ECE 4110/5110	Monday, 11/06/23

Lecture 19: MMSE and LLSE

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Related Reading

Grimmett and Stirzaker Section 9.2

MMSE and Conditional Expectation

Let there be two random vectors X and Y. Let's use $h(Y) = \hat{X}$ as an estimator of X. We adopt the minimal mean-squared-error (MMSE) criterion for optimality. In other words, we are looking for $\hat{X}$, which minimizes $E(\|X - \hat{X}(Y)\|^2$.

The optimal solution is $\hat{X}(Y) = E(X|Y)$.

LLSE and Orthogonality Principle

$E(X|Y)$ depends on $f(X|Y)$, which is usually hard to obtain. We instead often restrict our attention to linear least mean-squared-error (LLSE or LLMSE) estimator.

Given X and $Y_1, \ldots, Y_m$, and assume $E(X) = 0$, $E(Y_i) = 0$, $C_{ij} = E(Y_i, Y_j)$, and $C_{XY_j} = E(X, Y_j)$. How to construct $\hat{X} = \sum_{i=1}^{m} a_i Y_i$ to minimize mean-squared-error?

Orthogonality Principle: if error $e = X - \hat{X}$ is orthogonal to $Y_1, \ldots, Y_m$, then $\hat{X}$ is LLSE. Mathematically, the condition means that for any $j = 1, \ldots, m$, we have

$$E((X - \sum_{i=1}^{m} a_i Y_i) Y_j) = 0 \tag{1}$$