Project 3: LEO - Wyndor

Group 1:

Jacob Andreesen
Jeff Chen
Miao Xu
Yiyi Wang

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VISION

The Problem:

Production:

- The Wyndor Glass Co manufactures two different types of **doors** (**y_A , y_b**)
  - **Three plants** used to produce doors; each w/ different production limits

Marketing Strategy:

- Wyndor advertises their doors using both **TV (x_1) and Radio (x_2) ads.**
  - A max **200 ad slots** can be used; each ad carries a different cost

Covariates:

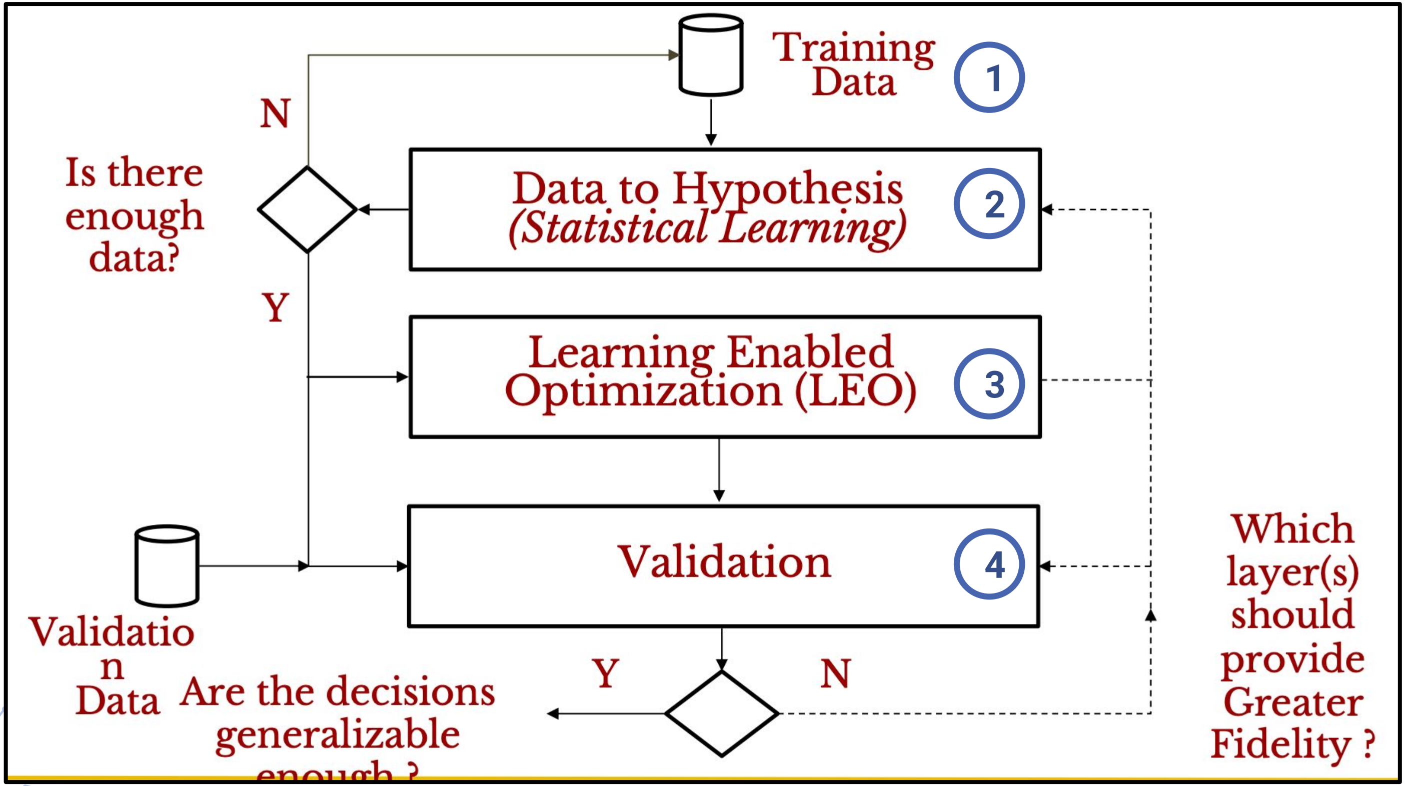
- **Potential Door Sales (ω) is dependent on the marketing strategy**

Goals:

Determine the **optimal marketing strategy** and **production schedule** to **maximize Wyndor's expected profit**.

To solve this Predictive Stochastic Programming problem, we will utilize the **Learning Enabled Optimization (LEO)** methodology

LEO Protocol:



02

TECHNICAL APPROACH

Approach:

1. 2-Stage Linear Program
2. Data Prep. & Error Values
3. Statistical Modeling & Optimization
4. Model Validation

Stochastic Linear Programming

First Stage:
Advertising

$$\begin{gathered} Max\ -0.1x_1\ -0.5x_2 + E[h(x, W|Z)] \\ x_1 + x_2 \leq 200 \\ x_1 - 0.5x_2 \geq 0 \\ L_1 \leq x_1 \leq U_1, \qquad L_2 \leq x_2 \leq U_2 \end{gathered}$$

Second Stage:
Production Schedule

$$\begin{aligned} h(x,(Z_i, W_i)) = \text{Max} \quad & 3y_A + 5y_B \\ \text{s.t.} \quad & y_A \leq 8 \\ & 2y_B \leq 24 \\ & 3y_A + 2y_B \leq 36 \\ & y_A + y_B \leq W_i|Z_i \\ & y_A, y_B \geq 0. \end{aligned}$$

Wyndor Covariates & Obj:

W: Response (Sales)
Z : Predictors (Ad. Strategy)
h = *Profit(x,W|Z)*: Wyndor Obj. Function

First Stage Decisions:

X1: Television Ad. Slots
X2: Radio Ad. Slots

Second Stage Decisions:

y_A: # of Door A produced
y_B: # of Door B produced

Data Prep: Linear Reg. & Error

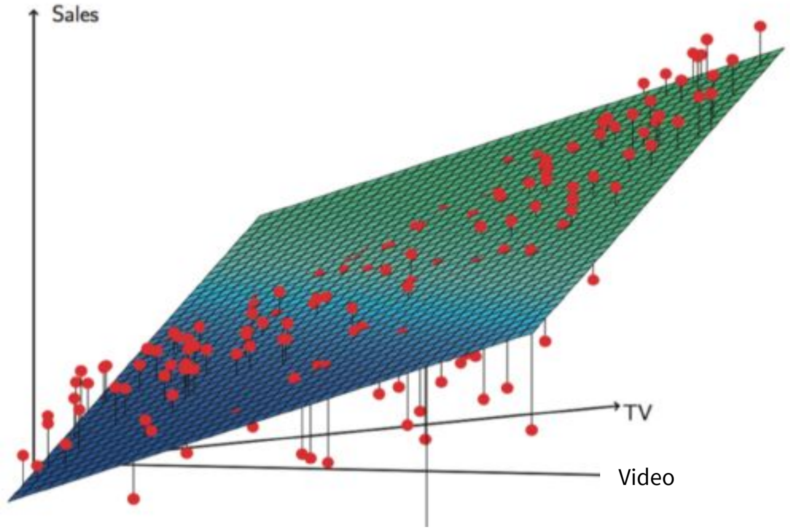
Multiple Linear Regression:

$$m(Z_i, \varepsilon) = \beta_0 + \sum_j \beta_j Z_{ij} + \varepsilon$$

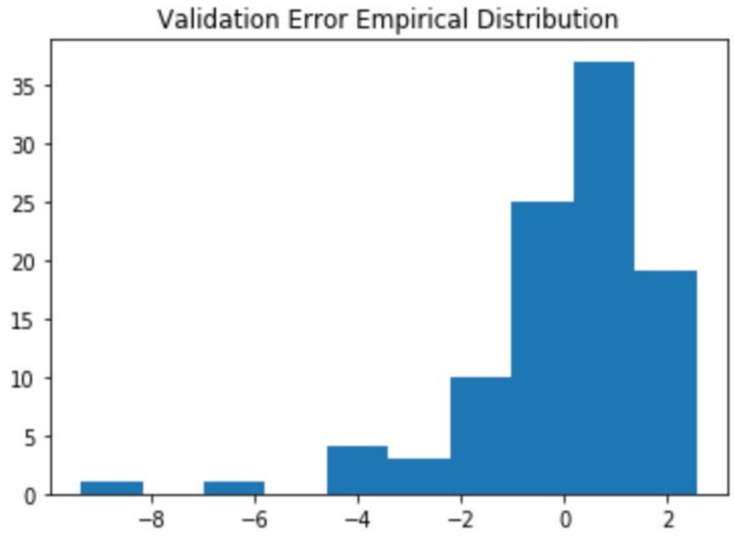
Train & Val. Error:

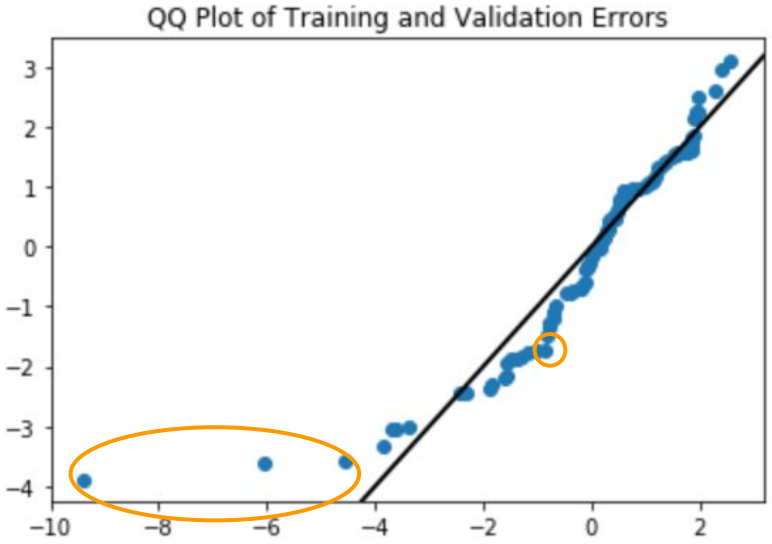
$$\varepsilon_{ti} = \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

$$\varepsilon_{vi} = \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

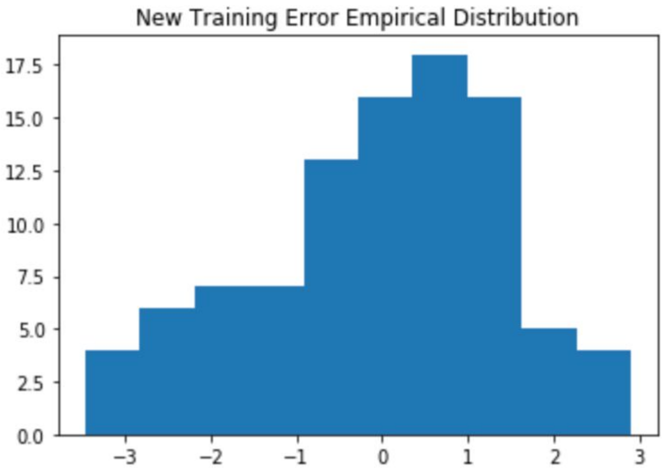


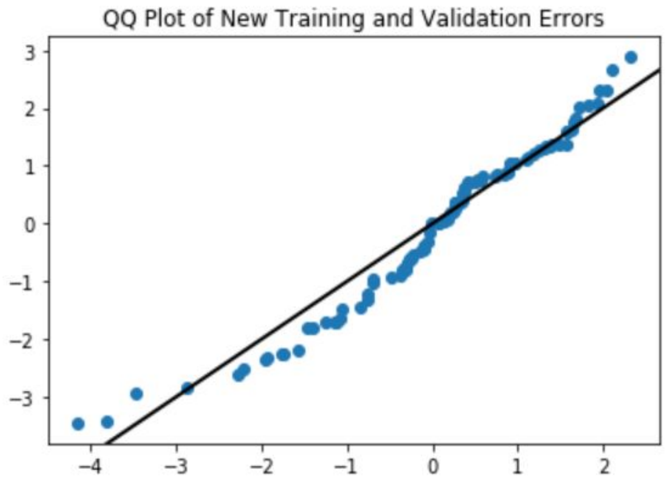
Data Prep: QQ Plot & Outliers





Data Prep: Refit Data





Regression Coefficients

$\beta_0 = 3.103$
$\beta_1 = 0.0422$
$\beta_2 = 0.2099$

Deterministic Forecast

$$\mathbb{E}[\tilde{\omega}] = \mathbb{E}[\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \tilde{\varepsilon}] = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

DF Model

Main Idea:
Prediction error assumed to be Zero

$$\begin{aligned}
\text{Max} \quad & -0.1x_1 - 0.5x_2 + \boxed{3y_A + 5y_B} \\
\text{s.t.} \quad & x_1 + x_2 \leq 200 \\
& x_1 - 0.5x_2 \geq 0 \\
& y_A \leq 8 \\
& 2y_B \leq 24 \\
& 3y_A + 2y_B \leq 36 \\
& -\beta_1 x_1 - \beta_2 x_2 + y_A + y_B \leq \beta_0 \\
& y_A, y_B \geq 0 \\
& L_1 \leq x_1 \leq U_1, \quad L_2 \leq x_2 \leq U_2
\end{aligned}$$

Linear Program Integration

EAE/SAA Model

$$\omega_i := \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \boxed{\varepsilon_{ti}}$$

EAE/SAA Model

$$\begin{array}{llllll}
\text{Max} & -0.1x_1 - 0.5x_2 + \boxed{\frac{1}{N}\sum_{i=1}^{N} 3y_{Ai} + 5y_{Bi}} & & & & \\
\text{s.t.} & x_1 \quad + x_2 & & \leq 200 & & \\
& x_1 - 0.5x_2 & & \geq 0 & & \\
& & y_{Ai} & \leq 8 & i = 1, \ldots, N & \\
& & 2y_{Bi} & \leq 24 & i = 1, \ldots, N & \\
& & 3y_{Ai} + 2y_{Bi} & \leq 36 & i = 1, \ldots, N & \\
& -\beta_1 x_1 - \beta_2 x_2 + & y_{Ai} + y_{Bi} & \leq \beta_0 + \varepsilon_{ti} & i = 1, \ldots, N & \\
& L_1 \leq x_1 \leq U_1, \quad L_2 \leq x_2 \leq U_2, y_{Ai}, y_{Bi} \geq 0. & & & &
\end{array}$$

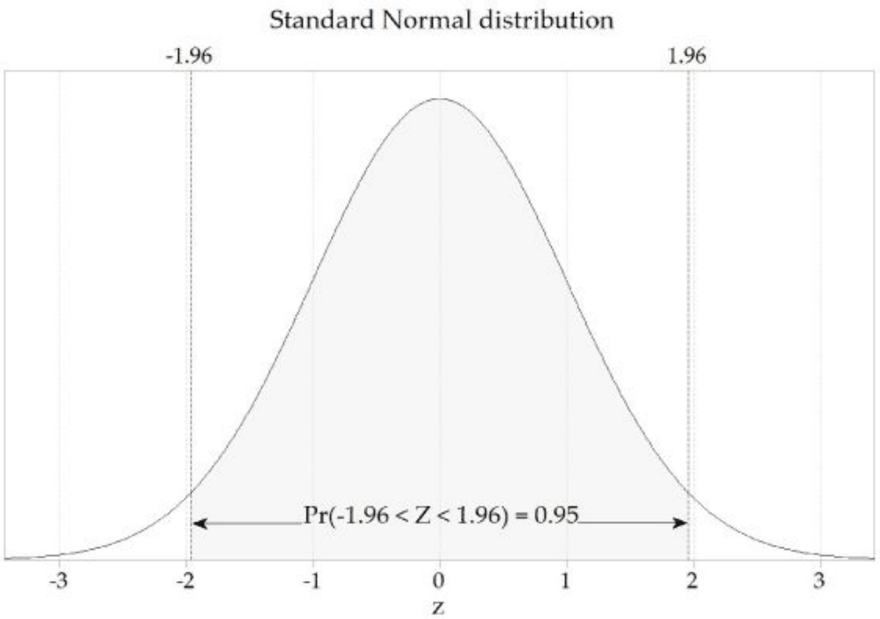
Linear Program Integration

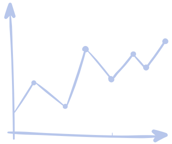
Main Idea:
Utilizes generic outcome of random error variable

Validation: MVSAE

MVSAE Procedure:

1. Solve model ***M*** times using validation error set & $\hat{x}$
2. Calculate mean and SD for optimal solution
3. Construct 95% confidence interval





Results

	Methodology	x1	x2	(in $K) MPO	(in $K) MVSAE
0	Deterministic LP	173.434077	26.565923	41.373631	[39.811, 40.136]
1	SLP with SAA	180.173138	19.826862	40.340003	[40.252, 40.723]

(Sampled N = 200; Replicated M = 1000 times)

Extension: NDU Model

$$E[\tilde{\omega}] = (\beta_0+\xi_0) + (\beta_1+\xi_1)x_1 + (\beta_2+\xi_2)x_2$$

NDU Model

$x = (x_1, x_2)$ Generic Outcome Vector

(ξ_0, ξ_1, ξ_2) Generic Error Outcomes

Vectors

$$\min_{\xi_i} \quad (\xi_i)^\top \Sigma_\beta^{-1}(\xi_i)$$

$$(\hat{\beta}_0 + \xi_{i0}) + (\hat{\beta}_1 + \xi_{i1})Z_{i1} + (\hat{\beta}_2 + \xi_{i2})Z_{i2} = W_i$$

Solve for error outcomes

03

SOFTWARE & RESOURCES

Supporting Software

Programming & Data Prep.	Regression & Modeling	Solvers
python™	PYOMO	GLPK
pandas	scikit learn	CBC

04

RESPONSIBILITIES & TIMELINE

Responsibilities

Data preparation and DF modeling

Jacob

Jeff

DF Modeling and validation

DF and ERE/SAA modeling & Validation

Miao

Yiyi

Report and extension

OUR PROCESS

4/10 — Model building with DF & EAE/SAA

4/12 — Data Preparation

4/15 — Solve the model with Pyomo

4/20 — Model Validation

4/29 — Extension and report

THANK YOU

Fight On