ECE 4110/5110 Wednesday, 10/11/23

Lecture 13: Markov Chain Monte Carlo

Dr. Kevin Tang Handout 14

Related Reading

Grimmett and Stirzaker Section 2.6 and Section 6.14 till the end of Metropolis algorithm

General Concept of Monte Carlo Simulation

Relate the quantity that is of interest to the average outcome of a random experiment. Perform the random experiment, measure the outcome, and obtain an approximation of the quantity, which may be hard to compute.

MCMC

To construct a Markov Chain with the following two properties:

- The chain has the target distribution as its stationary distribution. Therefore the long term behavior of chain will give the correct approximation.
- The transition probabilities of the chain admit a simple form. Therefore it is easy to simulate the chain.

Metropolis-Hastings Algorithm

There are two steps.

- Let H be the proposal matrix. Set $P(Y = j|X_n = i) = h_{ij}$. This is for easiness.
- Let A be the acceptance matrix. Set $X_{n+1} = j$ with probability a_{ij} and $X_{n+1} = X_n$ with probability $1 - a_{ij}$. This is for correctness.

With the choice of setting $a_{ij} = \min(1, \frac{\pi_j h_{ji}}{\pi_i h_{ij}})$, the following detailed balance equations hold for any (i, j).

$$\pi_i p_{ij} = \pi_j p_{ji} \tag{1}$$