Q1. Determine if the following degree sequences are graphical and, if so, draw a graph that realises the degree sequence.

(i) $3 \geq 3 \geq 2 \geq 2 \geq 2$

(ii) $4 \geq 4 \geq 3 \geq 2 \geq 1$

(iii) $4 \geq 3 \geq 3 \geq 2 \geq 2$

(iv) $3 \geq 3 \geq 3 \geq 2 \geq 2$

Q2. For which values of $n \in \mathbb{N}$ does there exist a 3-regular graph of order n?

Q3. Prove that a graph of order n and size e must have at least $n - e$ connected components.

Q4. Find the least number of colours required to colour the vertices of the Petersen graph such that no two vertices with the same colour are connected by an edge.

Q5. Let X be a set (you may assume that it is finite in this exercise) and let $\mathcal{X} \subseteq \mathcal{P}(X)$. Define the **intersection graph** of $\mathcal{X}$ to be the graph $G_{\mathcal{X}} = (\mathcal{X}, E_{\mathcal{X}})$ such that

$$E_{\mathcal{X}} = \{\{x, y\} \in \mathcal{P}_2(\mathcal{X}) \mid x \cap y \neq \emptyset\}$$

(i) Let $H = (V, F)$ be a finite graph. Prove that there exists a set X and $\mathcal{X} \subseteq \mathcal{P}(X)$ such that $H \cong G_{\mathcal{X}}$.

(ii) For a finite graph H define

$$s(H) = \min\{|X| \mid \text{there exists } \mathcal{X} \subseteq \mathcal{P}(X) \text{ with } H \cong G_{\mathcal{X}}\}$$

Compute $s(H)$ when H is a cycle of order n.

Q6. Let $G = (V, E)$ be a graph. Define $L(G) = (E, F)$ by

$$F = \{\{e_1, e_2\} \in \mathcal{P}_2(E) \mid e_1 \cap e_2 \neq \emptyset\}$$

(i) Can you describe $L(G)$ when G is a cycle graph?

(ii) Can you describe $L(G)$ when G is a complete graph?

Q7. Let G be a graph of order 10 and size 15.

(i) Is it the case that $\Delta(G) \geq 3$?

(ii) Is it the case that $\delta(G) \geq 2$?

Q8. Let $G = (V, E)$ be a graph. Define the *complement* of G, denoted G^c, by

$$G^c = (V, \mathcal{P}_2(V) \backslash E)$$

We say that a graph G is *self-complementary* if $G \cong G^c$.

(i) Is there a self-complementary graph of order 10?

(ii) Is there a self-complementary graph of order 11?

(iii) Is there a self-complementary graph of order 12 such that the number of vertices with degree 6 is odd?

(iv) Is there a self-complementary graph that is disconnected?