Homework 5

1. **Electron–Hole Recombination Under Strong Injection.** Consider electron–hole recombination under conditions of strong carrier-pair injection such that the recombination lifetime can be approximated by $\tau = 1/\mathfrak{r}\,\Delta n$, where $\mathfrak{r}$ is the recombination parameter of the material and Δn is the injection-generated excess carrier concentration. Assuming that the source of injection R is set to zero at $t = t_0$, find an analytic expression for $\Delta n(t)$, demonstrating that it exhibits power-law rather than exponential behavior.

2. ***Quasi-Fermi Levels of a Pumped Semiconductor***

(a) Under ideal conditions at $T = 0$ K, when there is no thermal electron–hole pair generation [see Fig. 16.1-3(*a*)], show that the quasi-Fermi levels are related to the concentrations of injected electron–hole pairs Δn by

$$E_{fc} = E_c + (3\pi^2)^{2/3}\frac{\hbar^2}{2m_c}(\Delta n)^{2/3} \tag{16.1-8a}$$

$$E_{fv} = E_v - (3\pi^2)^{2/3}\frac{\hbar^2}{2m_v}(\Delta n)^{2/3}, \tag{16.1-8b}$$

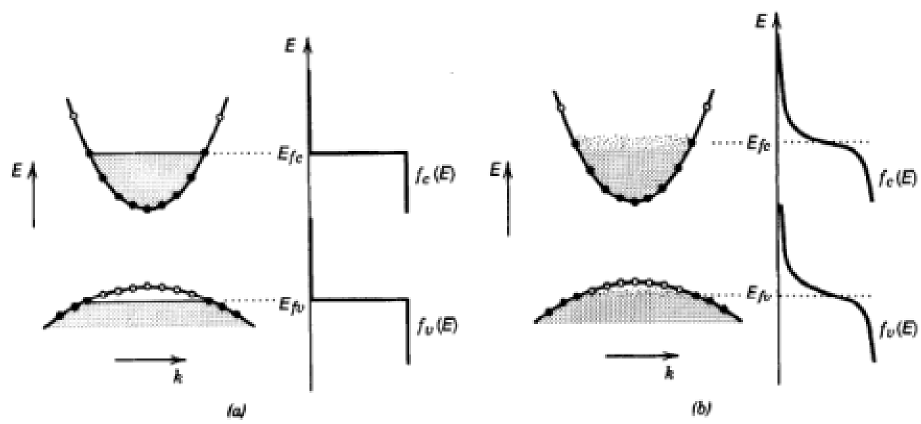

Figure 16.1-3 Energy bands and Fermi functions for a semiconductor in quasi-equilibrium (*a*) at $T = 0$ K, and (*b*) at $T > 0$ K.

so that

$$E_{fc} - E_{fv} = E_g + (3\pi^2)^{2/3}\frac{\hbar^2}{2m_r}(\Delta n)^{2/3}, \tag{16.1-8c}$$

where $\Delta n \gg n_0, p_0$. Under these conditions all Δn electrons occupy the lowest allowed energy levels in the conduction band, and all Δp holes occupy the highest allowed levels in the valence band. Compare with the results of Exercise 15.1-2.

(b) Sketch the functions $f_e(\nu)$ and $r_{sp}(\nu)$ for two values of Δn. Given the effect of temperature on the Fermi functions, as illustrated in Fig. 16.1-3(*b*), determine the effect of increasing the temperature on $r_{sp}(\nu)$.

Spectral Density of Injection Electroluminescence Under Weak Injection. For sufficiently weak injection, such that $E_c - E_{fc} \gg k_B T$ and $E_{fv} - E_v \gg k_B T$, the Fermi functions may be approximated by their exponential tails. Show that the luminescence rate can then be expressed as

$$\boxed{r_{sp}(\nu) = D(h\nu - E_g)^{1/2} \exp\left(-\frac{h\nu - E_g}{k_B T}\right), \qquad h\nu \geq E_g,} \tag{16.1-9a}$$

where

$$D = \frac{(2m_r)^{3/2}}{\pi \hbar^2 \tau_r} \exp\left(\frac{E_{fc} - E_{fv} - E_g}{k_B T}\right) \tag{16.1-9b}$$

is an exponentially increasing function of the separation between the quasi-Fermi levels $E_{fc} - E_{fv}$. The spectral density of the spontaneous emission rate is shown in Fig. 16.1-4; it has precisely the same shape as the thermal-equilibrium spectral density shown in Fig. 15.2-9, but its magnitude is increased by the factor $D/D_0 = \exp[(E_{fc} - E_{fv})/k_B T]$, which can be very large in the presence of injection. In thermal equilibrium $E_{fc} = E_{fv}$, so that (15.2-20) and (15.2-21) are recovered.

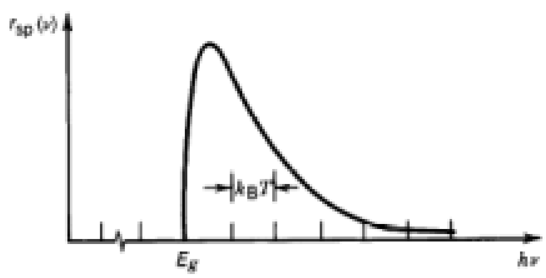

Figure 16.1-4 Spectral density of the direct band-to-band injection-electroluminescence rate $r_{sp}(\nu)$ (photons per second per hertz per cm^3), versus $h\nu$, from (16.1-9), under conditions of weak injection.

3.

Electroluminescence Spectral Linewidth

(a) Show that the spectral density of the emitted light described by (16.1-9) attains its peak value at a frequency ν_p determined by

$$\boxed{h\nu_p = E_g + \frac{k_B T}{2}.} \tag{16.1-10}$$

Peak Frequency

(b) Show that the full width at half-maximum (FWHM) of the spectral density is

$$\boxed{\Delta\nu \approx \frac{1.8 k_B T}{h},} \tag{16.1-11}$$

Spectral Width (Hz)

(c) Show that this width corresponds to a wavelength spread $\Delta\lambda \approx 1.8\lambda_p^2 k_B T/hc$, where $\lambda_p = c/\nu_p$. For $k_B T$ expressed in eV and the wavelength expressed in μm, show that

$$\Delta\lambda \approx 1.45\lambda_p^2 k_B T. \tag{16.1-12}$$

4. (d) Calculate $\Delta\nu$ and $\Delta\lambda$ at $T = 300$ K, for $\lambda_p = 0.8$ μm and $\lambda_p = 1.6$ μm.

5. **Transit Time.** Referring to Fig. 17.1-3, assume that a photon generates an electron–hole pair at the position $x = w/3$, that $v_e = 3v_h$ (in semiconductors v_e is generally larger than v_h), and that the carriers recombine at the contacts. For each carrier, find the magnitudes of the currents, i_h and i_e, and the durations of the currents, τ_h and τ_e. Express your results in terms of e, w, and v_e. Verify that the total charge induced in the circuit is e. For $v_e = 6 \times 10^7$ cm/s and $w = 10$ μm, sketch the time course of the currents.