Ve501 Probability and Random Processes

2021 Fall

Homework 3

Due: November 9, 2021, in the class

Submission Instructions

1. Follow the JI Honor Policies.
2. Write down the key intermediate steps, instead of simply giving the final answers.
3. Submit your homework in A4 papers. Neat and tidy handwriting is allowed.
4. No late homework submission is allowed.

1. Let X and Y be ternary random variables taking values 1, 2 and 3 with joint probabilities $p_{XY}(i,j)$ given by the matrix

$$\begin{bmatrix} \frac{1}{8} & 0 & \frac{1}{8} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{8} & 0 & \frac{1}{8} \end{bmatrix}$$

(a) Find $p_X(i)$ and $p_Y(j)$ for $i,j = 1,2,3$.
(b) Compute $P(X < Y)$.
(c) Determine whether or not X and Y are independent.

2. A rectangle of width W and height H is drawn where W is a uniform random variable on $(0,\omega]$ and H is a uniform random variable on $(0,h]$. For $0 < \lambda < 1$, find the probability that the area of the rectangle exceeds λ times the maximum possible area. Assume that W and H are independent.

3. Use the law of total probability to solve the following problems.
(a) Evaluate $E[\cos(X+Y)]$ if given $X = x$, Y is conditionally uniform on $[x-\pi, x+\pi]$.
(b) Evaluate $P(Y > y)$ if $X \sim \text{uniform}[1,2]$, and given $X = x$, Y is exponential with parameter x.
(c) Evaluate $E[Xe^Y]$ if $X \sim \text{uniform}[3,7]$, and given $X = x$, $Y \sim N(0,x^2)$.
(d) Let $X \sim \text{uniform}[1,2]$, and suppose that given $X = x$, $Y \sim N\left(0,\frac{1}{x}\right)$. Evaluate $E[\cos(XY)]$.

4. Find $P(X \leq Y)$ if X and Y are independent with $X \sim \exp(\lambda)$ and $Y \sim \exp(\mu)$.

5. Use the Auxiliary RV approach to derive an expression for the PDF of Z if $Z = 2X + Y$, assuming X and Y have some joint PDF, $f_{XY}(x, y)$. For dummy variable, let $W = Y$. You may leave your answer in terms of an integral.