上 海 交 通 大 学 试 卷

（ 2019~ 2020 Academic Year/Summer Semester ）

Class No.________________ _ Name in English or Pinyin: _____________________

Student ID No._______________ Name in Hanzi(if applicable): _____________________

VE230 Electromagnetics I

Midterm Exam 1

2020.06.11 20:00-21:40

The exam paper has 5 pages in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

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Exercises No. 题号	Points 得分	Grader's Signature 流水批阅人签名
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total 总分		

Question 1

i) Find the gradients of the following functions:

(a) $f(x, y, z) = x^2 + y^3 + z^4$

(b) $f(x, y, z) = x^2y^3z^4$

(c) $f(x, y, z) = e^x sin(y) ln(z)$

ii) Suppose the function sketched in Fig.1(a) is $\mathbf{v}_a = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$ and that in Fig.1(b) is $\mathbf{v}_b = x\hat{\mathbf{y}}$. Calculate their curls.

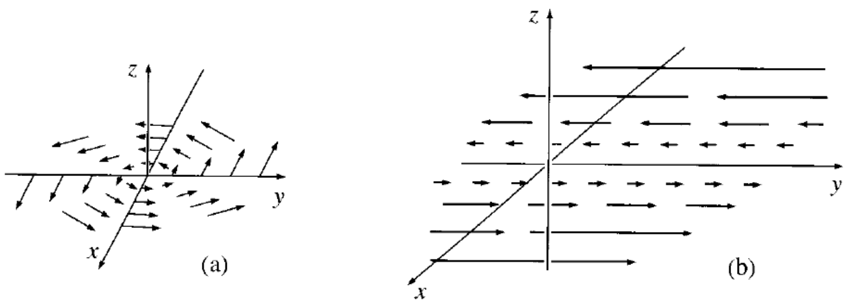

Figure 1: Question 1(ii)

Question 2

The height of a certain hill (in feet) is given by

$$h(x, y) = 10\left(2xy - 3x^2 - 4y^2 - 18x + 28y + 12\right)$$

where y is the distance(in miles) north, x the distance east of South Hadley.

(a) Where is the top of the hill located?

(b) How high is the hill?

(c) How steep is the slope(in feet per mile) at a point 1 mile north and 1 mile east of South Hadley? In what direction is the slope steepest, at that point?

Question 3

Divergence of vector $\mathbf{A}$ in spherical coordinates:

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta \sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Sketch the vector function

$$\mathbf{v} = \frac{\hat{\mathbf{R}}}{R^2}$$

and compute its divergence. The answer may surprise you...Can you explain it?

Question 4

Coulomb's Law:

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\hat{\mathbf{R}}$$

(a) Twelve equal charges, q, are situated at the corners of a regular 12-sided polygon (for instance, one on each numerical of a clock face). What is the net force on a test charge Q at the center?

(b) Suppose *one* of the 12 q's is removed (the one at "6 o'clock"). What is the force on Q?

(c) Now 13 equal charges, q, are placed at the center of a regular 13-sided polygon. What is the force on a test charge Q at the center?

(d) If one of the 13 q's is removed, what is the force on Q? Explain your reasoning.

Question 5

The electric field of a line charge:

$$\mathbf{E}(\mathbf{R}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{R}')}{R^2}\hat{\mathbf{R}}dl'^*$$

Find the electric field a distance z above the midpoint of a straight line segment of length 2L, which carries a uniform line charge λ (Fig.2).

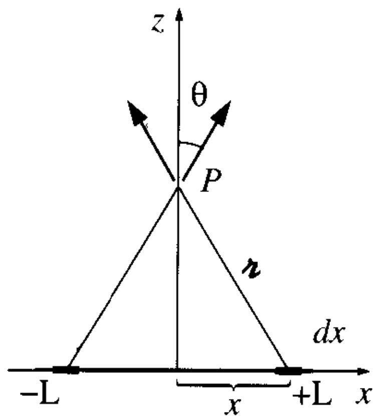

Figure 2: Question 5

Question 6

The potential inside and outside a uniformly charged solid sphere q with radius R is:

$$V(r) = \begin{cases} \text{Outside the sphere } (r > R) : \frac{q}{4\pi\epsilon_0}\frac{1}{r} \\ \text{Inside the sphere } (r < R) : \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right) \end{cases}$$

Find the energy stored in a uniformly charged solid sphere of radius R and charge q. Do it three different ways:

(a) Use $W = \frac{1}{2}\int \rho V dv$ and the above potential formula.

(b) Use $W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 dv$. Don't forget to integrate over *all space*.

(c) Use $W = \frac{\epsilon_0}{2} \left(\int_{\mathcal{V}} E^2 dv + \oint_{\mathcal{S}} V \mathbf{E} \cdot d\mathbf{a} \right)$. Take a spherical volume of radius a. Notice what happens as $a \to \infty$.