Ve501 Probability and Random Processes

2021 Fall

Homework 2

Due: October 21, 2021, in the class

Submission Instructions

1. Follow the JI Honor Policies.
2. Write down the key intermediate steps, instead of simply giving the final answers.
3. Submit your homework in A4 papers. Neat and tidy handwriting is allowed.
4. No late homework submission is allowed.

1. Find $VarX$ if X has probability generating function

$$G_X(z) = \frac{1}{6} + \frac{1}{6}z + \frac{2}{3}z^2.$$

2. A random variable X has generalized density

$$f_X(x) = \frac{1}{3}e^{-x}u(x) + \frac{1}{2}\delta(x) + \frac{1}{6}\delta(x-1),$$

where u is the unit step function and δ is the Dirac delta function.
(a) Sketch $f_X(x)$.
(b) Compute $P_r(X = 0)$ and $P_r(X = 1)$.
(c) Compute $P_r(0 < X < 1)$ and $P_r(X > 1)$.
(d) Use the above results to compute $P_r(0 \le X \le 1)$ and $P_r(X \ge 1)$.
(e) Compute $E[X]$.
(f) Find and sketch the cumulative distribution function $F_X(x)$.

3. If $X \sim uniform(0,1)$, show that $Y = \ln(1/X) \sim \exp(1)$ by finding its moment generating function for $s < 1$.

4. Compute $E[\sqrt{X}]$ if X has cdf

$$F_X(x) := \begin{cases} 0, & x < 0 \\ \sqrt{x}/4, & 0 \le x < 4 \\ (x+11)/20, & 4 \le x < 9 \\ 1, & x \ge 9 \end{cases}.$$

5. Suppose a RV X has a mean and variance of 3 and 9, respectively. Let $Z = 3X - 2$. Give the mean and variance of Z.

6. Use the characteristic function of the Gamma PDF, $\Phi_X(\omega) = (1 - j\omega\alpha)^{-\beta}$, to compute its second moment.