ECE 4110/5110 Monday, 10/30/23

Lecture 17: Stationary Processes

Dr. Kevin Tang Handout 18

Related Reading

Grimmett and Stirzaker Section 8.2, till the end of 8.2.6

Definitions

$\{X(t) : t \geq 0\}$ is *strongly stationary* if $\{X(t_1), X(t_2), \ldots, X(t_n)\}$ and $\{X(t_1+h), X(t_2+h), \ldots, X(t_n+h)\}$ have the same distribution for any t_1, t_2, $\ldots$, t_n and $h > 0$.

$\{X(t) : t \geq 0\}$ is *weakly stationary* if $E(X(t_1)) = E(X(t_2))$ and $Cov(X(t_1), X(t_2)) = Cov(X(t_1+h), X(t_2+h))$, for any t_1,t_2 and $h > 0$.

Therefore, $Cov(X(t), X(t+\tau))$ is only a function of τ, we denote as $C_{XX}(\tau)$ or even just $C(\tau)$ when there is no danger of confusion.

Properties

$$C(\tau) = C(-\tau) \tag{1}$$

$$C(0) \geq E^2(X(t)) \geq 0 \tag{2}$$

$$|C(\tau)| \leq C(0) \tag{3}$$

$C(\tau)$ is positive semi-definite. For any $n > 0$, λ_1, $\ldots$, λ_n, and t_1, $\ldots$, t_n

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j C(t_i - t_j) \geq 0 \tag{4}$$

Gaussian Process

$\{X(t), t \geq 0\}$ is a Gaussian process if for any n and t_1, $\ldots$, t_n, $X(t_1)$, $\ldots$, $X(t_n)$ are jointly Gaussian.

A Gaussian process is strongly stationary if and only if it is weakly stationary.