Non-excludable Bilateral Trade between Groups

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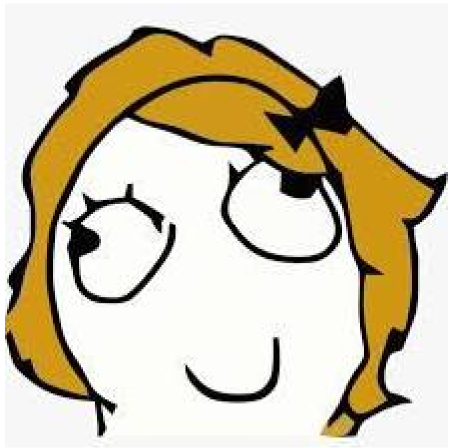
Bilateral Trade



Buyer
Private **value**: $v \sim F$

Seller
Private **cost**: $c \sim G$

Public information: F, G

Mechanism Design

- Based on **interactions** with the players, a mechanism decides:
  - Whether they should **trade** x
  - The **payment** of the buyer p
  - The **receipt** of the seller r

- Key difficulty: **truthfulness**

- **Revelation principle**: WLOG, **interactions** can be viewed as a sealed **bid** b from the buyer and a sealed **ask** a from the seller.

Mechanism Design

- Based on the players' **bid** b and **ask** a, a mechanism decides:
  - Whether they should **trade** $x(a, b)$
  - The **payment** of the buyer $p(a, b)$
  - The **receipt** of the seller $r(a, b)$

- **Utilities** of the players:
  - Buyer: $u_b(a, b) = v \cdot x(a, b) - p(a, b)$ (Obtained value - payment)
  - Seller: $u_s(a, b) = r(a, b) - c \cdot x(a, b)$ (Receipt - production cost)

Desiderata

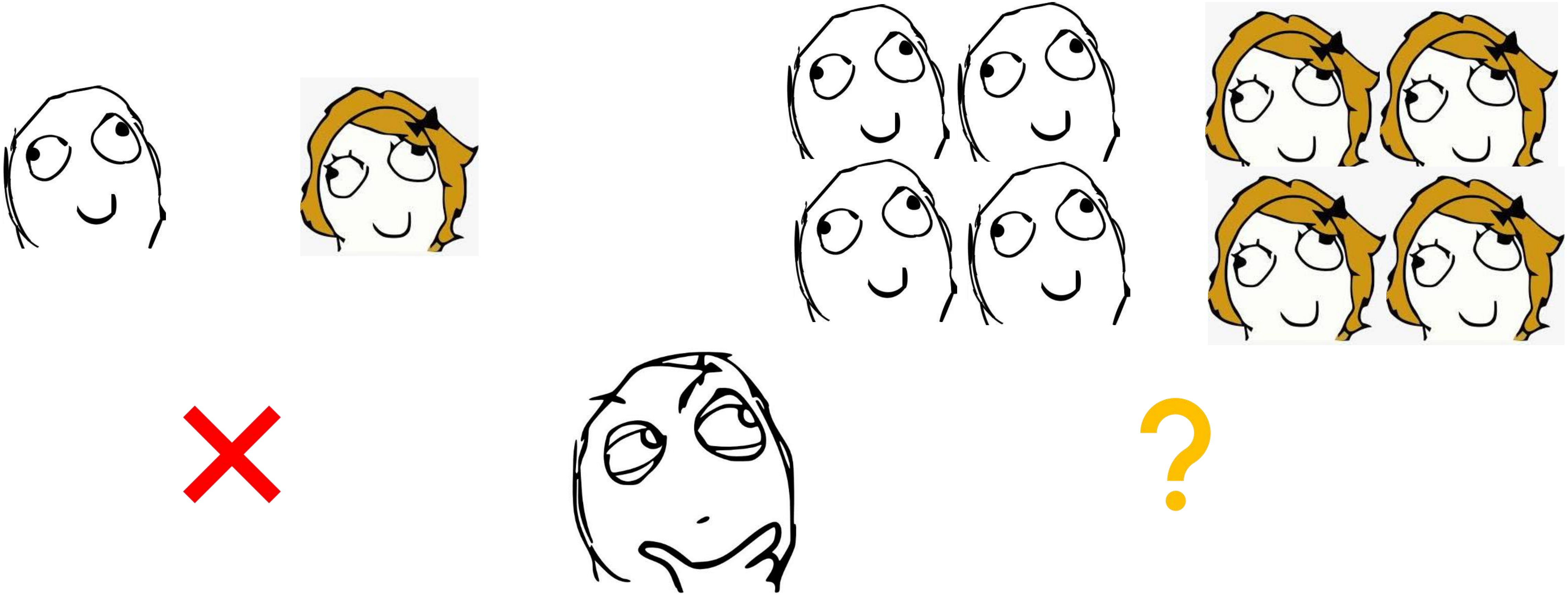
- **Incentive compatible (IC):** players bid/ask truthfully
- **Individually rational (IR):** players' utilities are non-negative
- **Budget balanced (BB):** buyer's payment ≥ seller's receipt
- **Efficient:** a trade happens whenever $v > c$

Myerson and Satterthwaite

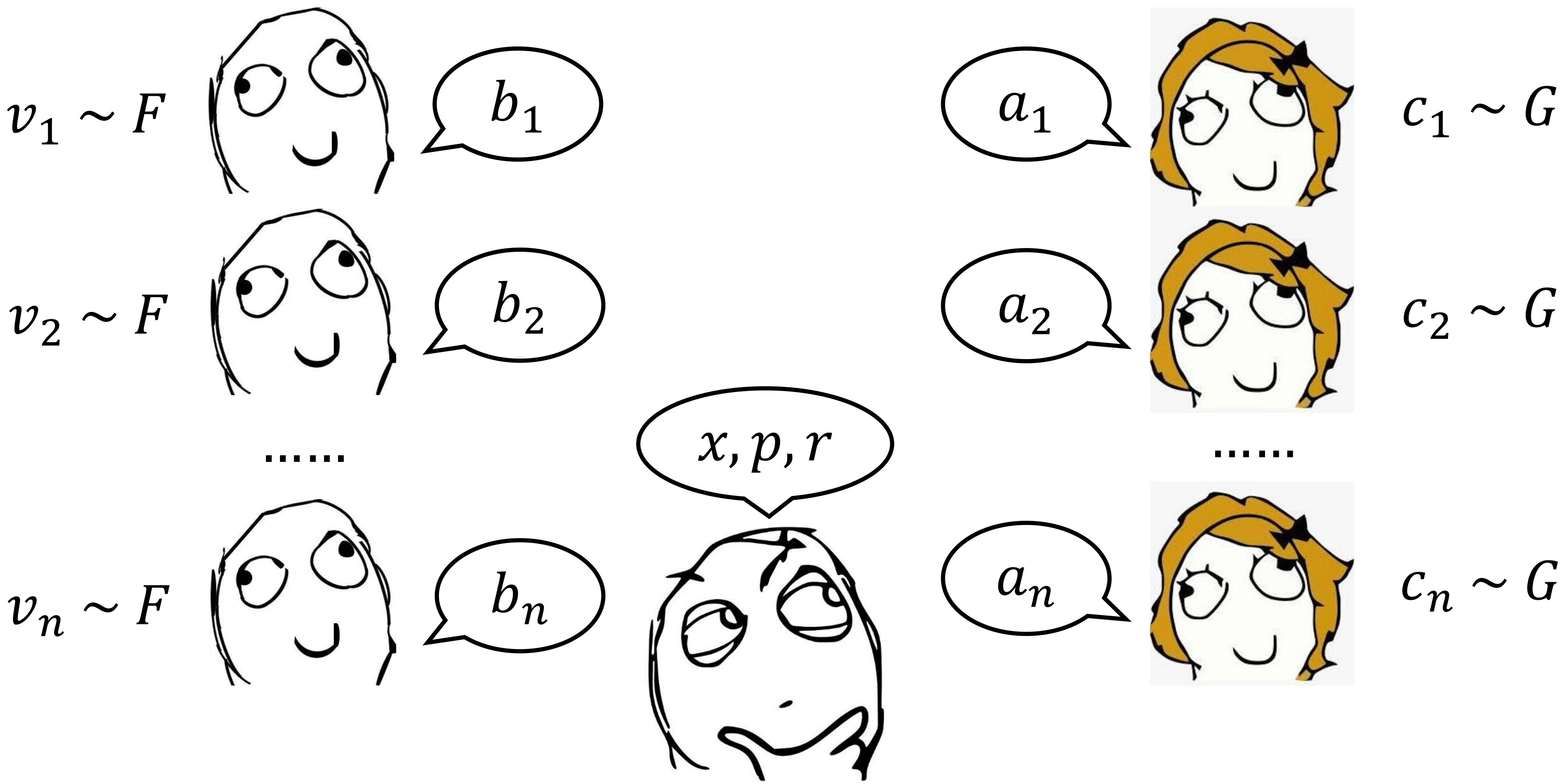
- A seminal impossibility by Myerson and Satterthwaite (1983):

- It is impossible to achieve all of {IC, IR, BB, Efficient} in bilateral trade, i.e, ***efficient bilateral trade cannot be implemented in a feasible way***.

Bypassing Myerson and Satterthwaite



Bilateral Trade Between Groups

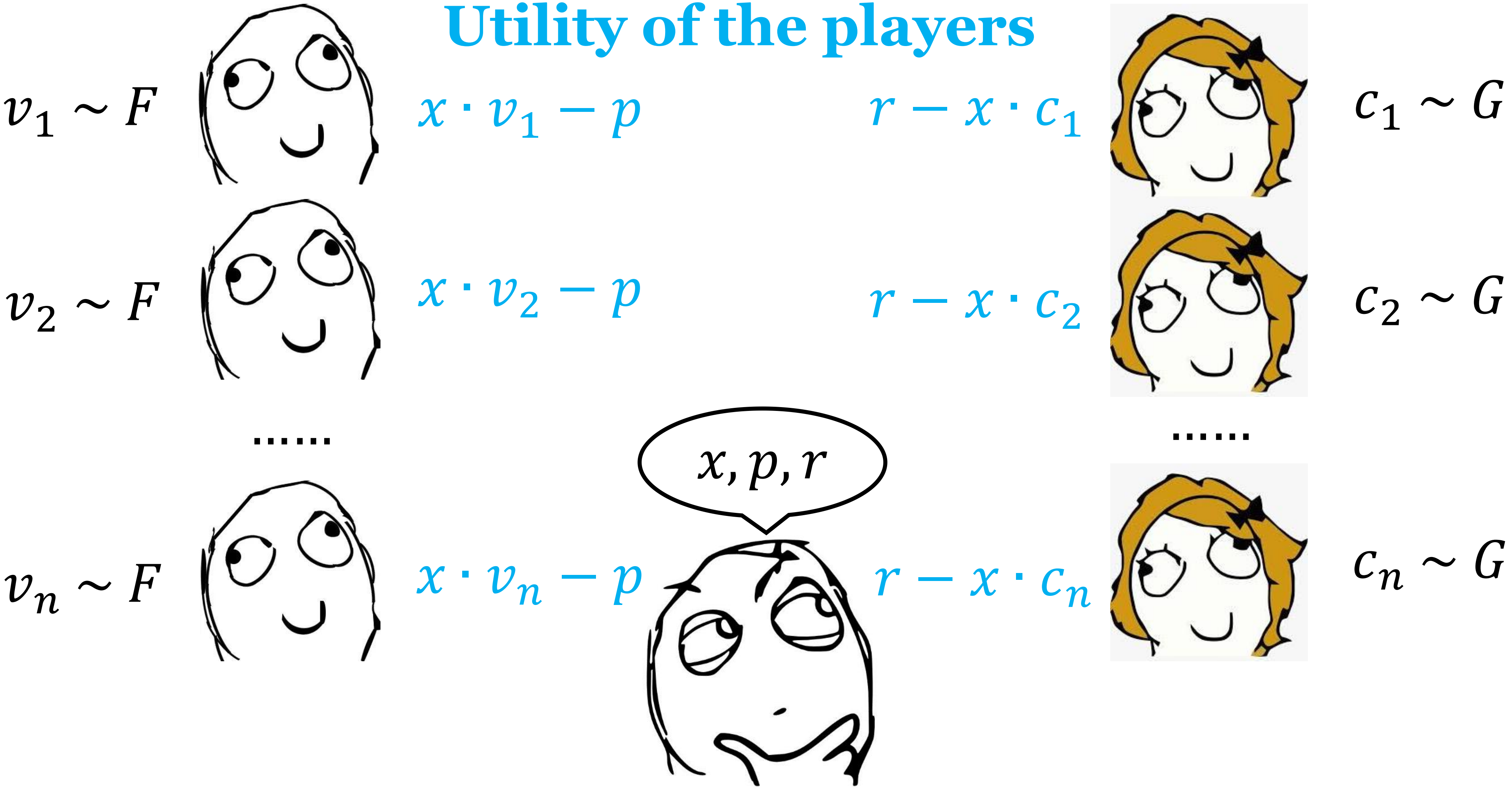


Non-Excludability

- **Non-Excludability:** the mechanism guarantees
  - The **players** **share** the same **allocation**
  - The **buyers** **share** the same **payment**
  - The **sellers** **share** the same **receipt**

- Based on the players' **bids** $\boldsymbol{b}$ and **asks** $\boldsymbol{a}$, a mechanism decides:
  - Whether **all** the players should **trade** $x(\boldsymbol{a}, \boldsymbol{b})$
  - The **payment** **shared** by the buyers $p(\boldsymbol{a}, \boldsymbol{b})$
  - The **receipt** **shared** by the sellers $r(\boldsymbol{a}, \boldsymbol{b})$

The Whole Picture



Desiderata

- **Incentive compatible (IC):** players bid/ask truthfully
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Desiderata

- **Incentive compatible (IC):** players bid/ask truthfully
- **Individually rational (IR):** groups' utilities are non-negative
- **Budget balanced (BB):** buyer's payment ≥ seller's receipt
- **Efficient (in the limit):** as $n \to \infty$, $\mathrm{GFT}/\mathrm{FB} \to 1$

Our Results in a Nutshell

- A **dichotomy** in the possibility of trading efficiently.

- In expectation:
  - If the buyers value the item (strictly) more than the sellers:
  - A mechanism achieving all desiderata in the limit is given

  - If the sellers value the item (weakly) more than the buyers:
  - No mechanisms can achieve all desiderata in the limit

Why Two Cases?

- Consider the **first best (FB)** in both cases.

- **Lemma 4.1.**
  - If $E_{v \sim F}[v] > E_{c \sim G}[c]$, then FB $= \Omega(n)$.
  - If $E_{v \sim F}[v] \leq E_{c \sim G}[c]$, then FB $= O(\sqrt{n})$.

- Lemma 4.1 naturally divides the problem into two cases.
  - When **the sellers value item more**, even FB goes to zero (per agent).
  - It is only possible to gain much when **the buyers value item more**.

Deterministic Mechanisms

- **Deterministic Mechanisms:** allocation $x(\boldsymbol{b}, \boldsymbol{a}) \in \{0,1\}$

- Our results for deterministic mechanisms:
  - A **characterization** of IC mechanisms (**Theorem 4.1, 4.2**)
  - A **positive result** when $E_{v\sim F}[v] > E_{c\sim G}[c]$ (**Theorem 4.3**)
  - A **negative result** when $E_{v\sim F}[v] \leq E_{c\sim G}[c]$ (**Theorem 4.4**)

Characterization of IC Mechanisms

- **Theorem 4.1.** Allocation $x(\boldsymbol{b}, \boldsymbol{a})$ can be implemented by an **IC** deterministic mechanism **if and only if:**
  - (a). For any $\boldsymbol{a}$, there is $\tau_{\boldsymbol{a}}$ and a monotone Boolean function $f_{\boldsymbol{a}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = f_{\boldsymbol{a}}(\mathbf{1}[b_1 \geq \tau_{\boldsymbol{a}}], \mathbf{1}[b_2 \geq \tau_{\boldsymbol{a}}], \ldots, \mathbf{1}[b_n \geq \tau_{\boldsymbol{a}}])$
  - (b). For any $\boldsymbol{b}$, there is $\theta_{\boldsymbol{b}}$ and a monotone Boolean function $g_{\boldsymbol{b}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = g_{\boldsymbol{b}}(\mathbf{1}[a_1 \leq \theta_{\boldsymbol{b}}], \mathbf{1}[a_2 \leq \theta_{\boldsymbol{b}}], \ldots, \mathbf{1}[a_n \leq \theta_{\boldsymbol{b}}])$

- A mechanism should decide in a **voting-like** way.

Characterization of IC Mechanisms

- **Theorem 4.1.** Allocation $x(\boldsymbol{b}, \boldsymbol{a})$ can be implemented by an **IC** deterministic mechanism **if and only if:**
  - (a). For any $\boldsymbol{a}$, there is $\tau_{\boldsymbol{a}}$ and a monotone Boolean function $f_{\boldsymbol{a}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = f_{\boldsymbol{a}}(\mathbf{1}[b_1 \geq \tau_{\boldsymbol{a}}], \mathbf{1}[b_2 \geq \tau_{\boldsymbol{a}}], \ldots, \mathbf{1}[b_n \geq \tau_{\boldsymbol{a}}])$
  - (b). For any $\boldsymbol{b}$, there is $\theta_{\boldsymbol{b}}$ and a monotone Boolean function $g_{\boldsymbol{b}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = g_{\boldsymbol{b}}(\mathbf{1}[a_1 \leq \theta_{\boldsymbol{b}}], \mathbf{1}[a_2 \leq \theta_{\boldsymbol{b}}], \ldots, \mathbf{1}[a_n \leq \theta_{\boldsymbol{b}}])$

- **Theorem 4.2.** Allocation $x(\boldsymbol{b}, \boldsymbol{a})$ can be implemented by an **IC and SBB** deterministic mechanism **if and only if:**
  - There is τ and a monotone Boolean function f, such that $x(\boldsymbol{b}, \boldsymbol{a}) = f\,(\mathbf{1}[b_1 \geq \tau], \ldots, \mathbf{1}[b_n \geq \tau], \mathbf{1}[a_1 \leq \tau], \ldots, \mathbf{1}[a_n \leq \tau])$

Buyers Value More: Positive Result

- **Algorithm 1:**
  - Always trade at price $\frac{1}{2}(E_{v\sim F}[v] + E_{c\sim G}[c])$
  - $x(\boldsymbol{b},\boldsymbol{a}) = 1, p(\boldsymbol{b},\boldsymbol{a}) = r(\boldsymbol{b},\boldsymbol{a}) = \frac{1}{2}(E_{v\sim F}[v] + E_{c\sim G}[c])$

- **Theorem 4.3.** Algorithm 1 is **IC** and **SBB**. When $E_{v\sim F}[v] > E_{c\sim G}[c]$, w.p. $1 - e^{-\Omega(n)}$, it is **IR**, and its **efficiency** is $1 - e^{-\Omega(n)}$.
  - Informally, Algorithm 1 achieves all desiderata in the limit.

Sellers Value More: Negative Result

- **Theorem 4.4.** When $E_{v\sim F}[v] \le E_{c\sim G}[c]$, **no deterministic IC** mechanisms can be **efficient in the limit**.

- Recall that in this case, $\mathrm{FB} = O(\sqrt{n})$ **(Lemma 4.1)**
  - There is no much to lose in the first place
  - Additively, Algorithm 1's loss is still $o(n)$

Randomized Mechanisms

- **Randomized Mechanisms:** allocation $x(\boldsymbol{b}, \boldsymbol{a}) \in [0,1]$

- We consider **smooth** randomized mechanisms
  - $x(\boldsymbol{b}, \boldsymbol{a})$ is **twice continuously differentiable**

- Our results for **smooth** randomized mechanisms:
  - A **characterization** of IC mechanisms (**Theorem 5.1**)
  - A **positive result** when $E_{v\sim F}[v] > E_{c\sim G}[c]$ (Same as deterministic)
  - A **negative result** when $E_{v\sim F}[v] \leq E_{c\sim G}[c]$ (**Theorem 5.2**)

Summary of Contributions

- We generalize bilateral trade to the multiplayer **setting**
  - This allows more positive results, bypassing Myerson & Satterthwaite
- We thoroughly study the new setting **theoretically**
  - We **characterize** the set of IC (truthful) mechanisms
  - We **give an efficient mechanism** when buyers value item more
  - We **show impossibility of efficiency** when sellers value item more
- We conduct **experiments** to show effect of our mechanism