Homework 4

1\.

Amplification of a Broadband Signal. The transition between two energy levels exhibits a Lorentzian lineshape of central frequency $\nu_0 = 5 \times 10^{14}$ with a linewidth $\Delta\nu = 10^{12}$ Hz. The population is inverted so that the maximum gain coefficient $\gamma(\nu_0) = 0.1\ \text{cm}^{-1}$. The medium has an additional loss coefficient $\alpha_s = 0.05\ \text{cm}^{-1}$, which is independent of ν. Approximately how much loss or gain is encountered by a light wave in 1 cm if it has a uniform power spectral density centered about ν_0 with a bandwidth $2\Delta\nu$?

2\.

The Two-Level Pumping System. Write the rate equations for a two-level system, showing that a steady-state population inversion cannot be achieved by using direct optical pumping between levels 1 and 2.

Hint: R2=-R1=R

3\.

Resonant Absorption of a Medium in Thermal Equilibrium. A unity refractive index medium of volume 1 cm^3 contains $N_a = 10^{23}$ atoms in thermal equilibrium. The ground state is energy level 1; level 2 has energy 2.48 eV above the ground state ($\lambda_o = 0.5\ \mu$m). The transition between these two levels is characterized by a spontaneous lifetime $t_{sp} = 1$ ms, and a Lorentzian lineshape of width $\Delta\nu = 1$ GHz. Consider two temperatures, T_1 and T_2, such that $k_B T_1 = 0.026$ eV and $k_B T_2 = 0.26$ eV.

(a) Determine the populations N_1 and N_2.

(b) Determine the number of photons emitted spontaneously every second.

(c) Determine the attenuation coefficient of this medium at $\lambda_o = 0.5\ \mu$m assuming that the incident photon flux is small.

Hint: there's no pumping, such that, N_0 is negative, which leads to gain coefficient becomes attenuation one.

4\.

Number of Longitudinal Modes. An Ar^+-ion laser has a resonator of length 100 cm. The refractive index $n = 1$.

(a) Determine the frequency spacing ν_F between the resonator modes.

(b) Determine the number of longitudinal modes that the laser can sustain if the FWHM Doppler-broadened linewidth is $\Delta\nu_D = 3.5$ GHz and the loss coefficient is half the peak small-signal gain coefficient.

5. **Threshold Population Difference for an Ar^+-Ion Laser.** An Ar^+-ion laser has a 1-m-long resonator with 98% and 100% mirror reflectances. Other loss mechanisms are negligible. The atomic transition has a central wavelength $\lambda_o = 515$ nm, spontaneous lifetime $t_{sp} = 10$ ns, and linewidth $\Delta\lambda = 0.003$ nm. The lower energy level has a very short lifetime and hence zero population. The diameter of the oscillating mode is 1 mm. Determine (a) the photon lifetime and (b) the threshold population difference for laser action.

6. **Rate Equations in a Four-Level Laser.** Consider a four-level laser with an active volume $V = 1$ cm^3. The population densities of the upper and lower laser levels are N_2 and N_1 and $N = N_2 - N_1$. The pumping rate is such that the steady-state population difference N in the absence of stimulated emission and absorption is N_0. The photon-number density is n and the photon lifetime is τ_p. Write the rate equations for N_2, N_1, N, and n in terms of N_0, the transition cross section $\sigma(\nu)$, and the times t_{sp}, τ_1, τ_2, τ_{21}, and τ_p. Determine the steady state values of N and n.