Homework 1

1. ***The Paraboloidal Wave and the Gaussian Beam.*** Verify that a paraboloidal wave with the complex envelope $A(\mathbf{r}) = (A_0/z)\exp[-jk(x^2+y^2)/2z]$ [see (2.2-16)] satisfies the paraxial Helmholtz equation (2.2-22). Show that the wave with complex amplitude $A(\mathbf{r}) = [A/q(z)]\exp[-jk(x^2+y^2)/2q(z)]$, where $q(z) = z + jz_0$ and z_0 is a constant, also satisfies the paraxial Helmholtz equation. This wave, called the Gaussian beam, is the subject of Chap. 3. Sketch the intensity of the Gaussian beam in the plane $z = 0$.

2. **Spherical Waves.** Use a spherical coordinate system to verify that the complex amplitude of the spherical wave (2.2-15) satisfies the Helmholtz equation (2.2-7).

3. **Reflection of a Spherical Wave from a Planar Mirror.** A spherical wave is reflected from a planar mirror sufficiently far from the wave origin so that the Fresnel approximation is satisfied. By regarding the spherical wave locally as a plane wave with slowly varying direction, use the law of reflection of plane waves to determine the nature of the reflected wave.

4. **Optical Path Length.** A plane wave travels in a direction normal to a thin plate made of N thin parallel layers of thicknesses d_q and refractive indices n_q, $q = 1, 2, \ldots, N$. If all reflections are ignored, determine the complex amplitude transmittance of the plate. If the plate is replaced with a distance d of free space, what should d be so that the same complex amplitude transmittance is obtained? Show that this distance is the optical path length defined in Sec. 1.1.

5. **Standing Waves.** Derive an expression for the intensity I of the superposition of two plane waves of wavelength λ traveling in opposite directions along the z axis. Sketch I versus z.

6. ***Parameters of a Gaussian Laser Beam.*** A 1-mW He–Ne laser produces a Gaussian beam of wavelength $\lambda = 633$ nm and a spot size $2W_0 = 0.1$ mm.

   (a) Determine the angular divergence of the beam, its depth of focus, and its diameter at $z = 3.5 \times 10^5$ km (approximately the distance to the moon).

   (b) What is the radius of curvature of the wavefront at $z = 0$, $z = z_0$, and $z = 2z_0$?

   (c) What is the optical intensity (in $\mathrm{W/cm^2}$) at the beam center ($z = 0, \rho = 0$) and at the axial point $z = z_0$? Compare this with the intensity at $z = z_0$ of a 100-W spherical wave produced by a small isotropically emitting light source located at $z = 0$.

Determination of a Beam with Given Width and Curvature. Assuming that the width W and the radius of curvature R of a Gaussian beam are known at some point on the beam axis (Fig. 3.1-9), show that the beam waist is located at a distance

$$z = \frac{R}{1 + \left(\lambda R / \pi W^2\right)^2} \tag{3.1-24}$$

7.

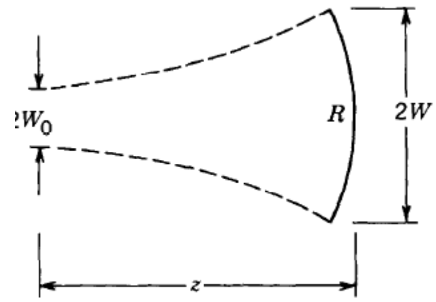

Figure 3.1-9 Given W and R, determine z and W_0.

to the left and the waist radius is

$$W_0 = \frac{W}{\left[1 + \left(\pi W^2 / \lambda R\right)^2\right]^{1/2}}. \tag{3.1-25}$$

Electric Field of Focused Light. (a) 1 W of optical power is focused uniformly on a flat target of size 0.1×0.1 mm^2 placed in free space. Determine the peak value of the electric field E_0 (V/m). Assume that the optical wave is approximated as a TEM plane wave within the area of the target. (b) Determine the electric field at the center of a Gaussian beam (a point on the beam axis at the beam waist) if the beam
8. power is 1 W and the beam waist radius $W_0 = 0.1$ mm. Refer to Sec. 3.1.

Conductivity and Absorption. In a medium with an electric current density $\mathcal{J}$, Maxwell's equation (5.2-4) is modified to $\nabla \times \mathcal{H} = \mathcal{J} + \epsilon\, \partial\mathcal{E}/\partial t$, with the other equations unaltered. If the medium is described by Ohm's law, $\mathcal{J} = \sigma\mathcal{E}$, where σ is the conductivity, show that the Helmholtz equation, (5.3-15), is applicable with a complex-valued k. Show that a plane wave traveling in this medium is attenuated,
9. and determine an expression for the attenuation coefficient α.

The Half-Wave Retarder. Linearly polarized light is transmitted through a half-wave retarder. If the polarization plane makes an angle θ with the fast axis of the retarder, show that the transmitted light is linearly polarized at an angle $-\theta$, i.e., rotates by an angle 2θ. Why is the half-wave retarder not equivalent to a polarization rotator?
10.

Reflectance of Glass. A plane wave is incident from air ($n = 1$) onto a glass plate ($n = 1.5$) at an angle of incidence 45°. Determine the intensity reflectances of the TE and TM waves. What is the average reflectance for unpolarized light (light
11. carrying TE and TM waves of equal intensities)?

12. **Refraction at the Brewster Angle.** Show that at the Brewster angle of incidence the directions of the reflected and refracted waves are orthogonal. The electric field of the refracted TM wave is then parallel to the direction of the reflected wave.