Ve 216: Introduction to Signals and Systems

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Outline

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Introduction

This chapter applied FT principles and tools to the analysis of communications systems (AM radios, FM radios, wireless phones, video, etc.)
All of it is based on the **modulation property** of the FT.

Overview

- Amplitude Modulation (AM radio, digital comm (modems))
- Synchronous demodulation
- Asynchronous demodulation
- Heterodyning tuner

We will only cover 8.1, 8.2, 8.3

Modulation Property

Modulation Property:

$$e^{j\omega_0 t} f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega - \omega_0)$$

$$f(t)\cos\omega_0 t \overset{\mathcal{F}}{\longleftrightarrow} \frac{F(\omega - \omega_0) + F(\omega + \omega_0)}{2}$$

Question

Why modulate signals in communication systems?

Antenna length requirement (1)

1. Antenna length requirement.
For efficient radiation, antenna lengths should be longer than $\lambda/10$, where λ is the wavelength of the signal to be radiated, given by

$$\lambda = \frac{c}{f_c}$$

where c is the speed of the light and f_c is the frequency of the signal.

Example

For a 30Hz frequency component, the wavelength is

$$\lambda = \frac{c}{f_c} = \frac{3 \cdot 10^8 m/s}{30 s^{-1}} = 10,000\text{km},$$

so $\lambda/10 = 1000$km. Obviously impractical.

Antenna length requirement (2)

1. Antenna length requirement.

After considering the wavelength formula,

$$\lambda = \frac{c}{f_c}$$

it becomes apparent that by increasing the frequency of the signal, we can decrease the antenna length required.

Question

OK, so let's say we forget antennas and stick to copper wire (like the cable company does). Now why should we modulate?

Interference from other signals

2. Interference from other signals.
   - If all radio stations transmitted baseband 20Hz-20kHz signals (over a cable for example), the receivers would receive all of those signals superimposed, and would have no way of separating them.
   - The solution to these problems is to modulate signals to different parts of the frequency spectrum.

FCC regulation

In the U.S., the frequency band allocation is controlled by the Federal Communications Commission (FCC).

- AM radio: 540-1600kHz
- FM radio: 88-108MHz

Note: lots of room for many non-overlapping channels each $\pm$20kHz wide.

(Human voice about 1kHz bandwidth)

Outline

DSB/SC-AM

- So we need to shift the audio signal from baseband to something much higher.
- We have already seen that the **modulation property** of the FT provides a mechanism for this.
- There are many variations on how to do this (see Ve 353 Introduction to Communication Systems).
- The simplest method is called **double sideband**, **suppressed carrier**, **amplitude modulation** or **DSB/SC-AM**.

Sinusoidal amplitude modulation (1)

Let $x(t)$ be the modulating signal (the audio signal) and let $c(t)$ be the carrier signal (a high-frequency cosinusoid). Specifically

$$c(t) = \cos(\omega_c t + \theta_c)$$

where ω_c is called the **carrier frequency**.

The transmitted signal is

$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t + \theta_c)$$

$$\stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(\omega) = \frac{1}{2}[e^{j\theta_c}X(\omega - \omega_c) + e^{-j\theta_c}X(\omega + \omega_c)]$$

Sinusoidal amplitude modulation (2)

Block diagram of modulation system:

$$\begin{array}{c} x(t) \rightarrow \bigotimes \rightarrow y(t) \rightarrow \text{ antenna} \\ \uparrow \\ \cos(\omega_c t + \theta_c) \end{array}$$

(Picture) in frequency domain (MIT, Lecture 13.9) (textbook, Figure 8.4, p. 586) This multiplication is also sometimes called **mixing**. Note that graphically it appears that all of the audio signal information is retained, it is just moved to a different part of the spectrum.

Sinusoidal amplitude modulation (3)

Since it is now a high frequency signal, it can be transmitted by a practical antenna, and different radio stations can use different ω_c's.

Question

How much should the carrier frequencies (the ω_c's) of different stations be separated?

Sinusoidal amplitude modulation (4)

Question

The carriers of AM radio stations are spaced by 10kHz! How?

Outline

Synchronous Demodulation

- The signal must be restored to baseband for our ears to hear it.
- Conceptually, this can be done by another multiplication by a $\cos(\omega_c t + \theta_c)$ signal, followed by lowpass filtering.

Block diagram of demodulation system

$$y(t) \rightarrow \underset{\substack{\uparrow \\ \cos(\omega_c t + \theta_c)}}{\otimes} \rightarrow w(t) \rightarrow \boxed{h(t)} \rightarrow x(t)$$

$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

Frequency response and lowpass filtering

$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

$$W(\omega) = \frac{1}{2}[e^{j\theta_c} Y(\omega - \omega_c) + e^{-j\theta_c} Y(\omega + \omega_c)]$$

$$= \boxed{\frac{1}{4}e^{2j\theta_c} X(\omega - 2\omega_c) + \frac{1}{2}X(\omega) + \frac{1}{4}e^{-2j\theta_c} X(\omega + 2\omega_c)}$$

$$Y(\omega) = \frac{1}{2}[e^{j\theta_c} X(\omega - \omega_c) + e^{-j\theta_c} X(\omega + \omega_c)]$$

Now pass through a lowpass filter to extract the baseband signal. ***(Picture)***(MIT, Lecture 13.10 and 13.11)

Practical problem

Question

Any practical problems?

Outline

Asynchronous Demodulation

A simple system and associated waveform for an **asynchronous demodulation** system. ***(Picture)***(MIT, Lecture 13.17)

- The envelop of $y(t)$ (a smooth curve connecting the peaks) is a reasonable approximation of $x(t)$.
- $x(t)$ could be approximately recovered through the use of the **envelop detector** that tracks these peaks to extract the envelop.

Two basic assumptions

Two basic assumptions are required, so that the envelop is easily tracked.

1. $x(t)$ be positive. Solution: $x(t) + A > 0$.
2. $x(t)$ vary slowly compared to ω_c.

Example

- $x(t)$: 15-20 kHz
- $\omega_c/2\pi \in [500K, 2M]$Hz.

DSB/WC-AM

Definition

Transmitting some of the modulation signal too (uses power) is called **double sideband**, **with carrier**, **amplitude modulation** or **DSB/WC-AM**.

Now the modulated signal (transmitted) is:

$$y(t) = (A + x(t))\cos(\omega_c t)$$

(Picture)of block diagram (MIT, Lecture 13.18) (textbook, Figure 8.12) Spectrum of the DSB/WC signal:

$$Y(\omega) = A\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)]$$

(Picture)of $Y(\omega)$spectrum (MIT, Lecture 13.19) (textbook, Figure 8.14)

Issues associated with the value of A

Positive and negative issues associated with the value of A.

As A is increased

- The relative amount of carrier present in the modulated output increases, which results in easier demodulation for the envelop tractor. ***(Picture)***(MIT, Lecture 13.18)
- Inefficiency of the system in terms of the power because A dose not carry any information about the signal.

Single crystal receiver

- Since the envelope of the DSB/WC signal describes the original audio signal, an envelop detector (***(Picture)***, MIT, Lecture 13.17) will recover the envelope.

$$y(t) = (A + x(t))\cos(\omega_c t)$$

$$m(t) = A + \hat{x}(t)$$

- Just eliminate the DC component and you recover the original signal.

$y(t) \rightarrow$ $\boxed{\text{Envelop detector}}$ $\rightarrow m(t) \rightarrow$ $\boxed{\text{DC blocking filter}}$ $\rightarrow \hat{x}(t)$

Outline

Frequency-division multiplexing (1)

- Different stations are allocated different carrier frequencies, separated by (at least) the allowed bandwidth of each station.
- Commercial AM: 10KHz spacing between carriers.
- The transmitted spectrum of the kth station occupies $\omega_k \pm \omega_B$.

Example

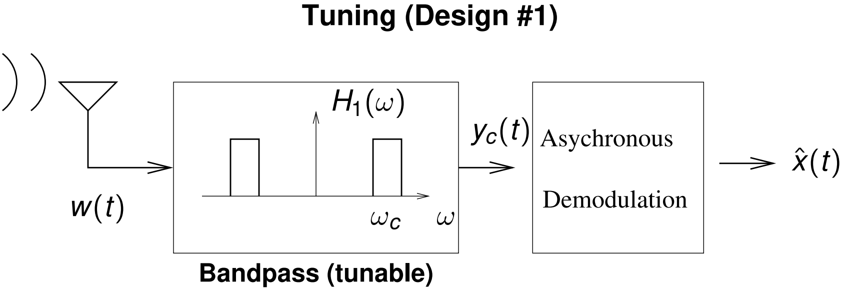
- ***(Picture)*** of block diagram multiple transmitters (MIT, Lecture 13.12)
- ***(Picture)*** of spectra illustrating (MIT, Lecture 13.13)
- The three signals appear simultaneously in time, but are separated in frequency.

Tunable bandpass filter (1)

Question

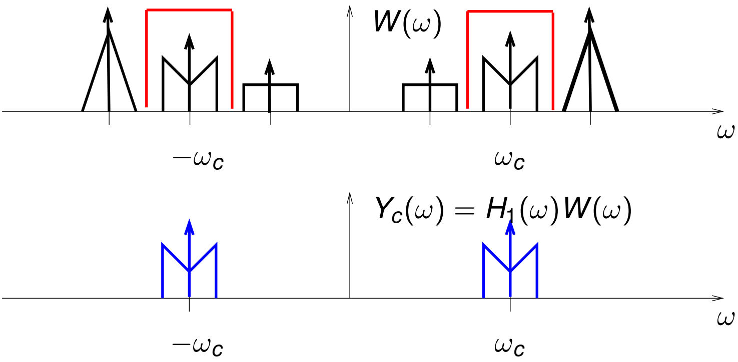
How can we “tune in” to our favorite station?

Design 1: a tunable bandpass filter.



Tunable bandpass filter (2)

(Frequency Division Multiplexing)



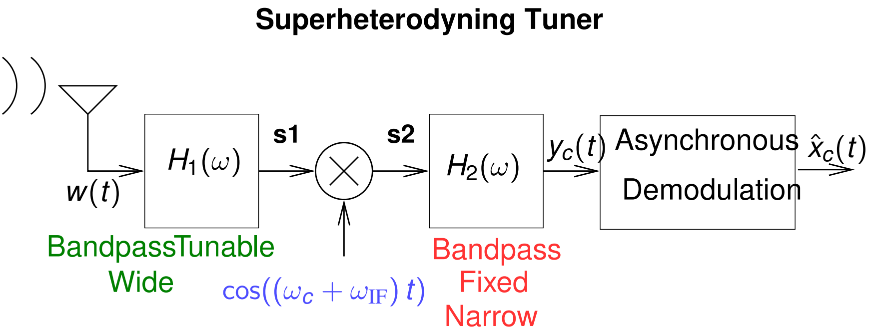
Tunable bandpass filter (3)

Question

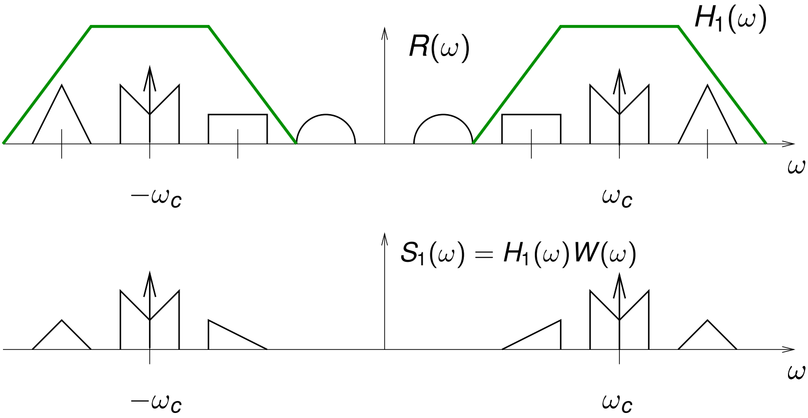
What are the practical requirements on the bandpass filter of this design?

Superheterodyning receiver

Design 2: **superheterodyning receiver** (avoids a tunable narrow bandpass filter)



Bandpass tunable wide



IF filter and mixing frequency

Definition

The fixed frequency bandpass filter called the **IF filter** for "intermediate frequency."

In commercial AM this is $\omega_{\mathrm{IF}}/2\pi = 455\mathrm{kHz}$.

Modulate the received RF signal by $\omega_0 = \omega_c + \omega_{\mathrm{IF}}$ to move the spectrum of the desired signal to be centered at ω_{IF}.

$$\begin{array}{c} s_1(t) \rightarrow \bigotimes \rightarrow s_2(t) \\ \uparrow \\ \cos(\omega_c + \omega_{\mathrm{IF}})\, t) \end{array}$$

When you "tune in" you are actually controlling the **mixing frequency** ω_0.

Mixing frequency: example

Example

If we want to tune in to the station at

$$\omega_c/2\pi = 1060\text{kHz}$$

then use

$$\omega_0/2\pi = (\omega_c + \omega_{\mathrm{IF}})/2\pi = 1060 + 455 = 1515\text{kHz}.$$

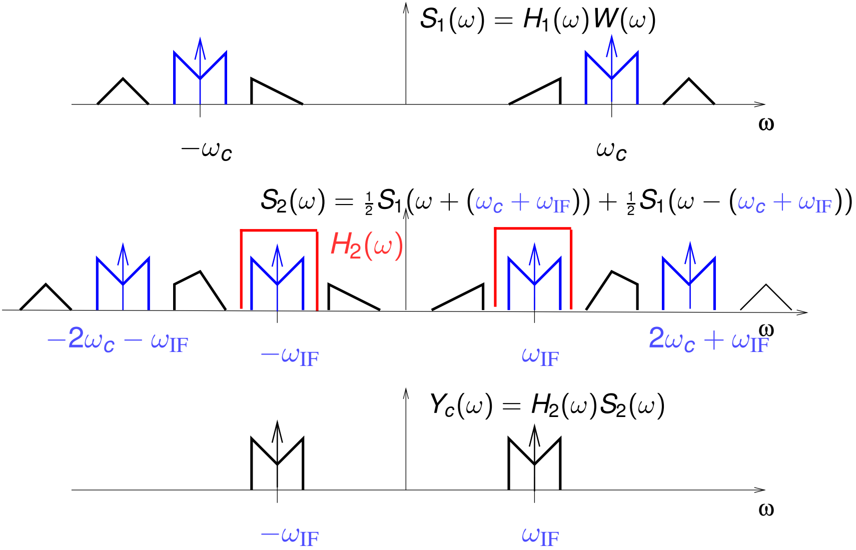
Before mixing, centered at $\pm$ 1060kHz.
After mixing by 1515kHz, centers at

$$\pm\omega_c \pm\omega_0 \Longrightarrow \pm\omega_{\mathrm{IF}} \text{ and } \pm(2\omega_c + \omega_{\mathrm{IF}})$$

$$\pm1060\pm1515\text{kHz} \Longrightarrow \pm455\text{kHz and } \pm2575\text{kHz}$$

.

Modulation and IF filter



Recovering the original signal

- After mixing with $\cos((\omega_c + \omega_{\mathrm{IF}})\, t)$, the desired station is centered in the passband of the IF filter.
- After the IF filter, asynchronous demodulation can be used to recover the original audio signal.

Question

One point we have omitted though. In the above example, what happens to the part of the received signal spectrum centered at

$$\omega_0/2\pi + \omega_{\mathrm{IF}}/2\pi = 1515 + 455 = 1970kHz?$$

Superheterodyning receiver

$$w(t) \rightarrow \boxed{\text{tunable bandpass at } \omega_c\text{, 900kHz wide}}$$

$$\rightarrow \boxed{\text{mix at } \omega_0 = \omega_c + \omega_{\mathrm{IF}}} \rightarrow \boxed{\text{bandpass at } \omega_{\mathrm{IF}}/2\pi\text{=455kHz} \pm \text{5kHz}}$$

$$\rightarrow \boxed{\text{asynchronous demodulate}} \rightarrow \hat{x}(t) \text{ audio}$$

A better approach?

Question

Is the following a better approach?

$$w(t) \rightarrow \boxed{\textit{mix at } \omega_0 = \omega_c + \omega_{\mathrm{IF}} \textit{ with } \omega_{\mathrm{IF}} = 0}$$

$$\rightarrow \boxed{\textit{lowpass with} \pm \textit{5kHz}} \rightarrow \hat{x}(t) \textit{audio}$$

Summary

- double sideband, suppressed carrier, amplitude modulation (DSB/SC-AM)
- double sideband, with carrier, amplitude modulation (DSB/WC-AM)
- synchronous demodulation
- asynchronous demodulation
- Frequency-division multiplexing (superheterodyning receiver)
- Systems-level analysis of communication system