Lecture 6: A Few Boundary Events of Simple Random Walk

Dr. Kevin Tang   Handout 7

Related Reading

Grimmett and stirzaker Sections 3.10 (Proof of (21) is not required), 5.3 ((1) - (4), pp. 162-164)

The First Return to the Origin

$$P_0(s) = 1 + P_0(s)F_0(s) \tag{1}$$

$$P_0(s) = \frac{1}{\sqrt{1 - 4pqs^2}} \tag{2}$$

$$\sum_{n=1}^{\infty} f_0(n) = 1 - |p - q| \tag{3}$$

The Maximum Value Attained

For $r \geq 1$ and $b < r$

$$P(M_n \geq r, S_n = b) = (\frac{q}{p})^{r-b} P(S_n = 2r - b) \tag{4}$$

Hitting Time

$$f_b(n) = \frac{|b|}{n} P(S_n = b) \tag{5}$$

The Last Return to the Origin

For a symmetric random walk

$$\alpha_{2n}(2k) = P(S_{2k} = 0)P(S_{2n-2k} = 0) \tag{6}$$