Analysis of Efficient Portfolio Construction and Market Models
ECO358 Assignment 2
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Introduction

In this analysis, we look at the intricate details of portfolio optimization in Canada's thriving financial market, concentrating on a few notable companies: Dollarama, Loblaw, and Canadian Tire. Using the S&P/TSX as a market benchmark, this study uses the Capital Asset Pricing Model (CAPM) to compute betas and estimate the cost of capital for these companies. Our goal is to build an efficient portfolio while negotiating the complex balance of risk and reward. By discovering the tangent portfolio that maximizes the Sharpe ratio, we hope to provide strategic insights into generating the optimal risk-adjusted returns while capturing the ideas of current portfolio theory in a dynamic market environment.

1.Beta and Cost of capital

a) For the analysis, I sourced five years of historical data for Dollarama, Loblaw, Canadian Tire, and the S&P/TSX index from March 20, 2019, to March 20, 2024, through Yahoo Finance. The data, downloaded in '.xls' format, provides the basis for calculating daily returns and applying the CAPM framework in our subsequent analyses.

b) Daily Returns Calculation For All
We used the formula:

$$R_i = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where P_t is the stock price on day t and P_t-1 is the stock price on the previous day.
We can use excel to automate the calculation for each trading day.

c) Beta Computation

	DOL-TSX	CTC-TSX	LOB-TSX	TSX-TSX
Covariance-TSX	0.01%	0.01%	0.01%	0.01%
Beta	0.59	1.07	0.51	1.00
Expected Returns	25.60%	6.55%	20.75%	7.74%
Volatility	23.97%	30.63%	20.89%	18.22%

For the beta computation, we calculated the covariance between each stock's daily returns and the S&P/TSX index's daily returns, and then divided that by the variance of the S&P/TSX index's daily returns. This approach yields the beta for each stock, reflecting its sensitivity to market movements. A beta greater than 1 indicates that the stock is more volatile than the market, while a beta less than 1 suggests it is less volatile. This metric is crucial for understanding the risk profile of each investment relative to the market as a whole.

d) Volatility-Return Graph

For our analysis, the daily expected returns were annualized by multiplying by 252, the number of trading days in a year. Similarly, to annualize volatility, which represents the standard deviation of returns, we multiplied the daily volatility by the square root of 252.
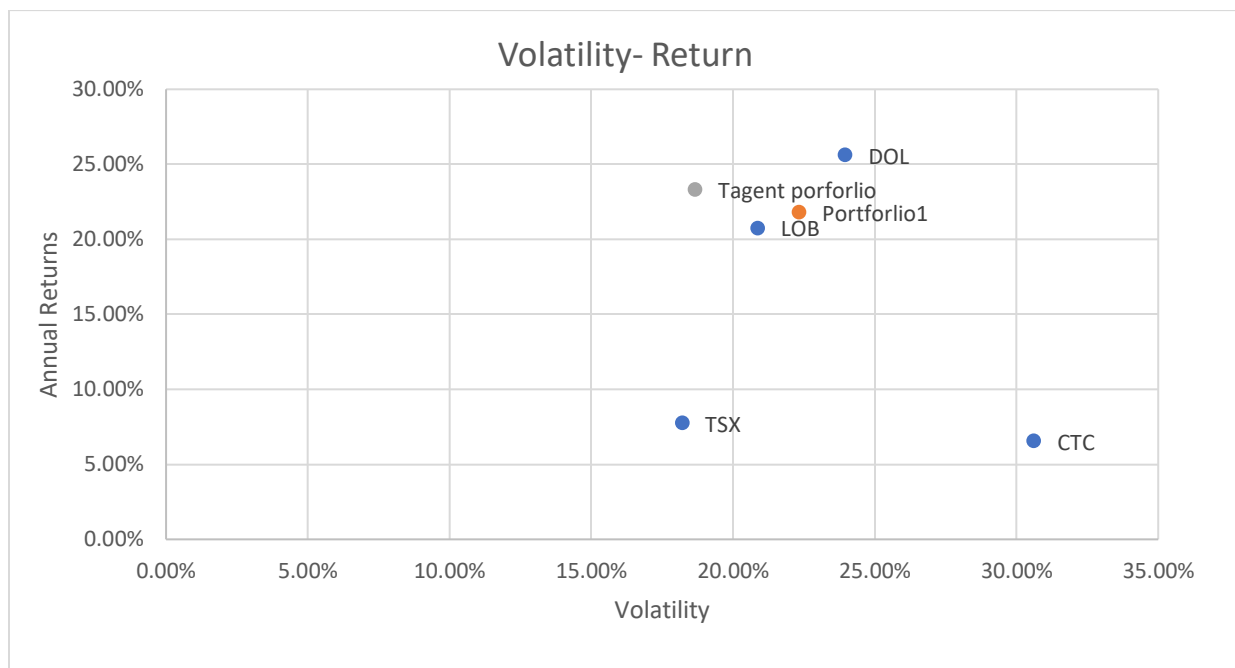
(Two points of portfolio that will be introduced later are merged in this plot)

e) CAPM

The Capital Asset Pricing Model (CAPM) formula is expressed as:

$$\text{Expected Return} = Rf + \beta \times (Rm - Rf)$$

(The risk-free rate used is the 3-5 year average yield of marketable bonds up to March 20, 2024, which is 3.57%.)

By applying this formula, we can estimate the cost of capital for Dollarama, Loblaws, and Canadian Tire, taking into account the risk-free rate of 3.57%, each stock's beta, and the expected market return derived from the S&P/TSX index.

	DOL	CTC	LOB
CAPM	6.04%	7.91%	6.59%

This cost of capital reflects the expected return necessary to compensate investors for the risk associated with each stock, compared to the risk-free alternative.

f) The intuition behind CAPM and the differences with DDM and DFCFM.

The CAPM provides a theoretical framework that relates the expected return of an investment to its risk, as compared to the overall market. The cost of capital calculated using the CAPM formula reflects the expected rate of return that compensates investors for bearing the risk of the investment, over and above the return on a risk-free asset.
When comparing the CAPM with other valuation models like the Dividend Discount Model (DDM) and the Discounted Free Cash Flow Model (DFCFM), CAPM stands out by directly linking the expected return to market risk (beta), offering a market-oriented perspective. While DDM focuses on expected dividends to value a stock and DFCFM considers the present value of future cash flows, CAPM uniquely incorporates the systematic risk component. Consequently, CAPM offers a broader market-based perspective on the cost of capital, which is especially relevant for investors diversifying their portfolios to mitigate unsystematic risk.

2. Create a portfolio and estimate its expected return and its volatility

a) Calculate the correlation and covariance matrices between our stocks

Correlation	Returns DOL	Returns CTC	Returns LOB
Returns DOL	100.000%	40.192%	36.782%
Returns CTC	40.192%	100.000%	30.682%
Returns LOB	36.782%	30.682%	100.000%

Covariance	Returns DOL	Returns CTC	Returns LOB
Returns DOL	0.023%	0.012%	0.007%
Returns CTC	0.012%	0.037%	0.008%
Returns LOB	0.007%	0.008%	0.017%

We calculated the correlation and covariance matrices between these stocks to understand the relationships and the degree of movement they share with each other. This step is crucial in portfolio construction as it helps in diversifying risk. Correlation measures the strength and direction of the relationship between the stock returns, whereas covariance assesses how the returns move together. By utilizing Excel's built-in functions, **CORREL** for correlation and **COVARIANCE.P** for covariance, we efficiently generated these matrices, providing a quantitative basis for understanding how the selected stocks might perform together under different market conditions.

- **Interpretation of Covariance**: If Dollarama and Loblaws have a high positive covariance, it suggests that when Dollarama's returns are above its average, Loblaw's returns are likely above its average as well, and vice versa. Conversely, a negative covariance between Dollarama and Canadian Tire would imply that when Dollarama's returns are above its average, Canadian Tire's returns are likely below its average.
- **Interpretation of Correlation**: A high positive correlation between Dollarama and Loblaws(close to 1) would mean their returns move strongly in the same direction. A negative correlation between Dollarama and Canadian Tire (close to -1) suggests their returns move in opposite directions. A correlation near 0 between any two stocks indicates that their returns are not linearly related.

b) Create 'Portfolio1' and compute the portfolio's expected returns, $E[RP]$, and its volatility, $SD[RP]$.

Portforlio1:

Name	Weights
DOL	80%
CTC	20%
LOB	0%
Sum	100%

	Returns DOL	Returns CTC	Returns LOB
Expected Daily Returns	0.10%	0.03%	0.08%

After calculating the correlation and covariance matrices, we constructed "Portfolio 1" by selecting specific weights for each stock.
The expected return of Portfolio 1 ($E[Rp]$) was calculated as a weighted average of the individual expected returns of the selected stocks.
Mathematically, the portfolio's expected return can be expressed as:

$$E[Rp]=w1E[R1]+w2E[R2]+w3E[R3]$$

Outcome:

Portfolio Daily Expected Returns	0.0865%

Then we annualize it by multiplying 252:

Portfolio Annual Expected return	21.79%

To calculate the portfolio's volatility (σp), we can apply a method that involves using the covariance matrix and the portfolio weights.
The mathematical representation of this process is:

$$\sigma_p = \sqrt{\mathbf{w}^T \mathbf{C} \mathbf{w}}$$

where:
- σ_p is the portfolio volatility,
- **w** is the column vector of portfolio weights for each asset,
- **C** is the covariance matrix of asset returns, and
- **w^***T* is the transpose of the weight vector.

We can do this simply in excel by one line code:
'=SQRT(MMULT(TRANSPOSE(H10:H12),MMULT(C10:E12,H10:H12)))'

Outcome:

Portfolio Daily StDev	1.4077%

Then we annualize it by multiplying 252:

Portfolio Annual StDev	22.35%

Interpretation:

A portfolio standard deviation of 22.35% represents the total risk of the portfolio in terms of the variability of its returns. This figure indicates that, over the period measured, the returns of the portfolio deviated from the average return by 22.35% in either direction.
In practical terms, a higher standard deviation suggests a higher level of risk, as the returns are more spread out over a wider range of outcomes, implying that the portfolio's return can be quite unpredictable. Conversely, a lower standard deviation would indicate that the returns are more tightly clustered around the average, suggesting less risk and more predictability in the returns you might expect from the portfolio.

c) Compute this portfolio's Beta.

The portfolio Beta (βp) was calculated using a weighted average of the individual Betas of Dollarama, Loblaws, and Canadian Tire, with the formula:

$$\beta_p = w_D\beta_D + w_L\beta_L + w_C\beta_C$$

Outcome:

Portfolio Beta	68.919%

Interpretation:

- **Relative Volatility**: The portfolio's price movements are expected to be about 68.9% as volatile as the market's movements. If the market increases or decreases, the portfolio is expected to increase or decrease by 68.9% of that market change, on average.
- **Risk Profile**: This beta value places the portfolio in a lower-risk category compared to the market index, which typically has a beta of 1. It would be considered more conservative and possibly suitable for risk-averse investors.

d) Add this portfolio to the volatility-returns graph from Q1

Please see graph in Q1 d.

f) Bonus: Find the portfolio allocation that minimizes variance for some given expected return.

Outcome Return	Outcome StDev
8.00%	28.21%
10.00%	25.20%
12.00%	22.64%
15.00%	19.98%
16.00%	19.35%
17.00%	18.82%
18.00%	18.42%
19.00%	18.15%
20.00%	18.01%
25.00%	22.07%

I select 10 expected returns which are shown above.

I created a solver with a set return and total weights and require it to minimize variance by changing weights to find Stdev and weights for each of them.

Detailed calculations and outcomes can be found in my Excel file.

Plot:

effcient frontier
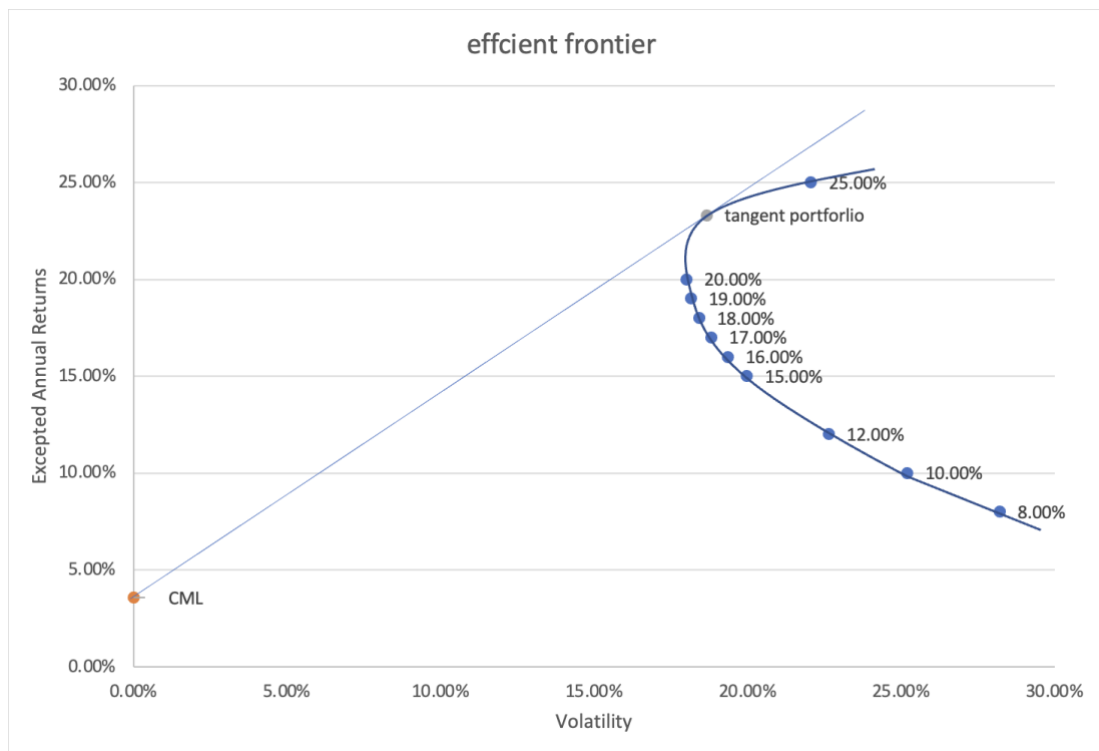
3. Find the tangent portfolio

a) Create a solver that maximizes the Sharpe Ratio by changing the weights of the portfolio.

Outcome of solver:

Optimized Weights	
DOL	52.59%
CTC	0.00%
LOB	47.41%
Sum	100.00%

Annual Expected Return	23.30%
Annual SdDev	18.67%

The Sharpe Ratio is calculated using the formula:

$$\text{Sharpe Ratio} = \frac{E[R_p] - R_f}{\sigma_p}$$

Where $E[Rp]$ is the expected return of the portfolio, R_f is the risk-free rate, and σp is the portfolio's standard deviation (volatility)

Outcome:

Sharp ratio	105.68%

Interpretation:

A tangent portfolio with a Sharpe ratio of 105.68% suggests an exceptional level of excess return per unit of risk, indicating that the portfolio is performing very well compared to the risk taken. This high Sharpe ratio implies that the portfolio's risk-adjusted returns are significantly above average, potentially making it an attractive option for investors who seek to maximize their returns for a given level of risk.

b) Find the tangent portfolio's Beta.

We can simply apply the same function we used in Q2 (c) to get tangent portfolio Beta again.

Outcome:

tangent portfolio beta	55.58%

Interpretation:

A beta of 55.58% indicates the portfolio is less volatile than the overall market. It suggests a conservative risk profile, with the portfolio experiencing less than proportional changes compared to market movements. This could be appealing to risk-averse investors looking for more stable returns.

c) Add the tangent portfolio to your volatility-return graph. How far is it from the index volatility-return profile you've found in Q1?

See graph in Q1 (d)

With a volatility of 18.67% and an annual return of 23.30%, the tangent portfolio is positioned as a point that has lower volatility compared to DOL, LOB and CTC but slightly higher than the TSX index. Its return is higher than all the individual stocks and the index, making it the most attractive option on the graph in terms of the risk-return trade-off.

d) Comment on the differences between your portfolio from Q2, this tangent portfolio, and the index you chose.

The Tangent Portfolio, with a beta of 0.5558, stands out for its superior risk-adjusted performance, offering the highest returns at 23.30% with lower volatility compared to both Portfolio 1 and the S&P/TSX index. Portfolio 1 shows a higher return than the market index at 21.79% but with moderately higher risk, indicated by its beta of 0.689 and higher volatility. The S&P/TSX index serves as the market standard with a beta of 1 and returns of 7.74%, demonstrating the benefits of targeted portfolio optimization in achieving returns that exceed the broader market while managing risk effectively.

4. Plot the Capital Market Line and Securities Market Line

a) Volatility-Return graph


Portforlio Optimization and CML
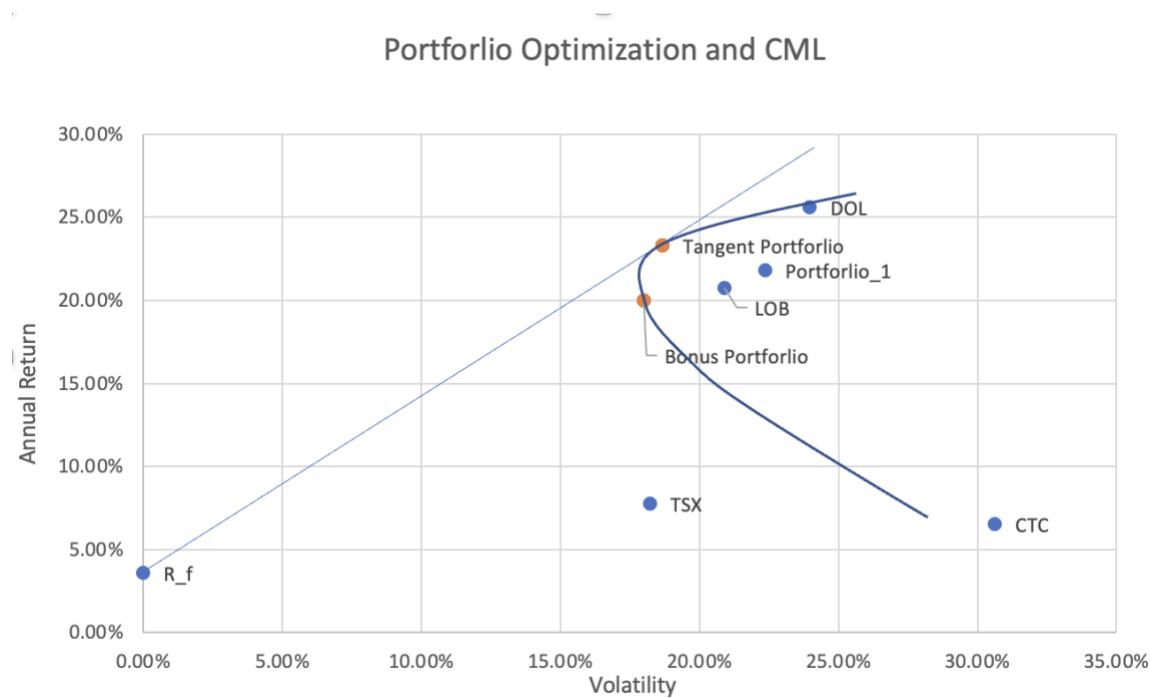
Note that our tangent portfolio lies above the market portfolio. This suggests that it has a higher expected return for a given level of risk. In theory, the tangent point on the efficient frontier in the CAPM context should be the market portfolio, which includes all investable assets. However, in practice, a portfolio manager can construct a portfolio that appears to be more efficient than the market portfolio, meaning it has a higher expected return for its level of risk, thereby shifting the tangent point. In our context, the market index reflects the broad market, while our tangent portfolio is concentrated factors that are expected to outperform the market based on certain risk premiums. We have previously maximized its Sharpe ratio using a solver.

It's also important to note that while a portfolio may currently appear to offer a better risk-return trade off than the market portfolio, it does not guarantee that this will persist in the future.

Market conditions, economic factors, and other variables can change, which may affect the portfolio's performance relative to the market index.

b) Beta-Return graph

SML
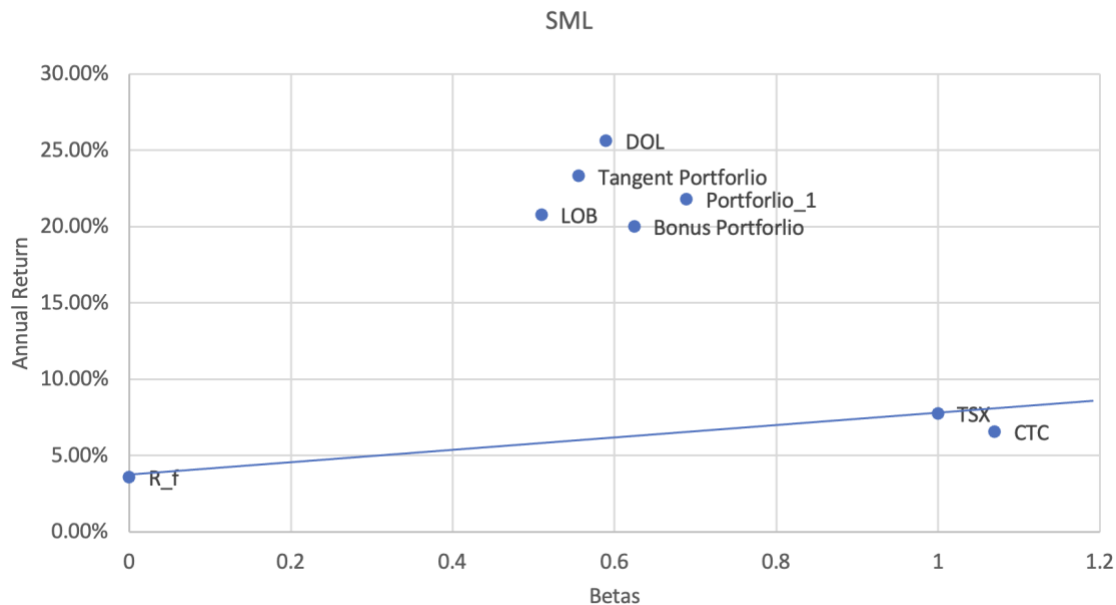
c) According to the Beta-Return graph above, points of my stocks, Portfolio 1, and the Tangent Portfolio do not fall on the Security Market Line. There are several reasons for this:

Non-Systematic Risk: The SML reflects only systematic risk (market risk), which is captured by beta. If there are other forms of risk affecting the assets (like specific company risk or industry risk), these won't be reflected by the SML but can influence returns.
Beta Inaccuracy: Beta is a historical measure and may not perfectly predict future market sensitivity. If the market conditions have changed or if the beta is calculated over a different time horizon than the return, this can lead to discrepancies.
Market Inefficiency: The SML assumes market efficiency. If some assets are overvalued or undervalued, they may not plot on the SML.
Differential Timing: Returns can be influenced by short-term events or conditions that are not reflected in the long-term beta calculation, causing temporary deviations from the SML.

d)
Assumptions of the SML:
- **Investors Make Rational Choices**: Investors are expected to make choices that maximize their returns for a given level of risk.

- **Markets are Efficient**: All securities are assumed to be fairly priced, reflecting all available information.
- **Homogeneous Expectations**: Investors are assumed to agree on the expected returns, volatilities, and correlations of all assets.
- **Single-Period Time Horizon**: The CAPM and SML are framed within a single-period model.

Empirical Justification:

My calculations have found evidence supporting the CAPM, suggesting that higher-beta assets have, on average, higher returns.

Conclusion

This report explored the complexity of establishing an efficient portfolio within Canada's financial landscape by doing rigorous analysis and applying the Capital Asset Pricing Model (CAPM) to Dollarama, Loblaw, and Canadian Tire. The Tangent Portfolio, which stands out for its better risk-adjusted performance, as indicated by its remarkable Sharpe ratio, was built and optimized as the main goal of my report. This portfolio not only exceeds its individual components and the larger S&P/TSX index in terms of return and volatility, but it also demonstrates the practical application of theoretical financial principles to generate optimal investing results. All in all, this report emphasizes the importance of strategic asset allocation and risk management in portfolio creation, offering useful insights for both theoretical comprehension and practical investing decisions.

Reference

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