9.24.

$$f(\theta) = \begin{cases} 0 & \theta \neq \frac{1}{2} \\ 1 & \theta = \frac{1}{2} \end{cases}$$

$$f(\theta|y) \propto f(y|\theta) f(\theta) = \begin{cases} 0 & \text{when } \theta \neq \frac{1}{2} \\ 1 & \text{when } \theta = \frac{1}{2} \end{cases}$$

$$\frac{f(y|\theta)f(\theta)\,|_{\theta=\frac{1}{2}}}{\int f(y|\theta)f(\theta)\,d\theta} = \frac{f(y|\theta)f(\theta)\,|_{\frac{1}{2}}}{f(y|\theta)\,|_{\frac{1}{2}}} = f(\theta)$$

1. Posterior is the same as your prior

2. It should rarely (never) be used: [Cromwell's rule] prior probability of 0/1 should be avoided.

3. Exception: 0/1 probability can apply to statements that are logically true/ false. E.g. 2+2=4

1. Beta distribution

$\theta \sim Beta(\alpha, \beta)$ $f(\theta) =$

*

$Beta(1, 1)$

$\Gamma(n+1)$

coin : $\theta \sim Beta(\alpha, \beta)$

Bernoulli $y_1, y_2, \dots y_i, \dots$

$f(\theta | \underset{\sim}{y}) =$

coin $\theta \sim \text{Beta}(\alpha, \beta)$

$X \sim \text{Bin}(n, \theta)$

$f(\theta | x)$

if $\alpha = 1$, $\beta = 1$

$f(\theta) = 1\{\theta \in [0,1]\}$

$f(x)$

If all possible coins or all possible probabilities are equally likely, then all possible outcomes X are equally likely

2. Normal distribution

$$\mu \sim N(\nu, w^2)$$

$$X|\mu \sim N(\mu, \sigma^2) \quad \sigma^2 \text{ is known}$$

exercise:

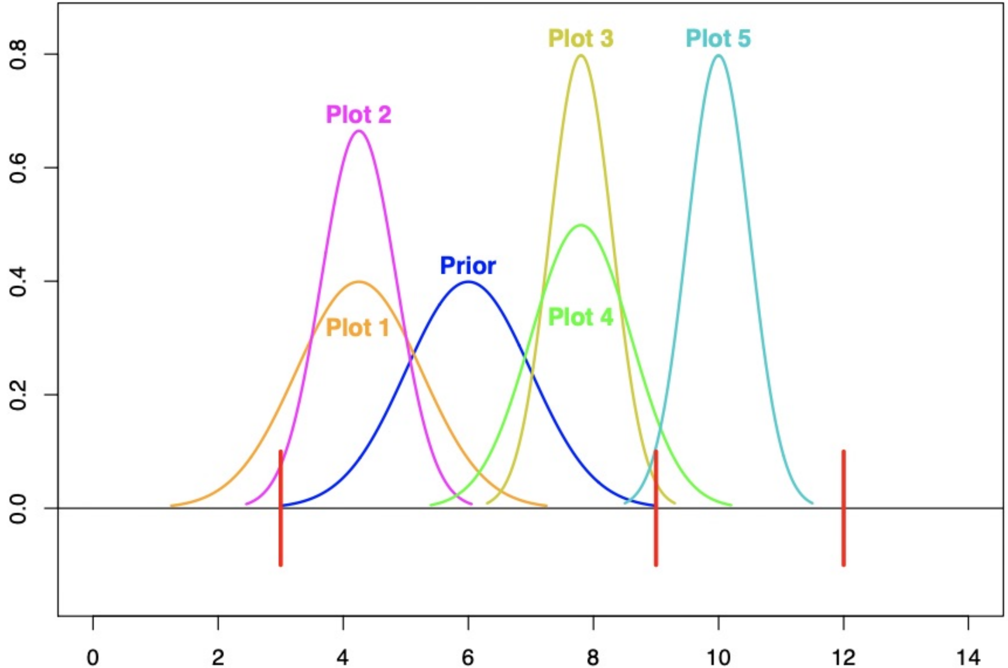
basketball team: $\theta \sim N(75, 6^2)$

individual player: $X|\theta \sim N(\theta, 4^2)$

Player A: made 85% of free throws in this year.

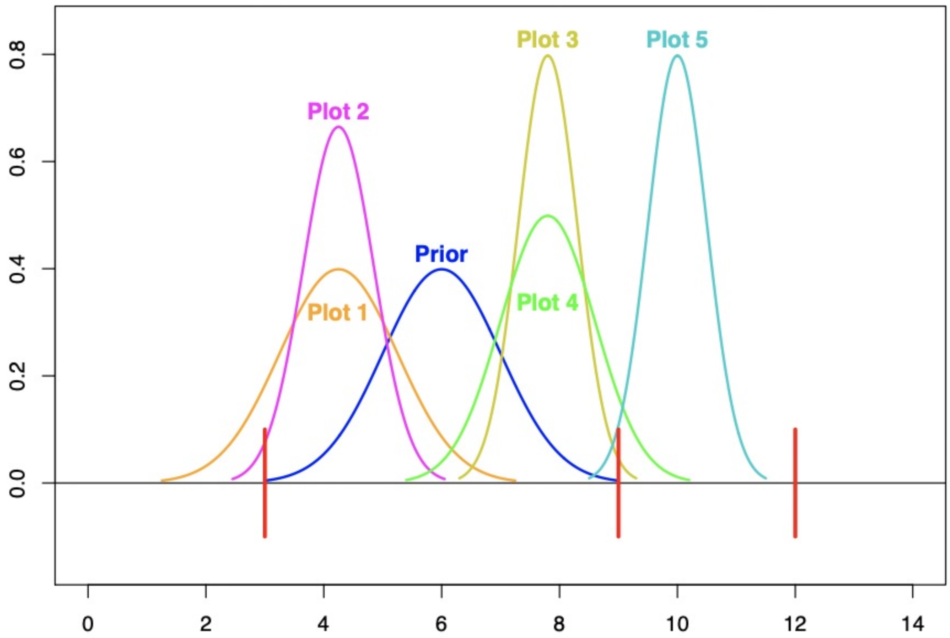
What is the posterior expected value of her career percentage?

Concept question: normal priors, normal likelihood



Blue graph = prior   Red lines = data in order: 3, 9, 12

(a) Which plot is the posterior to just the first data value?



Blue graph = prior Red lines = data in order: 3, 9, 12

(b) Which graph is posterior to all 3 data values?

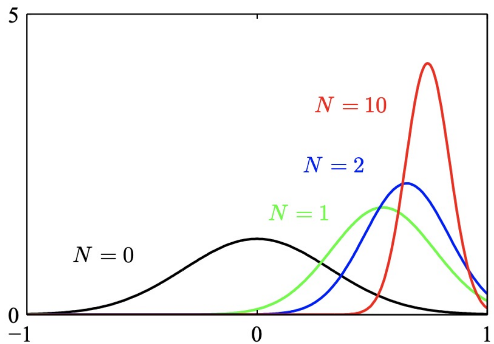


Figure 1: Sequentially updating a Gaussian mean starting with a prior centered on $\mu_0 = 0$. The true parameters are $\mu^* = 0.8$ (unknown), $(\sigma^2)^* = 0.1$ (known). Notice how the data quickly overwhelms the prior, and how the posterior becomes narrower. Source: Figure 2.12 [Bis06].

3. Gamma distribution

$Y_i \sim Pois(\lambda)$

e.g. Bus: $\lambda = 1/10$