ECE 4110/5110 Monday, 08/28/23

Lecture 3: Covariance and Correlation

Dr. Kevin Tang Handout 4

Related Reading

Bertsekas and Tsitsiklis Sections 4.2 and 4.3

Covariance

$$Cov(X,Y) = E((X - E(X))(Y - E(Y))) \tag{1}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) \tag{2}$$

$$Var(\sum_{i=1}^{n} X_i) = \sum_{(i,j)} Cov(X_i, X_j) = \sum_{i=1}^{n} Var(X_i) + \sum_{\{(i,j)|i \neq j\}} Cov(X_i, X_j) \tag{3}$$

Correlation Coefficient

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} \tag{4}$$

$$|\rho(X,Y)| \leq 1 \tag{5}$$

Law of Iterated Expectations:

$$E(E(X|Y)) = E(X) \tag{6}$$

Conditional Expectation

$$E((E(X|Y) - X)|Y) = 0 \tag{7}$$

$$E((E(X|Y) - X)E(X|Y)) = 0 \tag{8}$$

Law of Total Variance:

$$Var(X) = E(Var(X|Y)) + Var(E(X|Y)) \tag{9}$$

Multivariate Normal Distribution

$$M_{Y_1,Y_2,\ldots,Y_n}(s_1,s_2,\ldots,s_n) = \exp[\sum_{i=1}^{n} s_i\mu_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_is_jCov(Y_i,Y_j)] \tag{10}$$

$$f_{Y_1,Y_2,\ldots,Y_n}(y_1,y_2,\ldots,y_n) = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{\det B}}\exp\left(-\frac{1}{2}(\vec{y}-\vec{\mu})^TB^{-1}(\vec{y}-\vec{\mu})\right) \tag{11}$$