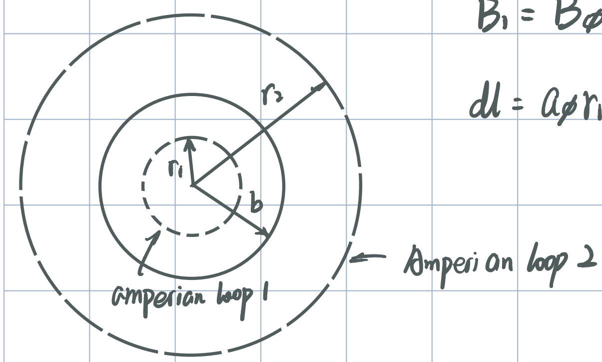
徐远泽

Problem 1.1

518370910020

1° Inside the conductor: $0<r_1<b$. we have an Amperian loop with radius r_1



$$B_1 = B_{\phi_1}\hat{\phi} \qquad \oint_{C_1} B_1\cdot dl = \int_0^{2\pi} B_{\phi_1} r_1\, d\phi = 2\pi r_1 B_{\phi_1}$$

$$dl = a_\phi r_1 d\phi$$

$$I_{encl} = \int_S \vec{J}\cdot d\vec{s} \qquad = \frac{I}{\pi b^2}$$

$$= \int_0^{r_1}\int_0^{2\pi} \frac{I}{\pi b^2}\,\hat{z}\cdot r_1\, d\phi\, dr_1\cdot\hat{z}$$

$$= \frac{I}{\pi b^2}\cdot\left(\frac{r_1^2}{2}\right)\cdot 2\pi \qquad = \frac{I r_1^2}{b^2}$$

By Amepre's circuital law,

$$B_1 = \hat{\phi}\cdot B_{\phi_1} = \hat{\phi}\cdot\frac{\mu_0 r_1 I}{2\pi b^2} \qquad 0<r_1\leq b$$

2° outside the conductor: $r_2\geq b$. we have an Amperian loop with radius r_2

By Ampere's law, $B_2 = \hat{\phi} B_{\phi_2}$ $\quad H_\phi(2\pi r_2) = I \Rightarrow H_\phi = \frac{I}{2\pi r_2}$

$$\vec{H} = \frac{I}{2\pi r_2}\hat{\phi} \quad\Rightarrow \vec{B} = \mu_0\vec{H} = \frac{\mu_0 I}{2\pi r_2}\hat{\phi}$$

$$r_2\geq b$$

Problem 2.

$$A = \frac{\mu_0 I}{4\pi}\int_{V'}\frac{J}{R}dv' \qquad B = \nabla\times A = \nabla\times\left[\frac{\mu_0 I}{4\pi}\int_{V'}\frac{J}{R}dv'\right]$$

$$= \frac{\mu_0 I}{4\pi}\int_{V'}\nabla\times\frac{J}{R}dv'$$

since $\nabla\times\frac{1}{R} = \hat{R}\cdot\frac{1}{R^2}$ $\longrightarrow$

$$= \frac{\mu_0 I}{4\pi}\int J\cdot\left(\nabla\times\frac{1}{R}\right)dv'$$

$$= \frac{\mu_0 I}{4\pi}\int_{V'}\frac{J\times a_R}{R^2}dv'$$

Using $\nabla \cdot B = 0$ and $\nabla \times B = \mu_0 J$ to prove $B = \frac{\mu_0}{4\pi} \int_{v'} \frac{J \times a_R}{R^2} dv'$

$\nabla \cdot B = 0$ and $\nabla \times B = \mu_0 J$, since $B = \nabla \times A$

$$\Rightarrow \nabla \times \nabla \times A = \nabla \cdot (\nabla \cdot A) - \nabla^2 A \qquad \nabla \cdot A = 0$$

since $H = -\nabla V_m$ when $J = 0$

$$J = \nabla \times H = \nabla \times (-\nabla V_m) = 0 \Rightarrow \nabla^2 V_m = 0$$

$$V = \int \frac{dQ}{4\pi \varepsilon_0 r}$$

$$\Rightarrow A = \int \frac{\mu_0 J dv}{4\pi R}$$

according to Bio-savart law

$$dH = \frac{I dl \times a_R}{4\pi R^3}$$

$$\Rightarrow H = \int_{v'} \frac{J dv \times a_R}{4\pi R^3} \Rightarrow \nabla V_m = \nabla \times A \text{ and } \nabla \times A = B$$

$$\Rightarrow B = \mu H, \quad B = \nabla V_m.$$

Therefore, $B = \frac{\mu}{4\pi} \int_{v'} \frac{J \times a_R}{R^2}$