Learning Coalition Structures with Games



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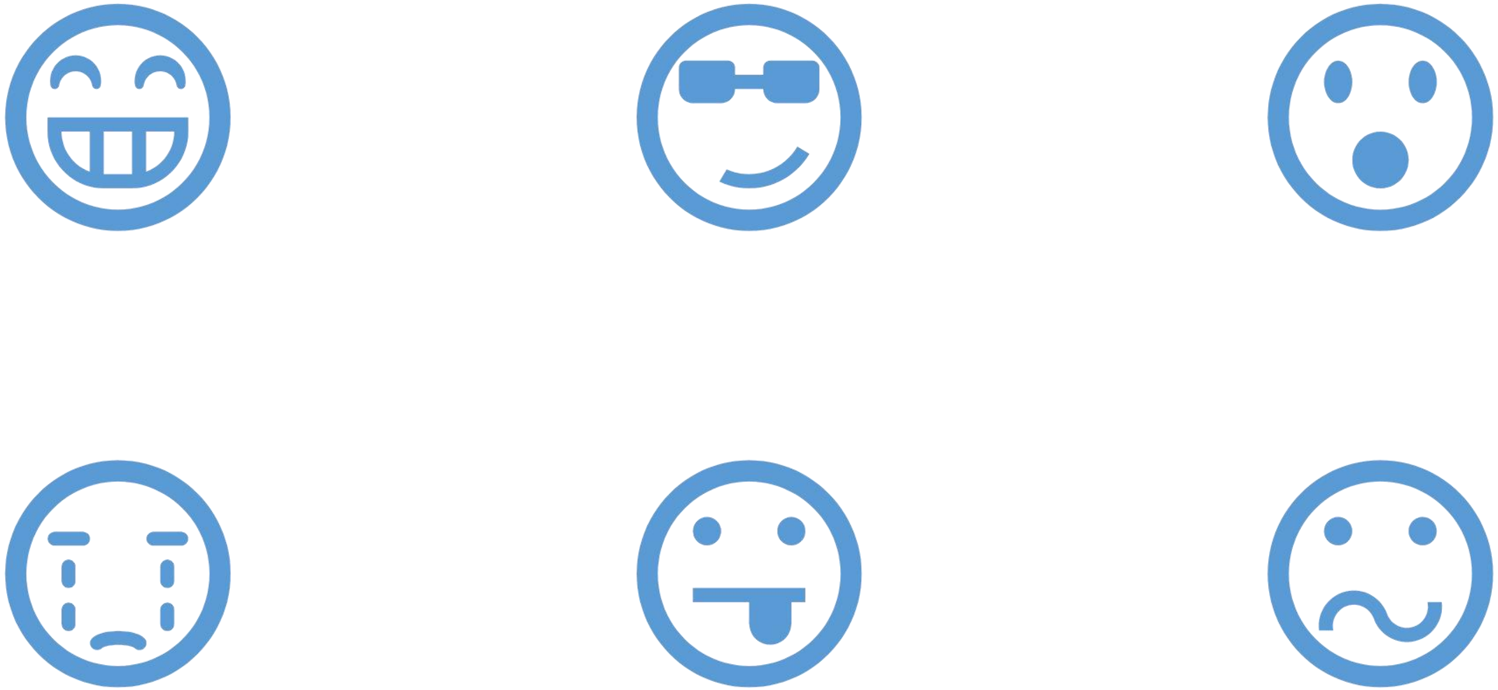
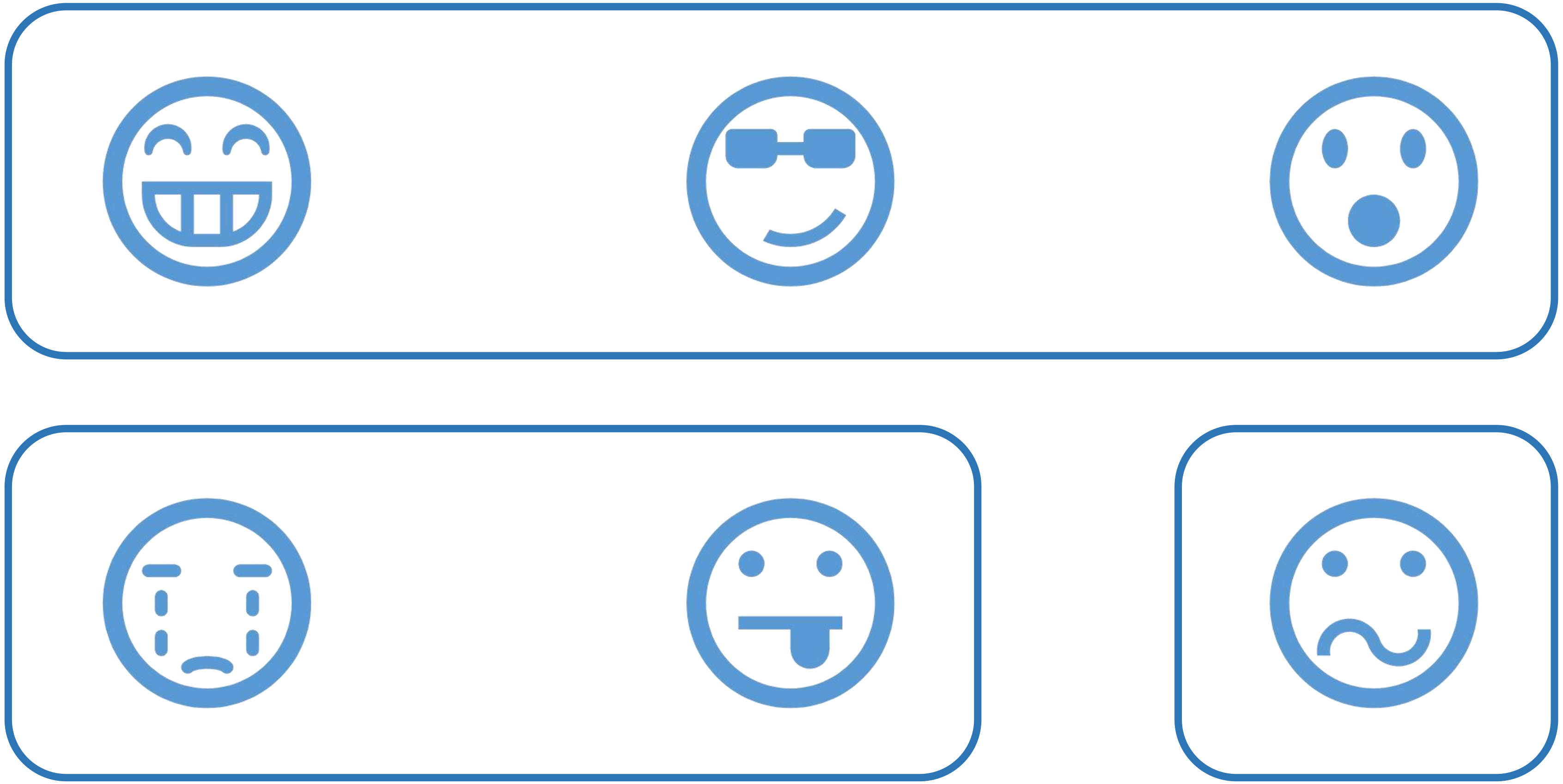
Coalition Structures



Coalition Structures



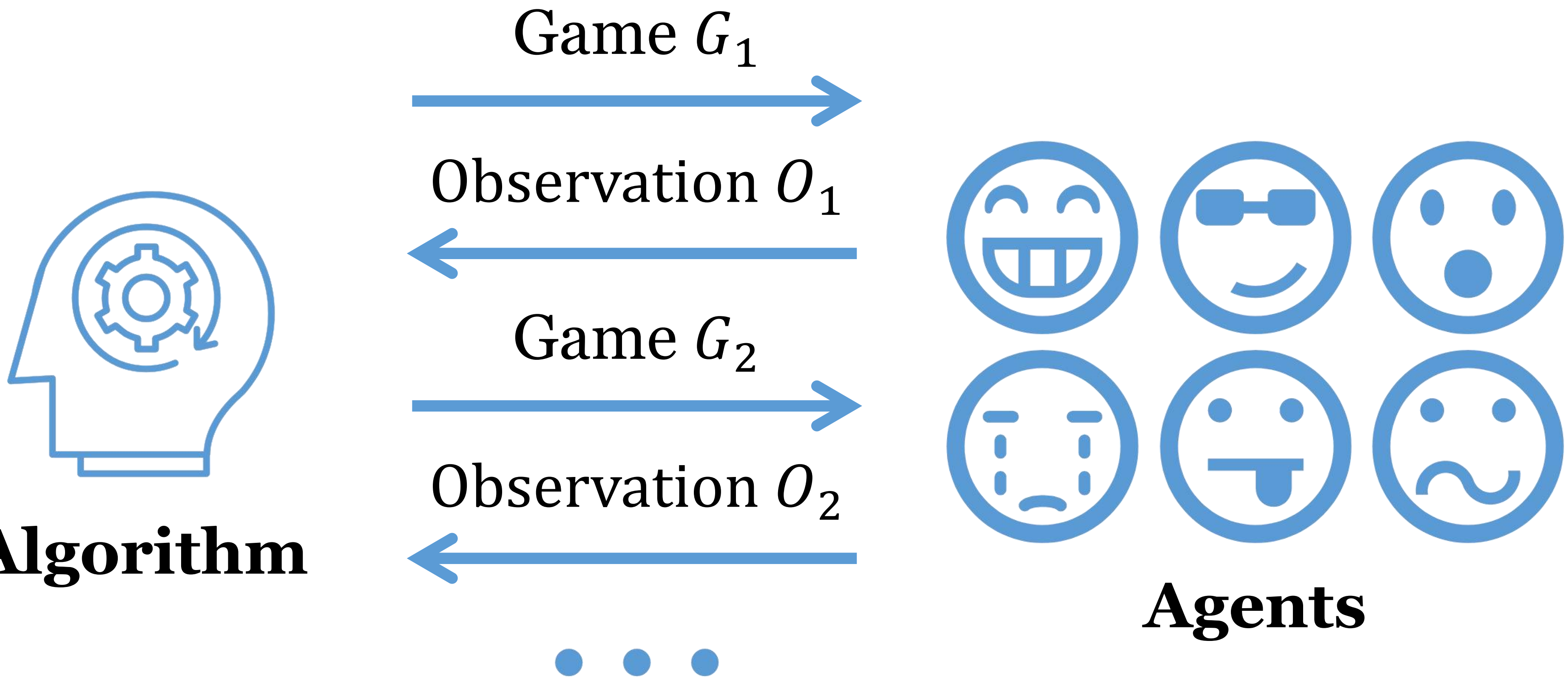
Coalition Structure Learning (CSL)

- **Coalition:** A nonempty subset of the agents, in which
  - The agents **coordinate their actions**
  - The agents **have common interests**

- **Coalition Structure:** A set partition of the agents $\{1, 2, \cdots, n\}$
  - Each set is a separate coalition
  - **Behavior Model in a Game:** Each coalition **act as a joint player** whose actual utility equals the **total utilities of its members**

- **Coalition Structure Learning (CSL):** Recover the unknown coalition structure by observing interactions in designed games

Interactive Model



Single-Bit Observation Oracle

- **Model:** The algorithm queries a game G and a strategy profile Σ, and the agents answer whether Σ is a **Nash Equilibrium** in G
  - The focus of this paper
  - Easy to compute for the agents
  - **One bit of information** per query

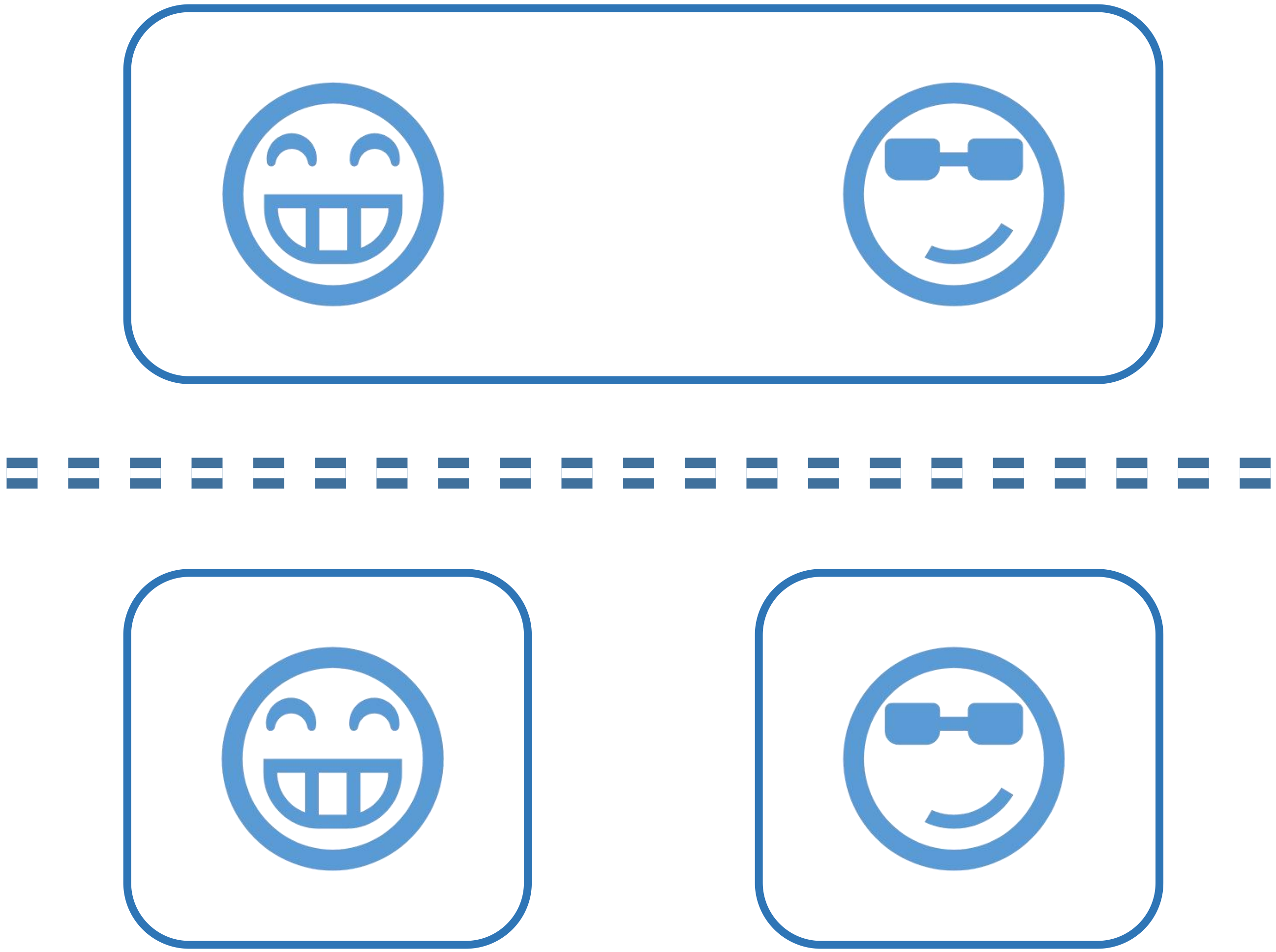
- **Theorem 3.1: Any algorithm** for CSL must interact **at least** $n \log_2 n - O(n \log_2 \log_2 n)$ rounds with the agents
  - We need this many bits of information to distinguish between answers

Types of Games

- What kind of games can the algorithm design?
  - Natural choice: **Normal form games**
    - The **most general** one, thus the **easiest** for the algorithm
  - Succinct games: **Congestion games**, **graphical games**
  - More related to practice: **Auctions**

- We study **all** the above settings in this paper
  - And show **asymptotically optimal algorithms** for all of them
  - We mainly focus on the **normal form game** setting in these slides

How to Distinguish Between the Two?



Normal Form Gadgets

- **Normal Form Gadgets:** A normal form game where a specific pair of agents (x, y) plays the **Prisoner's Dilemma**, and other agents only have one action that does not affect the game

	$\boldsymbol{C_y}$	$\boldsymbol{D_y}$
C_x	$(3, 3)$	$(0, 5)$
D_x	$(5, 0)$	$(1, 1)$

- **Lemma 3.1:** (D_x, D_y) is a **Nash Equilibrium** if and only if x and y are **not in the same coalition**

Product of Normal Form Gadgets

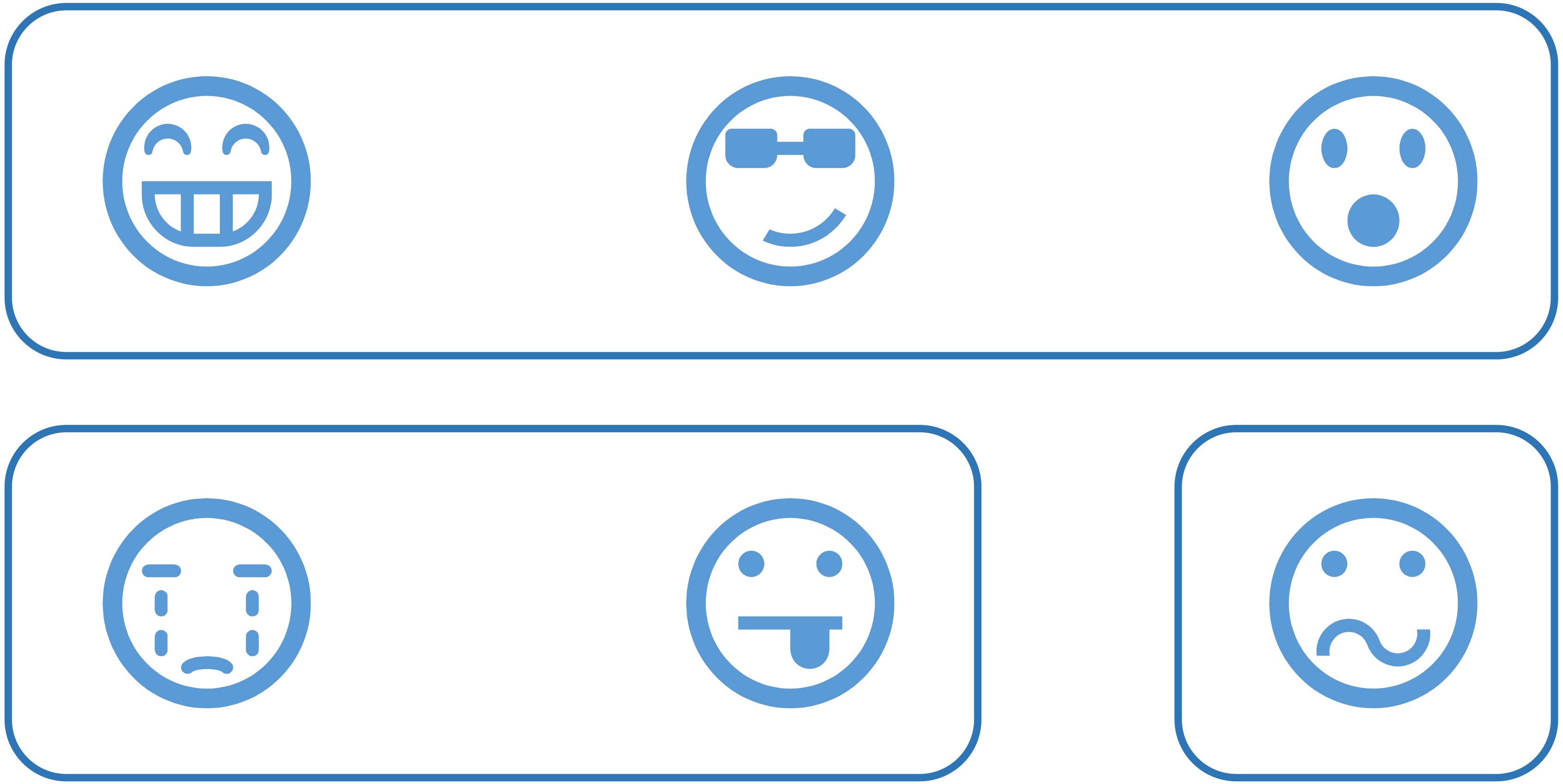
- **Product of Normal Form Gadgets:** Running several normal form gadgets simultaneously as **a single normal form game**
  - Agents **individually act** in each gadget
  - An agent's utility equals the **sum of the agent's utility** in each gadget

- **Lemma 3.2:** Always defect is a **Nash Equilibrium** iff the chosen pair are **not in the same coalition in each gadget**

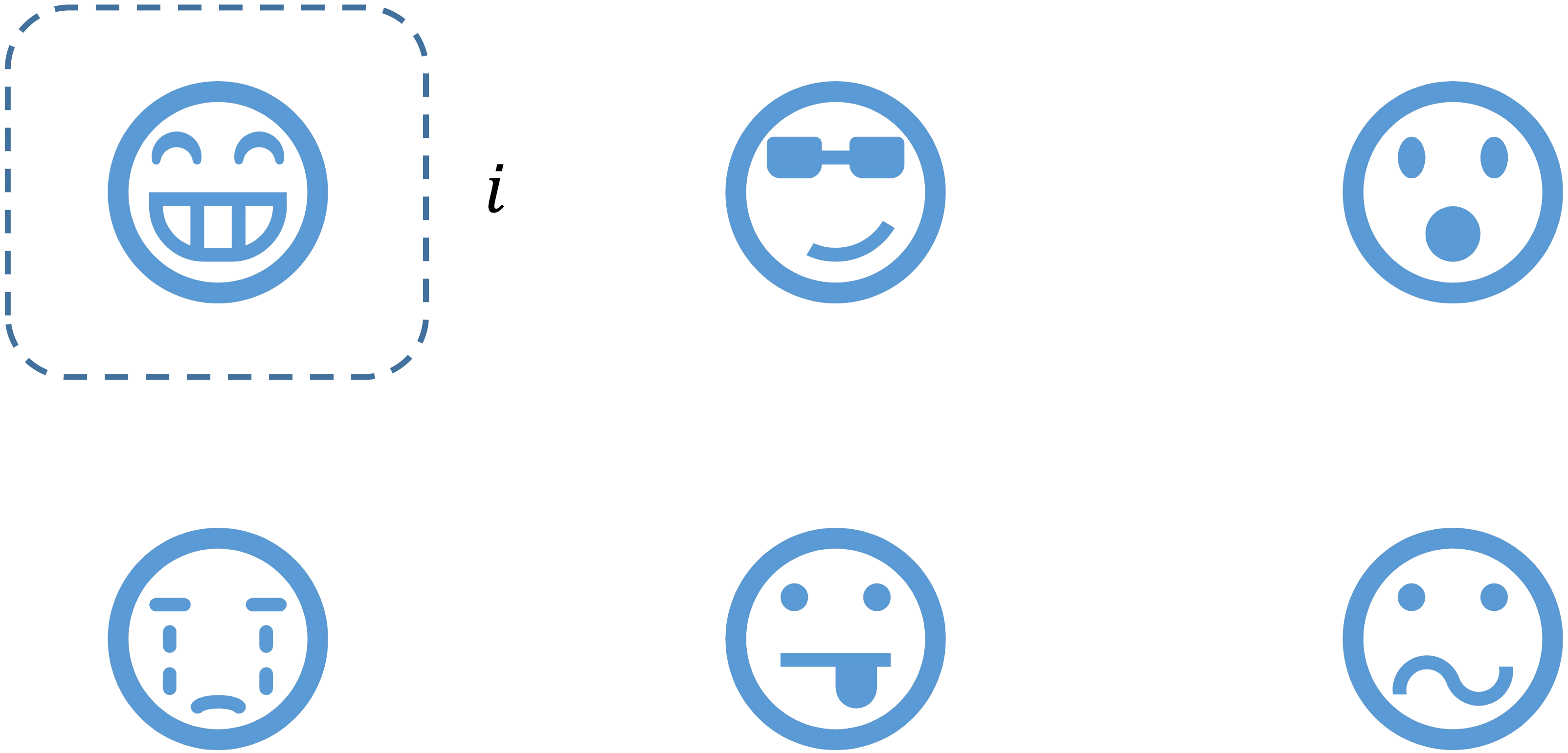
Iterative Grouping (IG)

- Determine each agent's coalition one by one
- For agent i, let all others play **normal form gadgets** with i
  - If always defect is an NE, then agent i has **no other teammates**
  - Otherwise, we know that **someone is in the same coalition** with i
- Run a **binary search** to locate one teammate j of i
  - **Merge** i and j as one joint player
  - Proceed iteratively until i's coalition is finalized

Example Execution



Example Execution



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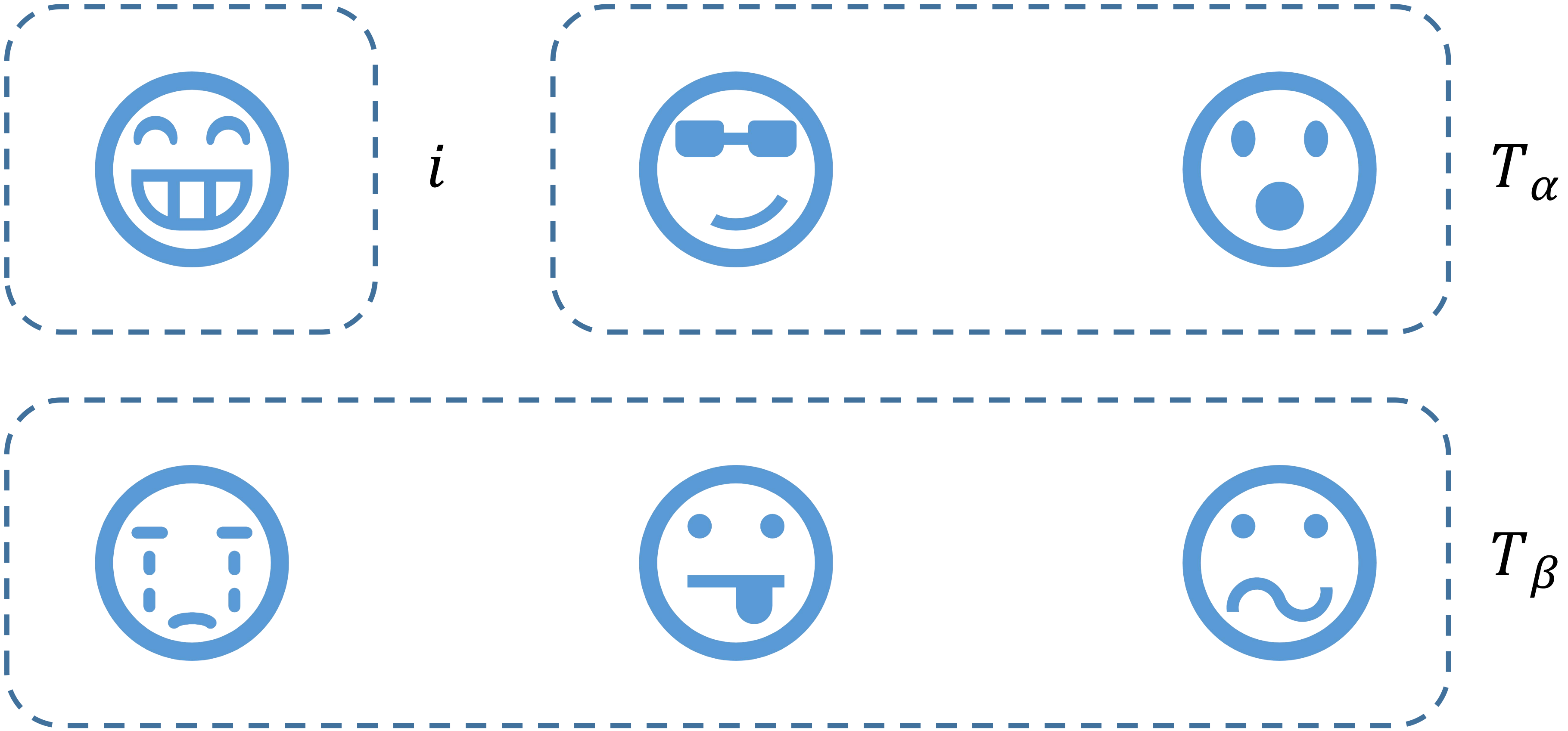

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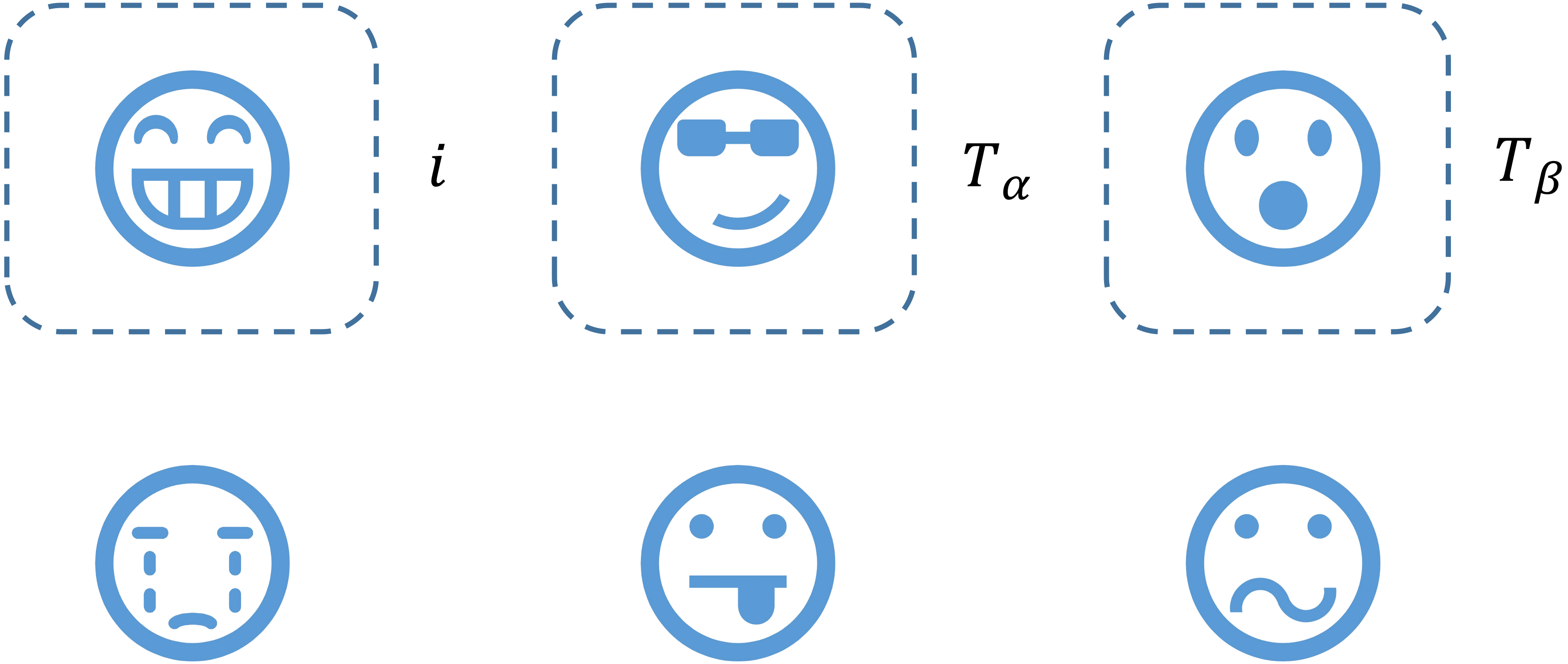

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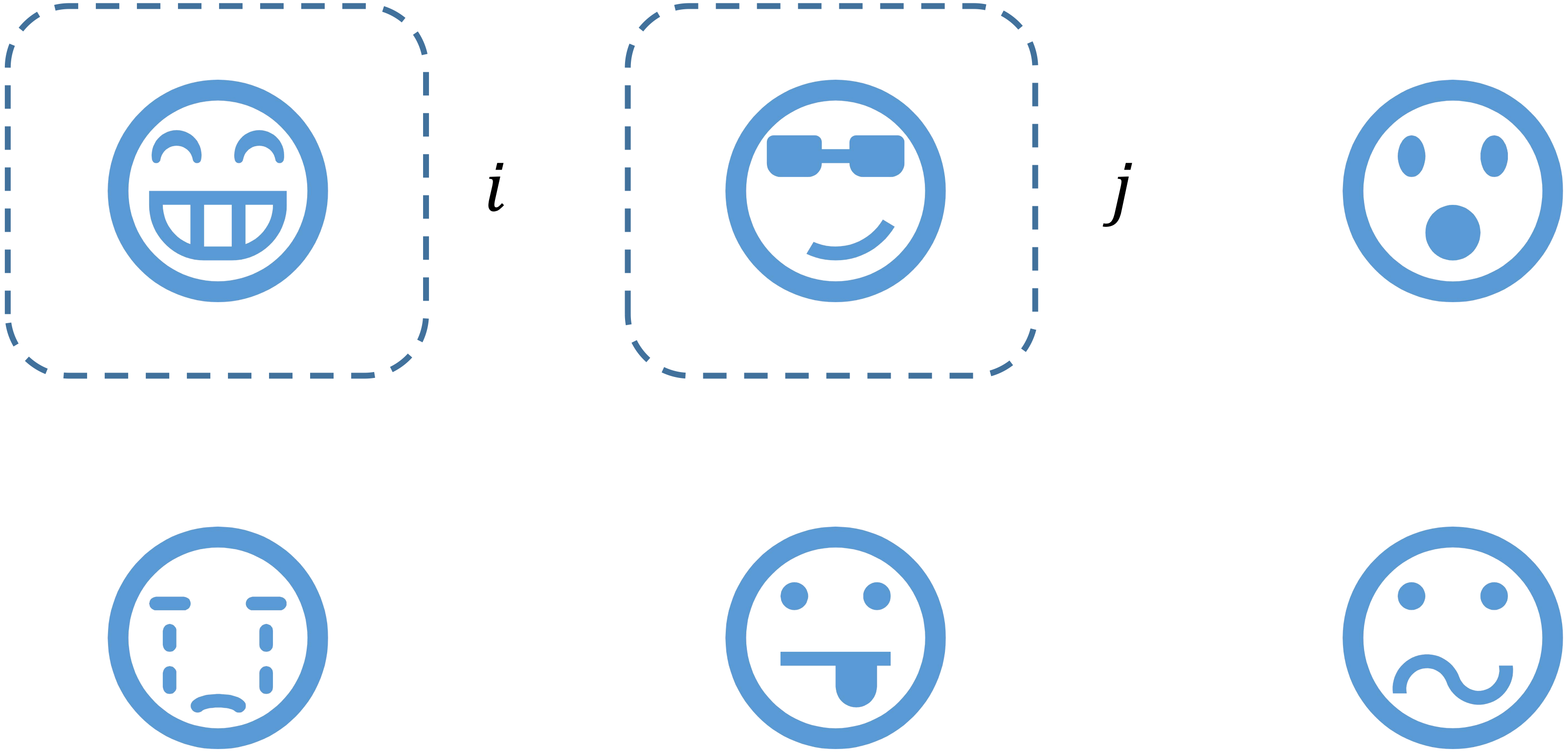

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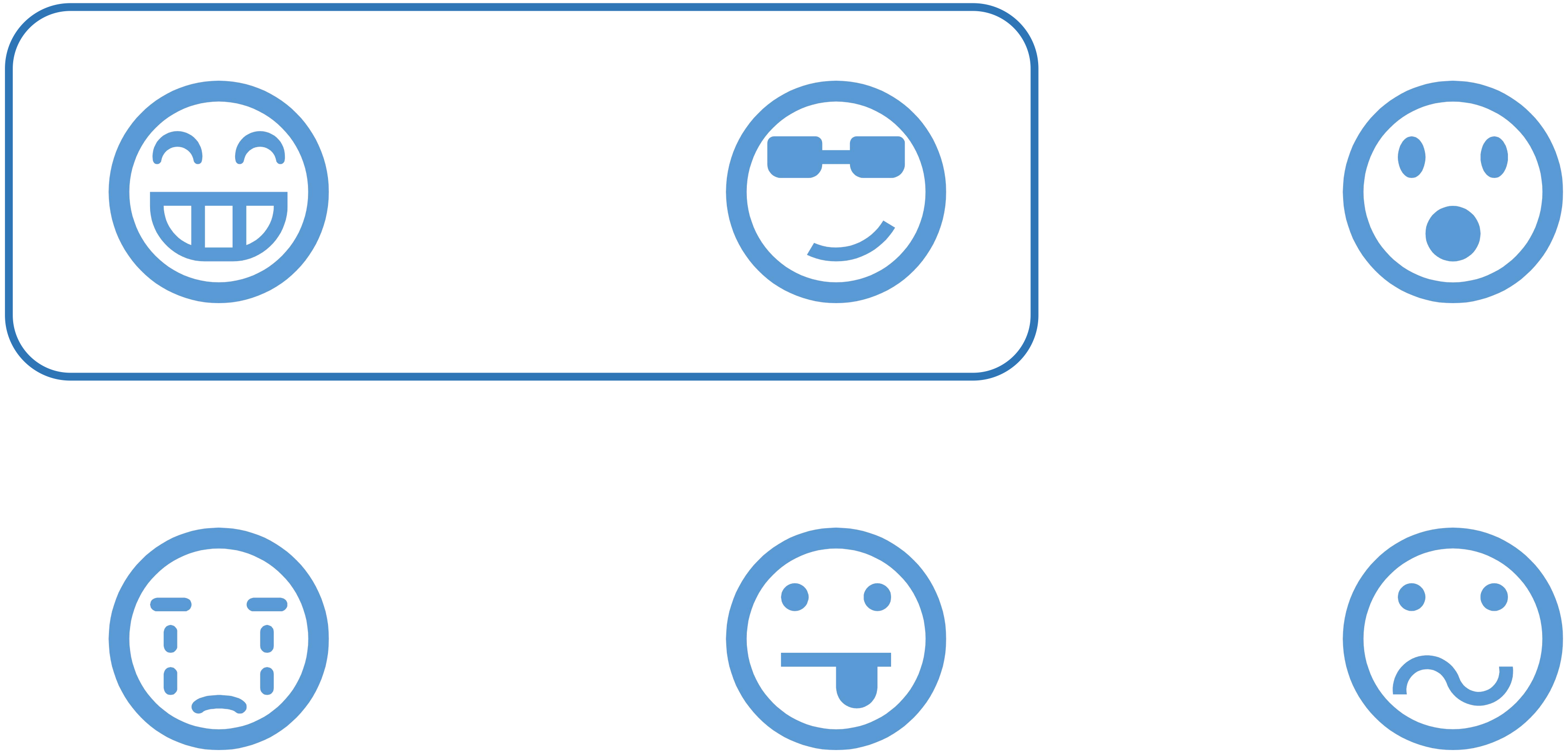

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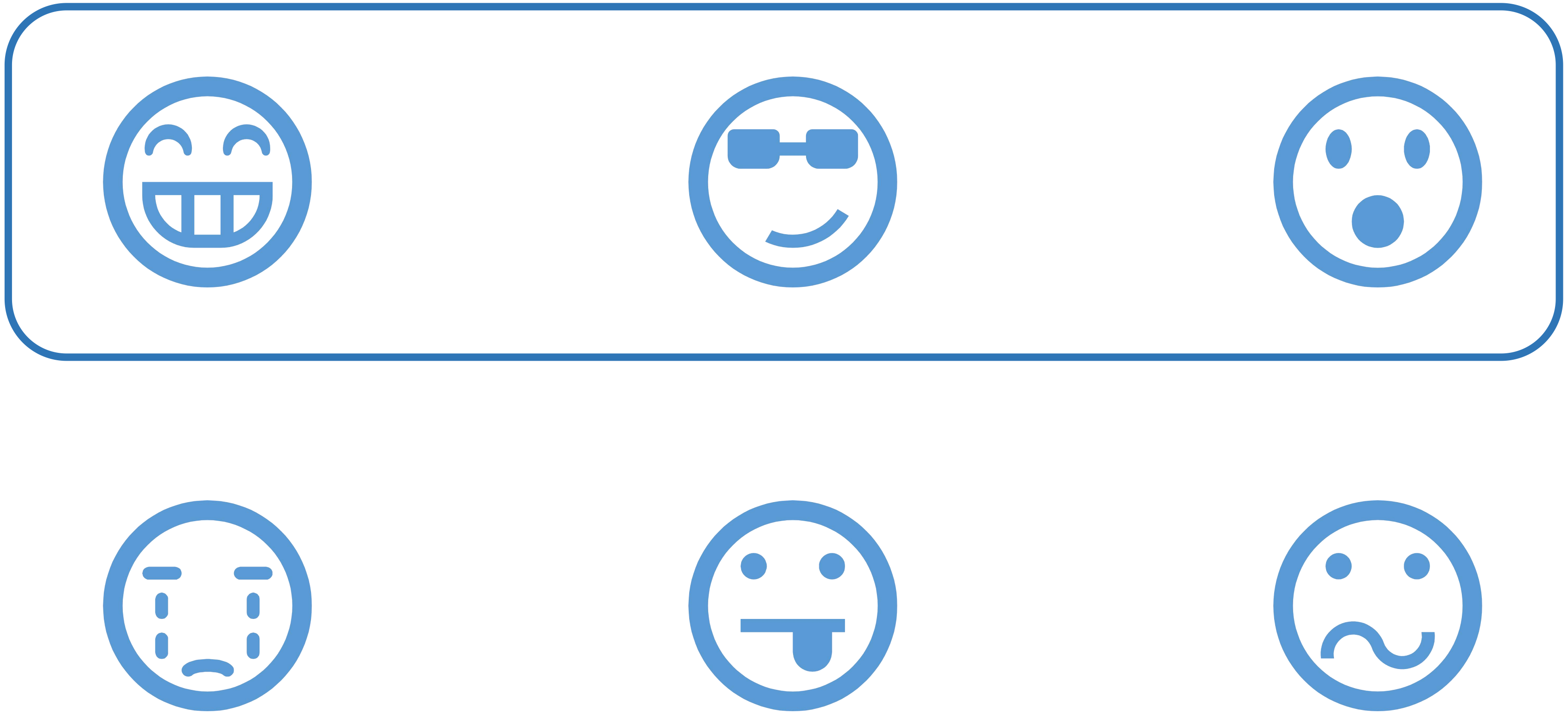

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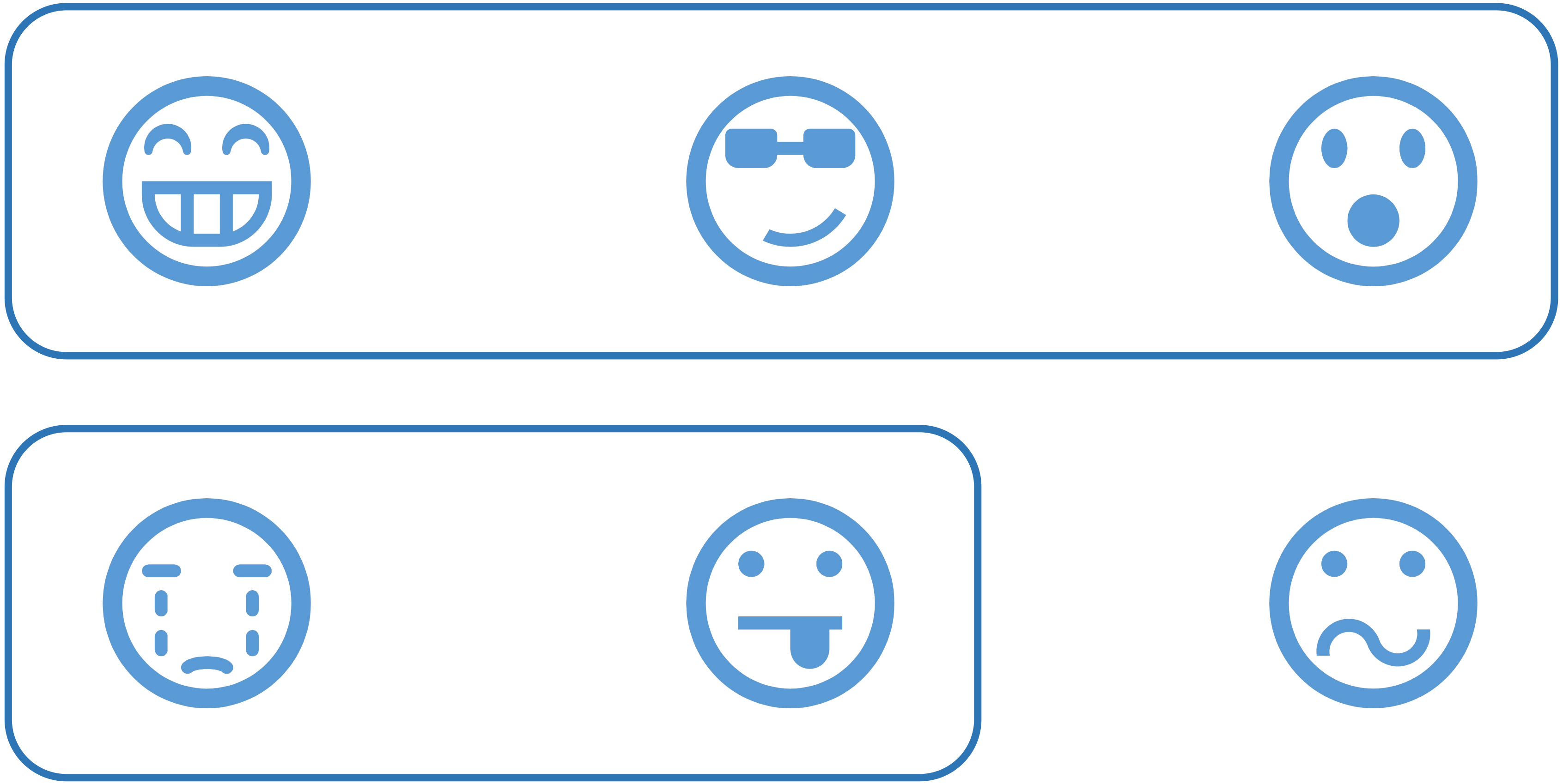

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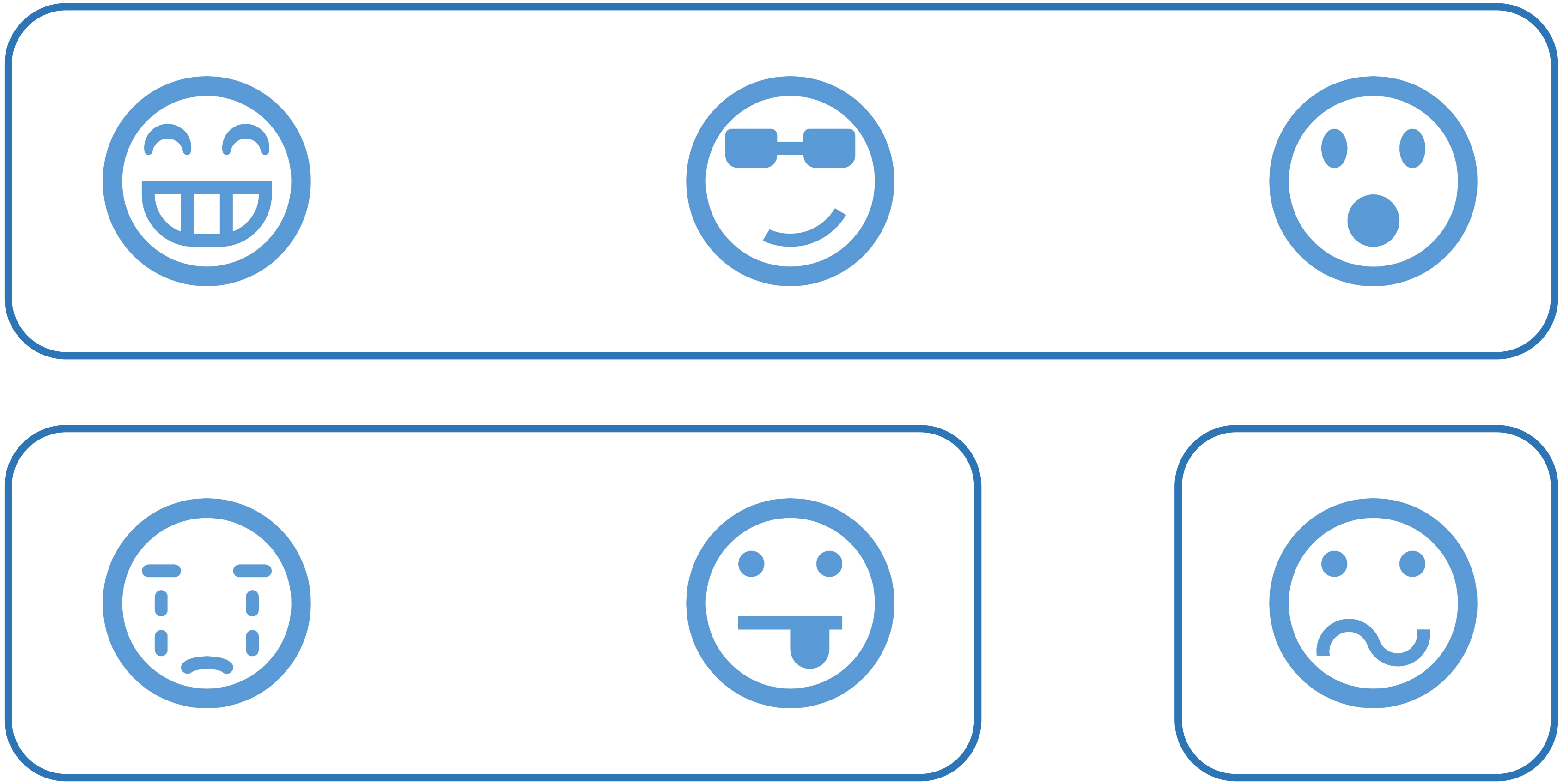


IG is Optimal

- **Theorem 3.2:** IG solves CSL with $n \log_2 n + 3n$ rounds
- **Recall Theorem 3.1:** Any algorithm for CSL must interact at least $n \log_2 n - O(n \log_2 \log_2 n)$ rounds with the agents
- IG is **optimal** up to low order terms

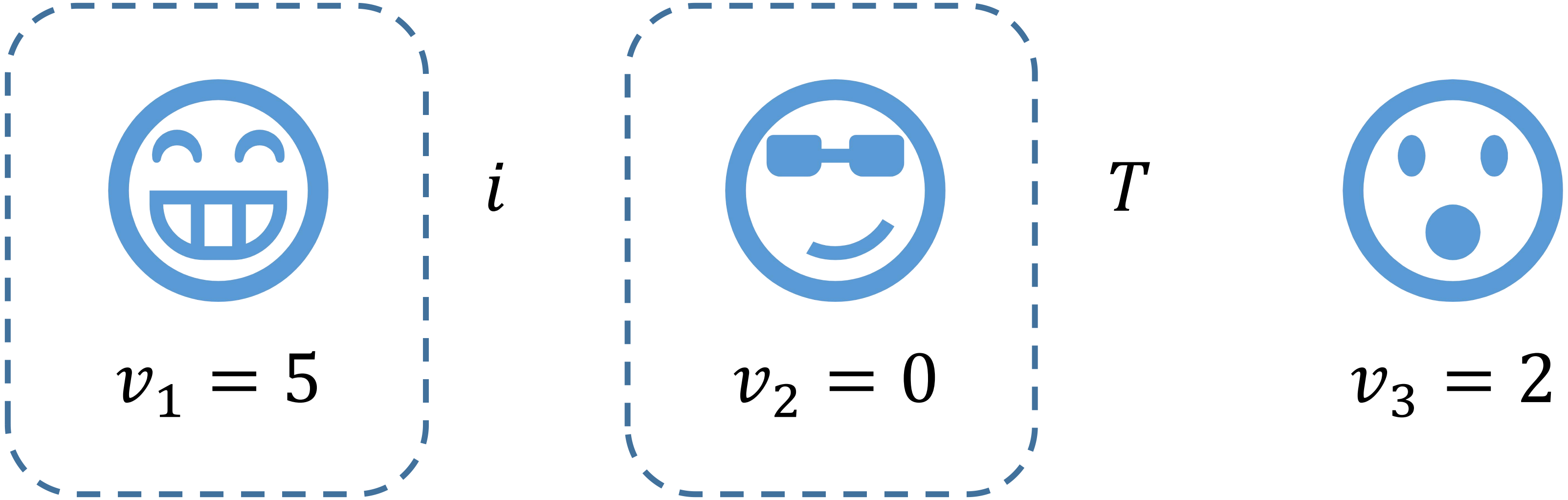
Extension to AuctionCSL

- **AuctionCSL:** The algorithm can only design auctions

- **Format:** Second-price auctions with personalized reserves
  - Each agent i have a **valuation** v_i and a **reserve price** r_i
  - The highest bidder wins, with $price = max\{second\ bid, reserve\ price\}$

- To better simulate the practice, we further restrict the algorithm
  - The algorithm can only design the **reserve prices**
  - The **valuations** are random each query, but the algorithm sees them

Auction Gadgets



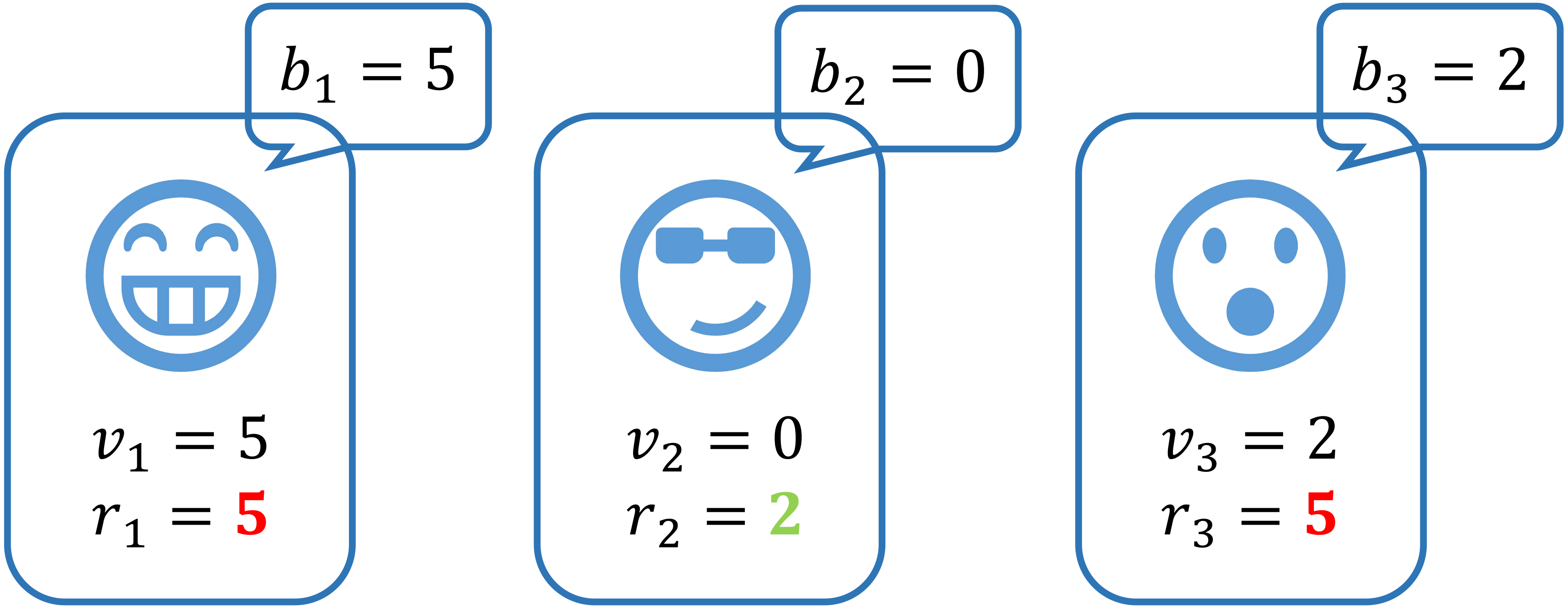
Auction Gadgets

$v_1 = 5$
$r_1 = \mathbf{5}$

$v_2 = 0$
$r_2 = \mathbf{2}$

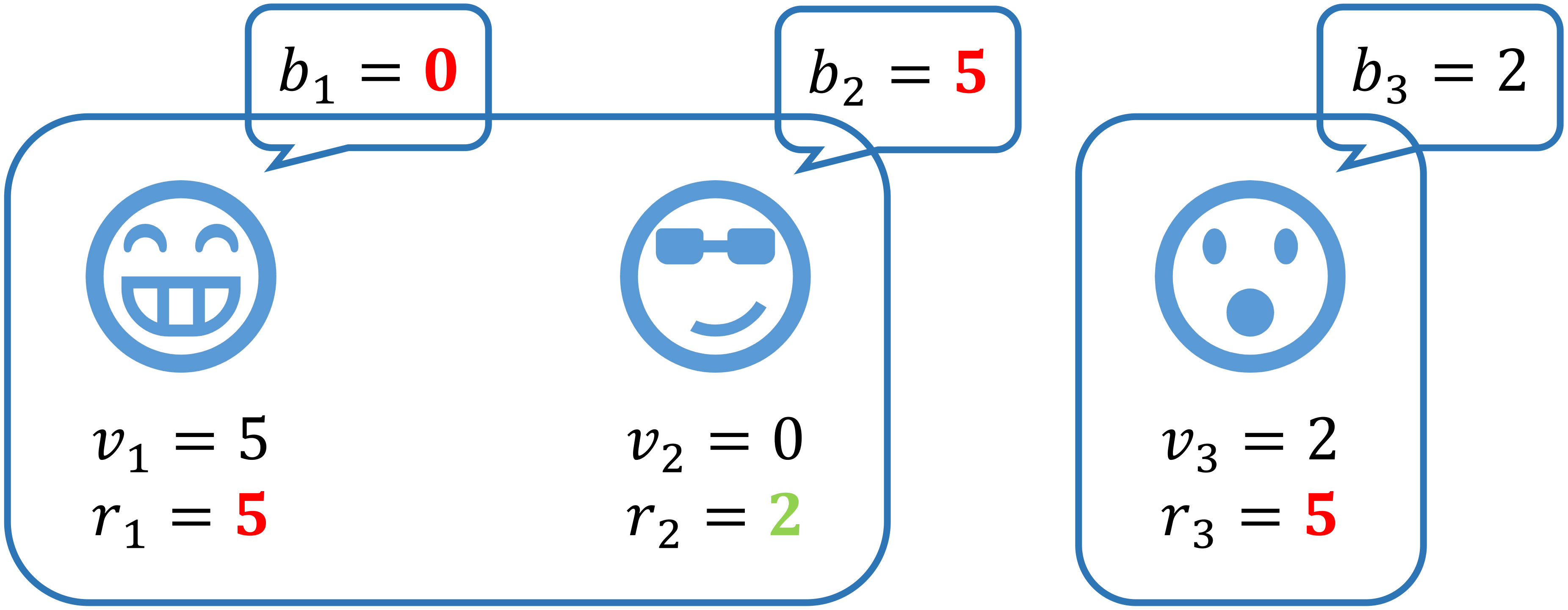
$v_3 = 2$
$r_3 = \mathbf{5}$

If Agent 1 is NOT Cooperating with 2



Truthful bidding IS a Nash Equilibrium

If Agent 1 IS Cooperating with 2



Truthful bidding is **NOT** a **Nash Equilibrium**

AuctionIG

- **AuctionIG:** Our algorithm built upon auction gadgets

- **Theorem 4.1: In expectation**, AuctionIG solves AuctionCSL with $(4.16 + o(1))n \log_2 n$ rounds

- AuctionIG is **optimal** asymptotically

Summary of Contributions

- We propose and formally **model** the CSL problem
- We study the single-bit observation setting **theoretically**
  - We propose an **optimal algorithm** in the normal form game setting
  - We extend the algorithm to other settings, including **graphical games**, **congestion games**, and **auctions**, while **preserving optimality**
- We conduct **experiments** to complement our theory

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