Portfolio Risk Analysis with PCA and GARCH

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Abstract

This project aims to evaluate the risk of a candidate portfolio by applying Principal Component Analysis (PCA) and GARCH models. The portfolio consists of six stocks: META, AMZN, AAPL, GOOG, V, and JNJ. The sample period is from January 4, 2010, to December 31, 2019. The proposed model uses PCA to identify factor structures and GARCH(1,1) to model individual factors' volatility. The results show that the model is effective in predicting portfolio risk, but further tuning is required to improve accuracy.

1 Introduction

1.1 Project Introduction

This project aims to utilize the techniques learned in Risk Management and Modelling to evaluate the risk of a candidate portfolio by applying various models and risk measures, including Principal Component Analysis (PCA) and Value-at-Risk (VaR). Finally, tests are conducted to check the models' validity in predictions for VaR.

1.2 Candidate Portfolio Introduction

The candidate portfolio consists of six stocks: META, AMZN, AAPL, GOOG, V, and JNJ. The sample period is from January 4, 2010, to December 31, 2019. The portfolio is equally weighted among all stocks, hence 1/6 for every stock.

2 Methodology

2.1 Principal Component Analysis (PCA)

In the proposed multivariate conditional volatility model, PCA is used to identify factor structure for the candidate portfolio. The first factor (PC1) accounts for about 52% of the total explained variance. The first four factors (PC1 – PC4) account for nearly 90% of the total explained variance. The details of the PCA composition are shown in Figure 1.

	PC1	PC2	PC3	PC4	PC5	PC6
FB	-0.3264	0.3544	0.8187	-0.3103	0.0313	-0.0182
Amazon	-0.6377	-0.7657	0.0606	-0.0389	0.0388	-0.0172
Apple	-0.3744	0.2449	-0.0758	0.3912	-0.8004	0.0224
Alphabet	-0.4914	0.3994	-0.5656	-0.4992	0.1599	-0.0649
Visa	-0.1304	0.1225	0.0184	0.4031	0.2698	-0.8558
J&J	-0.2969	0.2314	-0.0064	0.5809	0.5085	0.5121
Standard deviation	0.0288	0.0171	0.0134	0.0107	0.0095	0.0082
Proportion of Variance	0.5280	0.1854	0.1134	0.0726	0.0578	0.0429
Cumulative Proportion	0.5280	0.7134	0.8268	0.8994	0.9571	1.0000

Figure 1: PCA composition

2.2 GARCH(1,1) Model

GARCH(1,1) is applied to model each factor's volatility. GARCH allows for the capture of the persistence in volatility and unconditionally heavy tails, as well as fewer parameters compared with ARCH models.

3 Results and Discussion

3.1 Value-at-Risk (VaR)

The VaR is derived using the parametric method, where the portfolio's predicted conditional volatility calculated in the previous section is utilized. Furthermore, an assumption is made regarding the standardized returns, which is that the standardized returns are normally distributed. The predicted VaR is: 0.0216.

3.2 Backtesting

3.2.1 Rolling Predictions

To backtest the ability of the multivariate model and the parametric VaR method, a rolling prediction is introduced. The set estimation window length is ($5/5\% = 100$), where 5% is the required VaR estimate level. A rolling function in GARCH is directly applied to each factor (PC1-PC4), and hence the series of rolling predicted portfolio conditional volatility is derived. The series of VaR is constructed as follows: for VaR between T=1 and T=window length, VaR is calculated by the GARCH conditional volatility with the assumption that the standardized returns are normally distributed; for T larger than the window length, the rolling prediction of conditional volatility is used instead of the regular GARCH ones.

3.2.2 Hit Sequence and Violation Ratio

By the definitions of hit sequence and violation ratio, and the calculated rolling predictions, one could get the number of violations, non-violations, and could compare with the expected number of violations to see how the model works. It is shown that the proposed model has over forecasted VaR since the number of violations is less than the expected number of violations.

Metric	Value
Number of Violations	40
Number of Non-Violations	1775
Expected Number of Violations	90.75
Violation Ratio	0.441

Table 1: Hit Sequence and Violation Ratio

The following Figure 2 plots the rolling predicted VaR with the portfolio's daily demean return. From the graph, for most of the time, the predicted VaR can capture the tail behaviors. However, for some rather extreme volatile periods, the predicted VaR failed to capture the tail behaviors.

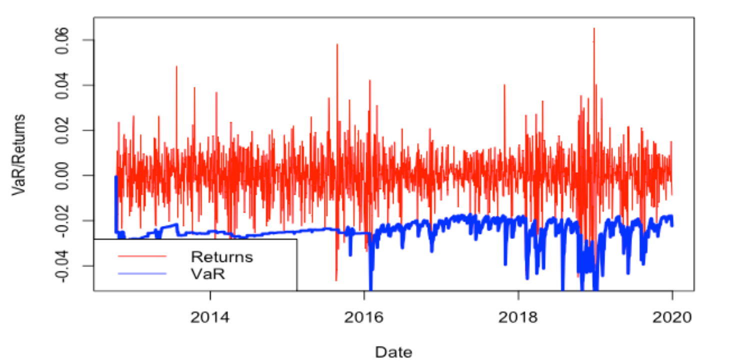

Figure 2: VaR estimate captures tail behaviours of returns

3.3 Conditional and Unconditional Coverage Ratio Tests

Both unconditional coverage ratio test and the conditional coverage ratio test are done under normal procedures and the conclusions are the following. For the unconditional test, we reject the null hypothesis, which means for the proposed model, the violations are not happening with the expected frequency (5%). For the conditional test, we could not reject the null hypothesis, which means that under the proposed model, violations are happening independently, and the likelihood of having a violation at time t does not depend on whether there is a violation at time (t-1).

4 Conclusion

The conditional coverage ratio test result has shown that the proposed multivariate model is relatively accurate while capturing tail behaviors. However, the model tends to overestimate VaR, as indicated by the violation ratio and the unconditional coverage test. This overestimation may be due to the assumption of normally distributed standardized returns, which does not fully capture the fat-tailed nature of financial data.

GARCH(1,1) and PCA could be good choices while evaluating the risk of the candidate portfolio, but the parameters may need to be more carefully tuned such that the model could produce a more accurate prediction. This could be done by rescaling the estimation window length so to produce a more accurate prediction for each factor's volatility.

Future work could explore alternative distributional assumptions (e.g., Student-t distribution) and more sophisticated GARCH models (e.g., EGARCH or GJR-GARCH) to improve the model's accuracy.