

## VE230 Chapter 3 Part 1

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### 1 Basic Concept

Electrostatics:

- i. electric charges are **at rest(not moving)**;
- ii. electric field **do not change with time**.

### 2 Electrostatics in Free Space

Static electric charges (source) in free space  $\rightarrow$  electric field

#### 2.1 Electric field intensity

$$\mathbf{E} = \lim_{q \rightarrow 0} \frac{\mathbf{F}}{q} \quad (\text{V/m})$$

#### 2.2 Fundamental Postulates of Electrostatics

- Differential form:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \times \mathbf{E} &= 0\end{aligned}$$

- Integral form:

$$\begin{aligned}\oint_S \mathbf{E} \cdot d\mathbf{s} &= \frac{Q}{\epsilon_0} \\ \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} &= 0\end{aligned}$$

$\mathbf{E}$  is **not solenoidal**, but **irrotational (conservative)**

### 3 Coulomb's Law

#### 3.1 Electric Field due to a System of Discrete Charges

- a single point charge (charge on the origin):

$$\mathbf{E} = \mathbf{a}_R E_R = \mathbf{a}_R \frac{q}{4\pi\epsilon_0 R^2} \quad (\text{V/m})$$

- a single point charge (charge is not on the origin):

$$\mathbf{E}_p = \frac{q (\mathbf{R} - \mathbf{R}')}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|^3} \quad (\text{V/m})$$

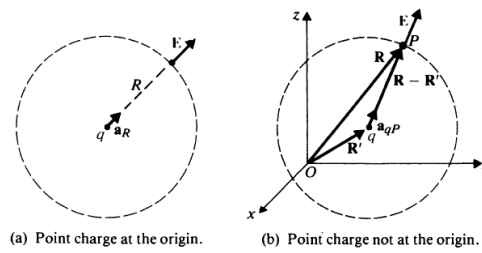


FIGURE 3-2  
Electric field due to a point charge.

### Example:

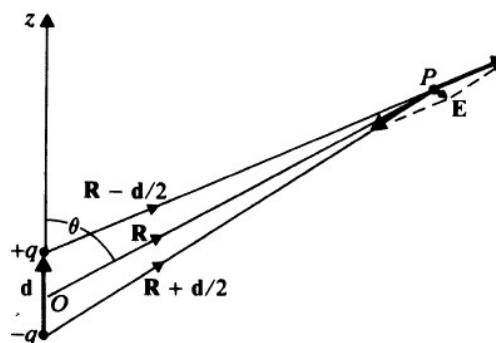
Determine the electric field intensity at  $P(-0.2, 0, -2.3)$  due to a point charge of  $+5(\text{nC})$  at  $Q(0.2, 0.1, -2.5)$  in air. (All dimensions are in meters).

- several point charges:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{q_k (\mathbf{R} - \mathbf{R}'_k)}{|\mathbf{R} - \mathbf{R}'_k|^3}$$

## 3.2 Electric Dipole

- Electric Field



general expression:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \left\{ \frac{\mathbf{R} - \frac{\mathbf{d}}{2}}{|\mathbf{R} - \frac{\mathbf{d}}{2}|^3} - \frac{\mathbf{R} + \frac{\mathbf{d}}{2}}{|\mathbf{R} + \frac{\mathbf{d}}{2}|^3} \right\}$$

if  $d \ll R$ :

$$\mathbf{E} \cong \frac{q}{4\pi\epsilon_0 R^3} \left[ 3 \frac{\mathbf{R} \cdot \mathbf{d}}{R^2} \mathbf{R} - \mathbf{d} \right]$$

- Electric Dipole Moment

Definition:

$$\mathbf{p} = q\mathbf{d}$$

$$\mathbf{p} = \mathbf{a}_z p = p (\mathbf{a}_R \cos \theta - \mathbf{a}_\theta \sin \theta)$$

$$\mathbf{R} \cdot \mathbf{p} = Rp \cos \theta$$

- Electric Field:

$$\mathbf{E} = \frac{p}{4\pi\epsilon_0 R^3} (\mathbf{a}_R 2 \cos \theta + \mathbf{a}_\theta \sin \theta) \quad (\text{V/m})$$

### 3.3 Electric Field due to a Continuous Distribution of Charge

- General Differential Element:

$$d\mathbf{E} = \mathbf{a}_R \frac{\rho dv'}{4\pi\epsilon_0 R^2}$$

- Line Charge:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{L'} \mathbf{a}_R \frac{\rho_\ell}{R^2} d\ell' \quad (\text{V/m})$$

- Surface Charge:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{S'} \mathbf{a}_R \frac{\rho_s}{R^2} ds' \quad (\text{V/m})$$

- Volume Charge:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{V'} \mathbf{a}_R \frac{\rho}{R^2} dv' \quad (\text{V/m})$$

**Example:**

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density  $\rho_\ell$  in air.

## 4 Gauss's Law and Application

### 4.1 Definition

The total outward flux of the E-field over any closed surface in free space is equal to **the total charge enclosed in the surface** divided by  $\epsilon_0$ .

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

### 4.2 Application

- **Conditions for Maxwell's Integral Equations:**

There is **a high degree of symmetry** in the charge distribution or in the electrical field (i.e., spherically symmetric, planar, line charge, etc).

**Example:**

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density  $\rho_\ell$  in air.

Determine the  $\mathbf{E}$  field caused by a spherical cloud of electrons with a volume charge density  $\rho = -\rho_o$  for  $0 \leq R \leq B$  (both  $\rho_o$  and  $b$  are positive) and  $\rho = 0$  for  $R > b$ .

### 4.3 Several Useful Models

**Note:** The charge distribution should be **uniform**.

different models	E(magnitude)
infinitely long, line charge	$E = \frac{\rho_\ell}{2\pi r\epsilon_0}$
infinite planar charge	$E = \frac{\rho_s}{2\epsilon_0}$
uniform spherical surface charge with radius R	$\begin{cases} E = 0 (r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0} (r > R) \end{cases}$
uniform sphere charge with radius R	$\begin{cases} E = \frac{Qr}{4\pi R^3} (r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0} (r > R) \end{cases}$
infinitely long, cylindrical charge with radius R	$\begin{cases} E = \frac{\rho_v r}{2\epsilon_0} (r < R) \\ E = \frac{\rho_v R^2}{2r\epsilon_0} (r > R) \end{cases}$

## 5 Electric Potential

- **Expression:**

$$\mathbf{E} = -\nabla V$$

- **Electric Potential Difference:**

$$V_2 - V_1 = - \int_{P_1}^{P_2} \mathbf{E} \cdot d\ell$$

- **Electric Potential due to a Charge Distribution**

- Line Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{L'} \frac{\rho_\ell}{R} d\ell' \quad (V)$$

- Surface Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{S'} \frac{\rho_s}{R} ds' \quad (V)$$

- Volume Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\rho}{R} dv' \quad (V)$$

- **Example:**

Obtain a formula for the electric field intensity and potential on the axis of a circular disk of radius b that carries a uniform surface charge  $\rho_s$ .