ECE 4110/5110 Monday, 09/18/23

Lecture 8: Discrete-Time Markov Chains

Dr. Kevin Tang Handout 9

Related Reading

Bertsekas and Tsitsiklis Section 7.1

Definition and Specification

Transition probability: For any i and j that belongs to the state space

$$p_{ij} = P(X_{n+1} = j | X_n = i) \tag{1}$$

Assumption: Markov Property

$$P(X_{n+1} = j | X_n = i, x_{n-1} = i_{n-1}, \dots, X_0 = i_0) = P(X_{n+1} = j | X_n = i) = p_{ij} \tag{2}$$

A Markov Chain is fully described by its transition probabilities, which is usually written as a transition probability matrix.

The Probability of a Path

$$P(X_1 = i_i, X_2 = i_2 \dots, X_n = i_n | X_0 = i_0) = p_{i_0 i_1} p_{i_1 i_2} \dots p_{i_{n-1} i_n} \tag{3}$$

n-Step Transition Probabilities

Chapman-Kolmogorov Equation: for any integer l with $1 \leq l \leq n-1$, we have

$$r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-l) r_{kj}(l) \tag{4}$$