ECE 4110/5110	Wednesday, 11/08/23

Lecture 20: Spectral Analysis of Weakly Stationary Processes

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Related Reading

Grimmett and Stirzaker Section 9.3

Spectral Representation of Weakly Stationary Processes

Assume $X(t)$ is weakly stationary, with $E(X(t)) = 0$ and $C_X(s) = Cov(X(t), X(t+s))$. Its spectral representation is

$$S_X(\omega) = \int_{-\infty}^{\infty} e^{-i\omega s} C_X(s) ds \tag{1}$$

where $S_X(\omega)$ is called the power spectrum density of $X(t)$.

Weakly Stationary Processs pass through an LTI system

Let X(t) pass through an LTI system with impulse response function $h(t)$. Let $Y(t)$ represent the output process. We then have $E(Y(t)) = 0$ and

$$Cov(Y(t_1), Y(t_2)) = C_Y(t_1 - t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_X(t_2 - t_1 + s_1 - s_2) h(s_1) h(s_2) ds_1 ds_2 \tag{2}$$

We can describe this relationship more compactly in the frequency domain where we have

$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega) \tag{3}$$