Quiz 1

1

Caculate the divergence of the followng vector functions:

(a)

$$\boldsymbol{v}_a = x^2\hat{\boldsymbol{x}} + 3xz^2\hat{\boldsymbol{y}} - 2xz\hat{\boldsymbol{z}}$$

(b)

$$\boldsymbol{v}_b = xy\hat{\boldsymbol{x}} + 2yzve\hat{c}ty + 3zxve\hat{c}tz$$

(c)

$$\boldsymbol{v}_c = y^2\hat{\boldsymbol{x}} + (2xy + z^2)\hat{\boldsymbol{y}} + 2yz\hat{\boldsymbol{z}}$$

2

Divergence Theorem: the volume integral of the divergence of a vector field equals the total outward flux of the vector through the surface that bounds the volume.

$$\int_V \boldsymbol{\nabla} \cdot \boldsymbol{A} dv = \oint_S \boldsymbol{A} \cdot d\boldsymbol{s}$$

Stokes's Theorem: the surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector along the contour bounding the surface.

$$\int_S (\boldsymbol{\nabla} \times \boldsymbol{A}) \cdot d\boldsymbol{s} = \oint_{\boldsymbol{C}} \boldsymbol{A} \cdot dl$$

(I) (a) Check the divergence theorem for the function $\boldsymbol{v}_1 = r^2\hat{\boldsymbol{r}}$, using as your volume the sphere of raduis R, centered at the origin.

(b) Do the same for $\boldsymbol{v}_2 = (1/r^2)\hat{\boldsymbol{r}}$.

(II) Test Stokes' theorem for the function $\boldsymbol{v} = (xy)\hat{\boldsymbol{x}} + (2yz)\hat{\boldsymbol{y}} + (3zx)\hat{\boldsymbol{z}}$, using the triangular shaded area of Fig 1.

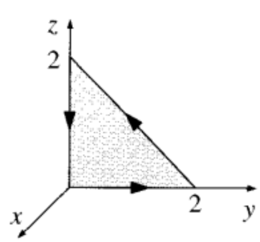

Figure 1: