ECE 4710J HW1 徐逸洋

1. (a) The store has 3 kinds of fruits. thus matrix B should be 4×3

$$B := \begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix}$$

(b) Since we have 4 kinds of fruit bowls. A should be a 3×4 matrix

$$A := \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

(c) $AB\vec{V_2} = \vec{x}$

$$AB = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 9 & 12 & 4 \\ 7 & 12 & 15 \\ 0 & 0 & 100 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 12 & 4 \\ 7 & 12 & 15 \\ 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \end{bmatrix} = \begin{bmatrix} 80 \\ 80 \\ 100 \end{bmatrix}$$

$$\vec{V_2} = (AB)^{-1}\vec{x} = \begin{bmatrix} 0.5 & -0.5 & 0.055 \\ -0.2917 & 0.3750 & -0.0446 \\ 0 & 0 & 0.01 \end{bmatrix} \begin{bmatrix} 80 \\ 80 \\ 100 \end{bmatrix} = \begin{bmatrix} 5.5 \\ 2.2083 \\ 1 \end{bmatrix}$$

2. (a) $\sigma(-x) = \dfrac{1}{1+e^{x}}$

$$= \frac{1+e^{x}-e^{x}}{1+e^{x}}$$

$$= 1 - \frac{e^{x}}{1+e^{x}}$$

$$= 1 - \frac{1}{e^{-x}+1}$$

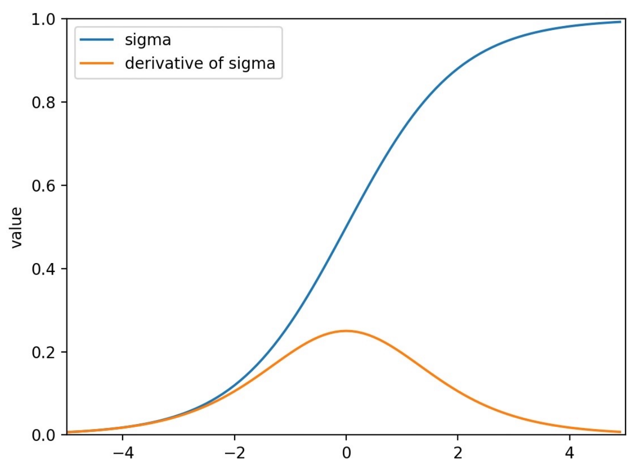
$$= 1 - \sigma(x) \qquad \square$$

(b) $\dfrac{d}{dx}\sigma(x) = \dfrac{-(1+e^{-x})'}{(1+e^{-x})^2} = \dfrac{-(-1)\cdot e^{-x}}{(1+e^{-x})^2} = \dfrac{e^{-x}}{(1+e^{-x})^2}$

R.H.S : $\sigma(x)(1-\sigma(x)) = \dfrac{1}{1+e^{-x}} \cdot \dfrac{e^{-x}}{1+e^{-x}} = \dfrac{e^{-x}}{(1+e^{-x})^2} \qquad \square$

$$\Rightarrow \frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$

(c)



3. minimize $f(c)$:= $\dfrac{d}{dc}f(c) = 0$ and $\dfrac{d^2}{dc^2}f(c) > 0$

$$\Rightarrow \frac{d}{dc}f(c) = 0 \qquad = -\frac{1}{n}\sum_{i=1}^{n} 2(x_i - c) = \frac{1}{n}\left(2nc - 2\sum_{i=1}^{n} x_i\right) = 0$$

$$\Rightarrow c = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Also, we need to make sure $\dfrac{d^2}{dc^2}f(c) > 0$ so that we get a min not max.

$$\Rightarrow \frac{d^2}{dc^2} f(c) = \frac{d}{dc}\left[\frac{1}{n}(2nc - 2\sum_{i=1}^{n} x_i)\right] = 2 > 0$$

Therefore, $f(c)$ has a minimum at $c = \frac{1}{n}\sum_{i=1}^{n} x_i$

4. TP: Test Positive ; H: Have breast cancer.
TN: Test Negative ; NH: Not Have cancer.

$P[H] = 1\%$ $\quad P[TP|H] = 80\%$ $\quad P[TP|NH] = 9.6\%$

Need to find out $P[H|TP]$

$$P[H|TP] = \frac{P[H]\cdot P[TP|H]}{P[H]\cdot P[TP|H] + P[NH]\cdot P[TP|NH]} = \frac{1\%\cdot 80\%}{1\%\cdot 80\% + 99\%\cdot 9.6\%}$$

$$= 7.764\%$$

5. According to the histogram, the probability of the test follows a binomial distribution. Therefore, the standard deviation $\sigma = \sqrt{np(1-p)}$ $= \sqrt{200\cdot 0.75\cdot 0.25} = 6.12$, so we choose b