

## Lecture 5: The Bernoulli Process and Random Walk

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Handout 6

**Related Reading**

Bertsekas and Tsitsiklis Section 6.1; Grimmett and stirzaker 3.9

**The Bernoulli Process**

- The number of successes in  $n$  trials is a binomial random variable.
- The number of trials up to (and including) the first success is a geometric random variable.
- The  $k$ th arrival time is a Pascal random variable.
- Splitting and merging of Bernoulli processes result in Bernoulli processes.

**Simple Random Walk**

$$S_n = S_0 + \sum_{i=1}^n X_i \quad (1)$$

Spatial homogeneity:

$$P(S_n = j | S_0 = a) = P(S_n = j + b | S_0 = a + b) \quad (2)$$

Temporal homogeneity:

$$P(S_n = j | S_0 = a) = P(S_{m+n} = j | S_m = a) \quad (3)$$

**Counting Sample Paths**

$$N_n(a, b) = \binom{n}{\frac{1}{2}(n+b-a)} \quad (4)$$

The Reflection Principle (assuming  $a > 0, b > 0$ )

$$N_n^0(a, b) = N_n(-a, b) \quad (5)$$