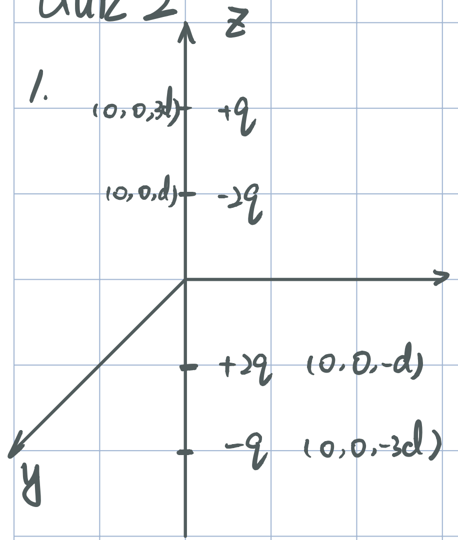
Quiz 2

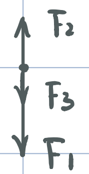
1.



Since xy plane is an grounded conductor potential = 0

There are three forces exerted on +q by other 3 charges.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q\cdot 2q}{(2d)^2} \quad (-2q \text{ on } +q)$$



$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q\cdot(2q)}{(4d)^2}$$

$$F_3 = \frac{1}{4\pi\epsilon_0}\cdot\frac{q\cdot q}{(6d)^2}$$

$$F = [F_2 - (F_1 + F_3)]\,\hat{z} = \frac{q^2}{4\pi\epsilon_0 d^2}\left(\frac{2}{4^2} - \frac{2}{2^2} - \frac{1}{6^2}\right)$$

$$= \frac{q^2}{4\pi\epsilon_0 d^2}\left(\frac{1}{8} - \frac{1}{2} - \frac{1}{36}\right)$$

$$= -\frac{29q^2}{288\pi\epsilon_0 d^2}\,\hat{z}$$

Question 2

Since it doesn't relate to z axis, so that the Laplace's equation is

$$\frac{\partial^2}{\partial x^2}V + \frac{\partial^2}{\partial y^2}V = 0$$ ⇒ we seperate V as a function of both x and y : $V(x,y) = X(x)Y(y)$, then $\nabla^2 V$ becomes

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} = 0$$

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$$

$$\Rightarrow \frac{1}{X}\frac{d^2}{dx^2}X = C_1 \qquad \frac{1}{Y}\frac{d^2}{dy^2}Y = C_2 \qquad C_1 + C_2 = 0$$

Let $C_1 = k^2 \Rightarrow C_2 = -k^2 \Rightarrow \frac{d^2X}{dx^2} = k^2X \qquad \frac{d^2Y}{dy^2} = -k^2Y$

we suppose the solutions of $X(x)$ and $Y(y)$ are

$$X(x) = Ae^{kx} + Be^{-kx} \quad , \quad Y(y) = C\sin ky + D\cos ky$$

$$\Rightarrow V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$$

According to the boundary conditions:

$y = 0$, $V = 0$

$y = a$, $V = 0$ $\qquad \Rightarrow V(x,0) = (Ae^{kx} + Be^{-kx})(D) = 0$

$x \to \infty$, $V = 0$ $\qquad \because Ae^{kx} + Be^{-kx} \neq 0 \Rightarrow D = 0$

$\Downarrow$

$$A = 0 \Rightarrow V(x,y) = e^{-kx}(C\sin ky + D\cos ky)$$

$$= e^{-kx} \cdot C \sin ky$$

according to $y = a$. $V = 0 \Rightarrow V(x,a) = 0 = 2A\cosh(kx)C\sin k \cdot a$

$$\Rightarrow ka = n\pi \Rightarrow k = \frac{n\pi}{a}$$

general solution is $V(x,y) = \sum_{n=1}^{\infty} C_n e^{-n\pi x/a} \sin\left(\frac{n\pi y}{a}\right)$

another boundary condition: $x = b$, $V(x,y) = V_0$

$$\Rightarrow V(0,y) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi y}{a}\right) = V_0(y)$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n \int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n'\pi y}{a}\right) dy = \int_0^a V_0(y) \sin\left(\frac{n'\pi y}{a}\right) dy$$

$$\Rightarrow \int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n'\pi y}{a}\right) = \begin{cases} 0 & n \neq n' \\ \frac{a}{2} & n = n' \end{cases}$$

$\underbrace{\text{only take}}_{\text{none 0 term}}$ so that $C_n = \frac{2}{a} \int_0^a V_0(y) \sin\left(\frac{n\pi}{a} \cdot y\right) dy$

$$\begin{cases} 3x + y - 6z = -10 \\ 2x + y - 5z = -8 \\ 6x - 3y + 3z = 0 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \\ 6 & -3 & 3 & 0 \end{array}\right]$$

① change order to make the left top as one.

$$\begin{array}{cc} \left[\begin{array}{ccc|c} 6 & -3 & 3 & 0 \\ 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \end{array}\right] & \begin{array}{c} \\ -1 \\ -1 \end{array} \end{array} \Rightarrow \begin{array}{cc} \left[\begin{array}{ccc|c} 1 & -5 & 14 & 18 \\ 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \end{array}\right] & \begin{array}{c} \\ -3 \\ -2 \end{array} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 14 & 18 \\ 0 & 16 & -48 & -64 \\ 0 & 11 & -33 & -44 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -5 & 14 & 18 \\ 0 & 1 & -3 & -4 \\ 0 & 1 & -3 & -4 \end{array}\right] = \begin{array}{cc} \left[\begin{array}{ccc|c} 1 & -5 & 14 & 18 \\ 0 & 1 & -3 & -4 \end{array}\right] & \times 5 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & -3 & -4 \end{array}\right] \Rightarrow \begin{cases} x + z = 2 \\ y - 3z = -4 \end{cases}$$

$$\begin{cases} x = 2 - z \\ y = -4 + 3z \end{cases}$$

infinite many solution sets.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}.$$