Homework 2

1. **Field Distribution.** (a) Show that a single TEM plane wave $E_x(y, z) = A\exp(-jk_y y)\exp(-j\beta z)$ cannot satisfy the boundary conditions, $E_x(\pm d/2, z) = 0$ at all z, in the mirror waveguide illustrated in Fig. 7.1-1.
(b) Show that the sum of two TEM plane waves written as $E_x(y, z) = A_1\exp(-jk_{y1}y)\exp(-j\beta_1 z) + A_2\exp(-jk_{y2}y)\exp(-j\beta_2 z)$ does satisfy the boundary conditions if $A_1 = \pm A_2$, $\beta_1 = \beta_2$, and $k_{y1} = -k_{y2} = m\pi/d$, $m = 1, 2, \ldots$.

Note that : for (b) find the conditions that the sum of waves satisfy the boundary conditions, not to "show" or "proof"， i.e. consider , even / odd modes

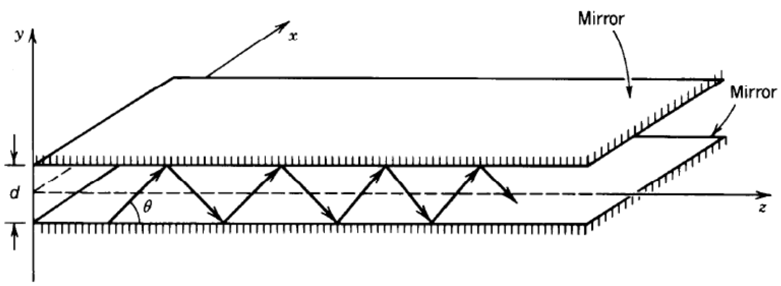

Figure 7.1-1 Planar-mirror waveguide.

2. **Modal Dispersion.** Light of wavelength $\lambda_o = 0.633\ \mu m$ is transmitted through a mirror waveguide of mirror separation $d = 10\ \mu m$ and $n = 1$. Determine the number of TE and TM modes. Determine the group velocities of the fastest and the slowest mode. If a narrow pulse of light is carried by all modes for a distance of 1 m in the waveguide, how much does the pulse spread as a result of the differences of the group velocities?

3. **Parameters of a Dielectric Waveguide.** Light of free-space wavelength $\lambda_o = 0.87\ \mu m$ is guided by a thin planar film of width $d = 2\ \mu m$ and refractive index $n_1 = 1.6$ surrounded by a medium of refractive index $n_2 = 1.4$.
(a) Determine the critical angle θ_c and its complement $\bar{\theta}_c$, the numerical aperture NA, and the maximum acceptance angle for light originating in air ($n = 1$).
(b) Determine the number of TE modes.
(c) Determine the bounce angle θ and the group velocity v of the $m = 0$ TE mode.

4. **Single-Mode Waveguide.** What is the largest thickness d of a planar symmetric dielectric waveguide with refractive indices $n_1 = 1.50$ and $n_2 = 1.46$ for which there is only one TE mode at $\lambda_o = 1.3\ \mu m$? What is the number of modes if a waveguide with this thickness is used at $\lambda_o = 0.85\ \mu m$ instead?

5. **Mode Cutoff.** Show that the cutoff condition for TE mode $m > 0$ in a symmetric slab waveguide with $n_1 \approx n_2$ is approximately $\lambda_o^2 \approx 8n_1\Delta n d^2/m^2$, where $\Delta n = n_1 - n_2$.

Modes of a Rectangular Dielectric Waveguide. A rectangular dielectric waveguide has a square cross section of area 10^{-2} mm^2 and numerical aperture NA = 0.1. Use (7.3-3) to plot the number of TE modes as a function of frequency ν. Compare your
6. results with Fig. 7.2-4.

Modes. A step-index fiber has radius $a = 5$ μm, core refractive index $n_1 = 1.45$, and fractional refractive-index change $\Delta = 0.002$. Determine the shortest wavelength λ_c for which the fiber is a single-mode waveguide. If the wavelength is changed to $\lambda_c/2$,
7. identify the indices (l, m) of all the guided modes.

Propagation Constants and Wavevector (Step-Index Fiber). A step-index fiber of radius $a = 20$ μm and refractive indices $n_1 = 1.47$ and $n_2 = 1.46$ operates at $\lambda_o = 1.55$ μm. Using the quasi-plane wave theory and considering only guided modes with azimuthal index $l = 1$:
8. (a) Determine the smallest and largest propagation constants.

Modal Dispersion in Step-Index Fibers. Determine the core radius of a multimode step-index fiber with a numerical aperture NA = 0.1 if the number of modes $M = 5000$ when the wavelength is 0.87 μm. If the core refractive index $n_1 = 1.445$, the group index $N_1 = 1.456$, and Δ is approximately independent of wavelength,
9. determine the modal-dispersion response time σ_τ for a 2-km fiber.