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Prelab 1

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4.1 (a) $RC\cdot\frac{dV_{out}(t)}{dt}+V_{out}(t)=V_{in}(t)$ (1).

$V_{out}(t)=(1-e^{-t/RC})u(t)$ $\quad$ $V_{in}(t)=u(t)$

$$\frac{dV_{out}(t)}{dt}=\begin{cases}\frac{d}{dt}(1-e^{-t/RC}) & t\geq 0\\ 0 & t<0\end{cases}=\begin{cases}\frac{1}{RC}e^{-t/RC} & t\geq 0\\ 0 & t<0\end{cases}$$

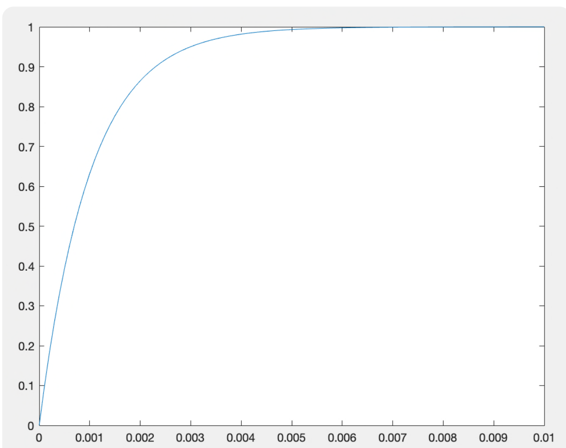
$=\frac{1}{RC}e^{-t/RC}u(t)$ substitute into (1). $\Rightarrow RC\cdot\frac{1}{RC}e^{-t/RC}u(t)+(1-e^{-t/RC})u(t)$

$=u(t)=V_{in}(t)$

Thus, we've verified that $y_{step}(t)=(1-e^{-t/RC})u(t)$

(b)

Vout



t

4.2 Since $u(t)\xrightarrow{LTI}y_{step}$

so that $\frac{d}{dt}u(t)\xrightarrow{LTI}\frac{d}{dt}y_{step}(t)$

$\frac{d}{dt}u(t)=\delta(t)\xrightarrow{LTI}h(t)=\frac{d}{dt}y_{step}(t)$

$$h(t)=\frac{d}{dt}y_{step}(t)=\delta(t)(1-e^{-\frac{t}{RC}})+\frac{1}{RC}e^{-\frac{t}{RC}}u(t)=\boxed{\frac{1}{RC}e^{-\frac{t}{RC}}u(t)}$$

4.3

(a) $\frac{b}{\Delta}u(t) \xrightarrow{LTI} \frac{b}{\Delta}y_{step}(t)$ $\qquad$ $\frac{b}{\Delta}u(t-\Delta) \xrightarrow{LTI} \frac{b}{\Delta}y_{step}(t-\Delta)$

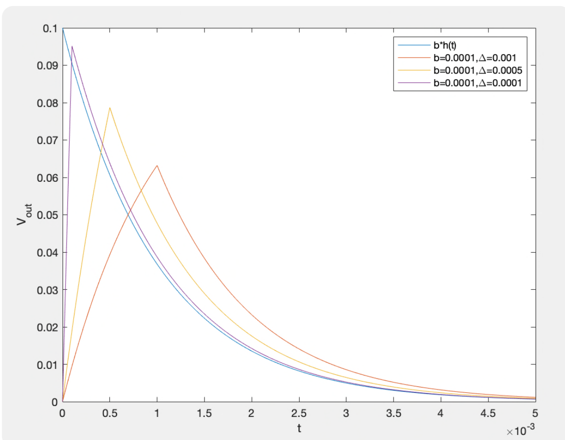
$$y_{b,\Delta}(t) = \frac{b}{\Delta}(y_{step}(t) - y_{step}(t-\Delta))$$

$$= \boxed{\frac{b}{\Delta}\left[(1-e^{-\frac{t}{RC}})u(t) - (1-e^{-\frac{t-\Delta}{RC}})u(t-\Delta)\right]}$$

(b) $$\lim_{\Delta\to 0} y_{b,\Delta}(t) = \lim_{\Delta\to 0}\frac{y_{step}(t)-y_{step}(t-\Delta)}{\Delta} = \frac{dy_{step}(t)}{dt} = \boxed{\frac{1}{RC}e^{-t/RC}u(t)}$$

This is the same result as $h(t)$ in 4.2

(c)



4.4 $$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

for $x(t) = tu(t)$, $$y(t) = \int_{-\infty}^{t}(t-\tau)\frac{1}{RC}e^{-\tau/RC}u(\tau)d\tau$$

$$= \frac{t}{RC}\int_0^t e^{-\tau/RC}d\tau - \frac{1}{RC}\int_0^t \tau\cdot e^{-\tau/RC}d\tau$$

$$= t(1-e^{-t/RC})u(t) + [te^{-t/RC} - RC + RCe^{-t/RC}]u(t)$$

$$= [RC(e^{-\frac{t}{RC}}-1)+t]u(t)$$

$$V_{out}(t) = (1-e^{-1000t})u(t) - (1-e^{-1000(t-0.01)})u(t-0.01) + 200[0.001e^{-1000(t-0.01)} + 0.005 - 0.001e^{-1000(t-0.016)}]$$

4.5

$$RC \frac{d}{dt} V_{out}(t) + V_{out}(t) = V_{in}(t)$$

$$RC\, j\omega V_{out}(\omega) + V_{out}(\omega) = V_{in}(\omega)$$

$$\Rightarrow \quad H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{RCj\omega + 1}$$

4.6

f_c (Hz)	$\lvert H(j2\pi f_c)\rvert$	$\angle \lvert H(j2\pi f_c)\rvert$ (degree)	T_d (ms)
50	0.954	−17.4	0.969
200	0.623	−51.5	0.715
500	0.303	−72.3	0.402
1000	0.157	−81.0	0.225
5000	0.032	−88.2	0.049