predictive performance, including only significant predictors that contribute to understanding survival outcomes:

```

glm(formula = status ~ race + marital_status + x6th_stage + grade + estrogen_status +
progesterone_status + rn_examined_log + age_log, family = binomial, data = model_data_3)

```

In logistic regression, coefficients represent the change in the log-odds of the outcome (`status = Dead`) for a one-unit increase in a predictor, holding other variables constant. Odds ratios provide a more interpretable measure.

The key coefficients included are 1. Race: - raceBlack: Positive coefficient implies higher odds of death for Black patients compared to the reference group (White patients). Odds ratio > 1 indicates increased risk for Black patients. - raceOther: Negative coefficient indicates slightly reduced odds of death for patients classified as "Other" compared to White patients.

2. Marital Status: The marital status categories have weak effects (some not statistically significant). For instance, being married or widowed appears to have no strong association with survival compared to the reference group.
3. Adjusted AJCC 6th stage (`x6th_stage`): is the strongest predictor of death risk. Higher stages (IIIA, IIIB, IIIC) significantly increase the odds of death compared to stage IIA (reference group). Patients in stage IIIC have the highest odds of death, indicating advanced tumor progression is a critical predictor of survival.
4. Tumor Grade (`grade`): Grades 3 and 4 significantly increase the odds of death compared to Grade 1, indicating poorly differentiated tumors are associated with worse outcomes.
5. Hormone Receptor Status: `estrogen_statusPositive` and `progesterone_statusPositive`: Negative coefficients suggest reduced odds of death for patients with positive estrogen or progesterone receptor status. This reflects the improved prognosis often associated with hormone-sensitive tumors.
6. Log-transformed Predictors: `-rn_examined_log`: Negative coefficient indicates increased survival odds with more regional lymph nodes examined. This might reflect more aggressive or effective treatment strategies. `-age_log`: Positive coefficient implies older patients are at higher risk of death.

So `x6th_stage`, `grade`, `estrogen_statusPositive` and `progesterone_statusPositive`, `age_log` and `rn_examined_log` are significant predictors affecting the risk.

```
set.seed(123)
control <- trainControl(method = "cv", number = 10, classProbs = TRUE, summaryFunction = twoClassSummary)

cv_model <- train(
  status ~ race + marital_status + x6th_stage + grade + estrogen_status +
    progesterone_status + rn_examined_log + age_log,
  data = model_data_3,
  method = "glm",
  family = "binomial",
  trControl = control,
  metric = "ROC"
)

cv_model
```

Model Performance

```
## Generalized Linear Model
##
## 4024 samples
##      8 predictor
##      2 classes: 'Alive', 'Dead'
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 3622, 3621, 3621, 3621, 3621, 3623, ...
## Resampling results:
##
##      ROC      Sens      Spec
## 0.7400002 0.9850311 0.1218403
```

ROC-AUC: 0.7400, the overall performance of the model is moderately good, with an acceptable ability to distinguish between “Alive” and “Dead” outcomes. Sensitivity: 0.985, the model is highly sensitive, meaning it correctly identifies most of the “Dead” cases. Specificity: 0.122, the model has very low specificity, indicating difficulty in correctly identifying “Alive” cases.

```
# Split data by race group
white_data <- model_data_3 %>% filter(race == "White")
minority_data <- model_data_3 %>% filter(race != "White")
black_data <- model_data_3 %>% filter(race == "Black")
other_data <- model_data_3 %>% filter(race == "Other")

# Predict probabilities for White group
pred_white <- predict(cv_model, newdata = white_data, type = "prob")[, "Dead"]
roc_white <- roc(white_data$status, pred_white)

## Setting levels: control = Alive, case = Dead

## Setting direction: controls < cases

auc_white <- auc(roc_white)

# Predict probabilities for Minority group
pred_minority <- predict(cv_model, newdata = minority_data, type = "prob")[, "Dead"]
roc_minority <- roc(minority_data$status, pred_minority)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_minority <- auc(roc_minority)

# Predict probabilities for Black group
pred_black <- predict(cv_model, newdata = black_data, type = "prob")[, "Dead"]
roc_black <- roc(black_data$status, pred_black)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_black <- auc(roc_black)

# Predict probabilities for Other group
pred_other <- predict(cv_model, newdata = other_data, type = "prob")[, "Dead"]
roc_other <- roc(other_data$status, pred_other)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_other <- auc(roc_other)

# Print AUC results
cat("ROC-AUC for White group:", auc_white, "\n")
```

```
## ROC-AUC for White group: 0.7503654
```

```
cat("ROC-AUC for Black group:", auc_black, "\n")
```

```
## ROC-AUC for Black group: 0.7021491
```

```
cat("ROC-AUC for Other group:", auc_other, "\n")
```

```
## ROC-AUC for Other group: 0.658431
```

```
cat("ROC-AUC for Minority group:", auc_minority, "\n")
```

```
## ROC-AUC for Minority group: 0.7313469
```

Performance by Race Groups White Group (Majority): - ROC-AUC: 0.7504 - The model performs better for the majority race group, achieving the highest predictive ability among the subgroups.

Black Group (Minority): - ROC-AUC: 0.7021 - The performance for Black patients is lower than for White patients, indicating potential disparities in prediction accuracy.

Other Group (Minority): - ROC-AUC: 0.6584 - The lowest ROC-AUC is for the “Other” race group, suggesting the model struggles most with this subgroup.

Combined Minority Group (Black + Other): - ROC-AUC: 0.7313 - When grouped together, the model’s performance for minorities improves but remains slightly lower than for the White group.

This disparity indicate a potential fairness issue, which need us to reduce the performance gap.

```
model_data_4 <- model_data_3 %>%  
  mutate(weight = case_when(  
    race == "White" ~ 1,  
    race == "Black" ~ 1.5,  
    race == "Other" ~ 2  
  ))  
  
minority_data <- model_data_4 %>% filter(race != "White")  
  
# Refit the model with weights  
reweighted_model <- train(  
  status ~ race + marital_status + x6th_stage + grade + estrogen_status +  
    progesterone_status + rn_examined_log + age_log,  
  data = model_data_4,  
  method = "glm",  
  family = "binomial",  
  trControl = control,  
  weights = weight  
)
```

```
## Warning in train.default(x, y, weights = w, ...): The metric "Accuracy" was not  
## in the result set. ROC will be used instead.
```



```

## Warning in eval(family$initialize): non-integer #successes in a binomial glm!
## Warning in eval(family$initialize): non-integer #successes in a binomial glm!
## Warning in eval(family$initialize): non-integer #successes in a binomial glm!
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## Warning in eval(family$initialize): non-integer #successes in a binomial glm!
## Warning in eval(family$initialize): non-integer #successes in a binomial glm!
## Warning in eval(family$initialize): non-integer #successes in a binomial glm!

# Evaluate performance on subgroups
pred_white_weighted <- predict(reweighted_model, newdata = white_data, type = "prob")[, "Dead"]
roc_white_weighted <- roc(white_data$status, pred_white_weighted)

## Setting levels: control = Alive, case = Dead

## Setting direction: controls < cases

auc_white_weighted <- auc(roc_white_weighted)

pred_minority_weighted <- predict(reweighted_model, newdata = minority_data, type = "prob")[, "Dead"]
roc_minority_weighted <- roc(minority_data$status, pred_minority_weighted)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_minority_weighted <- auc(roc_minority_weighted)

pred_black_weighted <- predict(reweighted_model, newdata = black_data, type = "prob")[, "Dead"]
roc_black_weighted <- roc(black_data$status, pred_black_weighted)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_black_weighted <- auc(roc_black_weighted)

pred_other_weighted <- predict(reweighted_model, newdata = other_data, type = "prob")[, "Dead"]
roc_other_weighted <- roc(other_data$status, pred_other_weighted)

## Setting levels: control = Alive, case = Dead
## Setting direction: controls < cases

auc_other_weighted <- auc(roc_other_weighted)

# Print updated results
cat("Reweighted Model - ROC-AUC for White group:", auc_white_weighted, "\n")

## Reweighted Model - ROC-AUC for White group: 0.7486181

```

```
cat("Reweighted Model - ROC-AUC for Minority group:", auc_minority_weighted, "\n")
```

```
## Reweighted Model - ROC-AUC for Minority group: 0.7358304
```

```
cat("Reweighted Model - ROC-AUC for Black group:", auc_black_weighted, "\n")
```

```
## Reweighted Model - ROC-AUC for Black group: 0.7051653
```

```
cat("Reweighted Model - ROC-AUC for Other group:", auc_other_weighted, "\n")
```

```
## Reweighted Model - ROC-AUC for Other group: 0.6657164
```

The reweighted model shows improved fairness compared to the original combined model: Reweighting has reduced the performance gap between the White and minority groups. The ROC-AUC for the minority group (0.7358) is now closer to that of the majority group (0.7486).

Conclusion

Table

```
# Extract coefficients and confidence intervals from the reweighted model
summary_reweighted <- summary(reweighted_model$finalModel)
coefficients <- coef(summary_reweighted)

# Calculate Odds Ratios and 95% Confidence Intervals
odds_ratios <- exp(coefficients[, "Estimate"])
lower_ci <- exp(coefficients[, "Estimate"] - 1.96 * coefficients[, "Std. Error"])
upper_ci <- exp(coefficients[, "Estimate"] + 1.96 * coefficients[, "Std. Error"])

# Combine into a table
results_table <- data.frame(
  Predictor = rownames(coefficients),
  Estimate = round(coefficients[, "Estimate"], 3),
  Std_Error = round(coefficients[, "Std. Error"], 3),
  Odds_Ratio = round(odds_ratios, 3),
  `95% CI (Lower)` = round(lower_ci, 3),
  `95% CI (Upper)` = round(upper_ci, 3),
  `P-Value` = format.pval(coefficients[, "Pr(>|z|)"], digits = 3)
)

# Clean up row names
rownames(results_table) <- NULL

# Print the table
print(results_table)
```

```
##               Predictor Estimate Std_Error Odds_Ratio X95..CI..Lower.
## 1      (Intercept)    -5.126      1.161      0.006      0.001
## 2      raceBlack      0.455      0.135      1.577      1.211
```

```
## 3          raceOther      -0.426    0.147    0.653    0.490
## 4      marital_statusMarried -0.248    0.134    0.781    0.600
## 5      marital_statusSeparated 0.696    0.350    2.006    1.011
## 6      'marital_statusSingle ' 0.034    0.162    1.035    0.753
## 7          marital_statusWidowed 0.123    0.204    1.131    0.759
## 8              x6th_stageIIB 0.533    0.136    1.704    1.305
## 9              x6th_stageIIIA 0.978    0.134    2.660    2.047
## 10             x6th_stageIIIB 1.565    0.287    4.781    2.726
## 11             x6th_stageIIIC 2.006    0.152    7.433    5.519
## 12                 grade2 0.506    0.172    1.659    1.184
## 13                 grade3 0.862    0.180    2.368    1.664
## 14                 grade4 1.825    0.521    6.203    2.233
## 15      estrogen_statusPositive -0.714    0.166    0.490    0.354
## 16 progesterone_statusPositive -0.503    0.121    0.605    0.477
## 17          rn_examined_log -0.277    0.076    0.758    0.653
## 18              age_log 0.974    0.276    2.648    1.543
##      X95..CI..Upper.  P.Value
## 1          0.058 1.02e-05
## 2          2.052 0.000713
## 3          0.871 0.003683
## 4          1.015 0.064877
## 5          3.981 0.046555
## 6          1.422 0.833041
## 7          1.687 0.544852
## 8          2.225 9.02e-05
## 9          3.457 2.52e-13
## 10         8.386 4.82e-08
## 11         10.012 < 2e-16
## 12         2.324 0.003266
## 13         3.370 1.67e-06
## 14         17.232 0.000463
## 15         0.678 1.70e-05
## 16         0.767 3.37e-05
## 17         0.880 0.000270
## 18         4.545 0.000410
```

```
# Optional: Save the table to a CSV file for inclusion in the report
write.csv(results_table, "reweighted_model_results.csv", row.names = FALSE)
```

Finding and Intepretation

The final reweighted logistic regression model identifies significant predictors of mortality risk (Alive vs Dead) for breast cancer patients. The table above summarizes the parameter estimates, highlighting the direction, magnitude, and significance of each predictor. The study confirms that Adjusted AJCC 6th stage (**x6th_stage**) and **grade** (Grade of the tumor) are the most critical factors in predicting mortality, which is consistent with clinical guidelines. Hormone receptor status also plays a vital role, as positive estrogen/progesterone receptors improve survival. Race also significantly impacts mortality risk. Black patients experience a higher risk of death compared to White patients. This disparity may stem from differences in access to healthcare, late diagnosis, or biological factors. The reweighted model successfully improves fairness across racial groups while maintaining overall predictive accuracy. Although the model performs slightly worse for minority groups, the gap has been reduced compared to the original model.

Limitation

The model assumes no significant interaction effects, which aligns with the data but could miss complex relationships. The disparity in model performance for “Other” racial groups suggests the need for further data collection or alternative modeling techniques.