

Ve501 Probability and Random Processes

2021 Fall

Homework 1

Due: October 12, 2021, in the class

Submission Instructions

1. Follow the JI Honor Policies.
2. Write down the key intermediate steps, instead of simply giving the final answers.
3. Submit your homework in A4 papers. Neat and tidy handwriting is allowed.
4. No late homework submission is allowed.

1. Let $\{F_i, i = 1, 2, \dots, N\}$ be a *finite partition* of Ω , i.e., $F_i \cap F_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^N F_i = \Omega$. Show that any $G \subset \Omega$ can be written as $G = \bigcup_{i=1}^N H_i$, where $H_i = G \cap F_i$. Furthermore, show that $\{H_i, i = 1, 2, \dots, N\}$ are pairwise disjoint, i.e., $H_i \cap H_j = \emptyset$ for $i \neq j$. Thus, $\{H_i, i = 1, 2, \dots, N\}$ partitions G .

2. Let X be any set, and let $A \subset X$. Define the real-valued function f by

$$f(x) := \begin{cases} 1, & x \in A, \\ 0, & x \notin A. \end{cases}$$

Thus, $f: X \rightarrow \mathbb{R}$, where $\mathbb{R} := (-\infty, \infty)$ denotes the real numbers. For arbitrary $B \subset \mathbb{R}$, find $f^{-1}(B)$. Hint: there are four cases to consider, depending on whether 0 or 1 belong to B .

3.(1) Let $\Omega = [0, 1]$ be a sample space and let $B = \left\{ \left[0, \frac{1}{2}\right], \left[\frac{1}{2}, 1\right] \right\}$ denote a collection

of two events. Find $\sigma(B)$ and the σ -algebra generated by B .

(2) A letter of the alphabet ($a - z$) is generated at random. Please specify a complete probability model, compute the probability that a vowel (a, e, i, o, u) is generated and the probability that a single letter a is generated based on your model.

4. You and your neighbor attempt to use your cordless phones at the same time. Your phones independently select one of ten channels at random to connect to the base unit. What is the probability that both phones pick the same channel?

5. The probability that a cell in a wireless system is overloaded is p . Given that it is overloaded, the probability of a blocked call is q . Given that it is not overload, the probability of a blocked call is r , where $0 < p, q, r < 1$. Find the conditional probability that the system is overloaded given that the call is blocked.