ECE 4110/5110	Wednesday, 09/27/23

Lecture 11: Continuous-Time Markov Chains

Dr. Kevin Tang Handout 12

Related Reading

Bertsekas and Tsitsiklis 7.5

Definition and Assumptions

- T_n is exponentially distributed
- p_{ij} is independent of the past history and when the next transition happens

Balance Equations

$$\pi_j \sum_{k \neq j} q_{jk} = \sum_{k \neq j} \pi_k q_{kj} \tag{1}$$

$$\sum_{k=1}^{m} \pi_k = 1 \tag{2}$$

The key concept here is q_{ij} which is the transition rate from i to j, which depends on both transition probability p_{ij} and arrival rate μ_i.

Birth-Death Process and Its Applications

$$q_{ij} = 0, \qquad |i-j| > 1 \tag{3}$$

So instead of forming a general transition graph, states can be linearly arranged and transitions can only happen between neighboring states. This significantly simplifies the balance equations, leading to the local balance equations,

$$\pi_j q_{ji} = \pi_i q_{ij} \tag{4}$$

If we use this model to queueing, we have

$$\pi_i = \rho^i \pi_0 = \frac{\rho^i}{\sum_{k=1}^{m} \rho^k} \tag{5}$$