Quiz 3

徐逸洋
5183209100 20

Question 1

(a) $J = \frac{I}{S}$ $V_{12} = \frac{l}{\sigma S} I \Rightarrow I = JS \Rightarrow V_{12} = \frac{lJS}{\sigma S} = \frac{lJ}{\sigma}$

since $E = \frac{V_{12}}{l}$ in a conductor

$\Rightarrow E = \frac{\frac{lJ}{\sigma}}{l} = \frac{J}{\sigma} \Rightarrow J = \sigma E$

(b) since $R = \frac{\rho l}{S}$, $R = \frac{l}{\sigma S} = \frac{l}{\sigma \pi r^2}$

For inner wire $R_1 = \frac{l}{\sigma \pi a^2}$ ($r = a$)

For outer wire, $R_2 = \frac{l}{\sigma S}$, $S = \pi(a+x)^2 - \pi a^2$,

x is the thickness.

$\Rightarrow S = \pi(x^2 + 2ax)$

$\Rightarrow R_2 = \frac{l}{(0.1\sigma)[\pi(x^2+2ax)]}$

then $R_1 = R_2 \Rightarrow \frac{1}{\sigma \pi a^2} = \frac{1}{(0.1\sigma)[\pi(x^2+2ax)]}$

$\Rightarrow a^2 = 0.1(x^2 + 2ax)$

$\Rightarrow x^2 + 2ax - 10a^2 = 0$

✓ $x_1 = -a + \sqrt{11}a$, $x_2 = -a - \sqrt{11}a$ (< 0)

$x = a(\sqrt{11} - 1)$

(C). $J = \frac{I}{S}$, since $R_1 = R_2 \Rightarrow I_1 = I_2 = \frac{1}{2}I$

in the core: $J_1 = \frac{\frac{I}{2}}{\pi a^2} = \frac{I}{2\pi a^2}$ (A/m^2)

$$E_1 = \frac{J}{\sigma} = \frac{I}{2\pi\sigma a^2} \quad (V/m)$$

in the coat: $J_2 = \frac{\frac{I}{2}}{\pi(x^2+2ax)} = \frac{\frac{I}{2}}{\pi(\sqrt{11}-1)^2a^2+2(\sqrt{11}-1)a}$

$$= \frac{I}{20\pi a^2} \quad (A/m^2)$$

$$E_2 = \frac{J_2}{0.16} = \frac{I}{2\pi a^2} \quad V/m$$

Question 2.

(a) $E_{1t} = E_{2t}$, $E_{1t} = E_1 \sin\alpha_1$

$E_{2t} = E_2 \sin\alpha_2$

$\Rightarrow E_1 \sin\alpha_1 = E_2 \sin\alpha_2$

$J_{1n} = \sigma_1 E_{1n}$ $J_{2n} = \sigma_2 E_{2n}$, $J_{1n} = J_{2n}$

$\sigma_1 E_{1n} = \sigma_1 E_1 \cos\alpha_1$

$\sigma_2 E_{2n} = \sigma_2 E_2 \cos\alpha_2$

$\Rightarrow E_{2n} = \frac{\sigma_1}{\sigma_2} E_1 \cos\alpha_1$, $E_{2t} = E_1 \sin\alpha_1$

$E_2 = \sqrt{E_{2n}^2 + E_{2t}^2} = \boxed{E_1 \sqrt{\sin^2\alpha + (\frac{\sigma_1}{\sigma_2}\cos\alpha_1)^2}}$

the direction: $\frac{E_1 \sin\alpha_1}{\sigma_1 E_1 \cos\alpha_1} = \frac{E_2 \sin\alpha_2}{\sigma_2 E_2 \cos\alpha_2}$

$\Rightarrow \frac{\sigma_2}{\sigma_1} \tan\alpha_1 = \tan\alpha_2$

$\Rightarrow \alpha_2 = \boxed{\tan^{-1}(\frac{\sigma_2}{\sigma_1}\tan\alpha_1)}$

(b) $\rho_s = D_{2n} - D_{1n} = \epsilon_2 E_{2n} - \epsilon_1 E_{1n}$

$\boxed{= (\frac{\sigma_1}{\sigma_2}\epsilon_2 - \epsilon_1) E_1 \cos\alpha_1}$

(C) perfect dielectrics : $\Rightarrow \quad \sigma_1 = \sigma_2 = 0$

since $\epsilon_1 E_1 \cos\alpha_1 = \epsilon_2 E_2 \cos\alpha_2$

$$\Rightarrow \quad \frac{E_1 \sin\alpha_1}{\epsilon_1 E_1 \cos\alpha_1} = \frac{E_2 \sin\alpha_2}{\epsilon_2 E_2 \cos\alpha_2}$$

$$\Rightarrow \quad \frac{\tan\alpha_1}{\epsilon_1} = \frac{\tan\alpha_2}{\epsilon_2} \quad \Rightarrow \quad \alpha_2 = \tan^{-1}\left(\frac{\epsilon_2}{\epsilon_1}\tan\alpha_1\right)$$

$$E_{2n} = \frac{\epsilon_1}{\epsilon_2}\cos\alpha_1 E_1 \qquad E_{2t} = \sin\alpha_1 E_1$$

$$\Rightarrow \quad E_2 = E_1\sqrt{\sin^2\alpha_1 + \left(\frac{\epsilon_1}{\epsilon_2}\cos\alpha_1\right)^2}$$