ECE 4110/5110	Wednesday, 08/23/23

Lecture 2: Transforms

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Related Reading

Bertsekas and Tsitsiklis Sections 4.4 and 4.5

Definition

$$M_X(s) = E(e^{sX}) \tag{1}$$

Properties

If $Y = aX + b$, then

$$M_Y(s) = e^{sb} M_X(sa) \tag{2}$$

It is often easy to compute moments based on transforms

$$E(X^n) = \frac{d^n M_x(s)}{ds^n}|_{s=0} \tag{3}$$

Some Special And Important Examples

- Poisson Random Variable: $M(s) = e^{\lambda}(e^s - 1)$
- Exponential Random Variable: $M(s) = \frac{\lambda}{\lambda - s}$ $(s < \lambda)$
- Normal Random Variable: $M(s) = e^{\frac{\sigma^2 s^2}{2} + \mu s}$

Inversion

In all cases that are of interest to this course, the transform $M_X(s)$ uniquely determines the CDF of X.

Sum of Fixed Number of Independent Random Variables

If $X_1, \ldots, X_n$ are independent random variables, and also $Z = \sum_{i=1}^{n} X_i$, where n is a fixed number, we have

$$M_z(s) = \prod_{i=1}^{n} M_{X_i}(s) \tag{4}$$

Sum of a Random Number of Independent Random Variables

If $X_1, \ldots, X_N$ are independent random variables, and also $Y = \sum_{i=1}^{N} X_i$, where N is a random variable and also independent of X_i's, we have

$$E(Y) = E(N)E(X) \tag{5}$$

$$Var(Y) = E(N)Var(X) + Var(N)(E(X))^2 \tag{6}$$

$$M_Y(s) = M_N(\log(M_X(s))) \tag{7}$$