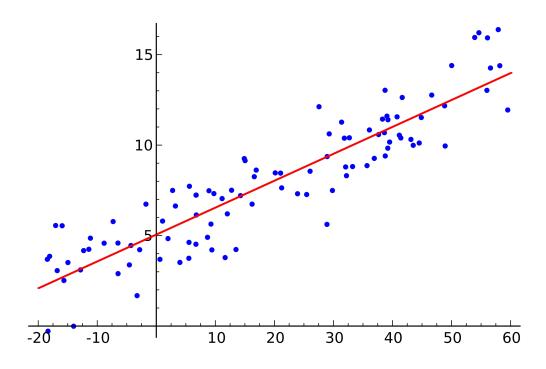


# Linear Regression

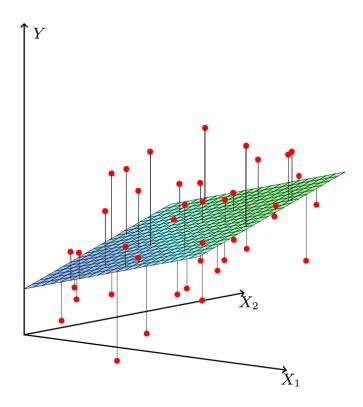
**Predicting Values** 

## Simple\* Linear Regression



<sup>\*</sup> Like, mathematicians refer to linear regression with one independent variable as "simple", not a comment on linear regression actually *being* simple

Chalkboard Example



**FIGURE 3.1.** Linear least squares fitting with  $X \in \mathbb{R}^2$ . We seek the linear function of X that minimizes the sum of squared residuals from Y.

### General Linear Regression Model

We have a vector of p attributes,  $X = (X_1, X_2, ..., X_p)$ 

We want to use these p attributes to predict a value, f(X)

We find p+1 values for beta  $(\beta_0, \beta_1, \beta_2, ..., \beta_p)$  based on our observed data.

$$f(X) = \beta_0 + \sum_{j=1}^p X_j \beta_j$$

#### Finding Beta Hat

• Step 1: Build a matrix X which will have dimensions  $m \times (p + 1)$ , where m is the number of observations in the dataset and p is the number of <u>attributes</u>.

$$\mathbf{X} = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & \dots & x_{1,p} \\ 1 & x_{2,1} & x_{2,2} & \dots & x_{2,p} \\ 1 & x_{2,1} & x_{3,2} & \dots & x_{3,p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{m,1} & x_{m,2} & \dots & x_{m,p} \end{bmatrix}$$

#### Finding Beta Hat

• Step 2 : Build the vector y which will have dimensions  $m \times 1$  that represents past examples of the value you want to predict based on the observations in the X matrix

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{bmatrix}$$

#### Finding Beta Hat

• Step 3 : Find the pseudo inverse of the X matrix and then multiply by the vector y . It will have dimensions ( p+1 )  $\times$  1,

$$\hat{\beta} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{y}$$

#### What is Beta Hat?

- Beta hat is a vector containing the least squares solution to the general linear regression model.
- This vector can now be used to predict any value given a set of p attributes.
- This is "The Model"

#### Find a Prediction, y hat

• Assume we have a new observation  $\dot{x}=(\dot{x}_1,\dot{x}_2,\ldots,\dot{x}_p)$ , we create the row vector of size 1 x ( p + 1 )

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & \dot{x}_1 & \dot{x}_2 & \dots & \dot{x}_p \end{bmatrix}$$

• Find a predicted value. The result, y hat, is a single value.

$$\hat{y} = \dot{x}\hat{\beta}$$