ECE 3100	Wednesday, 11/01/23

Lecture 18: Stationary Processes (II)

Dr. Kevin Tang	Handout 19

Related Reading

Grimmett and Stirzaker Section 9.6

Calculation of Autocovariance function

Example 1: $\{N(t), t \geq 0\}$ is a Poisson process with parameter λ, X_0 is a random variable that is independent of $N(t)$ with $P(X_0 = 1) = P(X_0 = -1) = 0.5$. Define $X(t) = X_0(-1)^{N(t)}$. Then $X(t)$ is strictly stationary, $E(X(t)) = 0$, and $C(\tau) = e^{-2\lambda|\tau|}$.

Furthermore, define $D(t) = \int_0^t X(s)ds$, we have E(D(t))=0 and $Var(D(t)) = \frac{1}{\lambda}(t - \frac{1}{2\lambda} + \frac{1}{2\lambda}e^{-2\lambda t})$.

Example 2: $\{N(t), t \geq 0\}$ is a Poisson process with parameter λ, and $Y(t)$ be the time from t until the next Possion arrival.

Then $Y(t)$ is strictly stationary, and for $\tau \geq 0$,

$$C(\tau) = Cov(Y(t), Y(t+\tau)) = \int_0^\tau ye^{-\lambda y}dy + \int_\tau^\infty y(y-\tau)\lambda e^{-\lambda y}dy \quad (1)$$

Gaussian Markov Processes

A continuous-time process $X(t)$, is a Markov process if for any $n > 0$, $t_1 < t_2 < \cdots < t_n$, x_1, x_2, $\ldots$, x_n,

$$P(X(t_n) \leq x|X(t_1) = x_1, \ldots, X(t_{n-1}) = x_{n-1}) = P(X(t_n) \leq x|X(t_{n-1}) = x_{n-1}) \quad (2)$$

For a stationary Gaussian Markov process $X(t)$, the following are necessarily true (assuming $E(X(t)) = 0$),

$$C(\tau) = C(0)e^{-\alpha|\tau|} \quad (3)$$

with some $\alpha > 0$. This is proved by showing the following equation holds for any $0 \leq s \leq s+t$,

$$C(0)C(s+t) = C(s)C(t) \quad (4)$$