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1\.

Amplification of a Broadband Signal. The transition between two energy levels exhibits a Lorentzian lineshape of central frequency $\nu_0 = 5 \times 10^{14}$ with a linewidth $\Delta\nu = 10^{12}$ Hz. The population is inverted so that the maximum gain coefficient $\gamma(\nu_0) = 0.1\ \text{cm}^{-1}$. The medium has an additional loss coefficient $\alpha_s = 0.05\ \text{cm}^{-1}$, which is independent of ν. Approximately how much loss or gain is encountered by a light wave in 1 cm if it has a uniform power spectral density centered about ν_0 with a bandwidth $2\Delta\nu$?

$\gamma(\nu)$ is also Lorentzian. $\gamma(\nu) = \gamma(\nu_0)\dfrac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2+(\Delta\nu/2)^2}$

$g = \dfrac{\int I(\nu)\gamma(\nu)\,d\nu}{\int I(\nu)\,d\nu}$, since $I(\nu) = I_0$ for $\nu_0-\Delta\nu < \nu < \nu_0+\Delta\nu$ and $I(\nu)=0$

$\Rightarrow g = \dfrac{\pi}{2}\Delta\nu\,\dfrac{\gamma(\nu_0)}{2\Delta\nu}$

$= \dfrac{\pi}{4}\gamma(\nu_0)$

$g_{net} = g - \alpha_s = \dfrac{\pi}{4}\cdot 0.1\ \text{cm}^{-1} - 0.05\ \text{cm}^{-1} = 0.0285\ \text{cm}^{-1}$

2\. **The Two-Level Pumping System.** Write the rate equations for a two-level system, showing that a steady-state population inversion cannot be achieved by using direct optical pumping between levels 1 and 2.

Hint: R2=-R1=R

$\dfrac{dN_2}{dt} = R_2 - \dfrac{N_2}{\tau_2}$

$\dfrac{dN_1}{dt} = -R_1 - \dfrac{N_1}{\tau_1} + \dfrac{N_2}{\tau_2}$

$\dfrac{dN_1}{dt} = -\dfrac{dN_2}{dt}$. suppose N_2 is the higher level. $N_2 = \dfrac{g_1 T(\frac{6}{\varepsilon}+6\phi)N}{1+(g_1+g_2)(\frac{6}{\varepsilon}+6\phi)T}$

since $N_2 > N_1$, $N_2+N_1 = N \Rightarrow N_2 > \frac{N}{2}$ $\Rightarrow$ let $A = T(\frac{6}{\varepsilon}+6\phi) \Rightarrow \dfrac{g_1 A}{1+(g_1+g_2)A} > \dfrac{1}{2}$

$\Rightarrow 2g_1 A > 1 + g_1 A + g_2 A$

$g_1 A > 1 + g_2 A$

which cannot be achived.

3. **Resonant Absorption of a Medium in Thermal Equilibrium.** A unity refractive index medium of volume 1 cm^3 contains $N_a = 10^{23}$ atoms in thermal equilibrium. The ground state is energy level 1; level 2 has energy 2.48 eV above the ground state ($\lambda_o = 0.5\ \mu m$). The transition between these two levels is characterized by a spontaneous lifetime $t_{sp} = 1$ ms, and a Lorentzian lineshape of width $\Delta\nu = 1$ GHz. Consider two temperatures, T_1 and T_2, such that $k_B T_1 = 0.026$ eV and $k_B T_2 = 0.26$ eV.

(a) Determine the populations N_1 and N_2.
(b) Determine the number of photons emitted spontaneously every second.
(c) Determine the attenuation coefficient of this medium at $\lambda_o = 0.5\ \mu m$ assuming that the incident photon flux is small.

Hint: there's no pumping, such that, N_0 is negative, which leads to gain coefficient becomes attenuation one.

(a). Since the system is in thermal equlibrium, we can get

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} \qquad E_2 - E_1 = 2.48\,eV$$

at T_1 $\Rightarrow \frac{N_2}{N_1} = e^{-(2.48/0.026)} = 3.76\times10^{-42}$

Also, $N_1 + N_2 = N_a = 10^{23}$

$N_2 + (3.67\times10^{-42} N_2) = 10^{23} \Rightarrow N_2 \approx 0,\ N_1 = 10^{23}$

at T_2: $\frac{N_2}{N_1} = e^{-2.48/0.26} = 8.3\times10^{-5}$

$\Rightarrow N_1 + 8.3\times10^{-5} N_1 = N_a = 10^{23}$

$\Rightarrow N_1 = 9.99\times10^{22},\ N_2 = 8.3\times10^{18}$

(b) $\frac{N_2}{t_{sp}} = \frac{8.3\times10^{18}}{1\times10^{-3}s} = 8.3\times10^{21}\ s^{-1}$

(c) $\alpha = \frac{h\nu}{c}(N_1 B_{12} - N_2 B_{21}) \qquad B_{21} = \frac{c^3}{2h\nu^3}A_{21} = \frac{c^2}{2h\nu^3} = \frac{c^2\lambda_0^3}{2h\ c^3} = \frac{\lambda_0^3}{2hc} = \frac{125\times10^{-19}}{2\times6.6\times10^{-34}\times3\times10^8}$

$= 3.15\times10^5$

$$\alpha = \frac{h}{\lambda_0}(9.99\times10^{22} - 8.3\times10^{18})B_{21}$$

$$= \frac{6.6\times10^{-34}}{5\times10^{-7}}\times3.15\times10^5\cdot9.98\times10^{22}$$

$$= 4.6\ m^{-1}$$

at T_1, $\alpha = \frac{hc}{\lambda_0 c}(N_1 B_{12})$ since $N_2 \approx 0$ $\quad \alpha = \frac{6.6\times10^{-34}}{5\times10^{-7}}\times3.15\times10^5\times10^{23} = 41.6\ m^{-1}$

Number of Longitudinal Modes. An Ar^+-ion laser has a resonator of length 100 cm. The refractive index $n = 1$.
(a) Determine the frequency spacing ν_F between the resonator modes.
(b) Determine the number of longitudinal modes that the laser can sustain if the FWHM Doppler-broadened linewidth is $\Delta\nu_D = 3.5$ GHz and the loss coefficient is half the peak small-signal gain coefficient.

4.

(a) $V_F = \frac{c}{2nL}$ $L = 1m,\ n = 1$ $\Rightarrow V_F = \frac{3\times10^8}{2} = 1.5\times10^8 Hz = 0.15 GHz$

(b) $N = \frac{\Delta V_D}{V_F} = \frac{3.5GHz}{0.15GHz} = 23.33$ modes

Threshold Population Difference for an Ar^+-Ion Laser. An Ar^+-ion laser has a 1-m-long resonator with 98% and 100% mirror reflectances. Other loss mechanisms are negligible. The atomic transition has a central wavelength $\lambda_o = 515$ nm, spontaneous lifetime $t_{sp} = 10$ ns, and linewidth $\Delta\lambda = 0.003$ nm. The lower energy level has a very short lifetime and hence zero population. The diameter of the oscillating mode is 1 mm. Determine (a) the photon lifetime and (b) the threshold population difference for laser action.

5.

(a) $\tau_p = \frac{1}{F(1-S)} = \frac{1}{\frac{c}{2nd}(1-S)} = \frac{2nd}{c(1-S)} = \frac{2\cdot n\cdot d}{c\cdot(1-R_1R_2)} = \frac{2\cdot1\cdot1}{3\times10^8(1-0.98\cdot1)} = 3.3\times10^{-7} s$

(b) $N_2 - N_1 = \frac{1}{c\cdot\tau_p\cdot\sigma}$

For Ar^+ Ion, $\sigma = 3\times10^{-12}$ $\Rightarrow N_2 - N_1 = \frac{1}{3\times10^8\cdot3.3\times10^{-7}\times3\times10^{-12}}$

$= 3.37\times10^9\ m^{-3}$

Rate Equations in a Four-Level Laser. Consider a four-level laser with an active volume $V = 1\ cm^3$. The population densities of the upper and lower laser levels are N_2 and N_1 and $N = N_2 - N_1$. The pumping rate is such that the steady-state population difference N in the absence of stimulated emission and absorption is N_0. The photon-number density is n and the photon lifetime is τ_p. Write the rate equations for N_2, N_1, N, and n in terms of N_0, the transition cross section $\sigma(\nu)$, and the times t_{sp}, τ_1, τ_2, τ_{21}, and τ_p. Determine the steady state values of N and n.

6.

1° $N = N_2 - N_1$, $N_0 = N\cdot n = (N_2 - N_1)n$

2° $\tau_p = \frac{n}{N_0}$ life time

3° transition cross section $\sigma(\nu) = V\cdot n = 1cm^3\cdot n$

4°

$$\tau_{sp} = \frac{\tau_p}{n} = \frac{1}{N_0}$$

$$\tau_1 = \frac{N_1}{n}$$

$$\tau_2 = \frac{N_2}{n}$$

$$\tau_{21} = \frac{(N_2 - N_1)}{n}$$

$$\tau_p = \frac{n}{N_0}$$