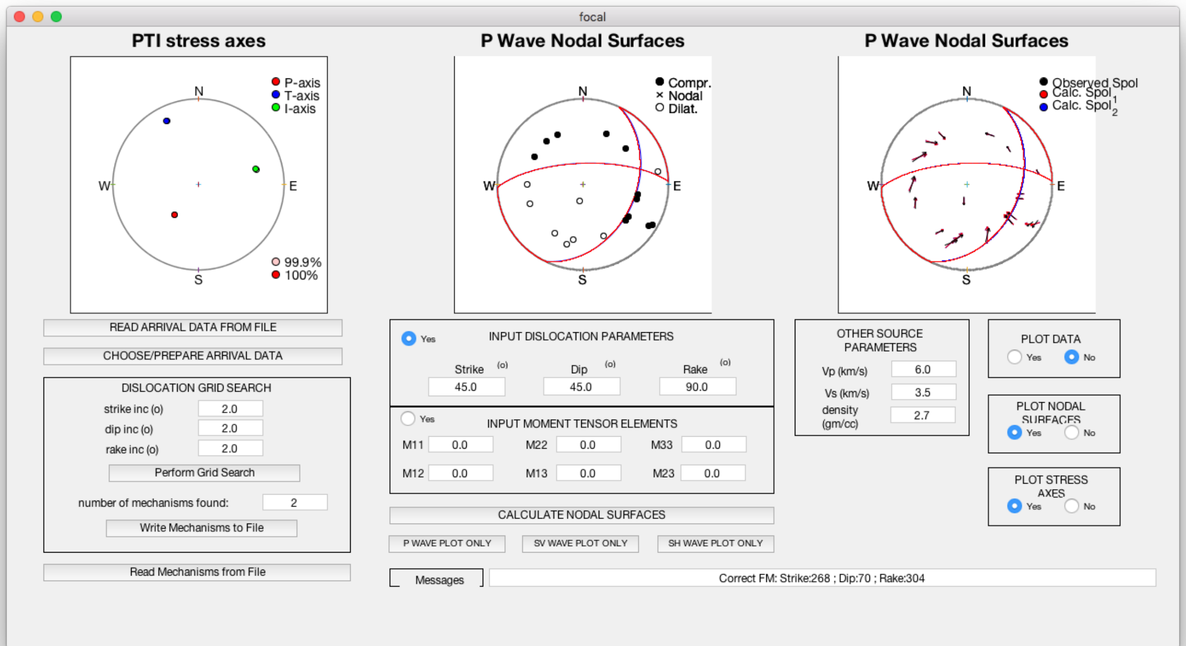
# Focal User Manual

November 26, 2018



# Contents

# 1 Overview

Focal is a MATLAB code that can be used to determine focal mechanisms of earthquakes. The code is based on a grid search algorithm with the P-, SV- and SH- wave polarities and amplitude ratios as constraints. The user has the ability to set relative weights to control the contributions from these two constraints (wave polarities and amplitude ratios). The amplitude ratios include $\frac{|SV|}{SH}$, $\frac{|SV|}{P}$, $\frac{SH}{P}$ and combinations of the amplitude ratios. Focal also allows manual inputs of the dislocation parameters or moment tensor elements to generate P-, SV- and SH-wave nodal surfaces that fit the input data.

The layout of Focal include three plots to display results depending on the method used. It has three panels. The first panel can be used to read in data parameters, perform the grid search method, write the results to file and perform error analysis. The second panel include include instruction for inputting focal mechanisms manually to determine nodal surfaces that best match input data. The third panel contains additional parameters needed for the manual input. This include the velocity and densities at the source depth and other options used in displaying the results.

For the grid search method, Focal has three plots showing the stress axes, P-wave nodal surfaces with the polarity data, and S-wave polarization with the nodal surfaces. The stress axes shows the locations of the pressure (P), tensional (T) and the intermediate (I) axes. The figure shows the results from a grid search method of an input synthetic data. The focal sphere are lower hemisphere projection (Fig. 1). When the dislocation parameters are given manually, the three plots will display the P-, SV- and SH-wave nodal surfaces superimposed (optional) on the input P-, SV- and SH- polarities (Fig. 2). Figure 2 shows the results of the synthetic data in Figure 1 using the manual-input option of code.

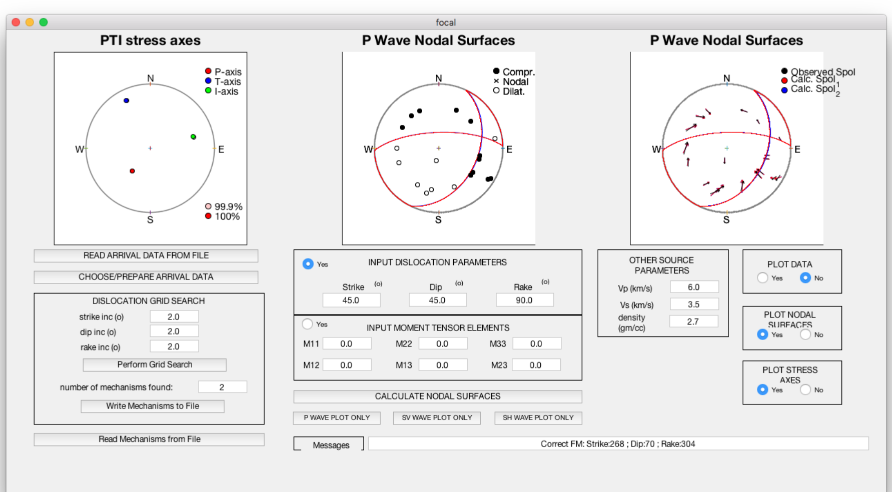



Figure 1: The results using a grid search method of an input synthetic data.

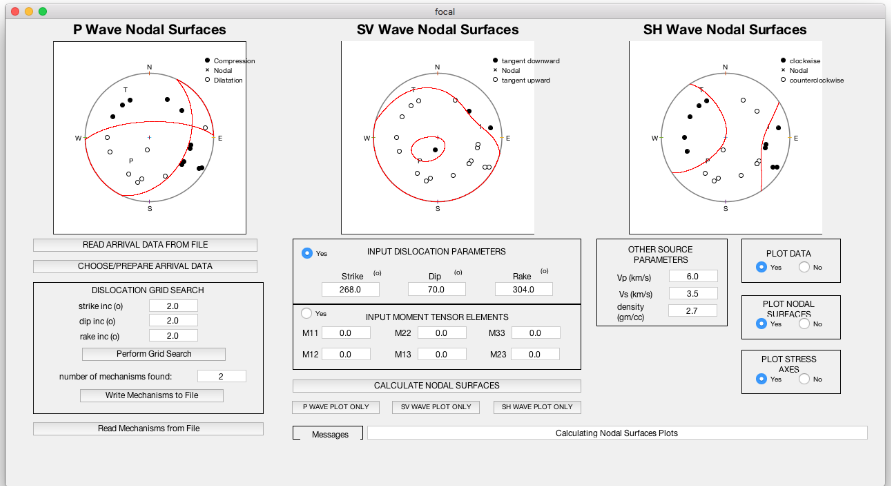



Figure 2: The results of the synthetic data in Figure 1 using the manual-input option of code.

# 2 Governing Equations

## 2.1 Forward Modeling: Generating Synthetic Data Sets

Synthetic wave displacements can be generated using the equation below:

$$u_i(\vec{x}, t) = G_{ij,k}[M_{jk}]$$

where $u_i(\vec{x}, t)$, $G_{ij,k}$ and $M_{jk}$ are displacement time series at location $\vec{x}$, derivative of the Love tensor ($G_{ij}$) with respect to k-direction, and moment tensor, respectively. The equation can be expanded using index notation and isolating the different source components as:

$$\begin{aligned}
u_i(\vec{x}, t) = G_{i1,1}[M_{11}] + G_{i2,2}[M_{22}] + G_{i3,3}[M_{33}] \quad & \Rightarrow \text{(isotropic + partial deviatoric source)} \\
+ G_{i1,2}[M_{12}] + G_{i2,1}[M_{21}] \quad & \Rightarrow \text{(vertical strike slip)} \\
+ G_{i1,3}[M_{13}] + G_{i3,1}[M_{31}] \quad & \Rightarrow \text{(vertical dip slip)} \\
+ G_{i2,3}[M_{23}] + G_{i3,2}[M_{32}] \quad & \Rightarrow \text{(vertical dip slip)}
\end{aligned}$$

- The derivatives of the Green's function, ignoring higher order of epicentral distance in the denominator, is:

$$G_{ij,k} = \frac{\delta G_{ij}}{\delta \gamma_k} \rightarrow G_{ij,k} \approx \frac{G_{ij}\gamma_k}{\alpha}$$

where

$$4\pi\rho G_{ij} = \frac{3\gamma_i\gamma_j - \delta_{ij}}{R^3}\int_{\frac{R}{\alpha}}^{\frac{R}{\beta}} \tau h(t-\tau)d\tau + \frac{\gamma_i\gamma_j h(t-\frac{R}{\alpha})}{\alpha^2 R} - \frac{(\gamma_i\gamma_j - \delta_{ij})h(t-\frac{R}{\beta})}{\beta^2 R}$$

with the following intermediate results:

$$\begin{aligned}
\frac{3\gamma_i\gamma_j - \delta_{ij}}{R^3} &= \frac{\partial^2(\frac{1}{R})}{\partial x_i \partial x_j} \\
-\frac{(\gamma_i\gamma_j - \delta_{ij})}{R} &= \frac{\partial \gamma_j}{\partial x_i} = \frac{\partial^2 R}{\partial x_i \partial x_j} \\
\gamma_i &= \frac{(x_i - \xi_i)}{R} = \frac{\partial R}{\partial x_i}
\end{aligned}$$

- Secondly, we decompose moment tensor into isotropic, strike slip, dip slip DC sources and the vertical CLVD components

$$M = M_1(\phi_S) + M_2(\phi_D) + M_{iso} + M_{v-clvd}$$

$$\begin{aligned}
\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} &= \begin{pmatrix} -\frac{1}{2}(M_{22} - M_{11}) & M_{12} & 0 \\ M_{21} & \frac{1}{2}(M_{22} - M_{11}) & 0 \\ 0 & 0 & 0 \end{pmatrix} && \Rightarrow \text{strike-slip DC} \\
&+ \begin{pmatrix} 0 & 0 & M_{13} \\ 0 & 0 & M_{23} \\ M_{31} & M_{32} & 0 \end{pmatrix} && \Rightarrow \text{Dip-slip DC} \\
&+ \frac{1}{3}(M_{11} + M_{22} + M_{33}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} && \Rightarrow \text{Isotropic source} \\
&+ \frac{1}{3}[\frac{1}{2}(M_{22} + M_{11}) - M_{33}] \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} && \Rightarrow \text{Vertical CLVD}
\end{aligned}$$

- Using the results from above equations, we can determine the P-, SV- and SH-wave displacements with the following formula:

$$\begin{aligned}
U_R &= \gamma_1 u_1 + \gamma_2 u_2 + \gamma_3 u_3 && \Rightarrow P \\
U_\theta &= u_1 \cos\theta \cos\phi + u_2 \cos\theta \sin\phi - u_3 \sin\theta && \Rightarrow SV \\
U_\phi &= -u_1 \sin\phi + u_2 \cos\phi && \Rightarrow SH
\end{aligned}$$

where

$$\gamma_1 = \sin\theta\cos\phi$$
$$\gamma_2 = \sin\theta\sin\phi$$
$$\gamma_3 = \cos\theta$$

– **P-wave Amplitude**

$$U_R = U_R(\phi_S) + U_R(\phi_D) + U_{Riso} + U_{Rv-clvd}$$

$$\text{con} = \frac{\pi}{180} \qquad a = \text{az} \times \text{con}$$
$$p = \frac{\sin(\text{inc} \times \text{con})}{\alpha_p} \qquad \eta_\alpha = \sqrt{\frac{1}{\alpha^2} - p^2}$$

$$\begin{aligned}
P_{\text{sph}} &= [\frac{(M_{11} + M_{22} + M_{33})}{3}] \times hr0 && \Rightarrow \text{(isotropic source)} \\
&+ [\frac{M_{22} - M_{11}}{2}\cos(2a) - M_{12}\sin(2a)] \times hr1 && \Rightarrow \text{(vertical strike-slip)} \\
&+ [M_{13}\cos(a) + M_{23}\sin(a)] \times hr2 && \Rightarrow \text{(vertical dip-slip)} \\
&+ [\frac{M_{11} + M_{22} - 2M_{33}}{6}] \times hr3 && \Rightarrow \text{(clvd source)}
\end{aligned}$$

where

$$con1 = \frac{10^5}{4\pi\rho\alpha} \qquad hr0 = \frac{\text{con1}}{\alpha^2} \qquad hr1 = \text{con1} \times p^2$$
$$hr2 = -2 \times \text{con1} \times eps \times p\eta_\alpha \qquad hr3 = -con1 \times (p^2 - 2\eta_\alpha^2)$$

– **SV-wave Amplitude**

$$U_\theta = U_\theta(\phi_S) + U_\theta(\phi_D) + U_{\theta iso} + U_{\theta v-clvd}$$

$$\text{con} = \frac{\pi}{180} \qquad a = \text{az} \times \text{con}$$
$$p = \frac{\sin(\text{inc} \times \text{con})}{\beta_p} \qquad \eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2}$$

$$\begin{aligned}
SV_{\text{sph}} &= [\frac{1}{2}(M_{22} - M_{11})\cos(2a) - M_{12}\sin(2a)] \times hr1 && \Rightarrow \text{(source)} \\
&+ [M_{13}\cos(a) + M_{23}\sin(a)] \times hr2 && \Rightarrow \text{(slip)} \\
&+ [\frac{M_{11} + M_{22} - 2M_{33}}{6}] \times hr3 && \Rightarrow \text{(source)}
\end{aligned}$$

where

$$con1 = \frac{10^5}{4\pi\rho\beta} \qquad hr1 = \text{con1} \times eps \times p\eta_\beta$$

$$hr2 = -con1 \times (\eta_\beta^2 - p^2) \qquad hr3 = -con1 \times 3 \times eps \times p\eta_\beta$$

– **SH-wave Amplitude**

$$U_\phi = U_\phi(\phi_S) + U_\phi(\phi_D) + U_{\phi iso} + U_{\phi v-clvd}$$

$$\text{con} = \frac{\pi}{180} \qquad a = \text{az} \times \text{con}$$

$$p = \frac{\sin(\text{inc} \times \text{con})}{\beta_p} \qquad \eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2}$$

$$\begin{aligned} SH_{\text{sph}} &= [\frac{M_{11} - M_{22}}{2} \sin(2a) - M_{12}\cos(2a)] \times hr1 && \Rightarrow \text{(source)} \\ &+ [M_{23}\cos(a) - M_{13}\sin(a)] \times hr2 && \Rightarrow \text{(slip)} \end{aligned}$$

where

$$con1 = \frac{10^5}{4\pi\rho\beta} \qquad hr1 = \frac{\text{con1} \times p}{\beta^2} \qquad hr2 = -\text{con1} \times eps \times \frac{\eta_\beta}{\beta}$$

- The moment tensor components can be determined using the strike (s), dip (d) and rake (r) data. All angles are in radians.

$$\begin{aligned} M_{11} &= \sin^2(s)\sin(r)\sin(2d) + \sin(2s)\cos(r)\sin(d) \\ M_{22} &= \cos^2(s)\sin(r)\sin(2d) - \sin(2s)\cos(r)\sin(d) \\ M_{33} &= -\sin(r)\sin(2d) \\ M_{12} &= -\cos(2s)\cos(r)\sin(d) - \frac{1}{2}\sin(2s)\sin(r)\sin(2d) \\ M_{13} &= \cos(s)\cos(r)\cos(d) + \sin(s)\sin(r)\cos(2d) \\ M_{23} &= \sin(s)\cos(r)\cos(d) + \cos(s)\sin(r)\cos(2d) \end{aligned}$$

- To compare displacements from radiation pattern discussed above with observed displacements, we need to converting $\text{P}_{\text{sph}}$, $\text{SV}_{\text{sph}}$ and $\text{SH}_{\text{sph}}$ to vertical (Z), radial (R) and tangential (T) components.

$$\begin{aligned} w &= -\eta_\alpha eps\Phi + p\Omega && \Rightarrow \text{(P-SV system)} \\ Q &= -p\Phi - \eta_\beta eps\Omega && \Rightarrow \text{(P-SV system)} \\ V &= p\chi && \Rightarrow \text{(SH system)} \end{aligned}$$

where:

$$\Phi \Rightarrow \text{(prad)} \; \Omega \Rightarrow \text{(svrad)} \qquad\qquad \chi \Rightarrow \text{(shrad)}$$

The above equations is for a coordinate system where z is positive downwards.

Since most seismograms have z positive upwards, we need to change the equations accordingly. For this, the equations are:

$$\begin{aligned}
w &= -(-\eta_\alpha eps\Phi + p\Omega) && \Rightarrow \text{(P-SV system)} \\
Q &= -p\Phi - \eta_\beta eps\Omega && \Rightarrow \text{(P-SV system)} \\
V &= p\chi && \Rightarrow \text{(SH system)}
\end{aligned}$$

For first motion displacements, we use:

– **P wave displacements** $\Rightarrow$

$$w = \eta_\alpha eps\Phi \Rightarrow \text{(z-comp of P wave)}$$

– **SV wave displacements** $\Rightarrow$

$$\begin{aligned}
w_{sv} &= -p \times \Omega && \Rightarrow \text{(z-comp of SV wave)} \\
Q_{sv} &= -\eta_\beta eps\Omega && \Rightarrow \text{(r-comp of SV wave)} \\
SV_{amp} &= sign(Q_{sv}) \times \sqrt{w_{sv}^2 + Q_{sv}^2}
\end{aligned}$$

In other words, the SV vector could either lean to the front or backwards based on the sign of its radial component.

– **SH wave displacements** $\Rightarrow$

$$V = p\chi \Rightarrow \text{(SH system)}$$

In summary, we can generate the synthetic datasets for Focal using the following steps:

- Decide the fault plane solutions to model in terms of strike, dip and rake, velocity model, source depth and the distances and azimuths of interested stations.
- Estimate $V_p$ and $V_s$ velocities, and density ($\rho$) at the source depth using the given velocity model,
- Calculate the incidence angles at each station including wave propagation information i.e., whether the wave is upgoing or downgoing. In Focal, we use a travel-time calculation algorithm (**travelt**) to determine the incidence angles.

- Determine moment tensor components using the given strike, dip and rake.
- Determine P-, SV- and SH- wave amplitudes at each station.
- Generate data for Focal in terms of p_pol, sv_pol and sh_pol. The p_pol, for example, consists of P-wave amplitudes, azimuth to the station, incidence angle, weight assigned to each amplitude, index, and eps [i.e., upgoing (-1) or downgoing (+1)].

$$\text{pol} = \begin{bmatrix}\text{amp} & \text{az} & \text{inc} & \text{weight} & \text{index} & \text{eps}\end{bmatrix}$$

- Focal use the pol data to plot the wave polarities on a focal sphere (optional). Focal create a circle with radius, R, and plot N, S, E and W markings. It then plots wave polarities using azimuths (az) and incident angle (inc). The pressure (P), tension (T) and intermediate (I) vectors were determined using eigenvectors of the moment tensor component. To plot each points, we use r, $x_p$ and $y_p$.

$$con = \frac{\pi}{180} \qquad r = \sqrt{2}R\sin(\frac{inc}{2}con)$$

$$x_p = R\sin(az \times con) \qquad y_p = R\cos(az \times con)$$

## 2.2 Inverse Method

Recall the forward problem formulation:

$$\text{G-matrix} \times \begin{bmatrix}\psi \\ \delta \\ \lambda \\ \alpha \\ \beta \\ \rho\end{bmatrix} = \begin{bmatrix}U_R(P) \\ U_\theta(SV) \\ U_\phi(SH)\end{bmatrix}$$

In other words, given the strike ($\psi$), dip ($\delta$), rake ($\lambda$), P- and S- wave velocities ($\alpha$ and $\beta$, respectively), and the density ($\rho$) at the source depth, we can generate the P, SV and SH displacements at the stations. In the inverse problem formulation using Grid Search Method, we use the forward problem to generate first motion amplitudes for all possible strike, dip and rake and compare the displacements with the observed at each station. We use the following steps:

- Load data in terms of $p_{pol}$, $sv_{pol}$ and $sh_{pol}$, including the $\alpha$, $\beta$ and $\rho$ at the source depth.

- Determine moment tensor components. i.e $M_{11}$, $M_{12}$ etc. for each combination of strike, dip and rake. Strike ranges from $0°$ to $(360° - \text{strike incr})$, dip ranges from $0°$ to $90°$, and rake ranges from $0°$ to (360° - rake incr) in steps specified by the increments.
- Determine P-, SV- and SH-wave displacements for all possible combinations of strike, dip and rake at each station.
- Convert the first-motion displacements to Z, R and T components as measured in the real data.

We use two main constraints in the grid search method. They are:

1. Wave polarities: P, SV and SH polarities.
2. Wave amplitudes: $\frac{|SV|}{P}$, $\frac{SH}{P}$ and $\frac{|SV|}{SH}$.

We use the absolute value of SV amplitude because of the possibility of a change in polarity of SV wave at the free surface.

#### 2.2.1 Grid search using wave polarities

- Convert observed and calculated displacements to polarities and form two vectors consisting of the P-, SV- and SH-wave polarities at each station. For example,

$$\text{Observed polarity} = \Big[\text{-1, 1, 1 (P)} \quad ... \quad \text{-1, 1, -1 (SV)} \quad ... \quad \text{1, 1, -1 (SH)}\Big]$$
$$\text{Calculated polarity} = \Big[\text{1, -1, 1 (P)} \quad ... \quad \text{1, 1, -1 (SV)} \quad ... \quad \text{1, 1, 1 (SH)}\Big]$$

- Apply the input amplitude weights to the observed polarities. The weights are based on the clarity of the amplitudes. We normalized the weight vector by 5 to make the highest polarity one (1). Apply the weight vector by the dot product of the observed polarities and the weight vector.

$$\text{weighted observed polarity} = [\text{Observed polarity}] \; .* \; [\frac{\text{weight vector}}{5}]$$

- Determine the accuracy of each strike, dip and rake using:

$$\epsilon_{pol} = \frac{[\text{weighted obs polarity}] \; .* \; [\text{calc polarities}]}{N_w} \times 100\%$$

where $N_w$ is number of non-zero weights for P, SV and SH data points. Note that the $\epsilon_{pol}$ is 100% (perfect solution using only wave polarity) because the dot product of the polarity vectors equals to the number of data points ($N_w$). If N is the number of P-, SV- and SH-wave amplitudes used and $N_w$ = N. Since the SV- wave amplitudes are sometimes indistinct on a seismogram and can change in polarity, there is a choice if the user does not want to use SV polarity in the focal mechanism solutions.

### 2.2.2 Grid search using wave amplitude ratios

In addition to the wave polarities, Focal adds amplitude ratios ($\frac{|SV|}{P}$, $\frac{SH}{P}$ and $\frac{|SV|}{SH}$) as constraints. Note that the SV amplitude is the radial component. Instead of the amplitude ratios explicitly, we calculate the angles the vector of each amplitude ratio makes with the horizontal for each station (from the north). e.g.

$\Omega(\frac{|SV|}{P})_{obs} = [67°, 32°, 21°]$; $\Omega(\frac{SV}{P})_{calc} = [50°, 20°, 15°]$;

$\Omega(\frac{SH}{P})_{obs} = [230°, 25°, 150°]$; $\Omega(\frac{SH}{P})_{calc} = [230°, 25°, 150°]$;

$\Omega(\frac{|SV|}{SH})_{obs} = [13°, 12°, 340°]$; $\Omega(\frac{SV}{SH})_{calc} = [13°, 12°, 340°]$;

For each amplitude ratio, we determine the difference in the calculated and observed angles (smallest angle between the two vectors) and apply the weight vectors. In other words, if the angle difference is more than 180°, we use 360 - $\Omega'$.

$$\Omega'(\frac{SV}{P}) = [\Omega(\frac{SV}{P})_{obs} - \Omega(\frac{SV}{P})_{calc}]$$

$$\Delta\Omega(\frac{SV}{P}) = \Omega'(\frac{SV}{P}).*\frac{\text{weight vector}_{min}}{5}$$

Note that the weight vector is the minimum weight between, for example, SV and P-wave amplitudes at each station. We determine the percentage of accuracy ($\epsilon_{ratios}$) using amplitude ratio as:

$$\Delta\Omega = [\Delta\Omega(\frac{SV}{P}); \Delta\Omega(\frac{SH}{P}); \Delta\Omega(\frac{SV}{SH})]$$

$$\epsilon_{ratios} = (1 - \frac{\sum_{n=1}^{N}\Delta\Omega}{179 * N}) \times 100\%$$

We use 179° because it is the maximum smallest angle (in 1° resolution) between two vectors.

### 2.2.3 Combining Wave Polarity and Wave Amplitude Ratios

After determining $\epsilon_{pol}$ and $\epsilon_{ratios}$, we then combine them using the weight the user specified for the two grid-search methods to determine $\epsilon$ for each combination of strike, dip and rake angles. Focal then determines the strike, dip and rake that minimizes $\epsilon$. The grid search based on polarity usually have more weight than amplitude ratio because it is less susceptible to wave propagation and free surface.

$$\epsilon = (W_{pol} \times \epsilon_{pol}) + (W_{ratios} \times \epsilon_{ratios})$$

where

$W_{pol}$ = weight for grid search using polarity data, and

$W_{ratios}$ = weight for grid search using amplitude ratio data.

# 3 Tutorial

## 3.1 Input Data

The data used in Focal include P-, SV- and SH-wave amplitudes, azimuths and incidence angle to seismic stations, weight assigned to each amplitude measurement, index, *eps* and velocity model. The *eps* indicate whether the wave is upgoing (*eps* = -1) or downgoing (*eps* = +1). The amplitude weight range from 1 (lowest weight) to 5 (very good amplitude measurement). Weight values of zero signify stations that will not be used in the focal mechanism solution. These information are supplied in the parameters.in file. We will describe the format of each input in the following sections for both synthetic and real data.

### 3.1.1 Synthetic Data

- A sample of Parameters.in file for synthetic datasets are as follows:

  ***% Set up source parameters to generate the desired fault plane solution data***

  strike = 268°; dip = 70°; rake = 304°;

  ***% Source depth and its error. The depth error will be used for error analysis***

  depth = 12.83; depth_error = 4; % km

  ***Enter the filename the synthetic data should be stored***

  filename = 'synth_arrival.dat';

  ***% Enter the distance and azimuth arrays***

  dist = [3.53 20. 3400.64 6000.33 160.78 280.56 4000.36 110.55 140.00 200. 932. 2000. 500.36 1001.55 1400.00 200. 8362. 2000. 3410. 724.];

  az = [10 15. 25. 50. 80. 105. 100. 120. 122. 130. 125. 130. 158. 190. 210. 250. 270. 300. 320. 333.];

  ***% Options to convert the P-, SV- and SH- displacements to Z, R and T components***

Table 1: Velocity model

| $V_p$ | $V_s$ | $\rho$ | depth | layer no |
|---|---|---|---|---|
| 6.08 | 3.51 | 2.732 | 0.0 | 1 |
| 6.25 | 3.60 | 2.756 | 6.0 | 1 |
| 6.55 | 3.70 | 2.789 | 12.0 | 1 |
| 7.20 | 4.20 | 2.90 | 40.0 | 1 |
| 8.00 | 4.60 | 3.20 | 40.0001 | 2 |
| 8.00 | 4.60 | 3.20 | 200.0 | 2 |

[1]**?**. syntax: vel_model = [vp vs rho depth layer_no]

z_displ = 0; r_displ = 0; t_displ = 0; % 1 is to convert and 0 otherwise.

***% Relative ratios between the grid search using polarity and amplitude ratios***

$w_{pol} = 0.8$; $w_{\text{ratio}} = 0.2$;

***% Some other options in the use of SV amplitudes in the grid search algorithm.***

$$\begin{aligned} \text{use\_sv\_pol} = 1; &\Rightarrow 0 - \text{do not use SV polarity} \\ &\Rightarrow 1 - \text{use SV polarity} \\ \text{use\_sv\_amp} = 3; &\Rightarrow 0 - \text{do not use SV at all} \\ &\Rightarrow 1 - \text{use only } \frac{|SV|}{SH} \\ &\Rightarrow 2 - \text{use only } \frac{|SV|}{P} \\ &\Rightarrow 3 - \text{use both } \frac{|SV|}{SV} and \frac{|SV|}{P} \end{aligned}$$

***% Enter the velocity model and indicate whether it is an halfspace or not.***

halfspace = 0;

*% syntax: vel_model = [vp vs rho depth layer_no]* e.g. Table 1.

- Lunch Focal to open focal Graphic User Interface (GUI). One way to lunch Focal is to type "focal" on the MATLAB terminal.
- Click on the **"READ ARRIVAL DATA FROM FILE"** button to load the parameters.in file (Fig. 3). This button prepares the parameters.in file in form of $p_{pol}$, $sv_{pol}$ and $sh_{pol}$ needed by Focal. In other words, It will determine incident angles of the direct P-, SV- and SH- waves to the stations, and also determine whether the wave is upgoing or downgoing.

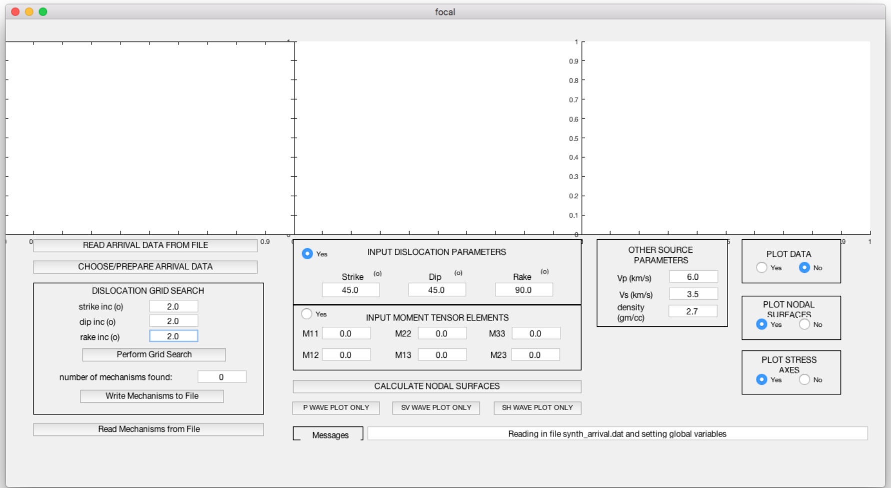



Figure 3:

### 3.1.2 Real Data

For the real data, the user need to edit another MATLAB file containing the information of each real datasets and reflect its filename in the parameters.in file. The use of a unique file for each real data will enable the user to use Focal for different earthquakes without overwriting the parameters.in file for each earthquake. For example, we include the inputs for a earthquake in real_data_parameters.in file. The real_data_parameters.in file is called from the main parameters.in file. The inputs are similar to those in the parameters.in file:

***% Source depth and its error for error analysis.***

depth = 12.83; depth_error = 4; % km

***% Set up distance and azimuth arrays including station number.***

sta_no = [1 2 3 4 5 6 7];

dist = [3.53 20.0 34.64 9.33 16.78 28.56 5.36 11.55 14.00 20.0 93262.0 2000.0 346410.0 72794.0];

az = [10. 15. 25. 50. 80. 105. 158. 190. 210. 250. 270. 300. 320. 333.];

***% Options to convert the P-, SV- and SH- displacements to Z, R and T components***

z_displ = 1; r_displ = 1; t_displ = 1; *% 1 is to convert and 0 otherwise.*

***% Relative ratios between the grid search using polarity and amplitude ratios***

$w_{pol} = 0.9$; $w_{\mathrm{ratio}} = 0.1$;

Table 2: Velocity model

| $V_p$ | $V_s$ | $\rho$ | depth | layer no |
|---|---|---|---|---|
| 6.08 | 3.51 | 2.732 | 0.0 | 1 |
| 6.25 | 3.60 | 2.756 | 6.0 | 1 |
| 6.55 | 3.70 | 2.789 | 12.0 | 1 |
| 7.2 | 4.20 | 2.90 | 40.0 | 1 |
| 8.0 | 4.60 | 3.20 | 40.0001 | 2 |
| 8.0 | 4.60 | 3.20 | 200.0 | 2 |

[1]**?**. syntax: vel_model = [vp vs rho depth layer_no]

***% Some other options in the use of SV amplitudes in the grid search algorithm.***

$$\begin{aligned}\text{use\_sv\_pol} = 1; &\Rightarrow 0 - \text{do not use SV polarity}\\ &\Rightarrow 1 - \text{use SV polarity}\\ \text{use\_sv\_amp} = 3; &\Rightarrow 0 - \text{do not use SV at all}\\ &\Rightarrow 1 - \text{use only } \frac{|SV|}{SH}\\ &\Rightarrow 2 - \text{use only } \frac{|SV|}{P}\\ &\Rightarrow 3 - \text{use both } \frac{|SV|}{SV} and \frac{|SV|}{P}\end{aligned}$$

***% Enter the velocity model and indicate whether it is an halfspace or not.***

halfspace = 0;

*% syntax: vel_model = [vp vs rho depth layer_no]* e.g. Table 1.

The amplitude data are specified using the format as in Table 3.

### 3.2 Focal Mechanism Determination

Focal mechanism that fit the datasets can be determined using two methods:

(1) grid search method, or

(2) by manually specifying the dislocation parameters that fits the input data.

Table 3: P-, SV- and SH- amplitudes and weights data

| Wave type | stat_no | amplitude | weight |
|---|---|---|---|
| p_data | 1 | -3.48406e2 | 5 |
| | 2 | -3.54463e2 | 5 |
| | 3 | 29.9481 | 5 |
| | 4 | 59.32707 | 5 |
| | 5 | 29.64097 | 5 |
| | 6 | -23.0 | 5 |
| | 7 | 7.10383e2 | 5 |
| sv_data | 1 | -66.43 | 5 |
| | 2 | 1343.46 | 5 |
| | 3 | 32.94 | 5 |
| | 4 | -395.7 | 5 |
| | 5 | 150.1 | 5 |
| | 6 | 60.0 | 5 |
| | 7 | 506.0 | 5 |
| s_data | 1 | 201.4 | 5 |
| | 2 | -2991.0 | 5 |
| | 3 | -373.9 | 5 |
| | 4 | 1310.0 | 5 |
| | 5 | 801.9 | 5 |
| | 6 | 217.9 | 5 |
| | 7 | 2737 | 5 |

[1]**?**. syntax: vel_model = [vp vs rho depth layer_no]

### 3.2.1 Dislocation Grid Search Method

The dislocation grid search section uses the amplitude information in the parameters.in file. The grid search is performed using the specified increments in strike, dip and rake angles by specifying the strike-, dip- and rake-increments in the DISLOCATION GRID SEARCH section of Focal.

After loading the data, click the **"Perform Grid Search"** button to determine focal mechanisms that fit the data sets. After the grid search algorithm is completed, Focal will display the P-wave nodal plane for the two best focal mechanism solutions. The best focal mechanism will be displayed on the message terminal (e.g. "*Correct FM: Strike: 268; Dip: 70; Rake: 304*"). Because there are many focal mechanisms that can fit the data, the grid search determines the degree of fitting in terms of percentage. A focal mechanism with 100% fits the data perfectly. Focal will also display the number of mechanisms found. These mechanisms can be written to file using the **"Write Mechanism to File"** button. An example of how focal displays the results is presented in figures 4 and 5 for the synthetic and real data, respectively. A sample of the text file containing the results is in Figure 6.

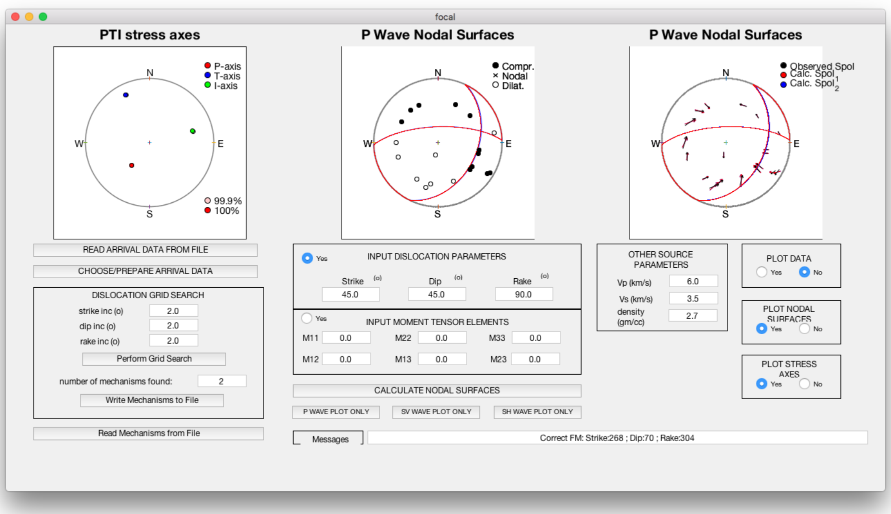



Figure 4: Result of synthetic data.

### 3.2.2 Manual Inputs

- Input Dislocation Parameters

  Focal allows the user to give the fault plane solutions manually instead of the grid-search method. Focal calculates the P-, SV- and SH-wave nodal surfaces superimposed on the polarity data. The user can then visually examine the nodal surfaces to fit the input data. We use the synthetic data set in section 3.1.1 for description. The synthetic data sets was generated using strike: 268°, dip: 70° and rake: 304°.

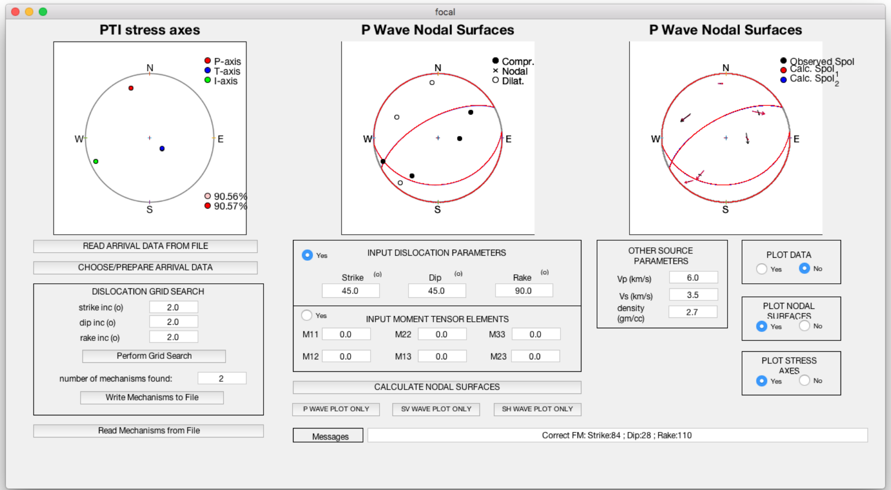



Figure 5: Result of real data.

For determine focal mechanisms using manual input of dislocation parameters, we follow the following steps:

– Read in the data (synthetic or real data) using the **"READ ARRIVAL DATA FROM FILE"** button.

– Manually input the strike, dip and rake under the **INPUT DISLOCATION PARAMETERS** in the middle panel of Focal instead of clicking the **Perform Grid Search**.

– Click the **CALCULATE NODAL SURFACES** button to generate P-, SV- and SH-wave nodal surfaces. The user can check **PLOT DATA** button to superimposed the polarity information on the nodal surfaces.

Examples:

(1) Using strike: 45°, dip: 45° and rake: 40°

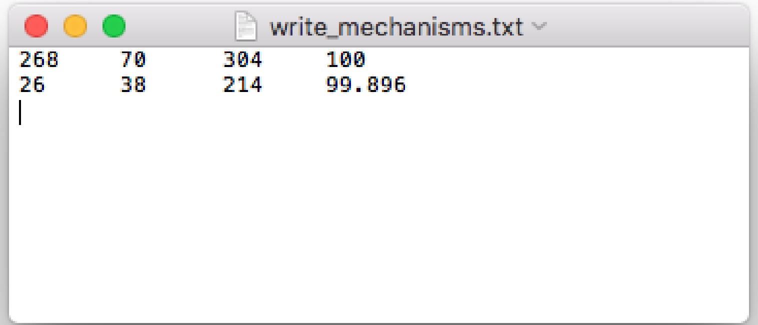
Though the nodal surfaces fit the polarity data, apart from few data points that doesn't fit, the fields of polarity is flipped (Fig. 7). For example, the SV- and SH-wave nodal surfaces place the tangential upward polarities (white) in the tensional field rather than pressure field.

(2) Using strike: 45°, dip: 38° and rake: 214°

This mechanism, with a correct rake angle, corrects the fields on the SV- and SH-wave nodal surfaces (Fig. 8). The tangential upward polarity now falls within the pressure field. There are still some data that falls outside the respective field.

(3) Using strike: 268°, dip: 70° and rake: 304° (Correct focal mechanism)

The correct focal mechanism fits all the data including the SV and SH- wave polarities (Fig. 9).




- Input Moment Tensor Components

  Focal also allows user to input the moment tensor components using the **INPUT MOMENT TENSOR COMPONENTS** instead of the dislocation parameters. The user can then click on the **CALCULATE NODAL SURFACES**.

#### 3.2.3 Error Analyses

In Focal, the ”READ MECHANISMS FROM FILE” button determines the error in the focal mechanisms. Focal addresses two sources of error in the focal mechanism. These sources are error in wave polarity and focal depth.

- **Error due to Wave polarity**

  The polarity of the first arrival can be unclear in waveforms with poor signal-to-noise ratio. This can introduce error in Focal because polarity information constrain the nodal planes. For example, when an unclear polarity is close to the nodal plane, it can affect the focal mechanism. Focal assesses this error by changing the polarity of each station and performs grid search at each iteration. Focal then plot an histogram showing the number of hits of strike, dip and rake for all the iteration. Figures 10 and 11 are error analyses for the synthetic and real data, respectively, presented in section 3.2.1. The histogram for the real data shows more variability than the one for the synthetic data due to the sparsity of the data coverage.

- **Error due to Focal depth**

  The second error analysis is to assess the error due to an error in the focal depth. Focal depth can affect the incident angle of the seismic rays to receivers (seismometers). These incident angles are important

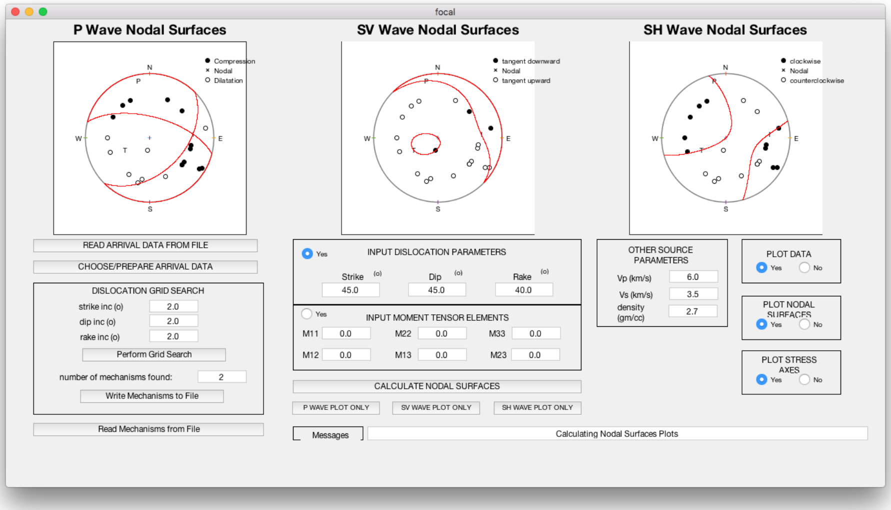



Figure 7: Manual Inputs of dislocation parameters: strike: 45°, dip: 45° and rake: 40°.

in plotting the polarity information on a focal sphere. Therefore, an error in the focal depth can affect the focal mechanism of the earthquake. Figures 12 and 13 show the variation of strike, dip and rake with respect to focal depth. The strike, dip and rake for the synthetic data flips to the auxiliary plane as focal depth changes. The figure for real data shows more variability in the solution due to error in focal depth.

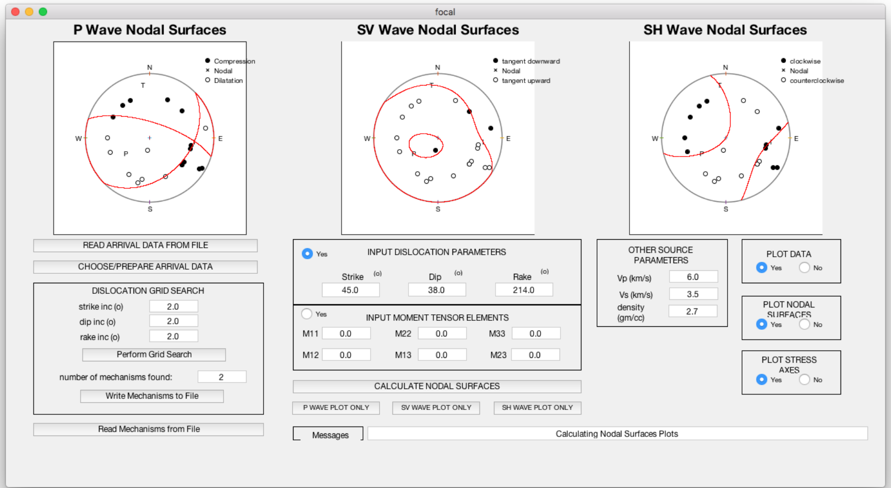



Figure 8: Manual Inputs of dislocation parameters: strike: 45°, dip: 38° and rake: 214°.

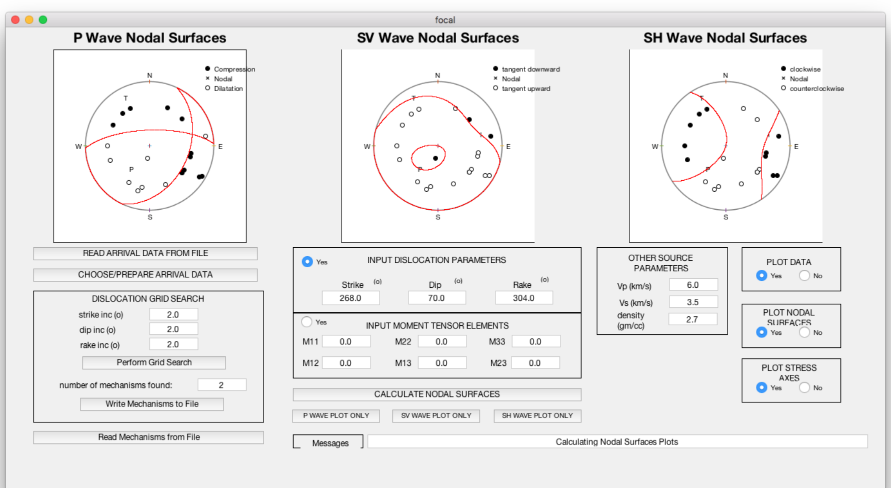



Figure 9: Manual Inputs of dislocation parameters: strike: 268°, dip: 70° and rake: 304° (Correct focal mechanism).

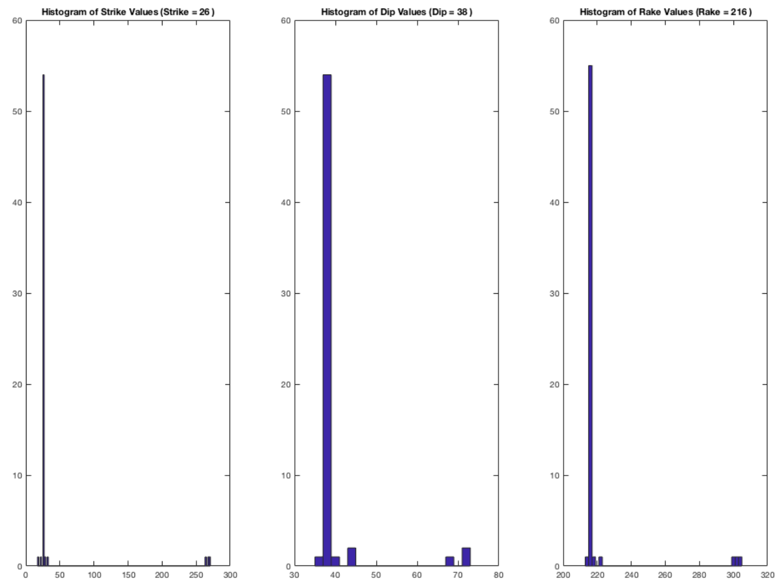



Figure 10: Error due to Wave polarity: synthetic data.

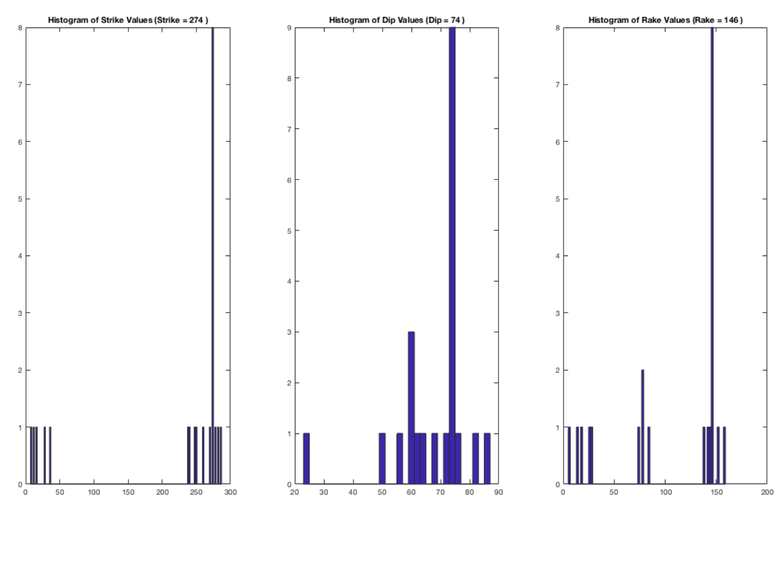



Figure 11: Error due to Wave polarity: real data.

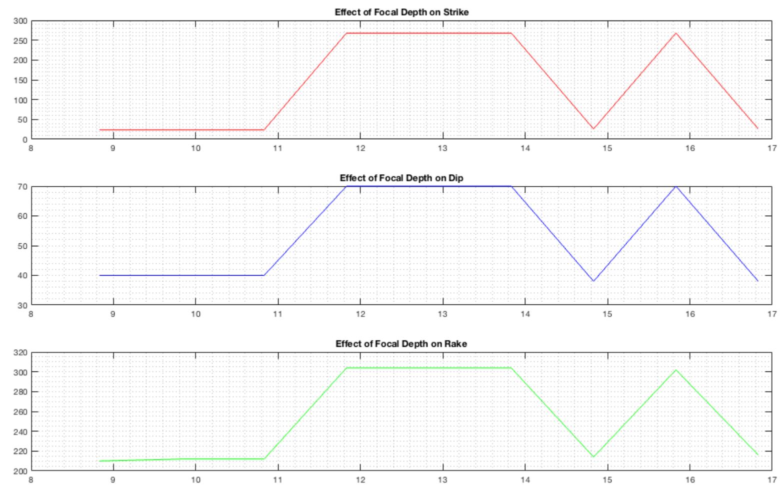



Figure 12: Error due to Focal depth: synthetic data.

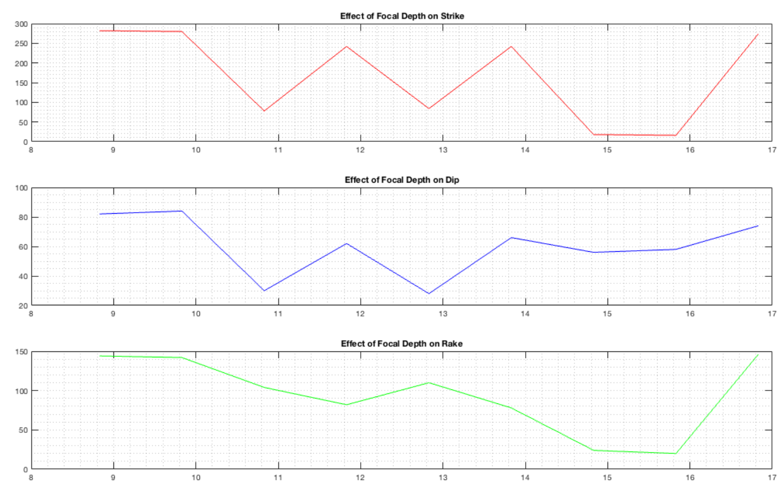



Figure 13: Error due to Focal depth: real data.