Supplementary Material for the Paper "Climate-adaptive Transmission Network Expansion Planning Considering Evolutions of Resources"

A. Transform the p-SDP into an MILP

Using the data-clustering-incorporated reformulation, the p-WDRO is recast into a p-SDP, as

$$\overline{R}^* = \min \overline{R} \tag{1a}$$

$$\text{s.t.: } Z_S^D(\overline{R}) \leq Z^{\max} \tag{1b}$$

$$Z_S^D(\overline{R}) = \min_{\substack{\alpha_m,\beta_m,\\ \tau_m,l_m}} \sum_{m=1}^{M} \alpha_m + \beta(d+\epsilon_{opt})^2 \tag{1c}$$

$$\text{s.t.:} \tag{1d}$$

$$\begin{bmatrix} \beta\mathbf{I} & -\beta\boldsymbol{\kappa}_m \\ -\beta\boldsymbol{\kappa}_m^T & \beta\boldsymbol{\kappa}_m^T\boldsymbol{\kappa}_m + S/G_m\cdot\alpha_m - 1 \end{bmatrix} \succeq \tau_m \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\overline{R}^2 \end{bmatrix} - \rho_m \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\overline{R}_{\max}^2 \end{bmatrix},\ \tau_m \geq 0, \rho_m \geq 0, \beta \geq 0 \tag{1e}$$

$$\begin{bmatrix} \beta\mathbf{I} & -\beta\boldsymbol{\kappa}_m \\ -\beta\boldsymbol{\kappa}_m^T & \beta\boldsymbol{\kappa}_m^T\boldsymbol{\kappa}_m + S/G_m\cdot\alpha_m \end{bmatrix} \succeq -l_m \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\overline{R}_{\max}^2 \end{bmatrix}, \quad l_m \geq 0, \beta \geq 0 \tag{1f}$$

which is also a bi-level problem with an SDP in the lower level.

Since the lower level $Z_S^D(\overline{R})$ is non-convex w.r.t. $\overline{R}$, we employ the piecewise linear approximation technique to transform the p-SDP into an MILP. First, the interval $[1, \overline{R}_{\max}]$ is evenly divided into H segments with a spacing of $\Delta\overline{R} = 0.1$. yielding $1 = \overline{R}_1 \leq \overline{R}_2 \leq \ldots \leq \overline{R}_{H+1} = \overline{R}_{\max}$. Second, Z_S^D is calculated at each $\overline{R}_h, h \in \{1,...,H\}$. Finally, (1) is recast into an MILP, as

$$\overline{R}^* = \min(\overline{R}) \tag{2a}$$

$$\text{s.t.: } Z_L^D(\overline{R}) \leq Z^{\max} \tag{2b}$$

$$Z_L^D(\overline{R}) = Z_S^D(\overline{R}_1) + \sum_{h=1}^{H}[Z_S^D(\overline{R}_{h+1}) - Z_S^D(\overline{R}_h)]\delta_h \tag{2c}$$

$$\overline{R} = 1 + \sum_{h=1}^{H} \Delta\overline{R}\delta_h \tag{2d}$$

$$u_h \in \{0,1\}, \quad u_h \leq \delta_h \leq 1 \quad \forall h \in \{1,...,H\} \tag{2e}$$

$$\delta_{h+1} \leq u_h \quad \forall h \in \{1,...,H-1\} \tag{2f}$$

where δ_h is the proportion of the hth segment covered by $\overline{R}$, and u_h indicates whether the hth segment has been fully used.

B. The overall C&CG algorithm

Recall the compact form of the TNEP is

$$\min_{\boldsymbol{v}_L} \boldsymbol{C}_L^T\boldsymbol{v}_L + \omega\cdot Q \tag{3a}$$

$$\text{s.t.: } \quad \boldsymbol{v}_L \in \{\mathbf{0},\mathbf{1}\}^{L_C} \tag{3b}$$

$$Q = \max_{\boldsymbol{P}_U\in\mathcal{U}^{\text{CUS}}} \min_{\boldsymbol{y},\Delta\boldsymbol{P}_U} \boldsymbol{C}_U^T\Delta\boldsymbol{P}_U \tag{3c}$$

$$\text{s.t.: } \mathbf{A}\boldsymbol{y} \leq \mathbf{b} + \mathbf{H}\boldsymbol{v}_L \quad (\boldsymbol{\varphi}) \tag{3d}$$

$$\mathbf{M}\,\boldsymbol{y} + \mathbf{E}_{N'U}\Delta\boldsymbol{P}_U = -\mathbf{E}_{N'U}\boldsymbol{P}_U \quad (\boldsymbol{\varpi}) \tag{3e}$$

C&CG algorithm [1] is adopted to solve the climate-adaptive TNEP, whose overall procedure can be seen in **Algorithm 1**. Therein, the (3) is decomposed into a master problem (MP) and a sub-problem (SP). In the τth iteration, the MP optimizes $\boldsymbol{v}_L$ with limited information about $\boldsymbol{P}_U$ and estimate the underlying Q by η, yielding a lower bound, denoted by LB, for the original model, as

$$LB = \min_{\boldsymbol{v}_L,\boldsymbol{x},\eta} \boldsymbol{C}_L^T\boldsymbol{v}_L + \omega\cdot\eta \tag{4a}$$

$$\text{s.t.: } \boldsymbol{v}_L \in \{0,1\}^{L_C},\ \eta \geq 0 \tag{4b}$$

$$\boldsymbol{x} = (\boldsymbol{x}_{\text{add}}^{(1)},...\boldsymbol{x}_{\text{add}}^{(t)},...;\boldsymbol{x}_{\text{add}}^{(\tau-1)})^T, \boldsymbol{x}_{\text{add}}^{(t)} = (\boldsymbol{y}^{(k)};\Delta\boldsymbol{P}_U^{(t)}) \tag{4c}$$

$$\boldsymbol{x}_{\text{add}}^{(t)} \in \mathcal{X}_{\text{add}}^{(t)} = \Big\{ \max\{\eta, Q^{\text{acc}}\} \geq \boldsymbol{C}_U^T\Delta\boldsymbol{P}_U^{(t)} \tag{4d}$$

$$\max\{\eta, Q^{\text{acc}}\} \geq Q^{*(t)} - \boldsymbol{\varphi}^{*(t)T}\mathbf{H}\big(\boldsymbol{v}_L - \boldsymbol{v}_L^{*(t)}\big) \tag{4e}$$

$$\mathbf{M}\,\boldsymbol{y}^{(t)} + \mathbf{E}_{N'U}\Delta\boldsymbol{P}_U^{(t)} = -\mathbf{E}_{N'U}\boldsymbol{P}_U^{*(t)} \tag{4f}$$

$$\mathbf{A}\boldsymbol{y}^{(t)} \leq \mathbf{b} + \mathbf{H}\boldsymbol{v}_L \Big\} \quad \forall t \in \{1,...\tau-1\} \text{ (if } \tau > 1) \tag{4g}$$

where $\boldsymbol{x}_{\text{add}}^{(t)}$ and $\mathcal{X}_{\text{add}}^{(t)}$ are added variables and constraints to deal with the worst cases $\boldsymbol{P}_U^{*(t)}$

Algorithm 1: Modified C&CG to solve the overall TNEP

Step 1: Initialization. Set $\tau = 1, UB = +\infty, LB = 0$;
Step 2: Solve the MP. Then, report LB and the optimal solution $\boldsymbol{v}_L^{*(\tau)}$;
Step 3: If $\tau > 1$ and $UB - LB \leq \Delta_1$, terminate;
Otherwise:
1) Solve SP with respect to $\boldsymbol{v}_L^{*(\tau)}$ using **Algorithm 2** in the main paper, which yields optimal value $Q^{*(\tau)}$ and associated optimal solution $(\boldsymbol{P}_U^{*(\tau)};\boldsymbol{\varphi}^{*(\tau)};\boldsymbol{\varpi}^{*(\tau)})$;
2) Update UB by $UB = \min\{UB, \boldsymbol{C}^T\boldsymbol{v}_L^* + \omega\cdot Q^{*(\tau)}\}$
Step 4: Update $\tau = \tau + 1$ and add new variables $\boldsymbol{x}^{(\tau)}$ and constraints set $\mathcal{X}_{\text{add}}^{(\tau)}$ to the MP.
Then, go back to **Step 2**.

C. Convergence of the overall C&CG algorithm

Convergence of the overall C&CG algorithm is discussed as follows. Suppose at iteration τ, $\boldsymbol{v}_L^{*(\tau)}$ is made by the MP, which leads to $Q^{*(\tau)}$ and $(\boldsymbol{P}_U^{*(\tau)},\boldsymbol{\varphi}^{*(\tau)},\boldsymbol{\varpi}^{*(\tau)})$ in the SP. Therefore, $LB \leq UB \leq \boldsymbol{C}_L^T\boldsymbol{v}_L^{*(\tau)} + \omega\cdot Q^{*(\tau)}$; If the $(\boldsymbol{\varphi}^{*(\tau)},\boldsymbol{\varpi}^{*(\tau)})$ has been identified in the previous iteration τ' with $1 \leq \tau' < \tau$. Then, $\boldsymbol{P}_U^{\tau*} = \boldsymbol{P}_U^{*\tau'}$ even if $\boldsymbol{v}_L^{*(\tau)} \neq \boldsymbol{v}_L^{*(\tau')}$, because the differences in $\boldsymbol{v}_L$ will not influence the optimal solution to $Q_{2,n}$. Then, the added variable $\boldsymbol{x}_{\text{add}}^{(\tau)}$ and constraint set $\mathcal{X}_{\text{add}}^{(\tau)}$ related to $(\boldsymbol{P}_U^{*(\tau)},\boldsymbol{\varphi}^{*(\tau)},\boldsymbol{\varpi}^{*(\tau)})$ will not bring about new information to the MP, so that in the $(\tau+1)$th iteration, we have $\boldsymbol{v}_L^{*(\tau)} = \boldsymbol{v}_L^{*(\tau+1)}$. It follows $LB = \boldsymbol{C}_L^T\boldsymbol{v}_L^{*(\tau)} + \omega\cdot\eta^* \geq \boldsymbol{C}_L^T\boldsymbol{v}_L^{*(\tau)} + \omega\cdot Q^{*(\tau)}$, if $\eta^* \geq Q^{\text{acc}}$; or $LB = \boldsymbol{C}_L^T\boldsymbol{v}_L^{*(\tau)} + \omega\cdot Q^{\text{acc}} \geq \boldsymbol{C}_L^T\boldsymbol{v}_L^{*(\tau)} + \omega\cdot Q^{*(\tau)}$

if $\eta^* < Q^{\text{acc}}$. In either case, we have $LB = UB$. Therefore, any repeated $(\boldsymbol{\varphi}^*, \boldsymbol{\varpi}^*)$ implies optimality. Plus that the dual feasibility set $\mathcal{W}$ has finite vertexes while $(\boldsymbol{\varphi}^*, \boldsymbol{\varpi}^*)$ is a vertex of $\mathcal{W}$. The overall C&CG algorithm is finitely convergent. Specifically, if there are no $\boldsymbol{P}_U^*$ found by SP yielding $Q^* \geq Q^{\text{acc}}$, we have $\eta^* < Q^{\text{acc}}$ and $LB = UB = \boldsymbol{C}_L^T \boldsymbol{v}_L^* + \omega \cdot Q^{\text{acc}}$; Otherwise, $LB = UB = \boldsymbol{C}_L^T \boldsymbol{v}_L^* + \omega \cdot \eta^*$, which means a trade-off between the investment cost $\boldsymbol{C}_L^T \boldsymbol{v}_L$ and security operation assessed by η is reached.

References

[1] B. Zeng and L. Zhao, "Solving two stage robust optimization problems using a column-and- constraint generation method," *Oper. Res. Lett.*, vol. 41, no. 5, pp. 457–461, 2013.