| ECE 4110/5110 | Wednesday, 09/20/23 |
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# Lecture 9: Steady State Behavior

| Dr. Kevin Tang | Handout 10 |
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**Related Reading**

Bertsekas and Tsitsiklis Sections 7.2 and 7.3

**Classification of States**

If $r_{ij}(n) > 0$ for some $n$, then we say $j$ is accessible from $i$.

Let $A(i)$ be the set of states that are accessible from $i$. If for any $j \in A(i)$, we have $i \in A(j)$, then $i$ is recurrent.

A state is called transient if it is not recurrent.

If i is a recurrent state, then A(i) is a recurrent class.

Markov Chain decoposition: one or more recurrent states plus possibly some transient states

A recurrent state is periodic if its states can be grouped in $d > 1$ disjoint subsets $S_1, \ldots, S_d$ so that all transitions from one subsets lead to the next subset.

A recurrent class is aperiodic if and only if thre exists a time $n > 0$, such that for any $i$ and $j$ in the class, we have $r_{ij}(n) > 0$.

**Steady State Balance Equation**

Consider a Markov Chain with a single aperiodic recurrent class, then the steady-state probabilities $\pi_j$ satisfy the following balance equations and the renormalization equation.

$$\pi_j = \sum_{k=1}^{m} \pi_k p_{kj} \tag{1}$$

$$\sum_{k=1}^{m} \pi_k = 1 \tag{2}$$