ECE 4110/5110	Wednesday, 10/25/23

Lecture 16: The Wiener process (II)

Dr. Kevin Tang	Handout 17

From Finite-dimensional Distribution to Conditional Distribution

Given a standard Wienerprocess $X(t)$,

$$P(X(1) > 0|X(0.5) = 1) = P(X(1) - X(0.5) > -1) = P(X(0.5) > -1) = \phi(\sqrt{2})$$

What about $P(X(0.5) > 0|X(1) = 1)$? In general, assuming $s < t$, we have

$$f_{X(s)|X(t)}(X(s) = x|X(t) = B) = K \exp\left(-\frac{(x - \frac{Bs}{t})^2}{2(t-s)}\right) \tag{1}$$

Therefore, $X(s|X(t))$ is a Gaussian random variable, with $E(X(s)|X(t) = B) = \frac{s}{t}B$, and $Var(X(s)|X(t) = B = \frac{s}{t}(t - s)$. We then find $P(X(0.5) > 0|X(1) = 1) = \phi(1)$.

Hitting Times

Let T_a be the first time the standard Wiener process $X(t)$ hits a, assume $a > 0$, we have

$$P(T_a \leq t) = \sqrt{\frac{2}{\pi}} \int_{\frac{a}{\sqrt{t}}}^{\infty} e^{-\frac{y^2}{2}} dy \tag{2}$$

Maximum Value Attained

$$P(\max_{0 \leq s \leq t} X(s) \geq a) = P(T_a \leq t) \tag{3}$$