Quiz 5

1

A metal bar slide over a pair of conducting rails in a uniform magnetic field $\boldsymbol{B} = \boldsymbol{a_z} B_0$ with a constant velocity $\boldsymbol{u}$, as shown in Fig 1.

a) Determine the open-circuit voltage V_0 that appears across terminals 1 and 2. *Hint: You can use* $V_0 = \oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l}$

b) Assuming that a resistance R is connected between the terminals, find the electric power dissipated in R.

c) Show that this electric power is equal to the mechanical power required to move the sliding bar with a velocity $\boldsymbol{u}$. Neglect the electric resistance of the metal bar and of the conducting rails. Neglect also the mechanical friction at the contact points. *Hint: You can use the formula for mechanical power required to move the sliding bar* $P_m = \boldsymbol{F} \cdot \boldsymbol{u}$

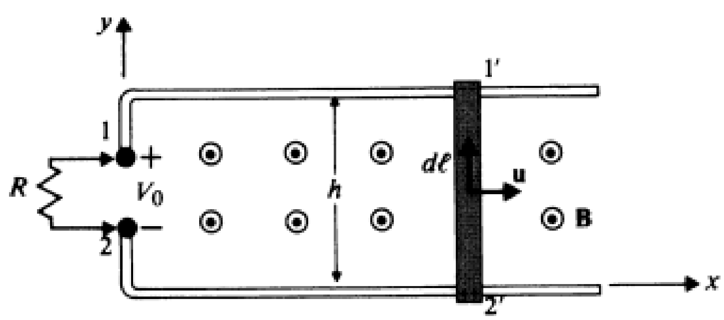

Figure 1: Figure for question 1

2

Source-free means that $\rho = 0, \boldsymbol{J} = 0, \sigma = 0$, under which the time-harmonic Maxwell's equations become:

$$\nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H},$$
$$\nabla \times \boldsymbol{H} = j\omega\epsilon \boldsymbol{E},$$
$$\nabla \cdot \boldsymbol{E} = 0,$$
$$\nabla \cdot \boldsymbol{H} = 0.$$

, where j is the imaginary number. Show that if $(\boldsymbol{E}, \boldsymbol{H})$ are solutions of source-free Maxwell's equations in a simple medium characterized by ϵ and μ, then so also are $(\boldsymbol{E'}, \boldsymbol{H'})$, where

$$\boldsymbol{E'} = \eta \boldsymbol{H}$$
$$\boldsymbol{H'} = -\frac{\boldsymbol{E}}{\eta}$$

In the above equations, $\eta = \sqrt{\mu/\epsilon}$ is called the **intrinsic impedance** of the medium.