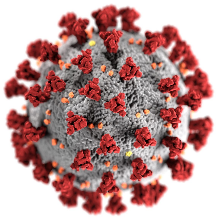

Project 4

Nationwide Ventilator Allocation during COVID-19 Pandemic

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VISION

The Problem:

The need for **ventilation services** during COVID-19 quickly overwhelmed many day-to-day operational capabilities. Many states in the U.S have faced **shortage** of ventilators. This project aims to optimize the **Nationwide Ventilator Allocation** during the COVID-19 pandemic.

Goals:

- Minimize **the unmet demand** for resources by optimally distributing them across all states. Find the **optimal allocation** of ventilators among the U.S.
- Implement **Deterministic/Simple Recourse** Model in Stochastic Optimization
- Solving the Stochastic Optimization problem by **Benders Decomposition**

02

TECHNICAL APPROACH

Approach:

1. Notation Definition
2. Optimization Setup
3. Solving Deterministic and SR
4. Performance Evaluation

Notation: Sets

- $\mathcal{S} = \{0, 1, ..., 50\}$, the index set of all states and Washington, D.C..
- $\mathcal{S}^+ = \mathcal{S} \cup \{51\}$, where 51 is the index for the Strategic National Stockpile. We assume that once SNS transfers the ventilators to a state, these ventilators belong to the state.
- $\mathcal{S}_j$ = a subset of $\mathcal{S}$, which includes the indexes of the states able to transfer resources to location j, $\forall j \in \mathcal{S}^+$.
- $\mathcal{T} = \{1, 2, ..., T\}$, the set of days which the plan covers.

Notation: Data Variables

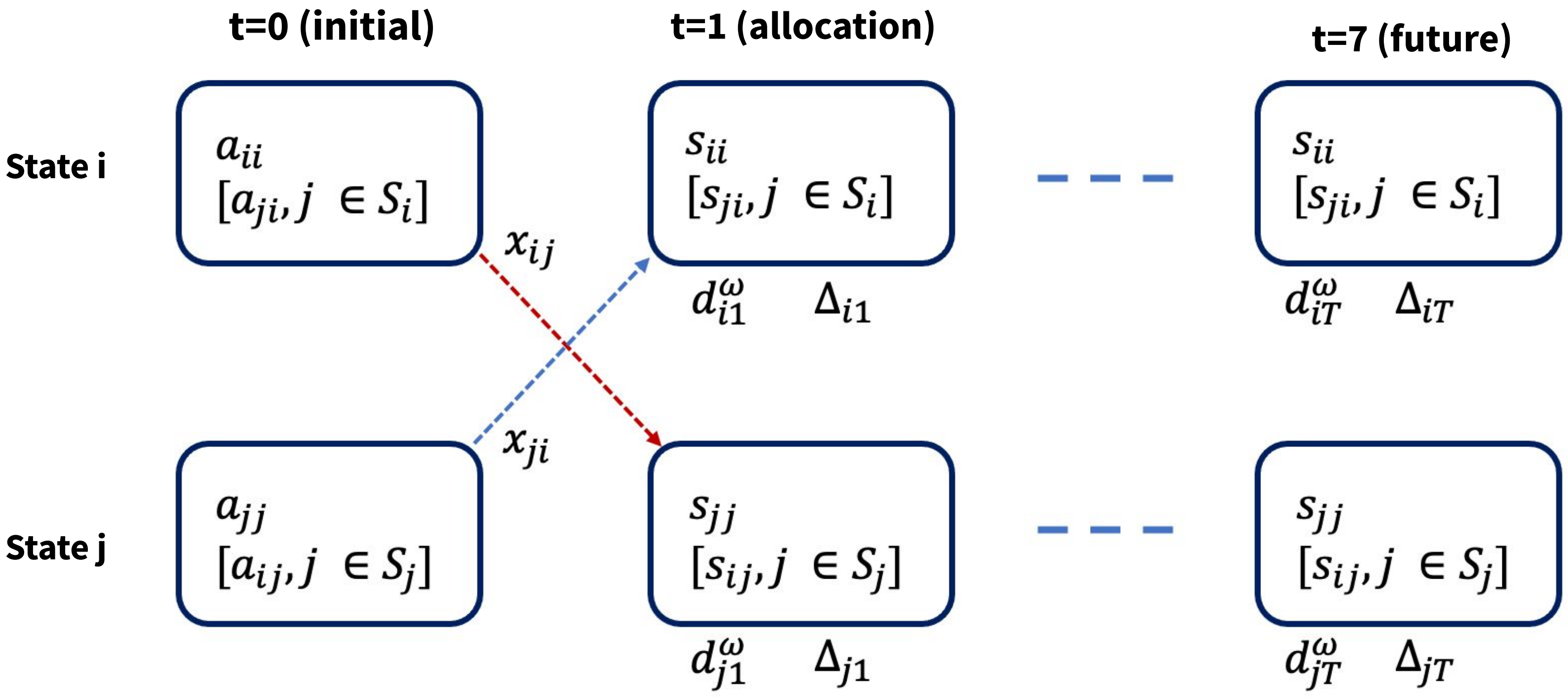
- a_{jj} = the initial total number of resources that are available at $j \in \mathcal{S}^+$, and owned by j.
- a_{ij} = the initial total number of resources that are available at $j \in \mathcal{S}^+$, and owned by $i \in \mathcal{S}_j$.
- d_{jt} = the demand for resources at $j \in \mathcal{S}$ on day $t \in \mathcal{T}$.
- U_{ji} = the maximum number of ventilators owned by $j \in \mathcal{S}^+$, and transferred to $i \in \mathcal{S}$.
- G_j = the minimum number of ventilators owned by state j which must be retained at location $j \in \mathcal{S}$.

Notation: Decision Variables

- x_{ij} = the number of ventilators shipped from location $i \in \mathcal{S}^+$ to location $j \in \mathcal{S}^+$ at time 0. The flow matrix is X, with x_{ij} as the element at (i, j).
- s_{jj} = the total number of ventilators that are available in $j \in \mathcal{S}^+$ after initial allocation, and owned by j.
- s_{ij} = the total number of ventilators that are available in $j \in \mathcal{S}^+$ after initial allocation, and owned by $i \in \mathcal{S}_j$.
- Δ_{jt} = unmet demand for ventilators at location $j \in \mathcal{S}$ on day $t \in \mathcal{T}$. The matrix representation is Δ.

Flow Diagram:

t = 7 days T=10 Weeks



Deterministic Model: Point Forecast

Main Idea:
InvVen_mean from "Prediction" dataset as the deterministic demand

$$\text{Min} \quad \sum_{j \in \mathcal{S}, t \in \mathcal{T}} \Delta_{jt} + \lambda \left(\sum_{j \in \mathcal{S}, i \in \mathcal{S}_j} s_{ij} \right)$$

s.t.

Flow Balance

$$s_{jj} = a_{jj} - \sum_{i \in \mathcal{S}_j} x_{ji} + x_{51,j}, \forall j \in \mathcal{S}$$

$$s_{51,51} = a_{51,51} - \sum_{j \in \mathcal{S}} x_{51,j}$$

$$s_{ij} = a_{ij} + x_{ij}, \forall j \in \mathcal{S}, i \in \mathcal{S}_j$$

Unmet Demand

$$\Delta_{jt} \geq \max\{0, d_{jt} - s_{jj} - \sum_{i \in \mathcal{S}_j} s_{ij}\}, \forall j \in \mathcal{S}, t \in \mathcal{T}$$

Policy Constraints

$$s_{jj} \geq G_j, \forall j \in \mathcal{S}^+$$

$$-a_{ij} \leq x_{ij} \leq U_{ij}, \forall i, j \in \mathcal{S}^+$$

Simple Recourse Model: Forecast uncertainty

First Stage:

$$\boxed{\text{Min} \quad \lambda \left(\sum_{j \in \mathcal{S}, i \in \mathcal{S}_j} s_{ij} \right) + \hat{\mathbb{E}}[h(s, X, \tilde{\omega})]}$$

$$s_{jj} = a_{jj} - \sum_{i \in \mathcal{S}_j} x_{ji} + x_{51,j}, \forall j \in \mathcal{S}$$

$$s_{51,51} = a_{51,51} - \sum_{j \in \mathcal{S}} x_{51,j}$$

$$s_{ij} = a_{ij} + x_{ij}, \forall j \in \mathcal{S}, i \in \mathcal{S}_j$$

$$s_{jj} \geq G_j, \forall j \in \mathcal{S}^+$$

$$-a_{ij} \leq x_{ij} \leq U_{ij}, \forall i, j \in \mathcal{S}^+$$

Second Stage:

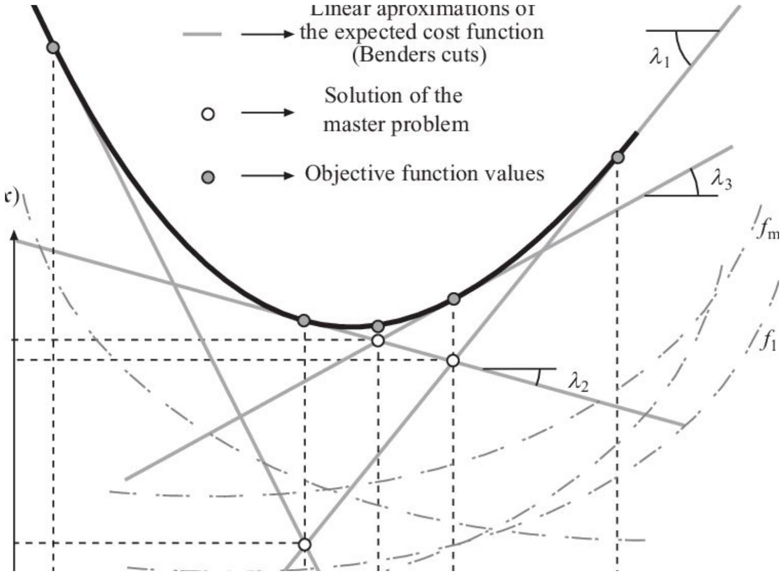
Forecast uncertainty

$$\boxed{h(s, X, \omega) = \text{Min} \sum_{j \in \mathcal{S}, t \in \mathcal{T}} \Delta_{jt}^{\omega}}$$

$$s.t. \quad \Delta_{jt}^{\omega} \geq \max\{0, d_{jt}^{\omega} - s_{jj} - \sum_{i \in \mathcal{S}_j} s_{ij}\}, \ \forall j \in \mathcal{S}, t \in \mathcal{T}$$

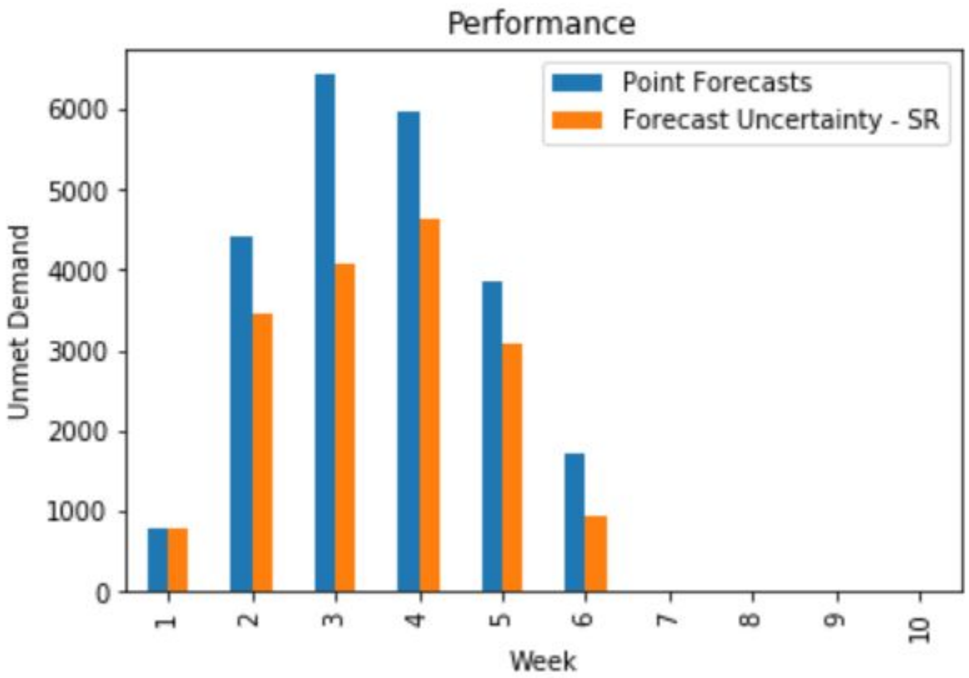
Benders Decomposition

1. **Solve First Stage:**
   Lower bound
2. **Solve Second Stage:**
   Upper bound and subgradient
3. **Add Benders Cuts and resolve First Stage:**
   Optimal solution for 1 week
4. **Resolve the problem:**
   Use the solution for last week as the initial stock of the new week



Results

Week	Point Forecasts	Forecast Uncertainty - SR
1	781	781
2	4421	3438
3	6427	4078
4	5949	4637
5	3855	3077
6	1706	923
7	0	0
8	0	0
9	0	0
10	0	0
Total	23139	16934



FUTURE WORK:

Option 1: Demand Forecast

Current Demand Distribution (Three Discrete Values)

↓

Utilize Regression b/w Covariates to Construct Regression

Predictors:
- *State Case Growth Rate (✓)*
- *Prior Unmet Ventilator Demand (✓)*
- *Vaccinated Population (✓)*

Option 2: Algorithm

Run with SGD instead of Benders and sample demand according to low:0.25, mean:0.5, high:0.25 demands

OR

All in one LP with SAA

03

SOFTWARE & RESOURCES

Supporting Software

Programming

python™

pandas

Determninistic & Simple Recourse Modeling

PYOMO

Solvers

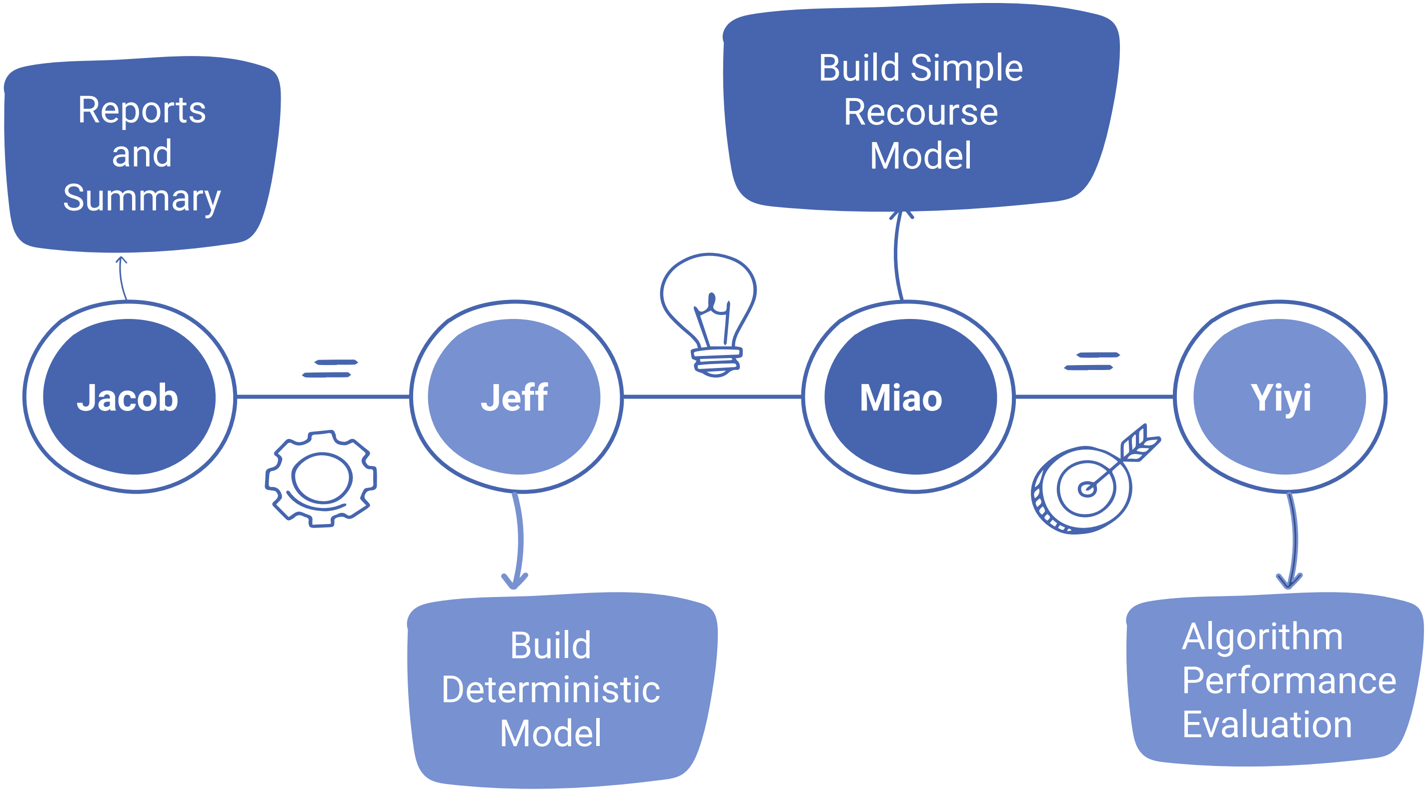
GLPK

CBC

04

RESPONSIBILITIES & TIMELINE

Responsibilities



OUR PROCESS

- **5/1** — Deterministic Model building
- **5/4** — SR Model building
- **5/9** — Solve the model with Pyomo
- **5/10** — Model Performance evaluation with “eval data”
- **5/11** — Extension and report

THANK YOU

Fight On