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IDENTIFICATION OF BASE PLANE PARAMETERS BY MEANS OF HOUGH TRANSFORM

Hough transform is a widely used method of recognizing shapes and objects from digital images. It was originally used for line detection from binary images, but together with the development of computational method, it was subsequently used for detecting more complex objects. In the paper, method of determination of geometrical parameters describing average machined or to-be-machined surfaces of casts, which can be created based on the clouds of points from coordinate machine measurements, was explained. The point cloud parameterization and its transformation into Hough space is explained. The identification of a shape is conducted by "voting". Any characteristic point "votes" i.e. adds a certain value to the value of points in Hough space which represent the surfaces on which that point lies. The paper describes the constraints of space accumulation due to the limitations of memory size. The method of its minimization was presented. The final set of values contained maxima which represented parameters of the surfaces from the original cloud of points. In addition, presented are the results for a cast which was a part of frame structure of machine tool.

Key words: Hough transform, coordinate measurements, machining

1. INTRODUCTION

Hough transform is a widely used method of detecting shapes such as circles, lines and planes from digital images [3]. The method originated to identify lines from binary images, but together with development of robust computational techniques, it helped to detect more complex parametrical objects [6]. Application of Hough transform into third dimension has recently gained a lot of attention. Currently, advanced sensor techniques allow to obtain detailed scans of complex large-scale surrounding, with millions of data points collected at high sampling rates [1, 8].

Application of Hough transform requires describing the detected shape parametrically as every point of the image is transformed into the parametric Hough space. Identification is performed by "voting". Every pixel (e.g. detected during

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contour recognition) votes, i.e. adds a certain value to already accumulated values of points in Hough space which represent parameters of the shape. As a result, a set is obtained, in which local maxima correspond to the parameters of the detected shapes in the original image.

Hough transform is a robust method of shape detection and is especially applied in the noisy images. Nonetheless, this method is not ideal as it requires a lot of memory due to the fact that all possible shapes have to be analyzed for every transformed point. However, there are methods which substantially decrease the computation time and hardware requirements [4]. The algorithm contains four major steps:

– determination of normal vectors for all points,
– transformation of points into Hough space,
– search for peaks,
– refinition of the results.

Methods for rapid automatic identification of particular geometric features have gained more attention, due to the development of visual inspection techniques and increased availability of CMMs. Shi et al. [9] presented a high-speed measurement algorithm for the position of holes in a large plane based on a flexible datum and the feature neighborhood model. Rahayem et al. [7] conducted a comparative study focused on fitting ellipses to 3D laser profile data. Ker et al. [5] developed Quick Hough Transform Parameter Search (QHTPS) algorithm to be implemented in a vision-based metrology system for in-process dimensional checking of nonpositioned parts and concentrated on part edge orientation. There is very little literature available referring to application of Hough Transform for automated quality inspection in terms of CMMs.

This article is an attempt to apply Hough transform into the automatic recognition of mating planes by analysis of point cloud obtained with coordinate measurements. The measured point cloud contains the information about the coordinates of particular points lying on the surface of the object. If there are many points available, a cloud is usually tessellated and transformed into a surface model which is then aligned and compared with the theoretical piece. However, when there is a limited number of measured points, the object cannot be easily reconstructed. In this article, the method of rapid automatic identification of particular planes from the unorganized cloud of points is described.

2. 3D HOUGH TRANSFORM FOR PLANE DETECTION

2.1. 2D example for line fitting into points

For a better understanding what Hough transform really is, let us consider a set of points into which a line can be fitted. Every line in 2D Cartesian coordinate system can be represented by the equation:

$$y = ax + b \tag{1}$$

Hough space for the 2D line can be described by two dimensions:

$$HS_{2D\text{line}} = (\theta, r) = [-\pi, \pi]^1 \times \mathbf{R} \tag{2}$$

Transformation of line into Hough space can be performed by applying the following formula:

$$r = x\cos\theta + y\sin\theta \tag{3}$$

Let us consider a line $y = x + 3$ and seven points that lie on the line: $\{-3,0\},\{-2,1\},\{-1,2\},\{0,3\},\{1,4\},\{2,5\},\{3,6\}$ (Figure 1a). Using equation (3) for every point, seven functions can be plotted in the continuous Hough space, which intersect at one point (Figure 2b).

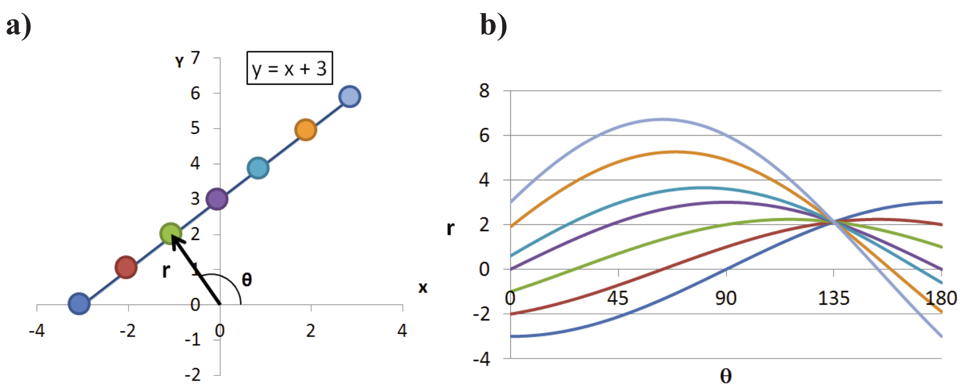


Figure 1. Selected points: a) lying on the line in the Cartesian coordinate system, b) represented in Hough space

The parameters of continuous Hough space calculated for the intersection point constitute a line equation in the 2D Cartesian system, namely:

$$\frac{3\sqrt{2}}{2} = x\cos 135° + y\sin 135° \Rightarrow y = x + 3 \tag{4}$$

If not all points lie on the line, Hough transform can be used for finding fitting line. For all points that were obtained e.g. from the measurement their transformation into Hough space is conducted by means of equation (3). For the considered set of seven points a small deviation was added in order to illustrate the problem (Figure 2a) and the transformation was performed into continuous Hough space (Figure 2b).

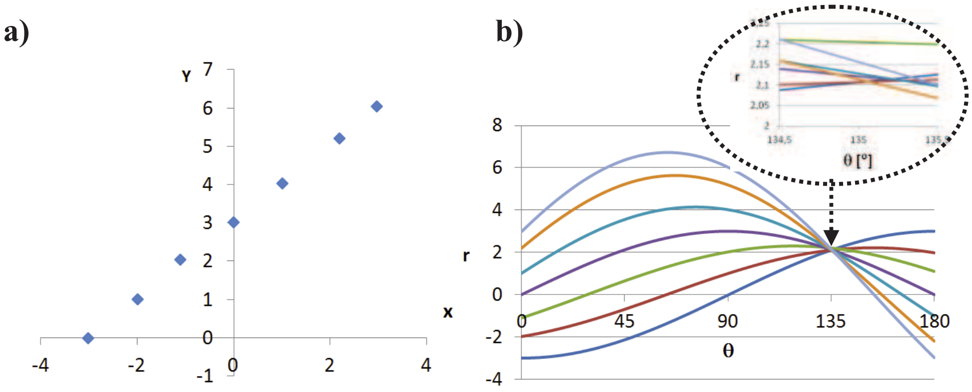


Figure 2. Selected points: a) lying on the line in the Cartesian coordinate system with added deviations, b) represented in Hough space

It can be noticed that all the lines do not intersect at one single point but there are many points of intersection. Thus, there are many lines which can be fitted into the points but one particular line crosses just through two points, what does not provide a good approximation. However, when we represent Hough space as discrete space with particular resolution of space parameters θ, r, then we might not receive as many intersection points as in the continuous space. The idea of Hough transform application for shape detection involves so called "voting". Each transformed point adds a single value to the discrete Hough space (Figure 3). The parameters θ, r obtained for the extreme values give an approximation of the line fitted into the seven points at the certain resolution. The increase of resolution of θ, r would in some cases lead to many local maxima in the accumulated Hough space. Therefore, the best solution out of those many would have to be chosen.

2.2. Plane in Hough space

In three-dimensional space a plane can be described by the following equation:

$$ax + by + cz + d = 0 \tag{5}$$

where a, b, c determine plane normal vectors and d is a distance between the plane and the origin of a coordinate system (0, 0, 0).

This definition, however, requires to use four-dimensional Hough space resulting in the enormous demand for memory used for transforming. Thanks to the normal vectors definition, Hough space can be described in this case by only three dimension as:

$$HS = (\alpha, \beta, d) = [-\pi, \pi]^1 \times [-\pi, \pi]^1 \times \mathbf{R} \tag{6}$$

where α, β are Euler angles related to the plane normal vector.

Hough transform defined in the above form (6) allows to save substantial amount of memory and reduce the computation time as the angular resolution is static and d is the only parameter that demands more memory (especially for surfaces that are located in a relatively huge space. In the standard approach, Hough transform involves analysis of all possible planes on which that individual point lies and accumulation of the plane parameters into Hough space.

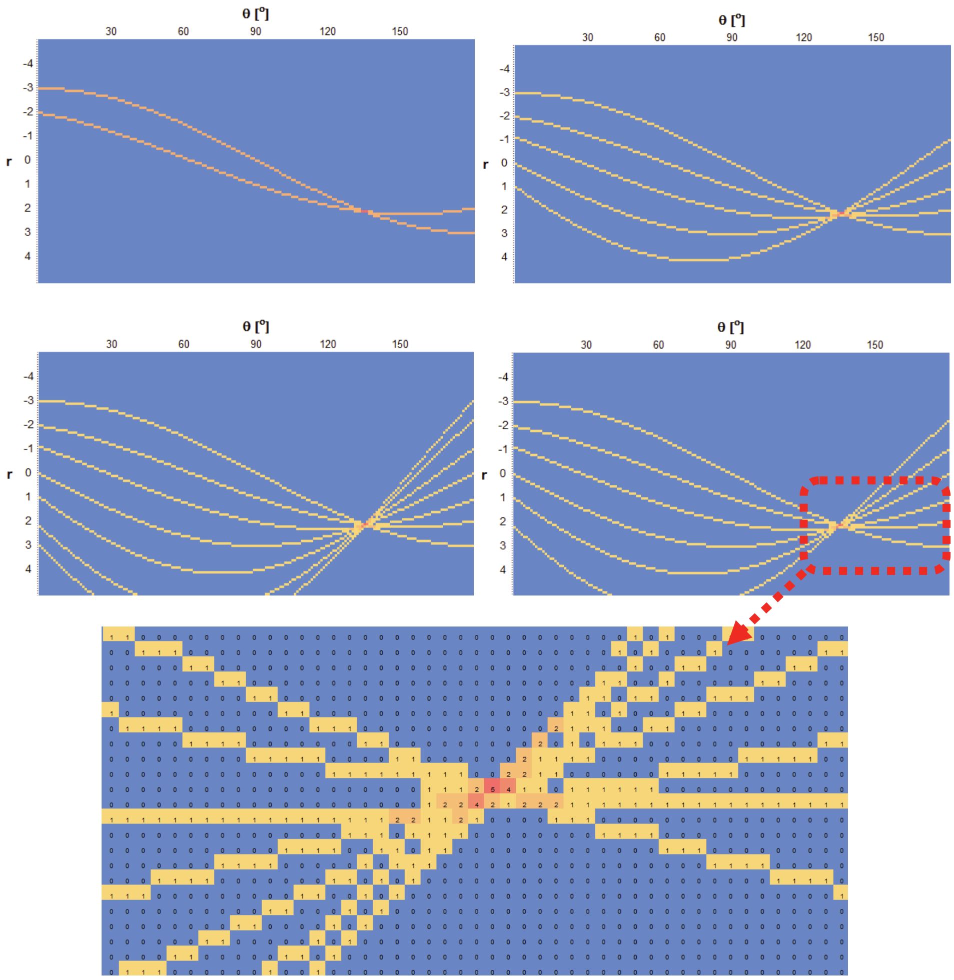


Figure 3. Discrete Hough space representation for 2D line identification

2.3. Normal vectors

Normal vectors are essential for initial determination of constraints of Hough space as they define potential orientation of particular planes to be detected. This approach allows to minimize the transformation and search space, decreasing the total processing time. For calculating normal vectors for individual point, one of three following methods can be applied [2]:

– Covariance matrix analysis – analysis of eigenvectors and eigenvalues calculated for the neighboring points,

– Average 3D gradient – analysis of average differences between neighboring points calculated in the vertical and horizontal direction,

– Average depth change – analysis of horizontal and vertical 3D differences from.

It usually occurs that normal vectors are calculated automatically by the measurement software dedicated for the particular coordinate measurement machine or device. The method of this calculation frequently remains a part of trade secret and is not disclosed publicly.

2.4. Transformation

Having surface normals from the point cloud, it is possible to estimate plane parameters for every point of the 3D Hough space. Coordinates of Hough space are calculated for an individual *i*-point by using normal vectors:

$$d_i = -(n_{xi}x_i + n_{yi}y_i + n_{zi}z_i) \tag{7}$$

where: $\vec{n}_i = (n_{xi}, n_{yi}, n_{zi})$ is the normal vector derived for the considered *i*-point and $\vec{p}_i = (x_i, y_i, z_i)$ is the considered *i*-point extracted from the point cloud.

Using equation (7) it is possible to compute all the required parameters of Hough space $h = (\alpha, \beta, d)$. The transformation of a particular point takes the following form:

$$h(p) = (\alpha, \beta, d) \tag{8}$$

2.5. Structure of Hough space

Hough space is represented by *n*-dimensional array, which dimensions are determined by the required resolution of the plane parameters. As an example, let us assume that the required angular resolution of α and β is 0,001 rad and the points are distributed in the 2 m × 2 m × 2 m space. Thus, the required size of the array should be at least 6284 × 6284 × 4000. It means that for the 32-bit floating point number representation, the memory requirements would exceed 600 GB, which is an extreme figure, even for the contemporary standards. Therefore, it is essential to significantly lower the computational demands. It can be done by conducting initial computation with rough accuracy in order to select potential areas of high density of values. Then for the areas, calculations are done with the higher accuracy (Figure 4). The process can be repeated iteratively until the requested accuracy is reached.

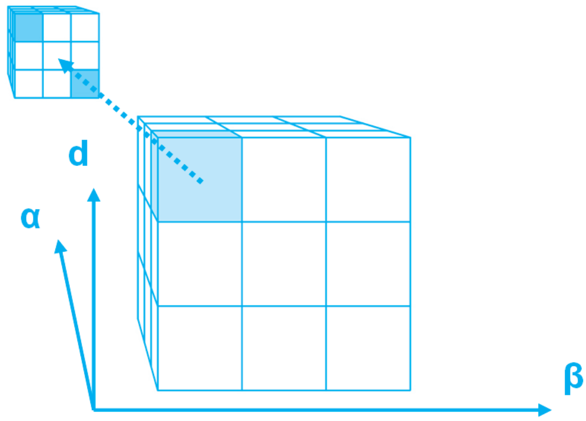


Figure 4. From rough to high parameters resolution

The aforementioned approach allows to save up to 99 % of the resources [4].

2.6. Accumulation

The final step of Hough space creation is accumulation of points which parameters refer to the estimated planes. If points coordinates are influenced by high measurement errors then the direct accumulation may lead to significant errors in further computations. Due to that fact, an individual point can be represented in 3D space smoothly by Gaussian function:

$$G(\bar{h},\bar{g},\bar{\sigma})=\frac{1}{\left(\sqrt{2\pi\sigma}\right)^{3}}\mathrm{e}^{-\frac{(\bar{h}-\bar{g})^{2}}{2\sigma^{2}}} \tag{9}$$

where: $\bar{h}(\alpha,\beta,d)$ is the projected point constituting a reference for the particular point $\bar{g}(\alpha,\beta,d)$ and σ is a standard deviation of the Gaussian distribution.

2.7. Finding extreme values

The next step in calculation parameters of approximated surface is to find local maxima in the accumulated Hough space. This process is extremely essential as identified maxima constitute potential parameters of the estimated plane. Due to the fact the Hough space can be relatively vast, it is vital to limit the search area to a few potential regions because most of planes are parallel or perpendicular to each other or inclined similarly to the corresponding planes of the measurement coordinate system.

If there is a significant value of measurement noise affecting the measurement point coordinates, it is suggested to take into analysis not only a particular point of interest but also its neighborhood in Hough space. Final value of point transformed into Hough space takes the following form:

$$m(x) = \frac{\sum_{p \in W(x)} h(p) \cdot p}{\sum_{p \in W(x)} h(p)} \tag{10}$$

where $W(x)$ is a set of neighboring points lying around point x and $h(p)$ is a value of Hough space in point x. A sliding window of a specified size is used to find the peaks. The nearest neighborhood is analyzed for an individual point represented in *HS* which allows to treat adjacent maxima as a one peak.

3. CASE STUDY

The point cloud obtained by means of coordinate measurement contains information about location of particular surface points of the measured object. If there are many points available the cloud is usually transformed into triangulated surface model which is then aligned and compared with theoretical model using e.g. best fit method. However, this approach requires a multitude of measured points and a specialized software. If there are few points available the interpretation of the cloud and identification of particular surface parameters is usually performed manually. Hough transform can be used to automate this process as it allows to determine which points refer to which surface.

In this study, two machined casting which were structural components of machine tool were examined. The part were measured by means of photogrammetric technique using GOM Tritop [10]. The markers were glued onto the particular surfaces and several photographs were captured. As a result, the point cloud was obtained which went under further investigations. Measurement system provided information about the normal vectors corresponding to the individual points.

Points were grouped according to the initial orientation in order to increase the accuracy and decrease the computational time. All the points were transformed into Hough space and smoothed by Gaussian kernel. Gaussian kernel in the Hough Space was always the same as standard deviation value was constant. Gaussian kernel could be pre-computed and added incrementally for the transformed points in the *HS*. For the identified maxima, the parameters of the potential planes were determined. Each point was attributed to the detected plane by finding the least distance. For the points that lied on the potentially identified plane a plane was fitted using 3D least squares method. The parameters of a plane:

$$z_i = ax_i + by_i + c \tag{11}$$

were calculated by solving following set of equations:

$$\begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i z_i \\ \sum_{i=1}^{n} y_i z_i \\ \sum_{i=1}^{n} z_i \end{bmatrix} \tag{12}$$

where: n is the number of points which constitute the surface. This approach allowed to increase the accuracy of the results as no fine scale refinement of *HS* is required in order to determine precise the orientation of the fitted plane. Hough Space is used for attributing the points to the potential plane, which is then precisely calculated by equation (12).

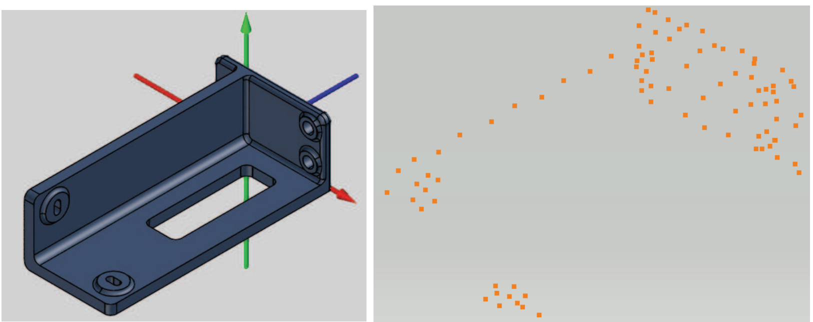
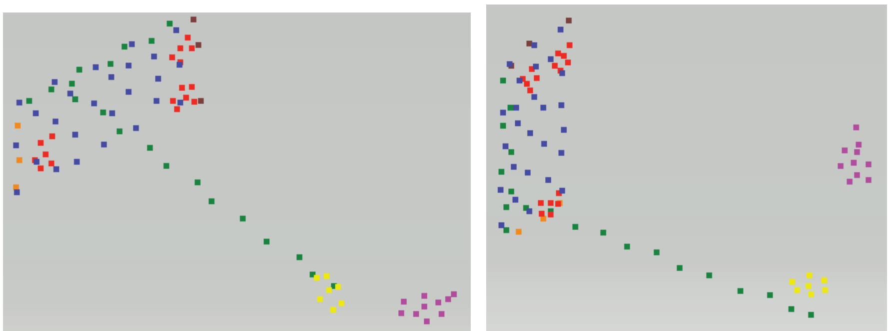


Figure 5. Measured first object and corresponding point cloud

a)



b)

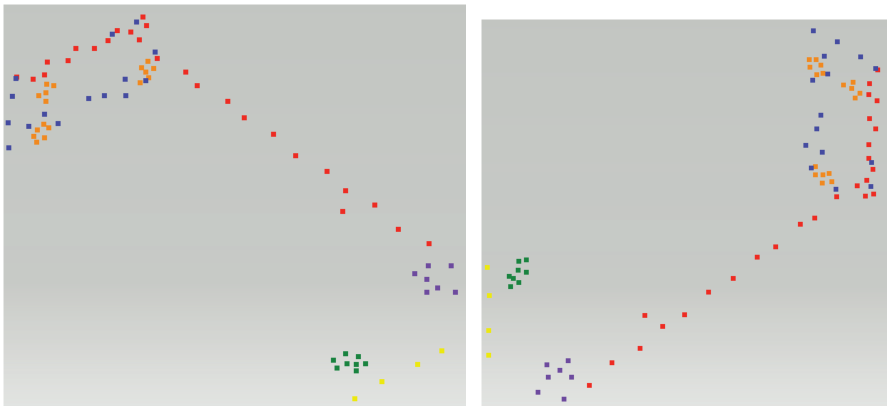


Figure 6. Color-coded image of points corresponded with planes they lie on: a) for part A and b) for part B

As a reference the real location of points was given and the 3D least squares planes were fitted by the same method. The overview of the point cloud for the first measured part is depicted in Figure 5. The results of computations are presented in Table 1 and in Figure 6.

Table 1

Results of detected planes for the measured castings A and B

Number of	Part A	Part B
Planes to be found	9	8
Detected planes	7	6

Table 2

Results of angular errors for the measured castings A and B

Plane	$\Delta\alpha$ [°]	$\Delta\beta$ [°]	$\Delta\gamma$ [°]	Number of points of real plane	Number of points of detected plane
Part A					
1	0	0	0	21	21
2	0	0	0	26	26
3	0	0	0	14	14
4	0	0	0	3	3
5	0	0	0	10	10
6	0	0	0	10	10
7	0	0	0	7	7
Part B					
1	0	0	0	25	25
2	1.483	1.421	0.342	11	13
3	0	0	0	15	15
4	0	0	0	7	7
5	0	0	0	4	4
6	0.568	0.674	1.14	4	6

It can be noticed that not all features were detected by using the proposed method. It happened that way in case two sets of points representing two separate surfaces (two flat features which would come in contact with two bolts) lie theoretically on the same plane (they have same theoretical x-, y- or z-value). By means of the described method, most of the planes were detected at the acceptable angular error level. Angular level is calculated as a difference between the orientation angles of a least squares plane normal vector: α – between plane and X-axis of the coordinate measurement system and β between plane and Y-axis of the coordinate measurement system and γ between plane and Z-axis of the coordinate measurement system (see Table 2).

4. SUMMARY

The article presented the idea of Hough transform and described its application into automatic fitting of planes into unorganized point cloud derived from coordinate measurement. In the described method, Hough transform is used to associate particular points with the potentially identified planes. The obtained sets are the base of further computation of best-fit plane fitted by means of 3D least squares method. The example of results shows that the good applicability for the limited sample problems when surface model cannot be easily derived from the cloud or there is no reference 3D model available.

Hough transform has gained a lot of attention lately due to the development of inexpensive measurements method as it allows robust interpretation of a obtained point cloud. The method can generalized for the other parameterized shapes such as cylinders or ellipses, which offers excellent prospects for the automatic interpretation of measurement data in the nearest future.

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IDENTYFIKACJA PARAMETRÓW POWIERZCHNI OBRÓBKOWYCH ODLEWÓW NA PODSTAWIE POMIARÓW WSPÓŁRZĘDNOŚCIOWYCH Z WYKORZYSTANIEM TRANSFORMACJI HOUGHA

S t r e s z c z e n i e

Transformacja Hougha jest metodą szeroko stosowaną do rozpoznawania kształtów i obiektów ze zdjęć. Początkowo była ona wykorzystywana do detekcji linii z obrazów binarnych, a wraz z rozwojem technik obliczeniowych zaczęto ją stosować do detekcji coraz bardziej skomplikowanych obiektów. W artykule przedstawiono metodę określania parametrów geometrycznych opisujących uśrednione powierzchnie obróbkowe odlewów otrzymane na podstawie analizy chmury punktów pomiarowych uzyskanych z pomiarów współrzędnościowych. Opisano sposób parametryzacji chmury punktów i ich przekształcenie do przestrzeni Hougha. Kształt był identyfikowany przez głosowanie. Charakterystyczny punkt „głosował", tj. dodawał pewną jednostkową liczbę do wartości tych punktów w zbiorze powierzchni, które reprezentowały powierzchnie przechodzące przez ten punkt. W artykule przedstawiono ograniczenia akumulacji przestrzeni ze względu na wielkość pamięci obliczeniowej oraz sposób minimalizacji tej przestrzeni. Wynikowy zbiór zawierał maksima reprezentujące powierzchnie wykryte w oryginalnej chmurze punktów reprezentującej odlew. Opisaną metodę zastosowano do automatycznej identyfikacji powierzchni obróbkowych odlewu stanowiącego fragment struktury nośnej obrabiarki.

Słowa kluczowe: transformacja Hougha, pomiary współrzędnościowe, obróbka skrawaniem