Static Electric Fields

1 Materials to Cover

- Recap of common mistakes in homework 1.
- Conductors/dielectrics in static electric field, electric flux density and dielectric constant
- boundary conditions for electrostatic field

2 Recap of common mistakes in homework 1

.

- When we introduced Cartesian coordinate, did we define θ, ϕ?

3 Conductors and dielectrics in static electric field

- conductors:
  - electrons migrate easily.
  - charges reach the surface and con-ductor redistribute themselves in a way that both the charge and the field vanish.
  - **static state conditions**:
    * inside the conductor:

      $$\rho = 0, \boldsymbol{E} = 0$$

      , where $\rho = 0$ represents no charge in the interior
    * on the conductor surface (boundary conditions)

      $$E_t = 0, E_n = \frac{\rho_s}{\epsilon_0}$$

- **Example. 3-11** A postive point charge Q is at the center of a spherical conducting shell of an inner radius R_i and an outer radius R_0. Determine $\boldsymbol{E}$ and V as functions of the radial distance R.

- electric field intensity tends to be higher at a point near the surface of a charged conductor with a larger curvature
  Example. 3-13 Consider two spherical conductors with radii b_1 and $b_2(b_2 > b_1)$ that are connected by a conducting wire. The distance of separation between the conductors is assumed to be very large in comparison to b_2 so that the charges on the spherical conductors may be considered as uniformly distributed. A total charge Q is deposited to the spheres. Find (a) the charges on the two spheres, and (b) the electric field intensities at the sphere surfaces.

- semiconductos:
  - relatively small number of freely movable charges.
- insulators(dielectrics):
  - electrons are confined to their orbits.
  - external electric field polarizes a dielectric material and create electric dipoles. The induced electric dipoles will modify the electric field both inside and outside the dielectric material, as shown in Fig 1.

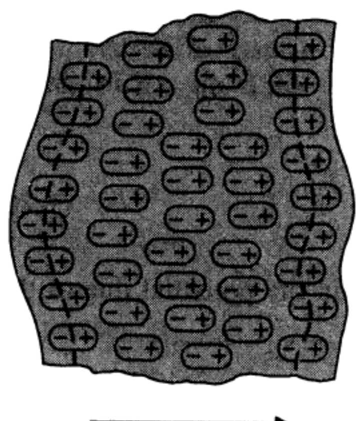

Figure 1: A cross section of a polarized dielectric medium

  - **polarization charge densities/ bound-charge densities:**
    * **polarization vector, $\boldsymbol{P}$**:

$$\boldsymbol{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{n\Delta v} \boldsymbol{p_k}}{\Delta v}$$

, where the numerator represents the vector sum of the induced dipole moment contained in a very small volume Δv.
    * charge distribution on surface density:

$$\rho_{ps} = \boldsymbol{P} \cdot \boldsymbol{a_n}$$

,
    * volume charge distribution density:

$$\rho_p = -\nabla \cdot \boldsymbol{P}$$

.

* **Derivation for these two polarization charge distribution densities.**

* **Show that divergence theorem still hold when we apply these two densities to an electric neutral dielectric**

4 Electric Flux Density and Dielectric Constant

- **electric flux density/electric displacement, $\boldsymbol{D}$**:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \quad (C/m^2)$$

- $$\nabla \cdot \boldsymbol{D} = \rho \quad (C/m^3)$$

  , where ρ is the volume density of free charges.

- Another form of **Gauss's law**:

$$\oint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q \quad (\boldsymbol{C})$$

  , the total outward flux of the electric displacement (the total outward electric flux) over any closed surface is equal to the total free charge enclosed in the surface.

- If the dielectric of the medium is **linear and isotropic**,

$$\boldsymbol{P} = \epsilon_0 \chi_e \boldsymbol{E}$$

$$\boldsymbol{D} = \epsilon_0(1+\chi_e)\boldsymbol{E} = \epsilon_0\epsilon_r\boldsymbol{E} = \epsilon\boldsymbol{E}$$

, where χ_e is a dimensionless quantity called electric susceptibility,

ϵ_r is a dimensionless quantity called as relative permittivity/ electric constant of the medium,

ϵ is the absolute permittivity/permittivity of the medium (F/m).

Example. 3-12 A postive point charge Q is at the center of a spherical dielectric shell of an inner radius R_i and an outer radius R_o. The dielectric constant of the shell is ϵ_r.

a) Describe the physical meaning of $\boldsymbol{D}$ and $\boldsymbol{P}$.

b) Determine $\boldsymbol{E}, V, \boldsymbol{D}, \boldsymbol{P}$ as functions of the radial distance R.

- For anisotropic,

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

,

For biaxial,

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

For uniaxial, $\epsilon_1 = \epsilon_2$, For isotropic, $\epsilon_1 = \epsilon_2 = \epsilon_3$ (the only kind of media we deal with in this course).

- dielectric breakdown: electric field is very strong, causes permanent dislocations and damage in the material.

  dielectric strength: the maximum electric field intensity that a dielectric material can withstand without breakdown.

5 Boundary Conditions for Electrostatic Fields

- the tangential component of an $\boldsymbol{E}$ field is continuous across an interface.

$$E_{1t} = E_{2t} \quad (V/m)$$

, or

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

- The normal component of $\boldsymbol{D}$ field is discontinous across an interface where a surface charge exists - the amount of discontinuity being equal to the surface charge density.

$$\boldsymbol{a_{n2}} \cdot (\boldsymbol{D_1} - \boldsymbol{D_2}) = \rho_s$$

, or

$$D_{1n} - D_{2n} = \rho_s \quad (C/m^2)$$

Example. 3-14 A lucite sheet ($\epsilon_r = 3.2$) is introduced perpendicularly in a uniform electric field $\boldsymbol{E_o} = \boldsymbol{a_x} E_o$ in free space. Determine $vect E_i, \boldsymbol{D_i}, \boldsymbol{P_i}$ inside the lucite.

Example. 3-15 Two dielectric media with permittivities ϵ_1 and ϵ_2 are separated by a charge-free boundary as shown in Fig 2. The electric field intensity in medium 1 at the point P_1 has a magnitude E_1 and makes an angle α_1 with the normal. Determine the magnitude and direction of the elctric field intensity at point P_2 in medium 2.

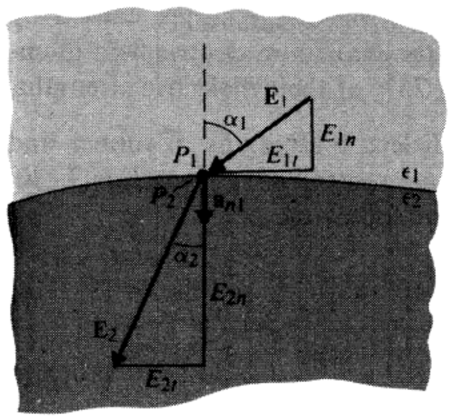

Figure 2: Boundary conditions at the interface between two dielectric media