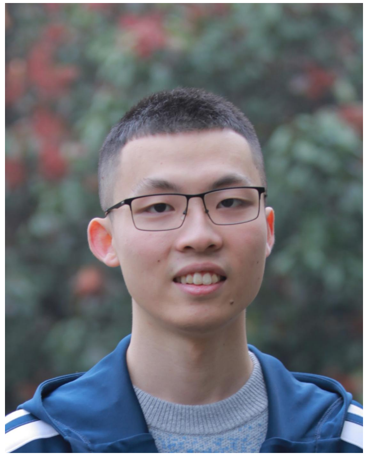
Bidder Selection Problem in Position Auctions:

A Fast and Simple Algorithm via Poisson Approximation





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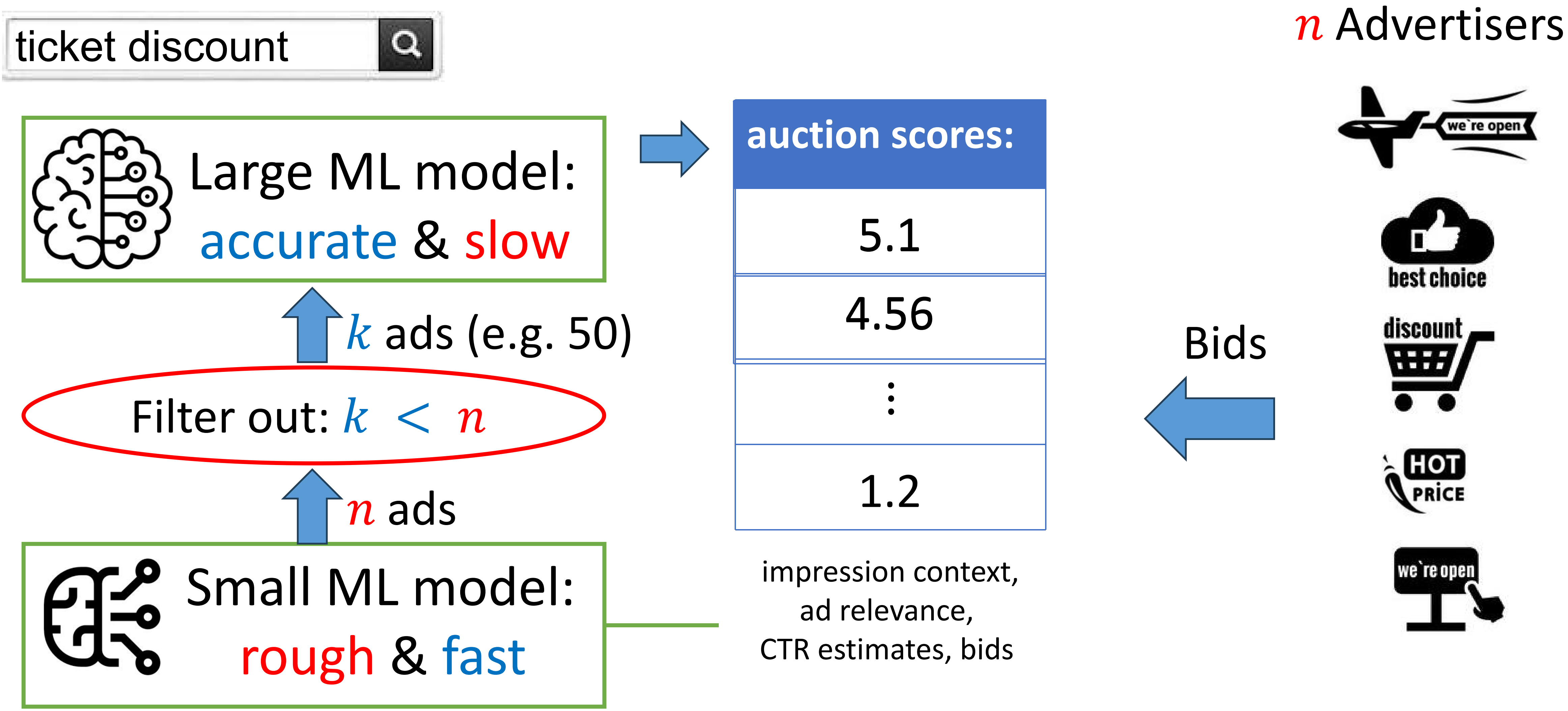
Bidder Selection Problem



Theoretical Model

k-max problem (<u>single-item auction</u>):

- INPUT: n independent random variables $X_1, \ldots, X_n$ and $k < n$
- OUTPUT: find k r.v. $S \subset [n]$ to $\max_{|S|=k} \mathbf{E}\left[\max_{i \in S} X_i\right]$

k-max for <u>position auctions</u>:

- generalizes single-item auction
- INPUT: r.v. $X_1, \ldots, X_n$ and $k < n$
- OUTPUT: k r.v. $\max_{|S|=k} \mathbf{E}\left[Welfare(S)\right]$

Our Results: Theory

New relaxation of k-max: $\max_{\|\boldsymbol{x}\|_1 \le k} f(\boldsymbol{x})$

1. $f(\boldsymbol{x})$ – nicely structured concave function $\Rightarrow$

   Easy to find $\boldsymbol{x}^* = \operatorname{argmax}_{\|\boldsymbol{x}\|_1 \le k} f(\boldsymbol{x})$

2. $f(\boldsymbol{x}^*) \to \max_{|S|=k} \mathbf{E}\left[\max_{i \in S} X_i\right]$ as k grows

3. Works for general position auctions.

$\Rightarrow$ Position auctions: Implementable PTAS

Our Results: Practice

- Homebrew implementation (python + standard convex libs):
  - Benchmarks: Greedy (Submodular Opt.) + Local Search
  - Instances as large as n = 1000, k = 200.
  - Good approximation on all instances.

Runtime: 1 day

Runtime: 45 sec.

Approx. to benchmarks: >99%

- Previous 2 EPTAS algorithms on k-max:
  - Unimplementable

Runtime: years $\left(2^{O(1/\varepsilon)^{O(1/\varepsilon)}}\right)$

n=3, k=2, ε=0.2