Ve501 Probability and Random Processes

2021 Fall

Homework 6

Due: December 14, 2021, in the class

Submission Instructions

1. Follow the JI Honor Policies.
2. Write down the key intermediate steps, instead of simply giving the final answers.
3. Submit your homework in A4 papers. Neat and tidy handwriting is allowed.
4. No late homework submission is allowed.

1. Let $\{X_n : n \geq 1\}$ be an i.i.d. process with Poisson marginal PMF $p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k \geq 0$, and let $N_i = \sum_{k=1}^{l} X_k$and where $N_0 = 0$. Is $\{N_l\}_{l\geq 0}$ a Markov process? Explain in detail.

2. Let $X[n]$ be a Markov chain on $n \geq 0$ taking values 1 and 2 with one-step transition probability matrix

$$\Pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

and state probability vector $\pi^{(n)} = \left[P([X[n] = 1]),\quad P([X[n] = 2])\right]$.

(a) Show that $\pi^{(n)} = \pi^{(0)}\Pi^n$.

(b) Draw a two-state transition diagram and label the branches with the one-step transition probabilities.

(c) Given that $X[0] = 1$, find the probability that the first transition to state 2 occurs at time n.

3. Sketch the state transition diagrams for the Markov Chains (MCs) with the following transition probability matrices. Classify the states of the Markov Chains.

(a) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$.

(b) $\begin{bmatrix} 0 & 0 & 1/3 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.