1. **Photon Energy.** (a) What voltage should be applied to accelerate an electron from zero velocity in order that it acquire the same energy as a photon of wavelength $\lambda_o = 0.87\ \mu m$?
(b) A photon of wavelength 1.06 μm is combined with a photon of wavelength 10.6 μm to create a photon whose energy is the sum of the energies of the two photons. What is the wavelength of the resultant photon? Photon interactions of

(a) $E = \frac{h \cdot c}{\lambda} = \frac{6.626\times10^{-34} \times 3\times10^{8}}{0.87\times10^{-6}} = \frac{1.99\times10^{-25}}{0.87\times10^{-6}} = 2.285\times10^{-19}\ J$

$= 1.426\ eV$

(b) $\lambda_1 = 1.06\times10^{-6}\ m. \quad \lambda_2 = 10.6\times10^{-6}\ m$

$E_1 = \frac{h\cdot c}{\lambda_1} = \frac{6.626\times10^{-34}\times3\times10^{8}}{1.06\times10^{-6}} = 1.875\times10^{-19}\ J$

$E_2 = \frac{h\cdot c}{10\cdot\lambda_1} = 1.875\times10^{-20}\ J$

$E_{tot} = E_1 + E_2 = 2.063\times10^{-19}\ J$

$\lambda_{tot} = \frac{hc}{E_{tot}} = \frac{6.626\times10^{-34}\times3\times10^{8}}{2.063\times10^{-19}} = 9.635\times10^{-7}\ m = 0.9635\ \mu m$

Photon Flux. Show that the power of a monochromatic optical beam that carries an average of one photon per optical cycle is inversely proportional to the squared wavelength.

2.

$P = \frac{E}{t}, \quad E = \frac{hc}{\lambda} \quad t = \frac{1}{f} = \frac{\lambda}{c}$

$\Rightarrow P = \left(\frac{hc}{\lambda}\right)/\left(\frac{\lambda}{c}\right) = \frac{\frac{hc}{\lambda}}{\frac{\lambda}{c}} = \frac{hc^2}{\lambda^2}$

Therefore $P \propto \frac{1}{\lambda^2}$

Comparison of Stimulated and Spontaneous Emission. An atom with two energy levels corresponding to the transition ($\lambda_o = 0.7\ \mu m$, $t_{sp} = 3$ ms, $\Delta\nu = 50$ GHz, Lorentzian lineshape) is placed in a resonator of volume $V = 100\ cm^3$ and refractive index $n = 1$. Two radiation modes (one at the center frequency ν_0 and the other at $\nu_0 + \Delta\nu$) are excited with 1000 photons each. Determine the probability density for stimulated emission (or absorption). If N_2 such atoms are excited to energy level 2, determine the time constant for the decay of N_2 due to stimulated *and* spontaneous emission. How many photons (rather than 1000) should be present so that the decay rate due to stimulated emission equals that due to spontaneous emission?

3.

$$P_{st} = \frac{Nc}{V} \sigma(\nu)$$

$$\sigma(\nu_0) = \frac{\lambda^2}{8\pi n^2} \cdot g(\nu) \quad , \quad g(\nu) = \frac{\frac{\Delta\nu}{2\pi}}{(\frac{\Delta\nu}{2})^2} = \frac{\frac{\Delta\nu}{2\pi}}{\frac{\Delta\nu^2}{4}} = \frac{2}{\pi\Delta\nu}$$

$$\sigma(\nu_0) = \frac{\lambda^2}{8\pi n^2} \cdot \frac{2}{\pi\Delta\nu}$$

$$= \frac{\lambda^2}{4\pi^2 t_{sp} \Delta\nu}$$

$$= \frac{(0.7\times10^{-6})^2}{4\times\pi^2\cdot 3\times10^{-3}\times 50\times10^9}$$

$$= 8.275\times10^{-23} \ m^2$$

$$= 8.275\times10^{-19} \ cm^2$$

$$P_{st} = \frac{1000\times 3\times10^{10}}{100} \times 8.275\times10^{-19}$$

$$= \underline{2.482\times10^{-13}}$$

If N_2 atoms are excited to energy level, $t_0 = \frac{1}{W_i + W_{sp}} = \frac{1}{3\times10^{-7} + \frac{1}{3\times10^{-3}}}$

$\approx 3ms$

stimulated emission = spontaneous emission

$$\Rightarrow W_i = \frac{Nc^3}{8\pi\nu_0^2 t_{sp} V} \left(\frac{12}{5\pi\Delta\nu}\right) = W_{sp} = \frac{1}{t_{sp}}$$

$$N = \frac{1}{t_{sp}} \cdot \frac{8\pi\nu_0^2 t_{sp} V (5\pi\Delta\nu)}{12 Nc^3}$$

$$= \frac{8\pi\cdot(4.3\times10^{14})^2\, 3\cdot10^{-3}\cdot 100\cdot 5\pi\cdot 50\times10^9}{12\cdot(3\times10^{10})^3}$$

$= \underline{1.13\times10^{12} \ \text{photons}}$

4. **Rate Equations for Broadband Radiation.** A resonator of unit volume contains atoms having two energy levels, labeled 1 and 2, corresponding to a transition of resonance frequency ν_0 and linewidth $\Delta\nu$. There are N_1 and N_2 atoms in the lower and upper levels, 1 and 2, respectively, and a total of $\bar{n}$ photons in each of the modes within a broad band surrounding ν_0. Photons are lost from the resonator at a rate $1/\tau_p$ as a result of imperfect reflection at the cavity walls. Assuming that there are no nonradiative transitions between levels 2 and 1, write rate equations for N_2 and $\bar{n}$.

Suppose N is the total amount of atoms $\Rightarrow N_1 + N_2 = N$

$$\frac{dN_2}{dt} = -\frac{N_2}{t_{sp}}$$

$$N_2(t) = N_2(0)\exp(-t/t_{sp})$$

$$\frac{dN_2}{dt} = N_1 W_i = \frac{\bar{n} N_1}{t_{sp}} = -N_2 W_i = -\frac{\bar{n} N_2}{t_{sp}}$$

$$= -\frac{N_2}{t_{sp}} + \frac{\bar{n} N_1}{t_{sp}} - \frac{\bar{n} N_2}{t_{sp}}$$

$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}$$

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} \qquad \frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$

Comparison of Stimulated and Spontaneous Emission in Blackbody Radiation. Find the temperature of a thermal-equilibrium blackbody cavity emitting a spectral energy density $\varrho(\nu)$, when the rates of stimulated and spontaneous emission from the atoms in the cavity walls are equal at $\lambda_o = 1\ \mu\text{m}$.

From wein's law. $\lambda T = 2898\ \mu\text{mk}$

$1\ \mu\text{m} \cdot T = 2898\ \mu\text{mK}$

$T = 2898\ \text{K}$