Homework 1: NeRF Assignment

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1. Implementation Details

1.1 Positional Encoding (`encoder.py`)

The `PositionalEncoder` class maps low-dimensional input coordinates to a higher-dimensional space using sinusoidal functions, enabling the MLP to learn high-frequency scene details.

Key Implementation:

```
if log_sampling:
    # Logarithmic sampling: 2^0, 2^1, ..., 2^(max_freq)
    freq_bands = 2. ** torch.linspace(0., max_freq, N_freqs)
else:
    # Linear sampling: evenly spaced between 2^0 and 2^max_freq
    freq_bands = torch.linspace(2. ** 0., 2. ** max_freq, N_freqs)

for freq in freq_bands:
    for p_fn in periodic_fns:
        embed_fn = lambda x, p_fn=p_fn, freq=freq: p_fn(x * freq)
        embed_fns.append(embed_fn)
        out_dim += d
```

The encoding transforms input `x` into:

$$\gamma(x) = [x, \sin(2^0 x), \cos(2^0 x), \sin(2^1 x), \cos(2^1 x), \ldots, \sin(2^{L-1} x), \cos(2^{L-1} x)]$$

This follows Section 5.1 of the NeRF paper, allowing the network to represent high-frequency variations in color and geometry.

1.2 NeRF MLP Architecture (`nerf.py`)

The `NeRF` class implements the MLP architecture from Figure 7 of the paper.

Key Implementation:

```
# 8 FC layers with skip connection at layer 5
self.pts_linears = nn.ModuleList(
    [nn.Linear(input_ch, W)] +
    [nn.Linear(W, W) if i not in self.skips else nn.Linear(W + input_ch,
W)
     for i in range(D - 1)]
)

# View-dependent branch
self.feature_linear = nn.Linear(W, W)      # Feature vector
self.alpha_linear = nn.Linear(W, 1)        # Volume density (σ)
self.rgb_linear = nn.Linear(W // 2, 3)     # RGB color
```

Architecture Details:

- **Position branch**: 8 fully-connected layers (256 channels each) with a skip connection at layer 5 that concatenates the input features
- **Density output**: View-independent σ predicted directly from position features
- **Color output**: View-dependent RGB predicted after concatenating position features with view direction encoding

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1.3 Volume Rendering (`renderer.py`)

The renderer implements classical volume rendering with three key components:

1.3.1 3D Point Sampling

```
# Parametric ray equation: p(t) = o + t*d
pts = rays_o[..., None, :] + rays_d[..., None, :] * z_vals[..., :, None]
```

1.3.2 Alpha Computation (Beer-Lambert Law)

```
# α = 1 - exp(-σ * δ)
alpha = 1. - torch.exp(-act_fn(raw) * dists)
```

1.3.3 Volume Rendering Integration

```
# Transmittance: T_i = Π_{j<i}(1 - α_j)
T = torch.cumprod(
    torch.cat([torch.ones((alpha.shape[0], 1), device=device), 1. - alpha + 1e-10], -1), -1
)[:, :-1]

# Weights: w_i = T_i * α_i
weights = alpha * T

# Final color: C = Σ w_i * c_i
rgb_map = torch.sum(weights[..., None] * rgb, -2)

# Expected depth: d = Σ w_i * z_i
depth_map = torch.sum(weights * z_vals, -1)
```

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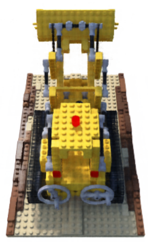
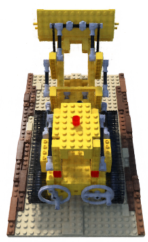
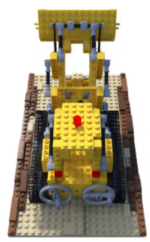
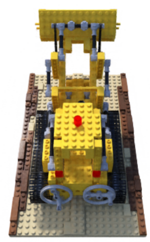
2. Training Results and Analysis

2.1 Lego Scene (Synthetic)

50K Iterations	100K Iterations	150K Iterations	200K Iterations
			

Quality Progression:

- **50K iterations**: Basic structure emerges, but details are blurry and colors are washed out. The overall shape of the lego bulldozer is recognizable but edges are soft.
- **100K iterations**: Significant improvement in sharpness and color saturation. Fine details like the treads and mechanical parts become more defined.
- **150K iterations**: High-frequency details are well-captured. Specular highlights and shadows appear more realistic.

- **200K iterations**: Convergence achieved with crisp edges, accurate colors, and proper view-dependent effects. The model successfully captures the metallic appearance and geometric complexity.

2.2 Fern Scene (Real-World LLFF)

50K Iterations	100K Iterations	150K Iterations	200K Iterations
			

Quality Progression: - **50K iterations**: The fern's overall structure is captured but with noticeable blur, especially in the intricate leaf patterns.

- **100K iterations**: Leaf edges become sharper, and the depth layering of overlapping fronds improves.
- **150K iterations**: Fine leaf textures and veins start appearing. The complex occlusion patterns are better resolved.
- **200K iterations**: The model achieves photorealistic quality with detailed leaf structures, proper depth-of-field effects, and accurate color reproduction of the natural scene.

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3. Novel View Synthesis Videos

The trained models generate smooth novel view animations along spiral camera paths:

- **Lego RGB**: `logs/blender_paper_lego/blender_paper_lego_spiral_200000_rgb.mp4`
- **Lego Depth**: `logs/blender_paper_lego/blender_paper_lego_spiral_200000_disp.mp4`
- **Fern RGB**: `logs/fern_test/fern_test_spiral_200000_rgb.mp4`
- **Fern Depth**: `logs/fern_test/fern_test_spiral_200000_disp.mp4`

These videos demonstrate the model's ability to synthesize consistent, high-quality views from novel camera positions not seen during training.

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4. Conclusion

The NeRF implementation successfully learns implicit 3D scene representations from 2D images. Key observations:

1. **Positional encoding** is critical for capturing high-frequency details
2. **Hierarchical sampling** (coarse + fine networks) improves rendering efficiency and quality
3. **View-dependent appearance** enables realistic specular and reflective effects
4. Training converges around 150K-200K iterations for both synthetic and real scenes