Bayesian Dynamic Linear Models for Strategic Asset Allocation

Jared Fisher
Carlos Carvalho, The University of Texas
Davide Pettenuzzo, Brandeis University

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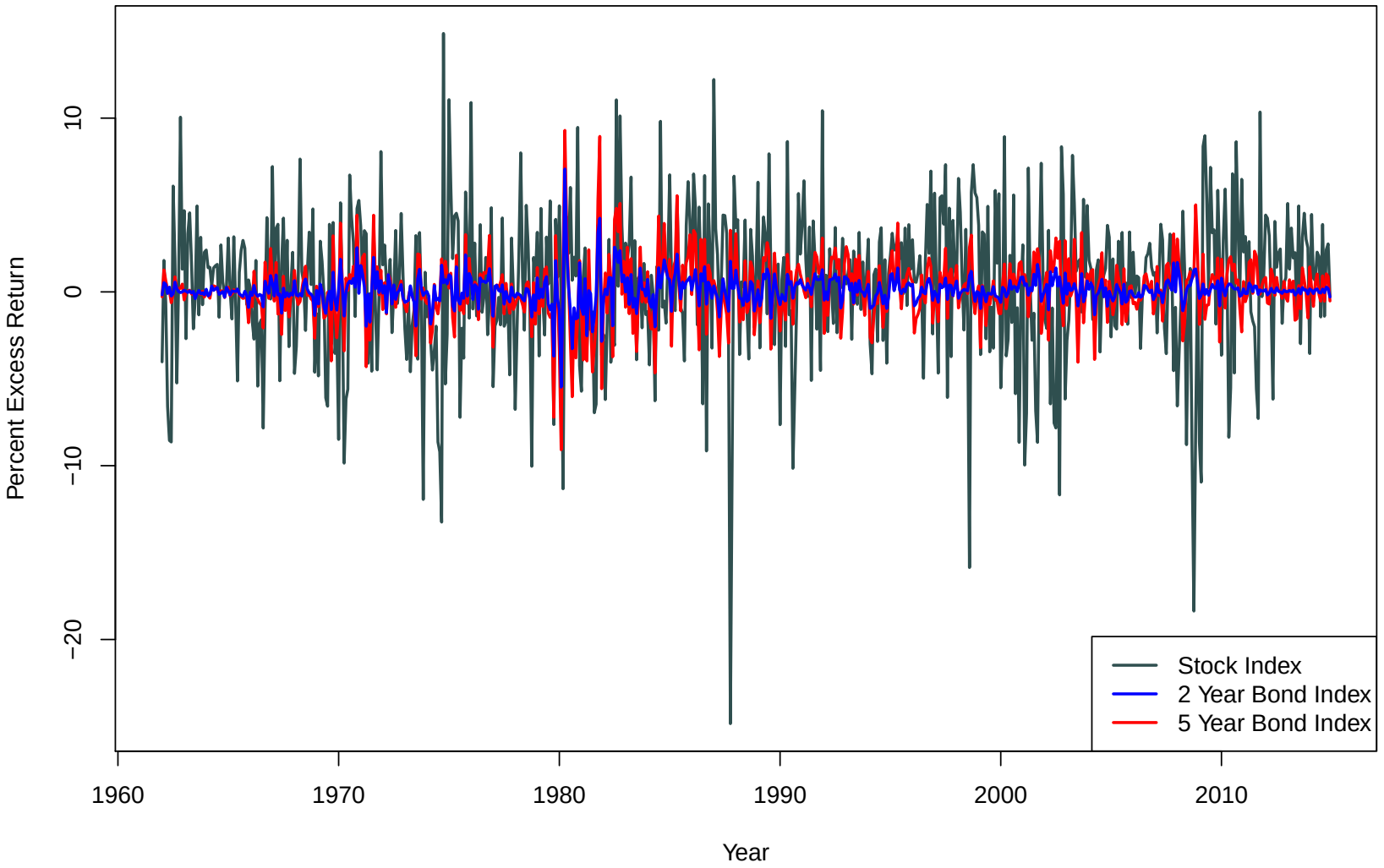
Excess Returns on an Index: is there Signal in the Noise?



Investing

- A portfolio consists of assets of varying riskiness.
- Return on a portfolio is a weighted sum of the individual assets' returns, where the weights are the proportions invested.
- Simple portfolio's return:

$$\begin{aligned} R_{p,t+1} &= w_t R_{1,t+1} + (1 - w_t) R_{f,t+1} \\ &= R_{f,t+1} + w_t (R_{1,t+1} - R_{f,t+1}) \end{aligned}$$

- $(R_{1,t+1} - R_{f,t+1}) \longrightarrow$ "excess return"
- How do we choose w_t?

Investing in Multiple Assets

- Return on multiple risky assets:

$$\begin{aligned} R_{p,t+1} &= (1 - \sum_i w_{i,t}) R_{f,t+1} + \sum_i w_{i,t} R_{i,t+1} \\ &= R_{f,t+1} + \sum_i w_{i,t} (R_{i,t+1} - R_{f,t+1}) \end{aligned}$$

- How do we choose $w_{i,t}$ for all i?
- Understand as much as we can about $(R_{i,t+1} - R_{f,t+1})$ at time t.

Making Investments

- For an investor with power utility and risk aversion γ
- Given forecasted $\hat{\mu}_t, \hat{\Sigma}_t$
- Portfolio weights vector is

$$w_t = \frac{1}{\gamma}\hat{\Sigma}_t^{-1}\left(\hat{\mu}_t + \frac{1}{2}diag(\hat{\Sigma}_t)\right)$$

Understanding Excess Returns

- What is the distribution of $Y_{i,t+1} = (R_{i,t+1} - R_{f,t+1})$, given what we know at time t?
- $E(Y_{i,t+1}|D_t)$ ("risk premium")
  - $= \mu$? (constant, no predictability)
  - $= \mu_t = f(X_t) = X_t'\beta$?
  - $= \mu_t = X_t'\beta_t$? ("time-varying parameters")
- $Var(Y_{i,t+1}|D_t)$
  - $= \sigma^2$? (constant volatility)
  - $= \sigma_t^2$? ("stochastic volatility")

Does Predictability Exist?

- Literature assumes linear relationship:
  $Y_{i,t+1} = X_t'\beta + \epsilon_{t+1}, Var(\epsilon) = \sigma^2$
- Tests are mostly in-sample, not out-of-sample (OOS).
- Welch and Goyal (2008) show that the good performance of popular variables in-sample don't hold OOS.
- More recently, authors show OOS predictability by deviating from the standard model.
  - Time-varying parameters (e.g. Dangl and Halling, 2012)
  - Stochastic volatility (e.g. Johannes, Korteweg and Polson, 2013)
  - Parameter uncertainty (Bayesian models)

Our Analysis

- Two research questions
  - Predictability: is there useful information in X?
  - Time-variation: are the parameter values (β and σ^2) constant with respect to time?
- Compare models with and without predictors and with and without variance discounting (of both regression coefficients and volatility)
- Benchmark: the constant model (i.e. $X_t = 1$)
  - Often called the expectation hypothesis model, it represents the efficient markets hypothesis/no predictability.

Data description

We will first look at portfolios of a risky asset (stock index or bond index) and a risk-free asset (3 month T-bill). We use the following data, spanning 1962-2014:

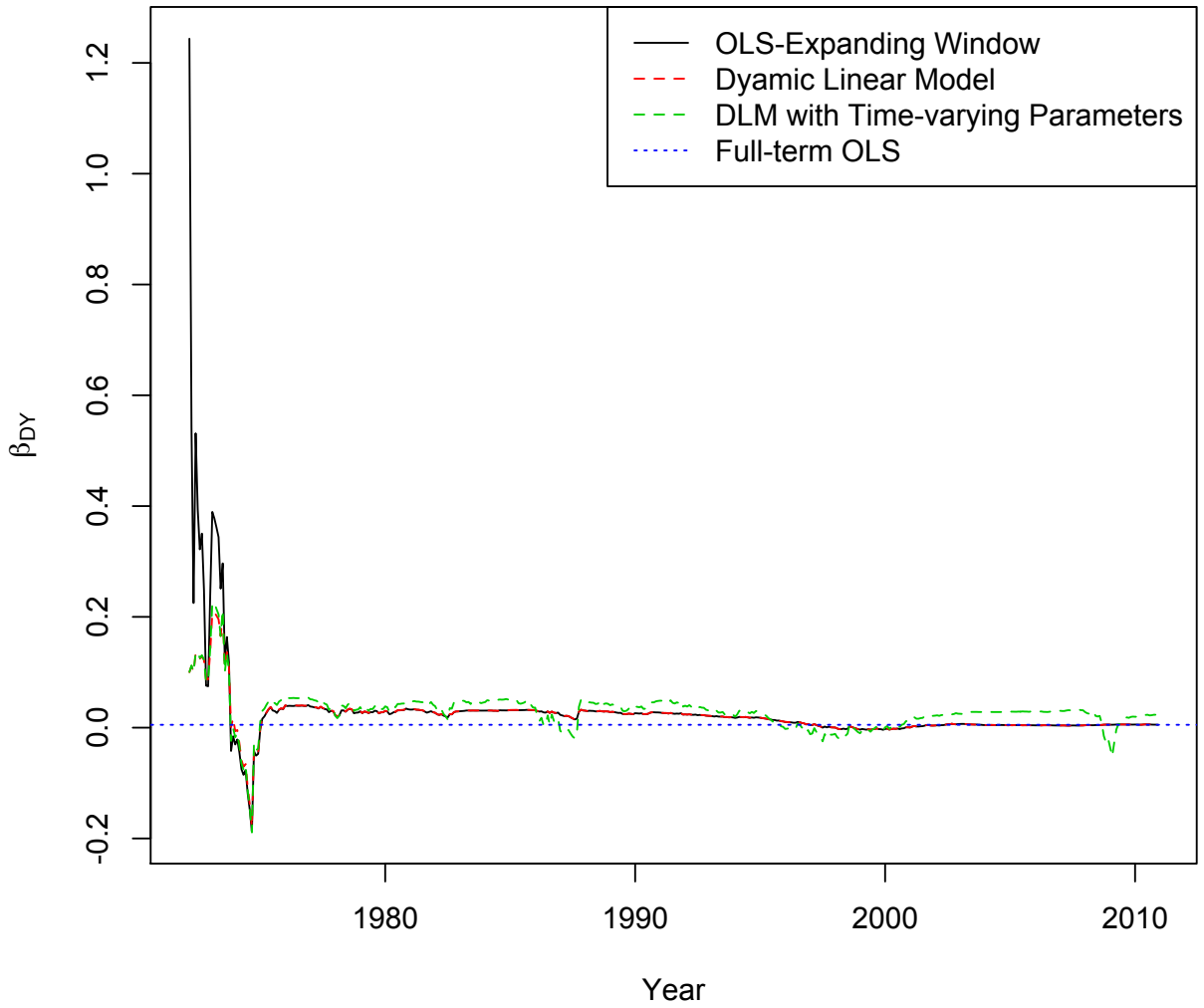
- Welch and Goyal's predictors of stock performance, updated to 2014,
- CRSP value weighted returns,
- Bonds data from Gargano, Pettenuzzo, and Timmermann (2015)
  - Bond index for 2-5 year maturities,
  - Cochrane and Piazzesi's (2005) linear combination of forward rates,
  - Fama and Bliss' (1987) forward spread,
  - Ludvigson and Ng's (2009) macro factor.

Our Univariate Model

$$
\begin{aligned}
Y_t &= X_{t-1}'\beta_t + \epsilon_t, \quad \epsilon_t \sim N(0, v_t/\phi_t) \\
\beta_t &= \beta_{t-1} + \omega_t, \quad \omega_t \sim N(0, w_t/\delta) \\
\phi_t &= \gamma_t \phi_{t-1}/\delta_v
\end{aligned}
$$

$$
\begin{aligned}
\beta_0 &\sim N(m_0, C_0) \\
\gamma_t &\sim Beta(\delta_v n_{t-1}/2, (1-\delta_v) n_{t-1}/2)
\end{aligned}
$$

(see West and Harrison, 1997)


OLS-Expanding Window
Dyamic Linear Model
DLM with Time-varying Parameters
Full-term OLS
βDY
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1980
1990
2000
2010
Year

Model Advantages

- Bayesian model without MCMC - saves computation time
- Bridges gap between Recursive model vs. Rolling-window model

Recursion

$$\vdots$$

$$\beta_{t-1}|D_{t-1} \sim N(m_{t-1}, C_{t-1}) \qquad R_t = \left(\frac{1}{\delta}\right) C_{t-1}$$

$$\beta_t|D_{t-1} \sim N(m_{t-1}, R_t) \qquad Q_t = X'_{t-1} R_t X_{t-1} + S_{t-1}$$

$$Y_t|D_{t-1} \sim N(f_t = X'_{t-1} m_{t-1}, Q_t) \qquad m_t = m_{t-1} + (Y_t - f_t)\, A_t, A_t = \frac{R_t X_t}{Q_t}$$

$$\beta_t|D_t \sim N(m_t, C_t) \qquad C_t = \frac{S_t}{S_{t-1}}(R_t - A_t A'_t Q_t)$$

$$\vdots \qquad S_t = h(\delta_v, Q_t, Y_t - f_t, S_{t-1})$$

Modeling Details

- Prior created on 1962-1971 data
- Models evaluated on 1985-2014
- Evaluated on both economic and statistical criteria.
  - Economic Measure: Certainty Equivalent Returns, using power utility (CRRA)
    - $U(wealth) = \frac{1}{1-\gamma}(wealth)^{1-\gamma}$
    - $\gamma = 5$
  - Statistical Prediction Measure: Mean Squared Prediction Error Ratio
  - Statistical Fit Measure: Average Log Score
- Restrict:
  - Portfolio weights $w_t \in [-2, 3]$
  - Coefficient variance discount factor $\delta \in [0.98, 1.0]$
  - Volatility discount factor $\delta_v \in [0.9, 1.0]$

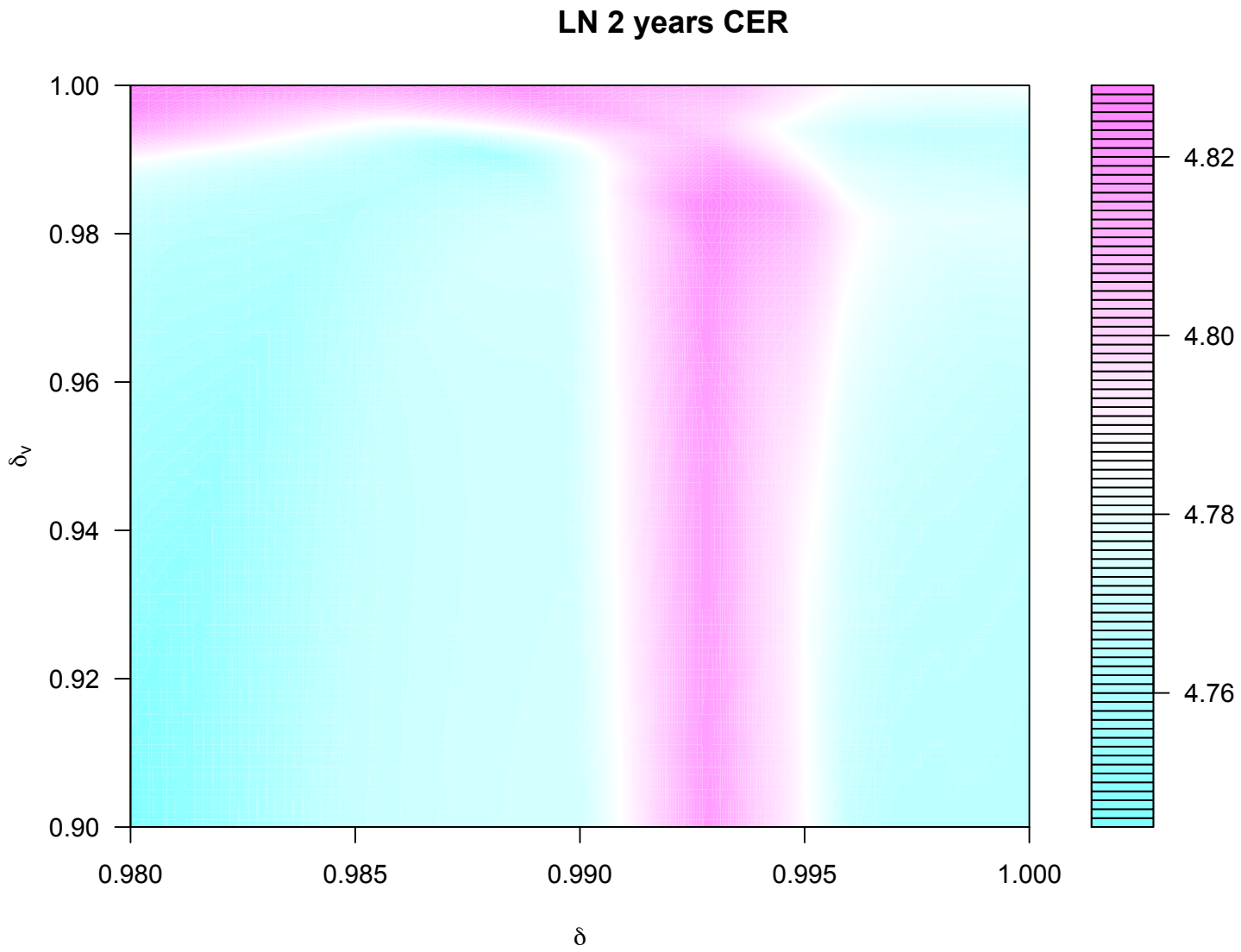
Results: No Discounting - Comparison to Literature

Stock Index				Bond Index				
Predictor	CER	ALS	MSE	Mat.	Pred.	CER	ALS	MSE
(none)	5.678	1.689	1.000	2	(none)	6.519	3.512	1.000
Log D/P	3.385	1.681	1.018	2	CP	5.838	3.517	1.049
Log D/Y	3.333	1.682	1.018	2	FB	6.902	3.519	**0.979**
Log E/P	3.884	1.685	1.012	2	LN	**8.535**	**3.531**	1.010
Smooth E/P	3.228	1.680	1.019	3	(none)	6.265	3.146	1.000
Log D/Payout	0.767	1.681	1.020	3	CP	5.853	3.150	1.028
B/M	3.133	1.680	1.020	3	FB	7.614	3.154	**0.975**
T Bill Rate	5.424	1.687	1.003	3	LN	**9.463**	**3.165**	0.981
LngTerm Yld	5.507	1.688	1.002	4	(none)	6.083	2.891	1.000
LngTerm Ret.	4.630	1.686	1.007	4	CP	5.827	2.895	1.015
Term Spread	2.764	1.683	1.010	4	FB	8.192	2.901	0.974
Def.Yld.Sprd	1.854	1.678	1.023	4	LN	**9.558**	**2.910**	**0.970**
Def.Ret.Sprd	4.199	1.692	0.999	5	(none)	5.910	2.694	1.000
Stock Var.	**6.426**	**1.701**	**0.977**	5	CP	5.882	2.697	1.007
Net Eqty Exp.	4.288	1.682	1.013	5	FB	8.416	2.704	0.974
Inflation	2.980	1.683	1.012	5	LN	**9.257**	**2.713**	**0.965**

Is There Predictability if we Discount Variances?

	CER Maximum			ALS Maximum			MSE Minimum		
Predictor:	Value	δ	δ_v	Value	δ	δ_v	Ratio	δ	δ_v
Mean	5.68	1.00	1.00	1.689	1.00	1.00	1.00	1.00	1.00
Mean - Dscnt	**6.50**	1.00	0.90	**1.736**	1.00	0.90	1.00	1.00	0.99
Log D/P	3.64	1.00	1.00	**1.730**	0.99	0.90	1.01	0.99	0.97
Log D/Y	3.63	1.00	1.00	**1.730**	0.99	0.90	1.01	0.99	0.96
Log E/P	5.43	1.00	0.90	**1.732**	1.00	0.90	1.01	1.00	1.00
Smooth E/P	3.59	1.00	1.00	**1.731**	0.99	0.90	1.01	0.99	0.97
Log D/Payout	0.77	1.00	1.00	**1.721**	1.00	0.90	1.02	1.00	1.00
Bk/Mkt	3.40	1.00	0.90	**1.729**	1.00	0.90	1.02	0.99	0.95
T Bill Rate	**5.81**	1.00	0.90	**1.734**	1.00	0.90	1.00	1.00	1.00
LngTrm Yld.	**6.05**	1.00	0.90	**1.736**	1.00	0.90	1.00	1.00	1.00
LngTrm Ret.	**6.49**	1.00	0.90	**1.740**	1.00	0.90	1.01	1.00	1.00
Term Spread	2.80	1.00	0.93	**1.727**	1.00	0.90	1.01	1.00	0.98
Def.Yld.Sprd	4.85	0.99	0.90	**1.730**	0.99	0.90	1.02	0.99	0.97
Def.Ret.Sprd	**6.74**	1.00	0.90	**1.735**	1.00	0.90	1.00	1.00	1.00
Stock Var.	**8.88**	1.00	0.90	**1.747**	1.00	0.90	**0.98**	1.00	1.00
Net Eqty. Exp.	**6.38**	0.99	0.92	**1.729**	0.99	0.90	1.00	0.99	0.95
Inflation	**5.85**	1.00	0.90	**1.730**	1.00	0.90	1.01	0.99	0.95

Discount Factor Heatmap



Average Over Models

- Many models beat the benchmark, given the correct discount factors.
- But, we don't know *a priori* how much to discount or which predictors will perform well.
- Solution: average and share strength across models.
  - For each time t, weight each of the models' prediction based on its performance up through time $t-1$.
  - Create different averaged models by weighting on utility and score, as well as an equal-weighted model.

$$w^U_{i,\tau+1} = \left(\frac{1}{\gamma} \frac{1}{\tau} \sum_{t=1}^{\tau} U_{i,t} \right)^{\frac{1}{1-\gamma}}$$

$$w^S_{i,\tau+1} = \left(\sum_{t=1}^{\tau} ln(score_{i,t}) \right) - \min_j \left(\sum_{t=1}^{\tau} ln(score_{j,t}) \right)$$

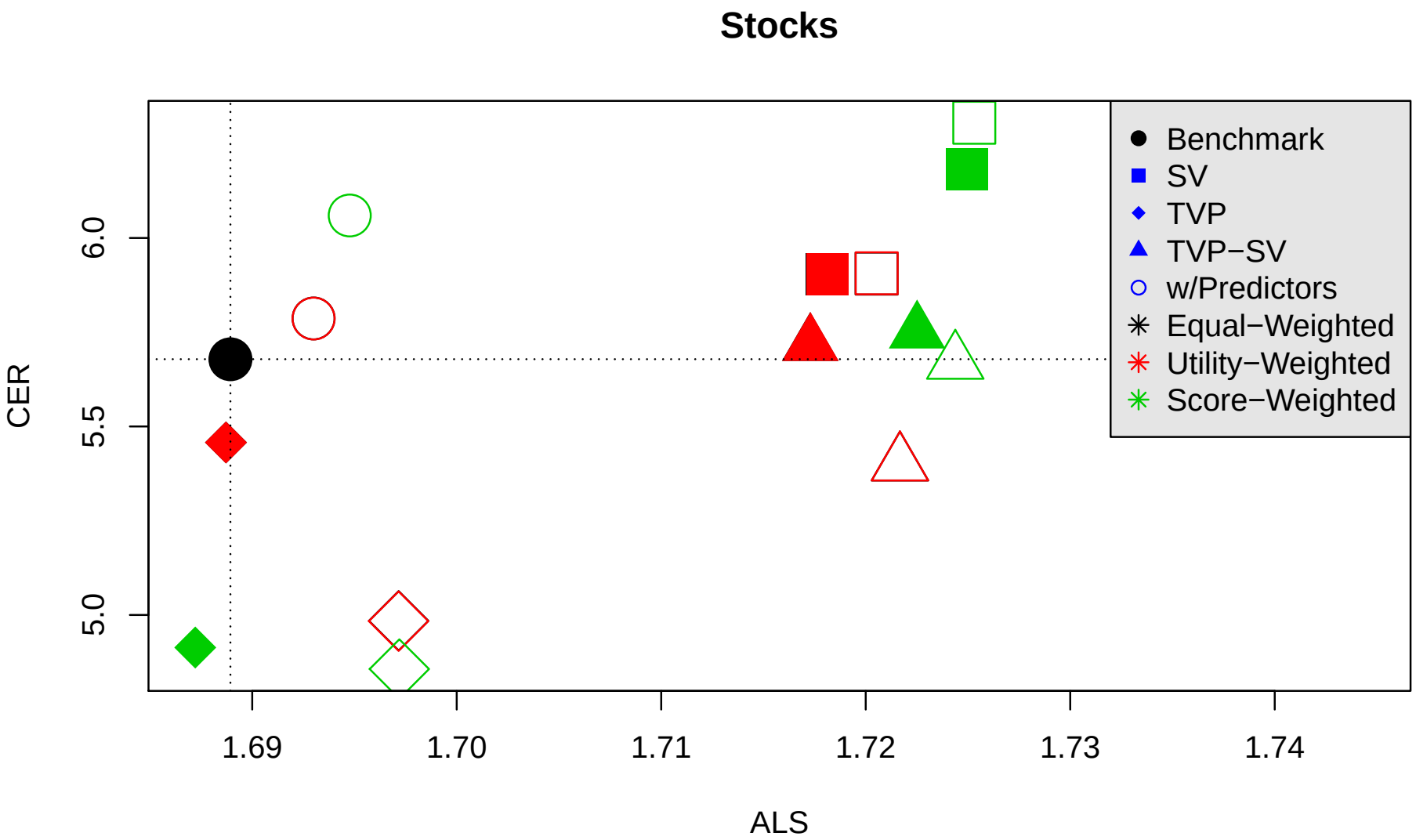
Modeling Details

- A model is fit for every combination of predictor, δ, and δ_v.
- 10 values of δ and δ_v are considered, equally spaced in the range $\delta \in [0.98, 1.0]$, $\delta_v \in [0.9, 1.0]$.

Model Averaging Results: Stocks

Pred	TVP	SV	Models	Weights	CER	ALS	MSE
0	0	0	1	(none)	5.678	1.689	1.0000
0	0	1	10	Equal	5.904	1.718	1.0000
0	0	1	10	Utility	5.904	1.718	1.0000
0	0	1	10	Score	6.182	1.725	1.0000
0	1	0	10	Equal	5.458	1.689	1.0005
0	1	0	10	Utility	5.458	1.689	1.0005
0	1	0	10	Score	4.914	1.687	1.0034
0	1	1	100	Equal	5.717	1.717	1.0005
0	1	1	100	Utility	5.717	1.717	1.0005
0	1	1	100	Score	5.750	1.723	1.0009
1	0	0	16	Equal	5.787	1.693	0.9992
1	0	0	16	Utility	5.786	1.693	0.9992
1	0	0	16	Score	6.060	1.695	**0.9967**
1	0	1	160	Equal	5.906	1.721	0.9992
1	0	1	160	Utility	5.906	1.721	0.9992
1	0	1	160	Score	**6.306**	**1.725**	0.9968
1	1	0	160	Equal	4.984	1.697	0.9999
1	1	0	160	Utility	4.984	1.697	0.9999
1	1	0	160	Score	4.857	1.697	0.9991
1	1	1	1600	Equal	5.400	1.722	0.9999
1	1	1	1600	Utility	5.400	1.722	0.9999
1	1	1	1600	Score	5.670	1.724	0.9982

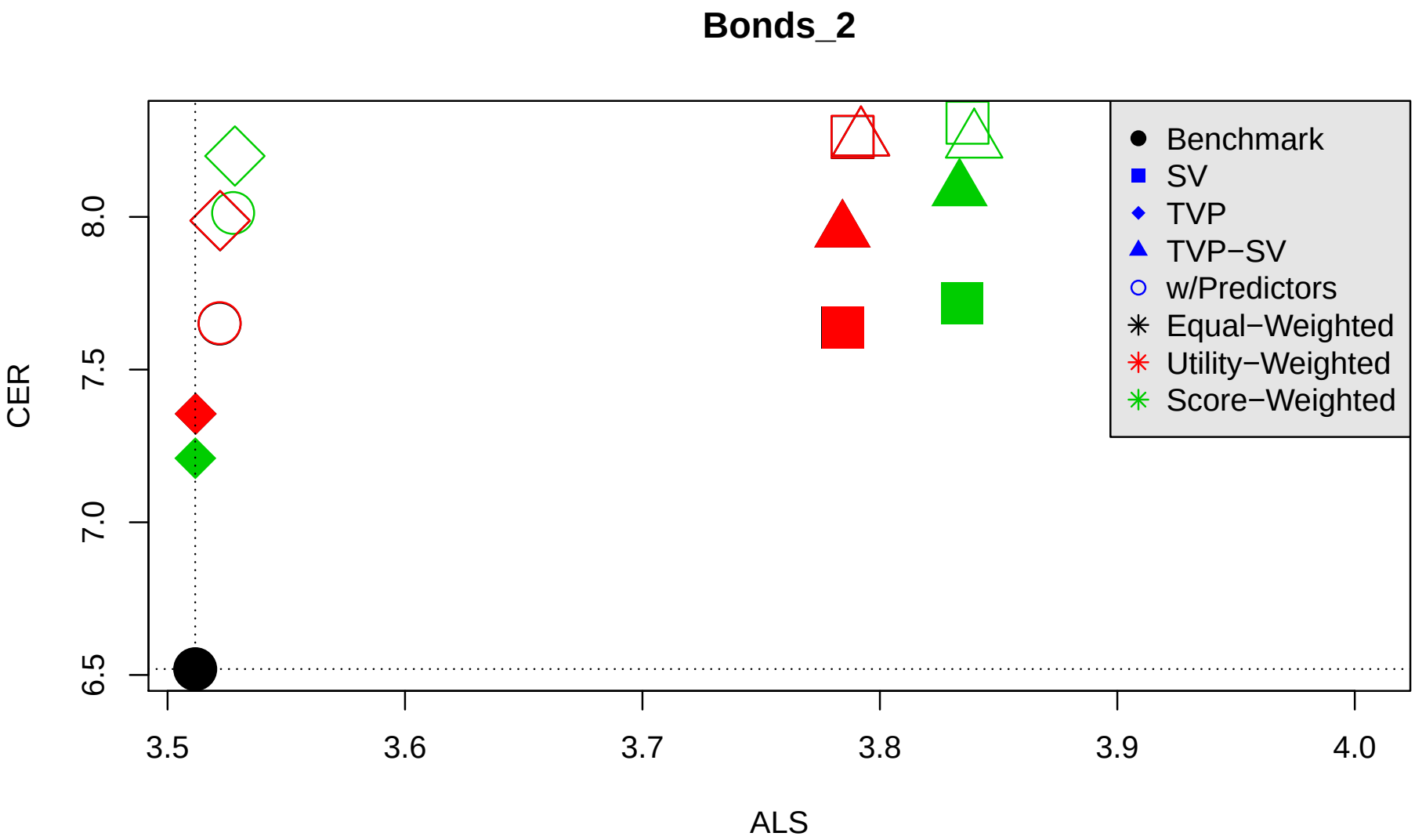
Model Averaging Results: Stocks



Model Averaging Results: Bonds, 2 Year Maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE
0	0	0	1	(none)	6.519	3.512	1.0000
0	0	1	10	Equal	7.637	3.784	1.0000
0	0	1	10	Utility	7.637	3.784	1.0000
0	0	1	10	Score	7.716	3.835	1.0000
0	1	0	10	Equal	7.355	3.512	0.9907
0	1	0	10	Utility	7.355	3.512	0.9907
0	1	0	10	Score	7.210	3.512	0.9957
0	1	1	100	Equal	7.953	3.784	0.9907
0	1	1	100	Utility	7.953	3.784	0.9907
0	1	1	100	Score	8.088	3.834	0.9908
1	0	0	4	Equal	7.651	3.522	0.9652
1	0	0	4	Utility	7.653	3.522	0.9652
1	0	0	4	Score	8.013	3.528	0.9829
1	0	1	40	Equal	8.261	3.788	0.9652
1	0	1	40	Utility	8.262	3.788	0.9652
1	0	1	40	Score	**8.308**	3.837	0.9641
1	1	0	40	Equal	7.987	3.522	0.9650
1	1	0	40	Utility	7.988	3.522	0.9650
1	1	0	40	Score	8.199	3.528	**0.9631**
1	1	1	400	Equal	8.254	3.792	0.9650
1	1	1	400	Utility	8.255	3.792	0.9650
1	1	1	400	Score	8.247	**3.840**	0.9644

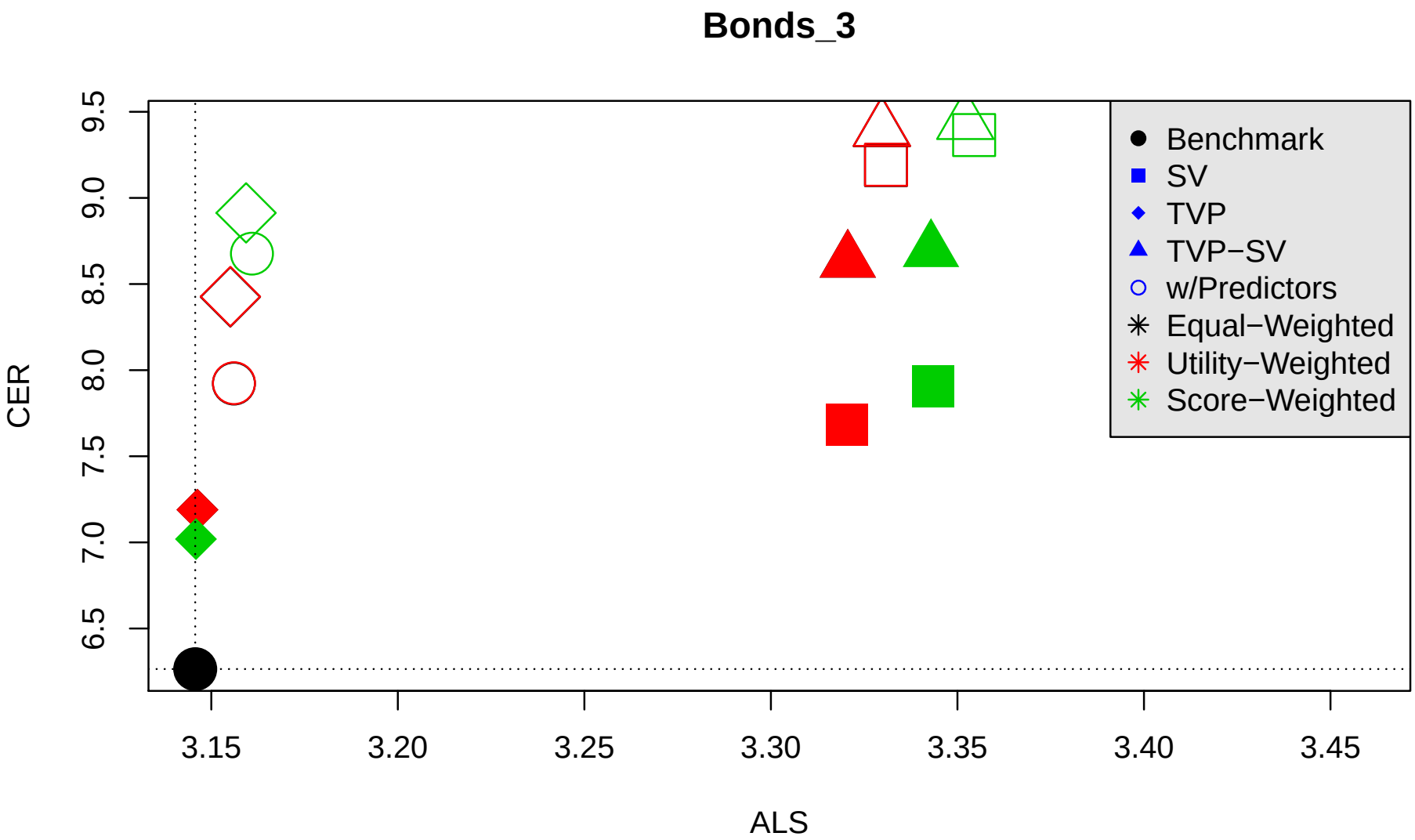
Model Averaging Results: Bonds, 2 Year Maturity



Model Averaging Results: Bonds, 3 Year Maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE
0	0	0	1	(none)	6.265	3.146	1.0000
0	0	1	10	Equal	7.685	3.320	1.0000
0	0	1	10	Utility	7.685	3.320	1.0000
0	0	1	10	Score	7.908	3.343	1.0000
0	1	0	10	Equal	7.189	3.146	0.9922
0	1	0	10	Utility	7.189	3.146	0.9922
0	1	0	10	Score	7.019	3.146	0.9967
0	1	1	100	Equal	8.632	3.321	0.9922
0	1	1	100	Utility	8.632	3.321	0.9922
0	1	1	100	Score	8.693	3.343	0.9923
1	0	0	4	Equal	7.922	3.156	0.9686
1	0	0	4	Utility	7.925	3.156	0.9685
1	0	0	4	Score	8.677	3.161	0.9734
1	0	1	40	Equal	9.191	3.331	0.9686
1	0	1	40	Utility	9.193	3.331	0.9685
1	0	1	40	Score	9.365	**3.354**	0.9680
1	1	0	40	Equal	8.425	3.155	0.9699
1	1	0	40	Utility	8.427	3.155	0.9698
1	1	0	40	Score	8.913	3.159	**0.9672**
1	1	1	400	Equal	9.395	3.330	0.9699
1	1	1	400	Utility	9.396	3.330	0.9698
1	1	1	400	Score	**9.437**	3.352	0.9697

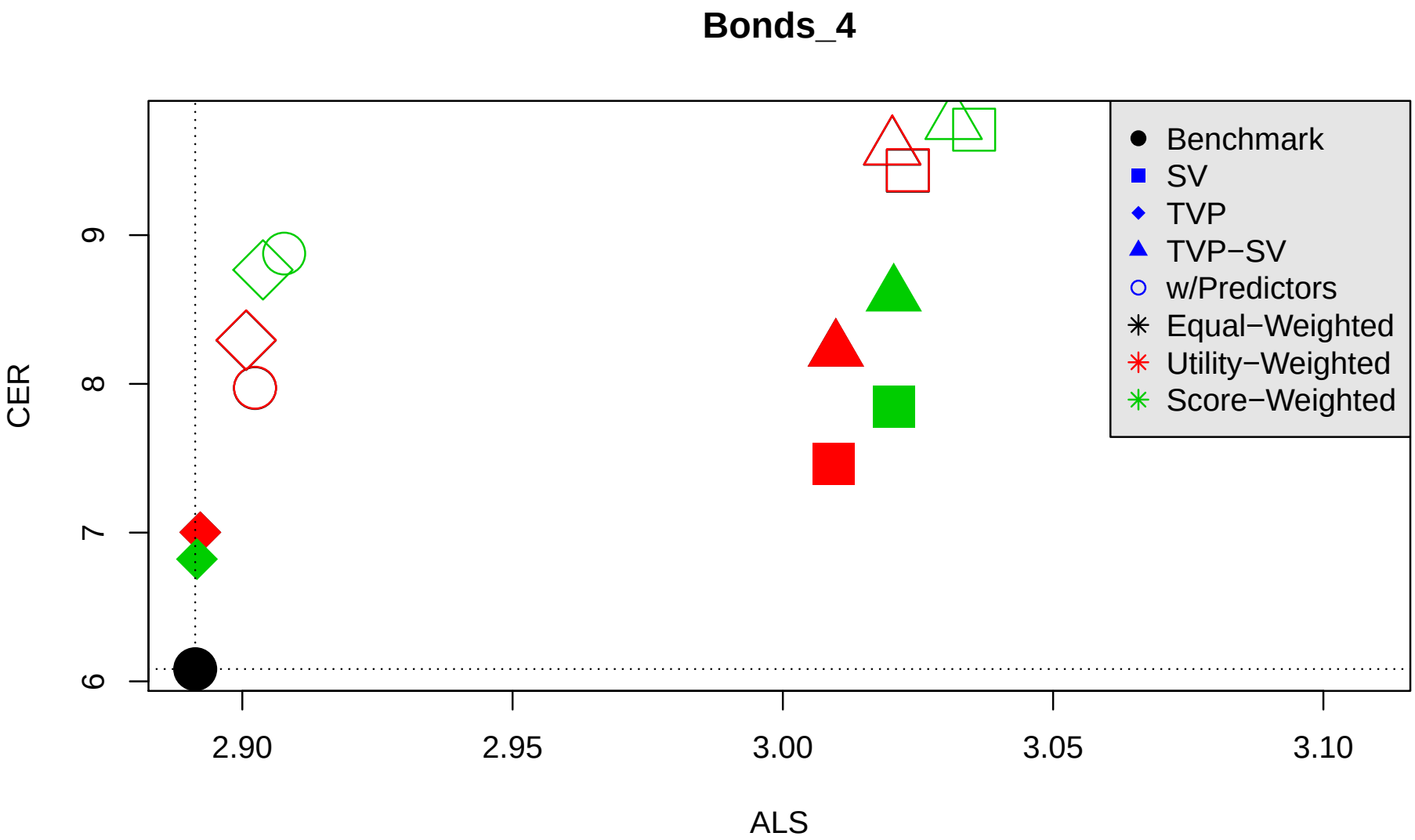
Model Averaging Results: Bonds, 3 Year Maturity



Model Averaging Results: Bonds, 4 Year Maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE
0	0	0	1	(none)	6.083	2.891	1.0000
0	0	1	10	Equal	7.461	3.009	1.0000
0	0	1	10	Utility	7.462	3.009	1.0000
0	0	1	10	Score	7.845	3.021	1.0000
0	1	0	10	Equal	7.002	2.892	0.9929
0	1	0	10	Utility	7.002	2.892	0.9929
0	1	0	10	Score	6.822	2.892	0.9972
0	1	1	100	Equal	8.226	3.010	0.9929
0	1	1	100	Utility	8.226	3.010	0.9929
0	1	1	100	Score	8.596	3.021	0.9930
1	0	0	4	Equal	7.972	2.902	0.9696
1	0	0	4	Utility	7.975	2.902	0.9695
1	0	0	4	Score	8.876	2.908	**0.9681**
1	0	1	40	Equal	9.433	3.023	0.9696
1	0	1	40	Utility	9.437	3.023	0.9695
1	0	1	40	Score	9.709	**3.035**	0.9693
1	1	0	40	Equal	8.293	2.901	0.9721
1	1	0	40	Utility	8.295	2.901	0.9720
1	1	0	40	Score	8.766	2.904	0.9689
1	1	1	400	Equal	9.583	3.020	0.9721
1	1	1	400	Utility	9.585	3.020	0.9720
1	1	1	400	Score	**9.756**	3.032	0.9721

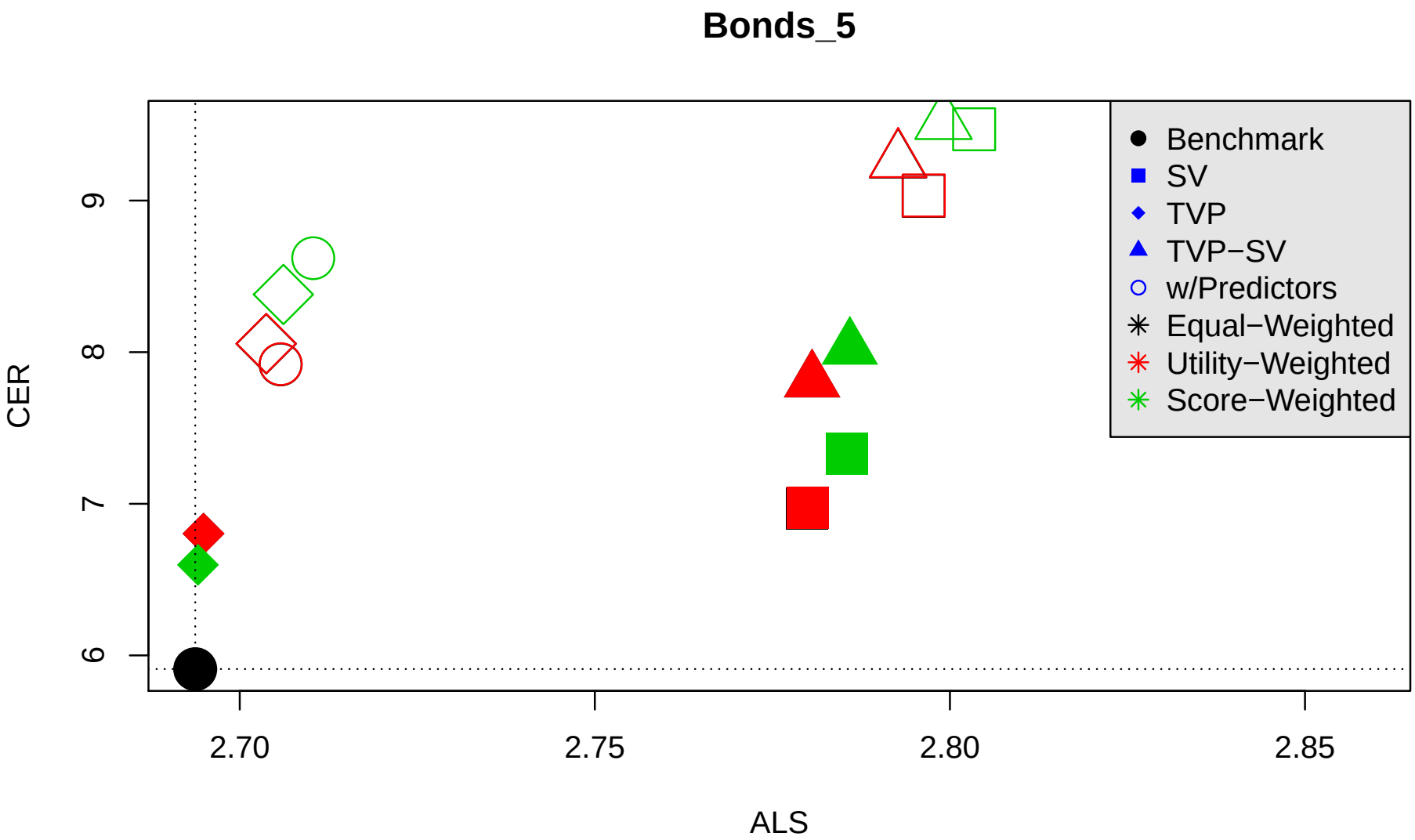
Model Averaging Results: Bonds, 4 Year Maturity



Model Averaging Results: Bonds, 5 Year Maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE
0	0	0	1	(none)	5.910	2.694	1.0000
0	0	1	10	Equal	6.971	2.780	1.0000
0	0	1	10	Utility	6.971	2.780	1.0000
0	0	1	10	Score	7.330	2.786	1.0000
0	1	0	10	Equal	6.803	2.695	0.9934
0	1	0	10	Utility	6.803	2.695	0.9934
0	1	0	10	Score	6.597	2.694	0.9977
0	1	1	100	Equal	7.809	2.781	0.9934
0	1	1	100	Utility	7.809	2.781	0.9934
0	1	1	100	Score	8.025	2.786	0.9934
1	0	0	4	Equal	7.920	2.706	0.9697
1	0	0	4	Utility	7.921	2.706	0.9696
1	0	0	4	Score	8.620	2.710	**0.9642**
1	0	1	40	Equal	9.031	2.796	0.9697
1	0	1	40	Utility	9.034	2.796	0.9697
1	0	1	40	Score	9.470	**2.803**	0.9697
1	1	0	40	Equal	8.055	2.704	0.9734
1	1	0	40	Utility	8.057	2.704	0.9734
1	1	0	40	Score	8.380	2.706	0.9705
1	1	1	400	Equal	9.259	2.793	0.9734
1	1	1	400	Utility	9.262	2.793	0.9734
1	1	1	400	Score	**9.513**	2.799	0.9737

Model Averaging Results: Bonds, 5 Year Maturity



Our Multivariate Model

- Ideal portfolio probably contains more than one risky asset.
- Use this same model, but fit for multiple risky assets.
- Portfolio of the stock index and a bond index, for a given maturity.
- Each model can include one stock predictor and one bond predictor

Our Multivariate Model

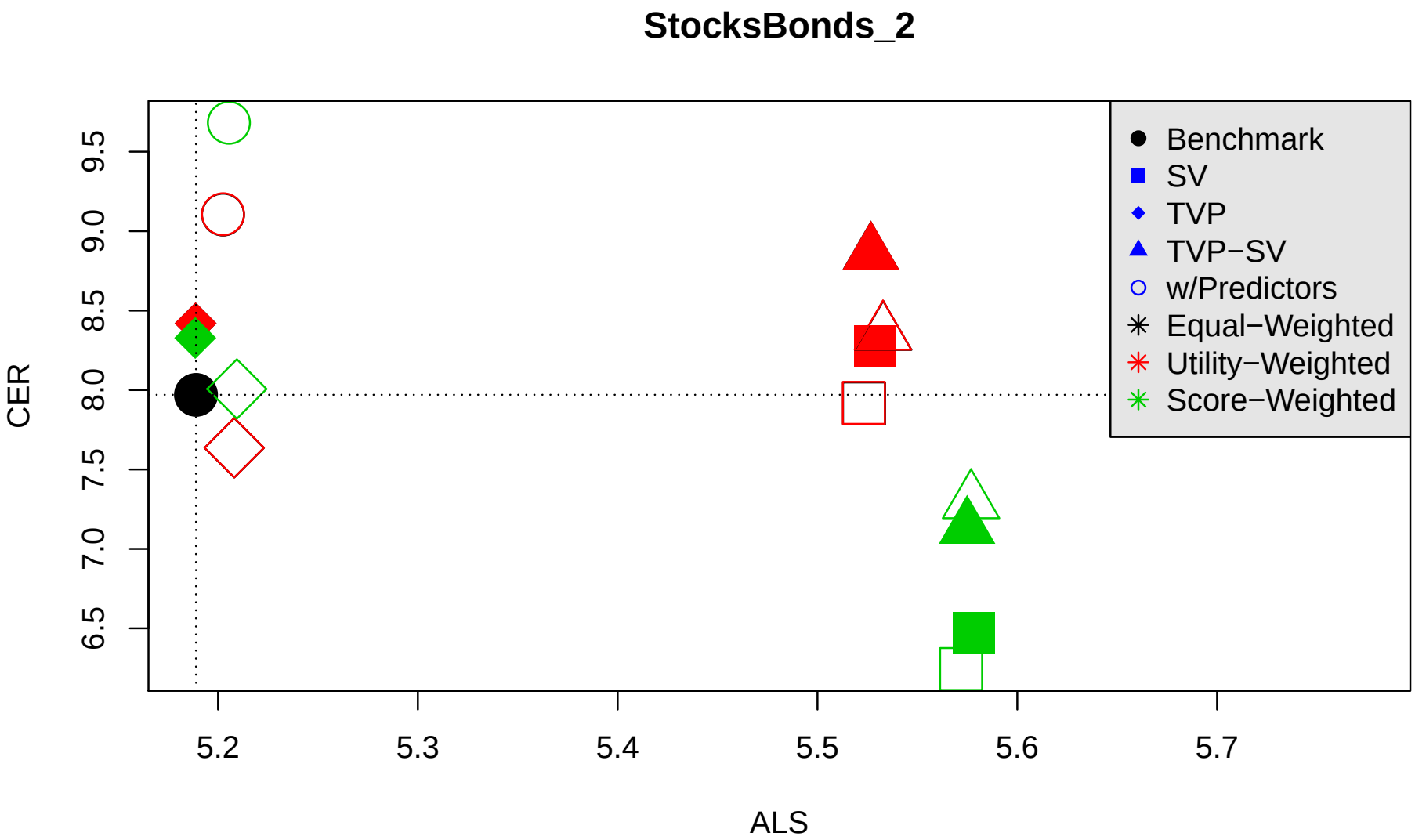
$$Y_t' = X_{t-1}' B_t + v_t'$$
$$B_t = B_{t-1} + \Omega_t$$

$$v_t \sim N(0, V_t \Sigma_t)$$
$$\Omega_t \sim N(0, \tfrac{1}{\delta} W_t, \Sigma_t)$$
$$(B_0, \Sigma_0 | D_0) \sim NW_{n_0}^{-1}(m_0, C_0, S_0)$$
$$\Sigma_t | D_{t-1} \sim W_{\delta_v n_{t-1}}^{-1}(S_{t-1})$$

Multivariate Model Averaging Results, 2 year maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE S.	MSE B.
0	0	0	1	(none)	7.970	5.189	**1.0000**	1.0000
0	0	1	10	Equal	8.276	5.529	**1.0000**	1.0000
0	0	1	10	Utility	8.275	5.529	**1.0000**	1.0000
0	0	1	10	Score	6.469	**5.578**	**1.0000**	1.0000
0	1	0	10	Equal	8.420	5.189	1.0005	0.9907
0	1	0	10	Utility	8.420	5.189	1.0005	0.9907
0	1	0	10	Score	8.328	5.189	1.0011	0.9939
0	1	1	100	Equal	8.862	5.527	1.0005	0.9907
0	1	1	100	Utility	8.861	5.527	1.0005	0.9907
0	1	1	100	Score	7.135	5.575	1.0005	0.9908
1	0	0	64	Equal	9.105	5.202	1.0014	0.9607
1	0	0	64	Utility	9.107	5.202	1.0014	0.9606
1	0	0	64	Score	**9.682**	5.205	1.0000	0.9538
1	0	1	640	Equal	7.916	5.523	1.0014	0.9607
1	0	1	640	Utility	7.919	5.523	1.0014	0.9606
1	0	1	640	Score	6.244	5.572	1.0008	0.9589
1	1	0	640	Equal	7.636	5.208	1.0017	0.9593
1	1	0	640	Utility	7.637	5.208	1.0017	0.9593
1	1	0	640	Score	8.007	5.209	1.0022	**0.9530**
1	1	1	6400	Equal	8.355	5.533	1.0017	0.9593
1	1	1	6400	Utility	8.357	5.533	1.0017	0.9593
1	1	1	6400	Score	7.296	5.577	1.0012	0.9582

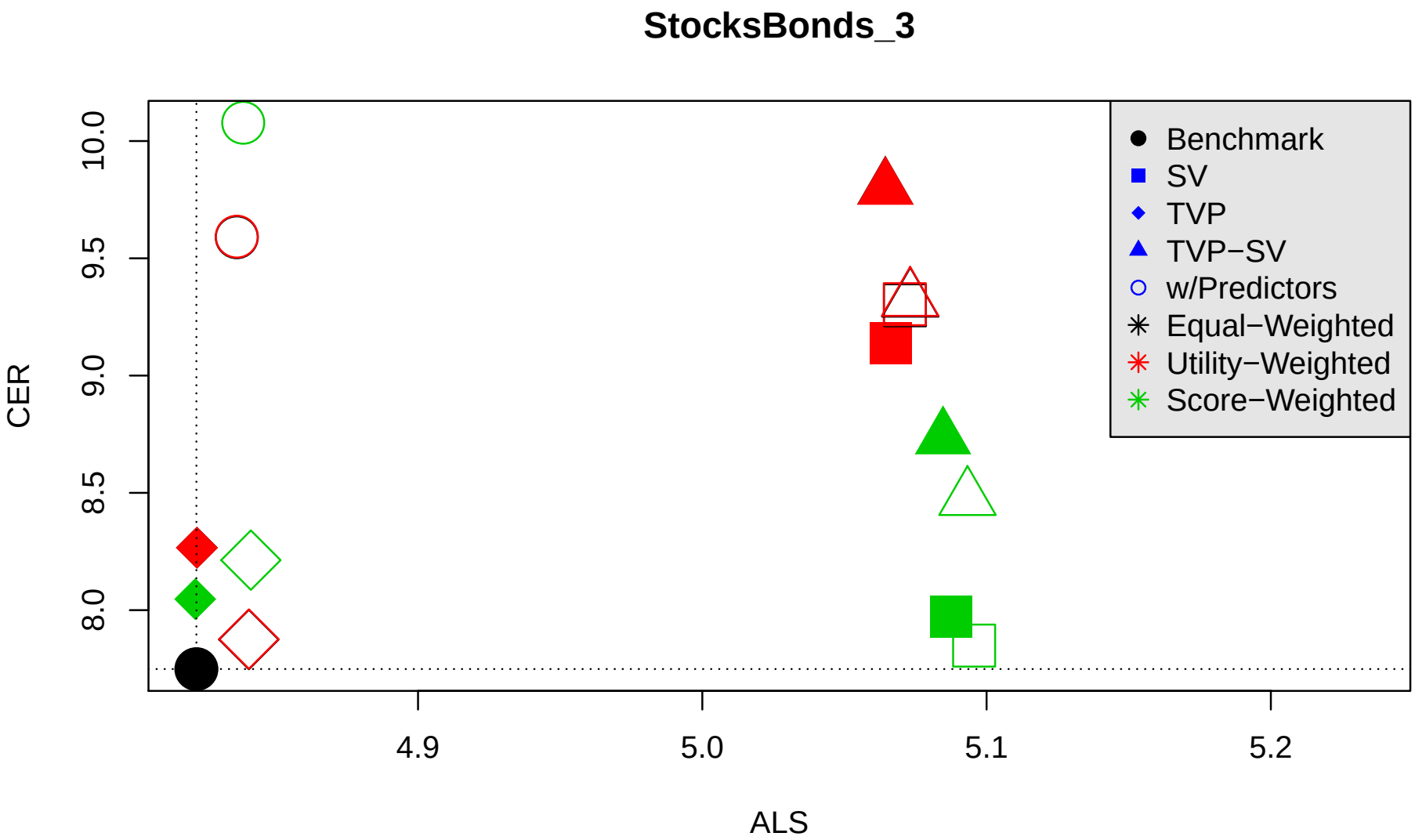
Multivariate Model Averaging Results, 2 year maturity



Multivariate Model Averaging Results, 3 year maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE S.	MSE B.
0	0	0	1	(none)	7.749	4.822	1.0000	1.0000
0	0	1	10	Equal	9.137	5.066	1.0000	1.0000
0	0	1	10	Utility	9.137	5.066	1.0000	1.0000
0	0	1	10	Score	7.971	5.088	1.0000	1.0000
0	1	0	10	Equal	8.266	4.822	1.0005	0.9922
0	1	0	10	Utility	8.266	4.822	1.0005	0.9922
0	1	0	10	Score	8.047	4.822	1.0016	0.9955
0	1	1	100	Equal	9.799	5.064	1.0005	0.9922
0	1	1	100	Utility	9.798	5.064	1.0005	0.9922
0	1	1	100	Score	8.734	5.085	1.0005	0.9923
1	0	0	64	Equal	9.590	4.836	1.0012	0.9644
1	0	0	64	Utility	9.593	4.836	1.0012	0.9644
1	0	0	64	Score	**10.078**	4.839	**0.9997**	**0.9593**
1	0	1	640	Equal	9.300	5.071	1.0012	0.9644
1	0	1	640	Utility	9.304	5.071	1.0012	0.9644
1	0	1	640	Score	7.849	**5.096**	1.0005	0.9635
1	1	0	640	Equal	7.875	4.840	1.0016	0.9656
1	1	0	640	Utility	7.877	4.841	1.0016	0.9655
1	1	0	640	Score	8.213	4.841	1.0021	0.9608
1	1	1	6400	Equal	9.321	5.073	1.0016	0.9656
1	1	1	6400	Utility	9.324	5.073	1.0016	0.9655
1	1	1	6400	Score	8.475	5.093	1.0010	0.9648

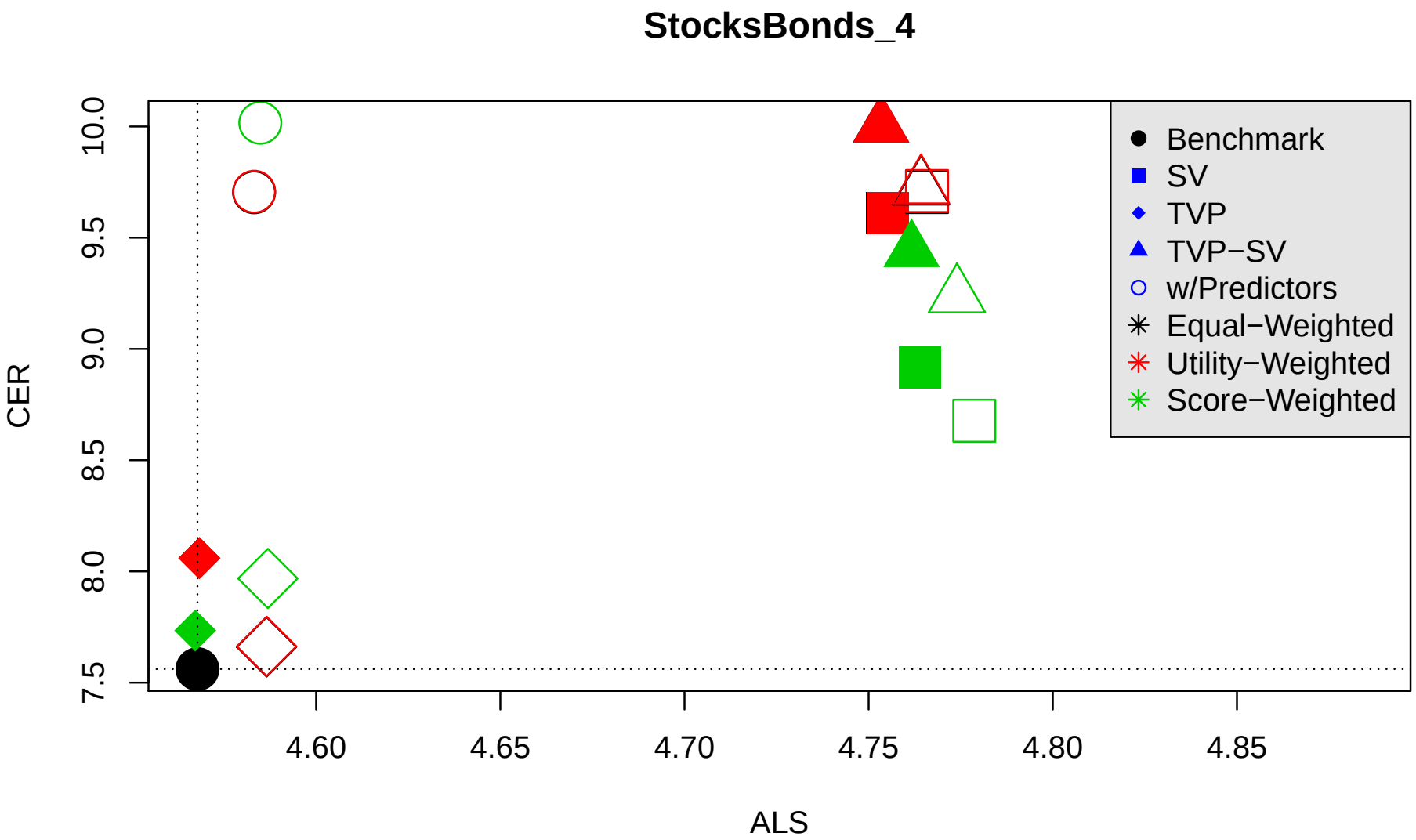
Multivariate Model Averaging Results, 3 year maturity



Multivariate Model Averaging Results, 4 year maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE S.	MSE B.
0	0	0	1	(none)	7.561	4.568	1.0000	1.0000
0	0	1	10	Equal	9.610	4.755	1.0000	1.0000
0	0	1	10	Utility	9.609	4.755	1.0000	1.0000
0	0	1	10	Score	8.916	4.764	1.0000	1.0000
0	1	0	10	Equal	8.060	4.568	1.0005	0.9929
0	1	0	10	Utility	8.060	4.568	1.0005	0.9929
0	1	0	10	Score	7.734	4.567	1.0020	0.9974
0	1	1	100	Equal	10.002	4.753	1.0005	0.9929
0	1	1	100	Utility	10.002	4.753	1.0005	0.9929
0	1	1	100	Score	9.441	4.762	1.0005	0.9930
1	0	0	64	Equal	9.704	4.583	1.0012	0.9660
1	0	0	64	Utility	9.707	4.583	1.0012	0.9660
1	0	0	64	Score	**10.017**	4.585	**0.9998**	**0.9623**
1	0	1	640	Equal	9.705	4.766	1.0012	0.9660
1	0	1	640	Utility	9.710	4.766	1.0012	0.9660
1	0	1	640	Score	8.677	**4.779**	1.0004	0.9655
1	1	0	640	Equal	7.661	4.587	1.0015	0.9690
1	1	0	640	Utility	7.663	4.587	1.0015	0.9689
1	1	0	640	Score	7.968	4.587	1.0020	0.9650
1	1	1	6400	Equal	9.722	4.764	1.0015	0.9690
1	1	1	6400	Utility	9.728	4.764	1.0015	0.9689
1	1	1	6400	Score	9.238	4.774	1.0009	0.9684

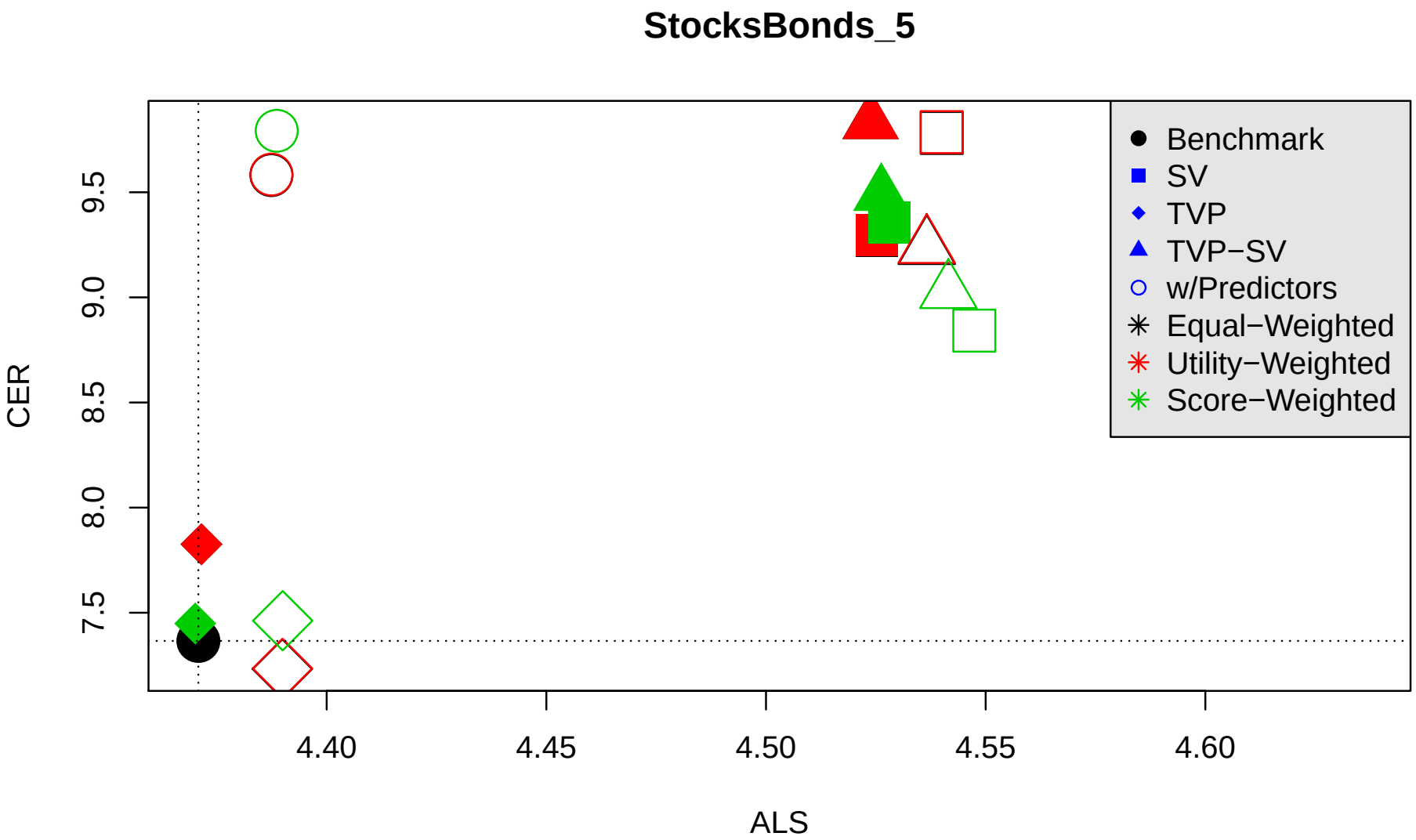
Multivariate Model Averaging Results, 4 year maturity



Multivariate Model Averaging Results, 5 year maturity

Pred	TVP	SV	Models	Weights	CER	ALS	MSE S.	MSE B.
0	0	0	1	(none)	7.366	4.371	1.0000	1.0000
0	0	1	10	Equal	9.294	4.525	1.0000	1.0000
0	0	1	10	Utility	9.295	4.525	1.0000	1.0000
0	0	1	10	Score	9.355	4.528	1.0000	1.0000
0	1	0	10	Equal	7.826	4.372	1.0005	0.9934
0	1	0	10	Utility	7.825	4.372	1.0005	0.9934
0	1	0	10	Score	7.449	4.370	1.0021	0.9985
0	1	1	100	Equal	9.831	4.524	1.0005	0.9934
0	1	1	100	Utility	**9.831**	4.524	1.0005	0.9934
0	1	1	100	Score	9.489	4.526	1.0005	0.9934
1	0	0	64	Equal	9.582	4.387	1.0012	0.9669
1	0	0	64	Utility	9.585	4.387	1.0012	0.9668
1	0	0	64	Score	9.793	4.389	**1.0000**	**0.9643**
1	0	1	640	Equal	9.782	4.540	1.0012	0.9669
1	0	1	640	Utility	9.786	4.540	1.0012	0.9668
1	0	1	640	Score	8.842	**4.547**	1.0004	0.9667
1	1	0	640	Equal	7.232	4.390	1.0016	0.9719
1	1	0	640	Utility	7.235	4.390	1.0016	0.9719
1	1	0	640	Score	7.462	4.390	1.0020	0.9689
1	1	1	6400	Equal	9.236	4.537	1.0016	0.9719
1	1	1	6400	Utility	9.242	4.537	1.0016	0.9719
1	1	1	6400	Score	9.027	4.542	1.0009	0.9716

Multivariate Model Averaging Results, 5 year maturity



Summary

- The best single risky asset models include predictors and stochastic volatility, perhaps with time-varying parameters for bonds.
- If optimizing statistical fit (ALS), the best models of multiple risky assets include stochastic volatility, usually with predictors.
- If optimizing economic significance (CER), the best models of multiple risky assets include
  - Predictors alone for shorter maturities.
  - Time-varying parameters and stochastic volatility with no predictors for larger maturities, equal or utility weighted (also the balanced choice).

Conclusions

- We demonstrate a Bayesian methodology that can quickly estimate a time-series model without requiring MCMC or another computation-intensive sampling algorithm.
- Time-varying parameters, stochastic volatility, and predictors generally show improvements over the benchmark model.
- Does predictability exist? Yes, the best averaged model in most cases include predictors.
- Is time variation important? Yes, especially stochastic volatility.

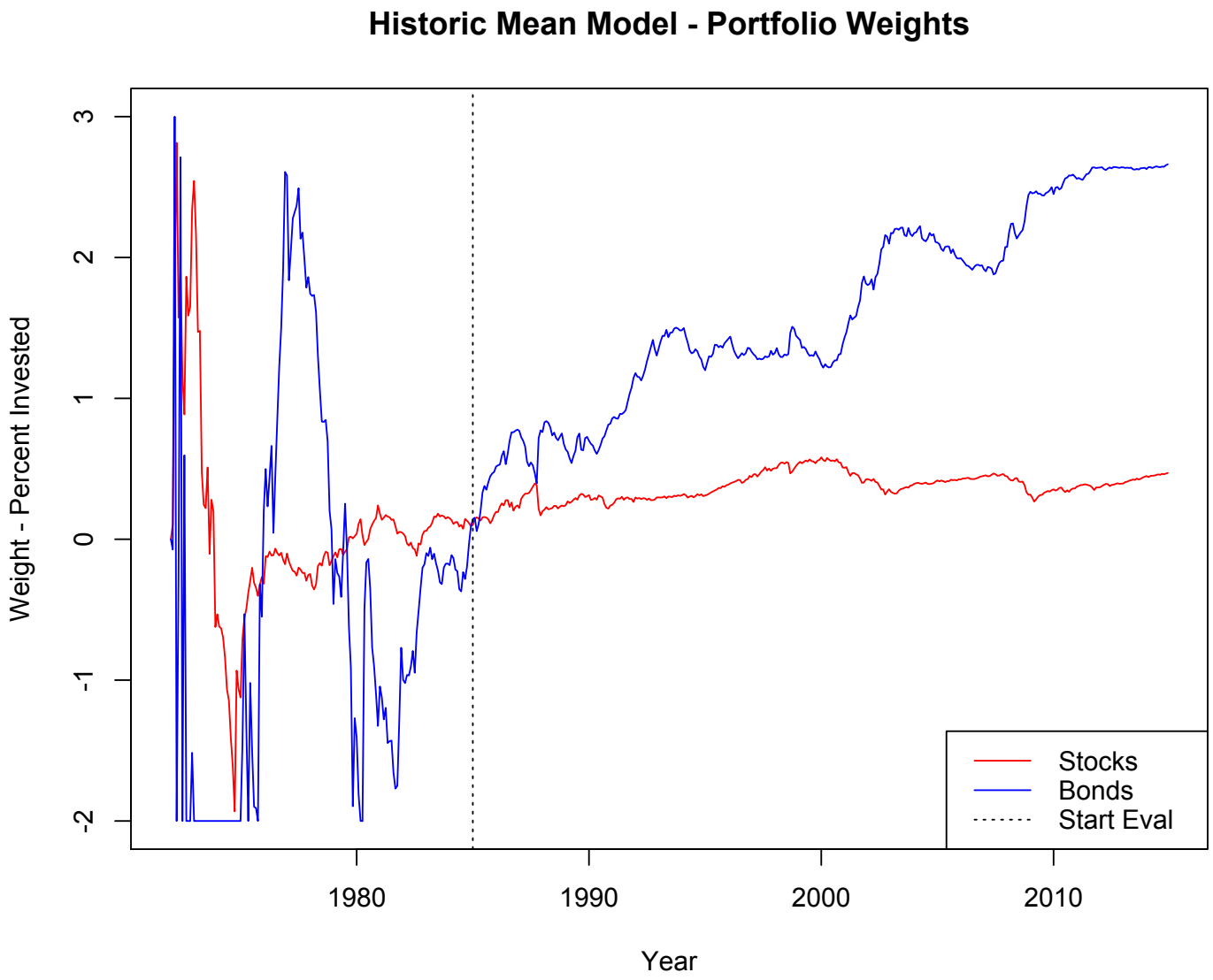
Questions, Comments?

Thank you!

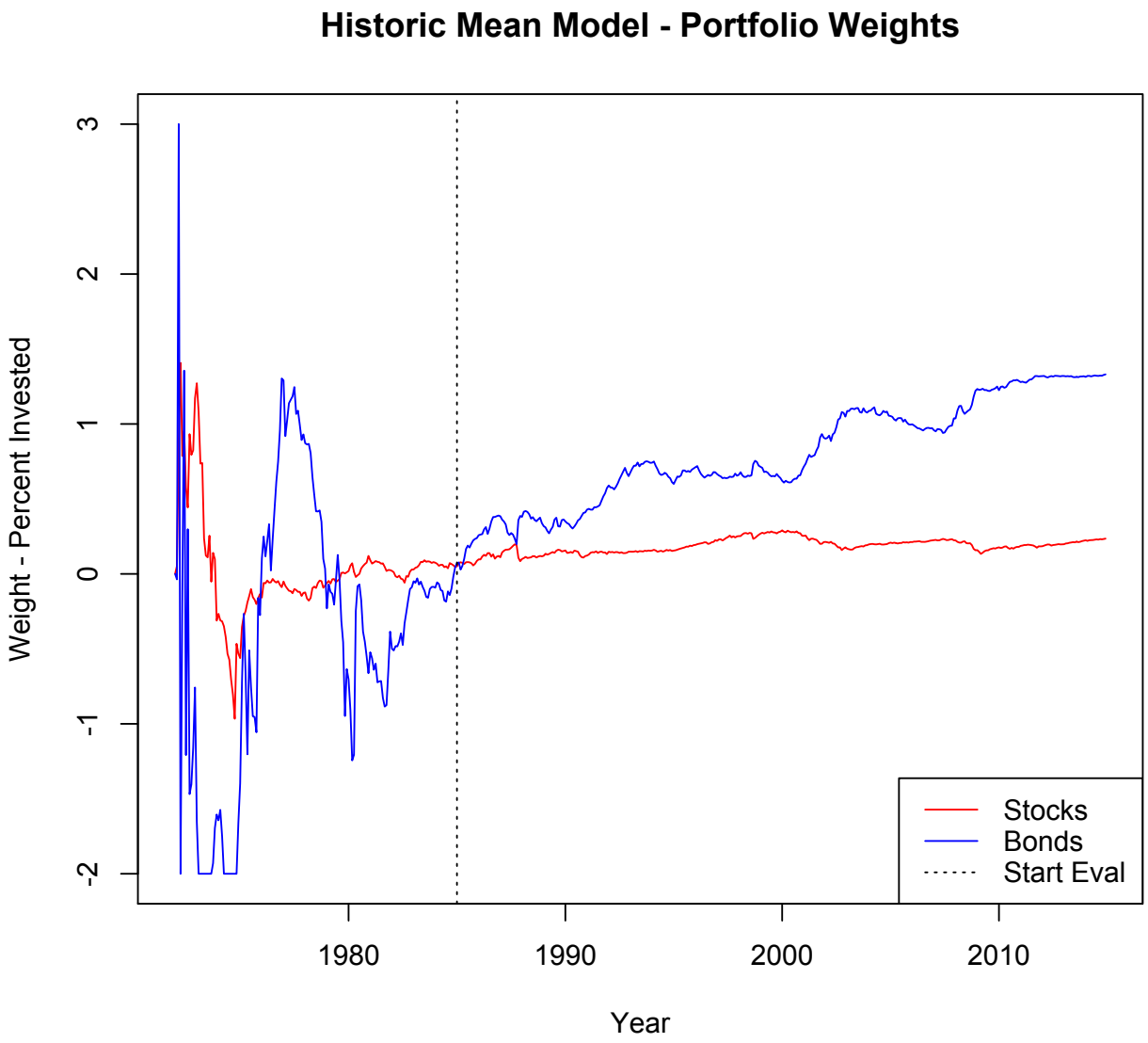
Different Risk Aversion

What if $\gamma = 10$?

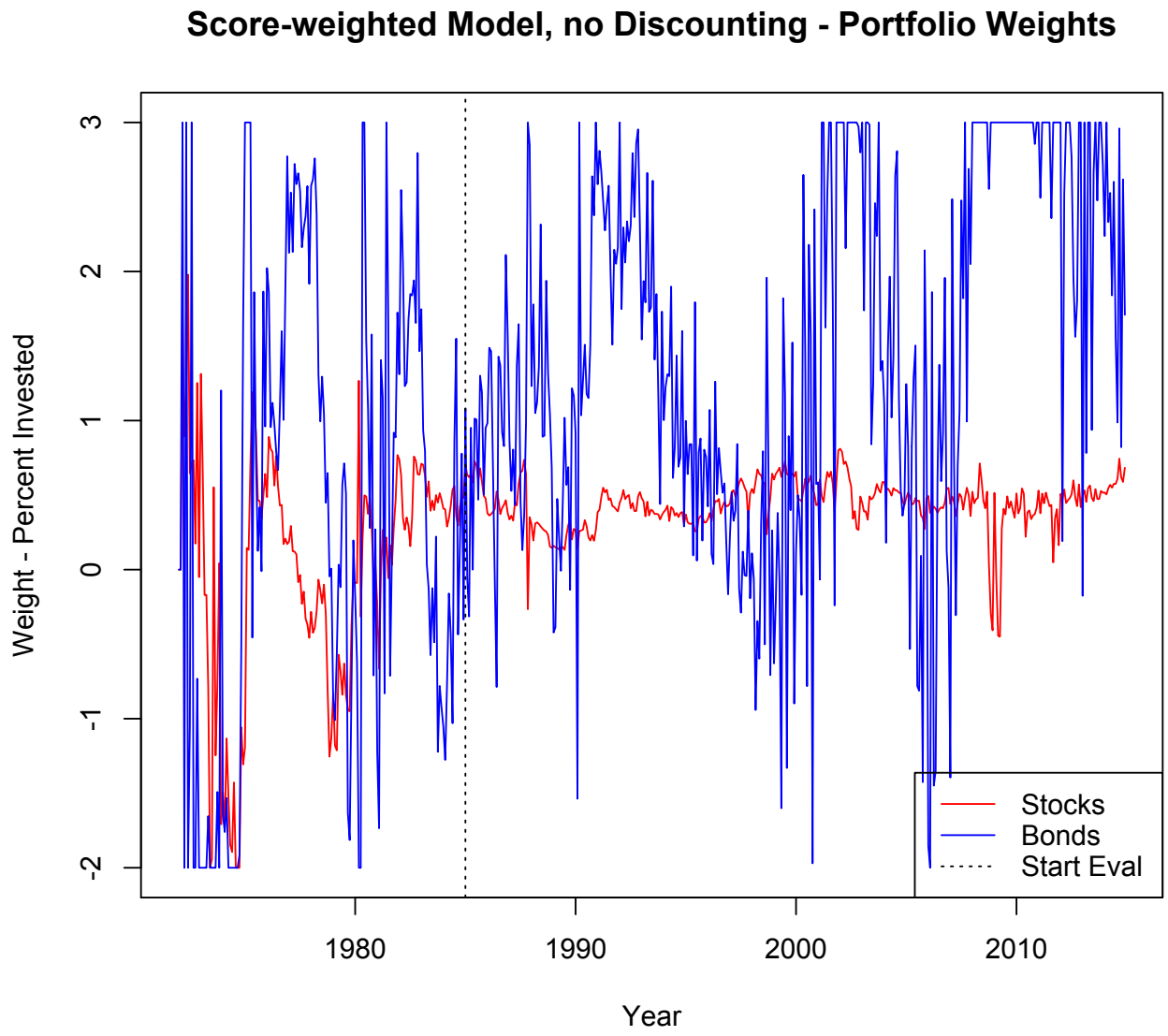
Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 5$



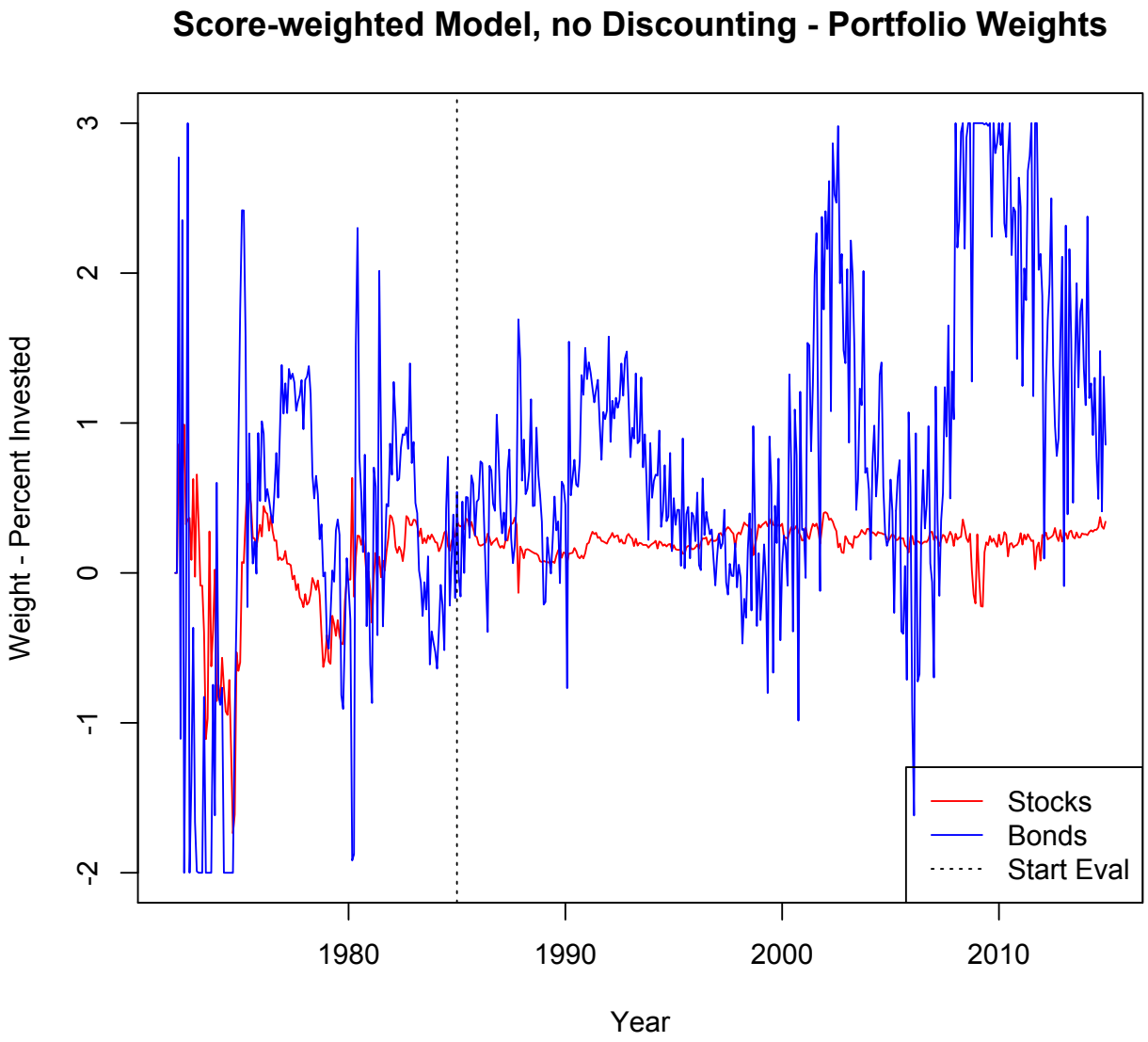
Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 10$



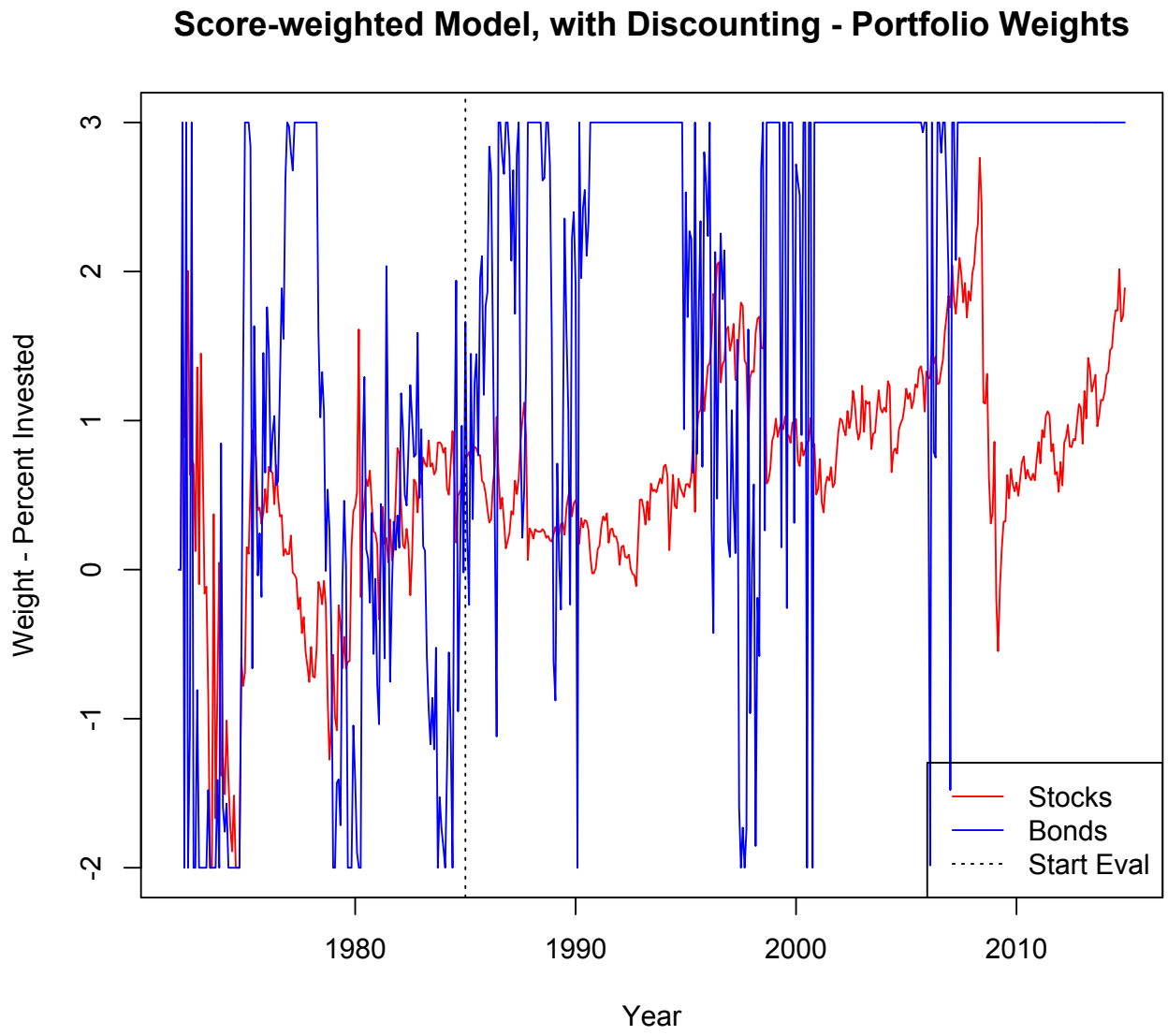
Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 5$



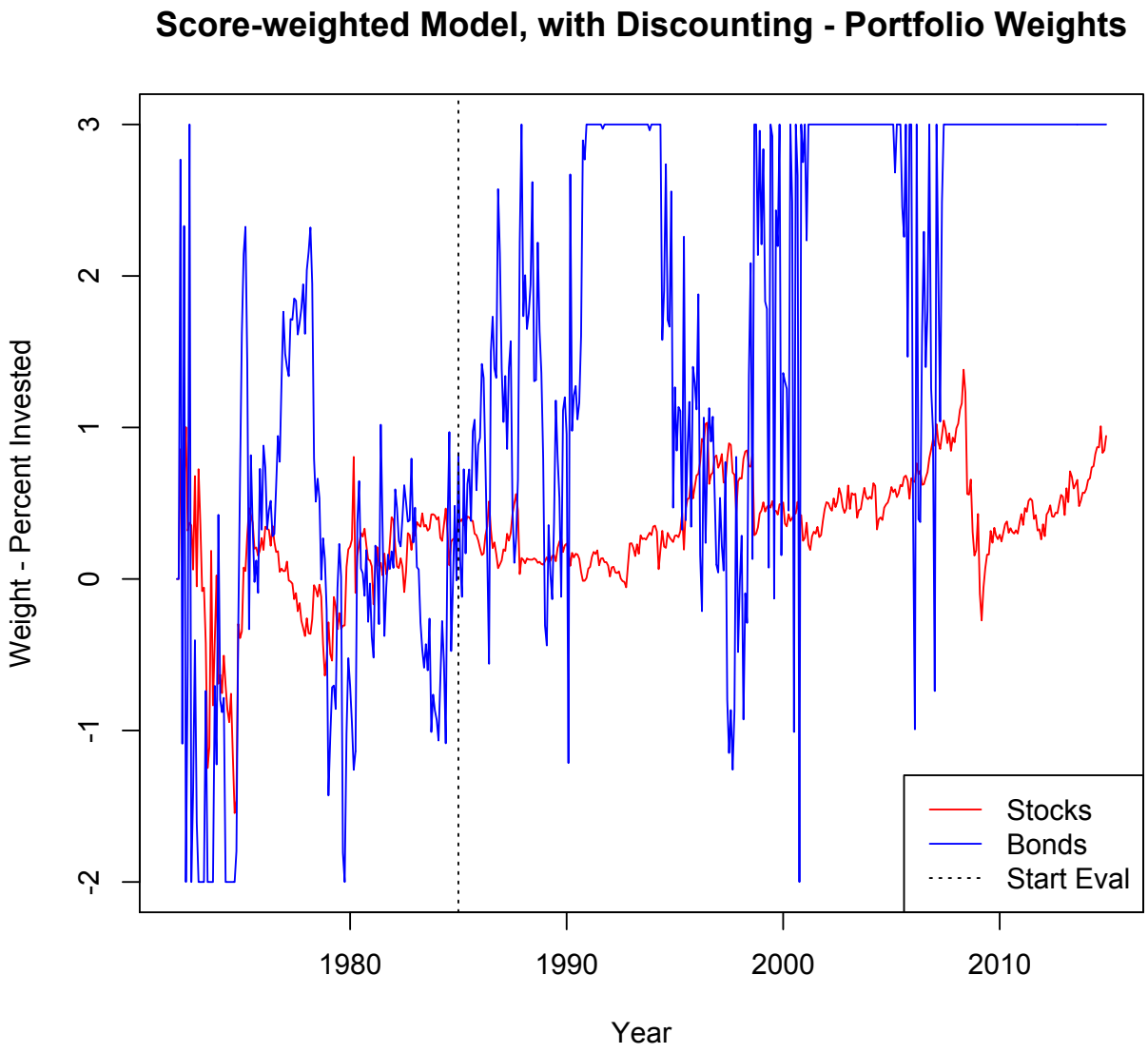
Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 10$



Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 5$



Multivariate Portfolio Weights, 2 Year Maturity, $\gamma = 10$



Intervention

Intervention: Expected risk premium should be non-negative if not positive.