MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Governments issue bonds to finance spending by borrowing from investors without permanently increasing the money supply, which helps avoid inflation from printing money.

   (b) Suppose the central bank keeps the current policy rate high, but markets expect rate cuts within 1–2 years; short yields stay high, while investors buy long bonds for expected future lower rates, pushing long yields down and flattening the long end.

   (c) Quantitative easing (QE) is large-scale central bank purchases of longer-maturity securities to lower long-term yields and support liquidity; since early 2020 the U.S. Fed used QE by buying Treasuries and agency MBS to stabilize markets and ease financial conditions.

2. I construct 0–5Y curves using ten nominal Government of Canada bonds (semi-annual coupons), chosen to be (i) same issuer and credit risk (GoC), (ii) actively traded benchmark-style issues, and (iii) spanning maturities so their cashflows provide coverage to bootstrap discount factors up to 5 years:

   - CAN 4.5 Feb 26 (ISIN CA135087R226; issued 11/1/2023; matures 2/1/2026)
   - CAN 0.25 Mar 26 (ISIN CA135087L518; issued 10/9/2020; matures 3/1/2026)
   - CAN 4 May 26 (ISIN CA135087R556; issued 2/12/2024; matures 5/1/2026)
   - CAN 1.5 Jun 26 (ISIN CA135087E679; issued 7/21/2015; matures 6/1/2026)
   - CAN 1 Sep 26 (ISIN CA135087L930; issued 4/16/2021; matures 9/1/2026)
   - CAN 4 Aug 26 (ISIN CA135087R978; issued 5/6/2024; matures 8/3/2026)
   - CAN 1 Jun 27 (ISIN CA135087F825; issued 8/3/2016; matures 6/1/2027)
   - CAN 3.25 Aug 27 (ISIN CA135087P733; issued 12/2/2022; matures 8/24/2027)
   - CAN 3.25 Sep 28 (ISIN CA135087Q491; issued 4/21/2023; matures 9/1/2028)
   - CAN 0.5 Dec 30 (ISIN CA135087L443; issued 10/5/2020; matures 12/1/2030)

3. For a covariance matrix of curve points, the eigenvectors give the main independent shapes of co-movement (principal components) and the eigenvalues give how much variance each shape explains; the largest eigenpair is the dominant factor driving day-to-day curve changes.

Empirical Questions - 75 points

4.

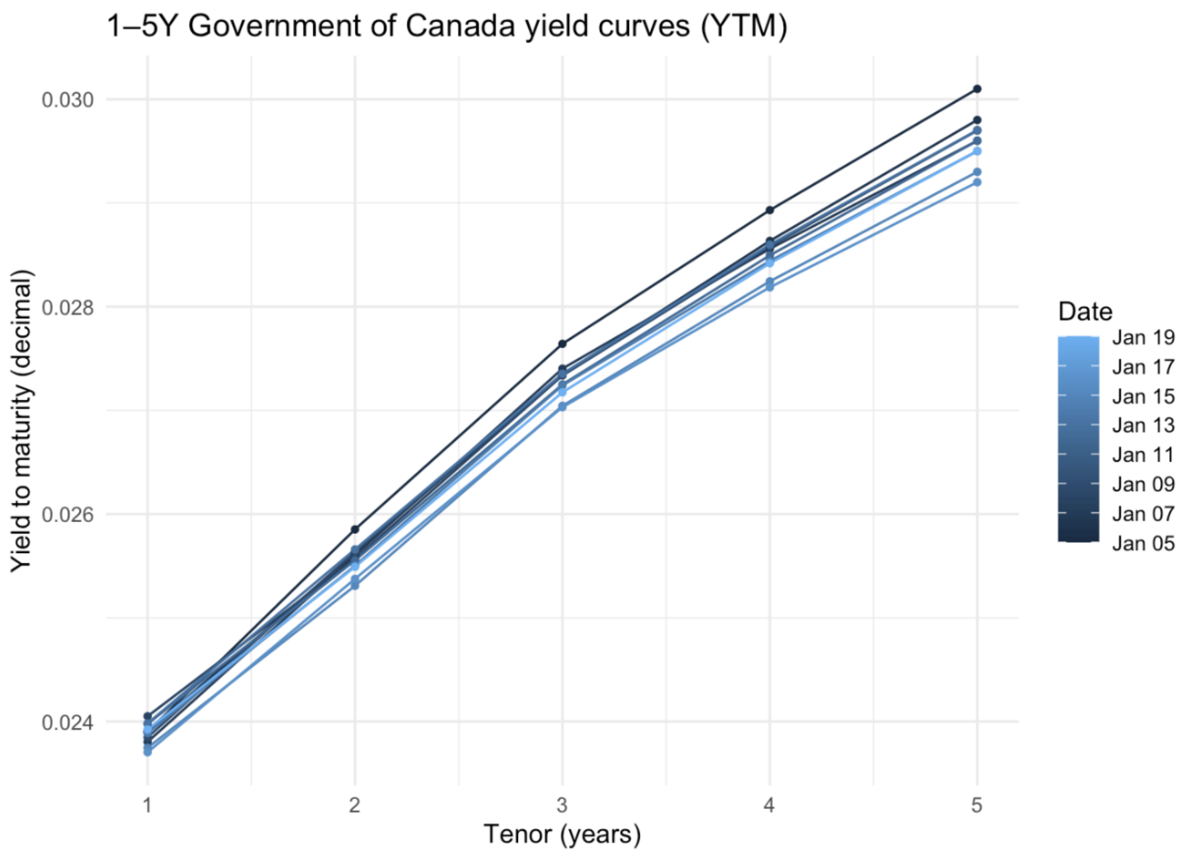

Figure 1: 1–5Y Government of Canada yield curves (YTM), daily (Jan 5–19, 2026).

(a)

(b) Pseudo-code to derive the 1–5Y spot curve (bootstrap, semi-annual coupons):

i. For each day, take the set of bond prices and cashflow schedules.

ii. Sort cashflow dates increasing. Initialize unknown discount factors $D(t)$.

iii. For the earliest maturity/cashflow date, solve $P = \sum CF_i D(t_i)$ for the first unknown $D(t)$ (all earlier $D(\cdot)$ known or none).

iv. Iterate: for each next cashflow date t_k, subtract PV of earlier cashflows using already-solved $D(t_i)$, then solve the remaining equation for $D(t_k)$.

v. Convert discount factors to spot rates $S(t)$ using the assignment compounding convention.

vi. Interpolate $S(t)$ to obtain spot rates at exactly 1,2,3,4,5 years.

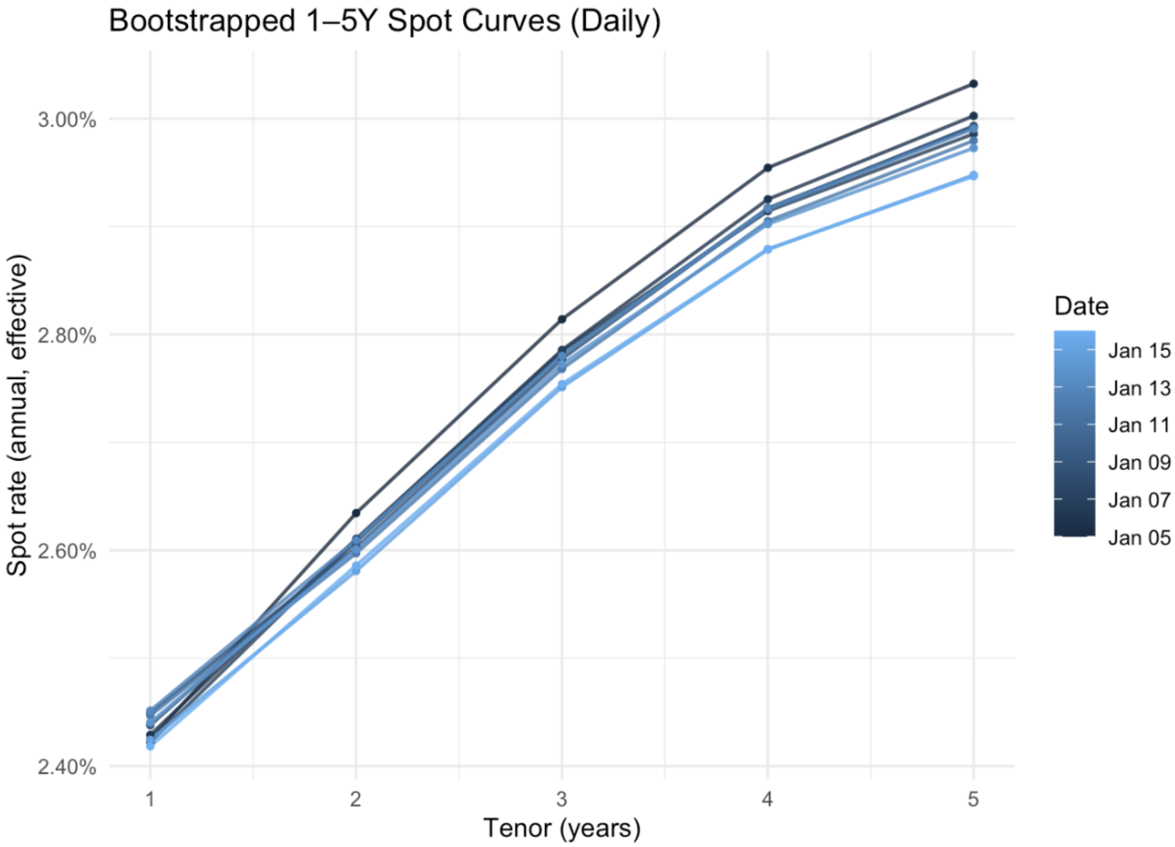

Figure 2: 1–5Y Government of Canada spot curves, daily (Jan 5–19, 2026).

(c) Pseudo-code to derive the 1-year forward curve (terms 2–5Y):

i. For each day, start from the spot curve $S(t)$ at $t = 1, 2, 3, 4, 5$.

ii. For each $n \in \{1, 2, 3, 4\}$, compute the 1Y-nY forward rate $F_{1,1+n}$ using the spot-to-forward identity under the assignment convention.

iii. Collect forward points: (1Y-1Y), (1Y-2Y), (1Y-3Y), (1Y-4Y) and plot across days.

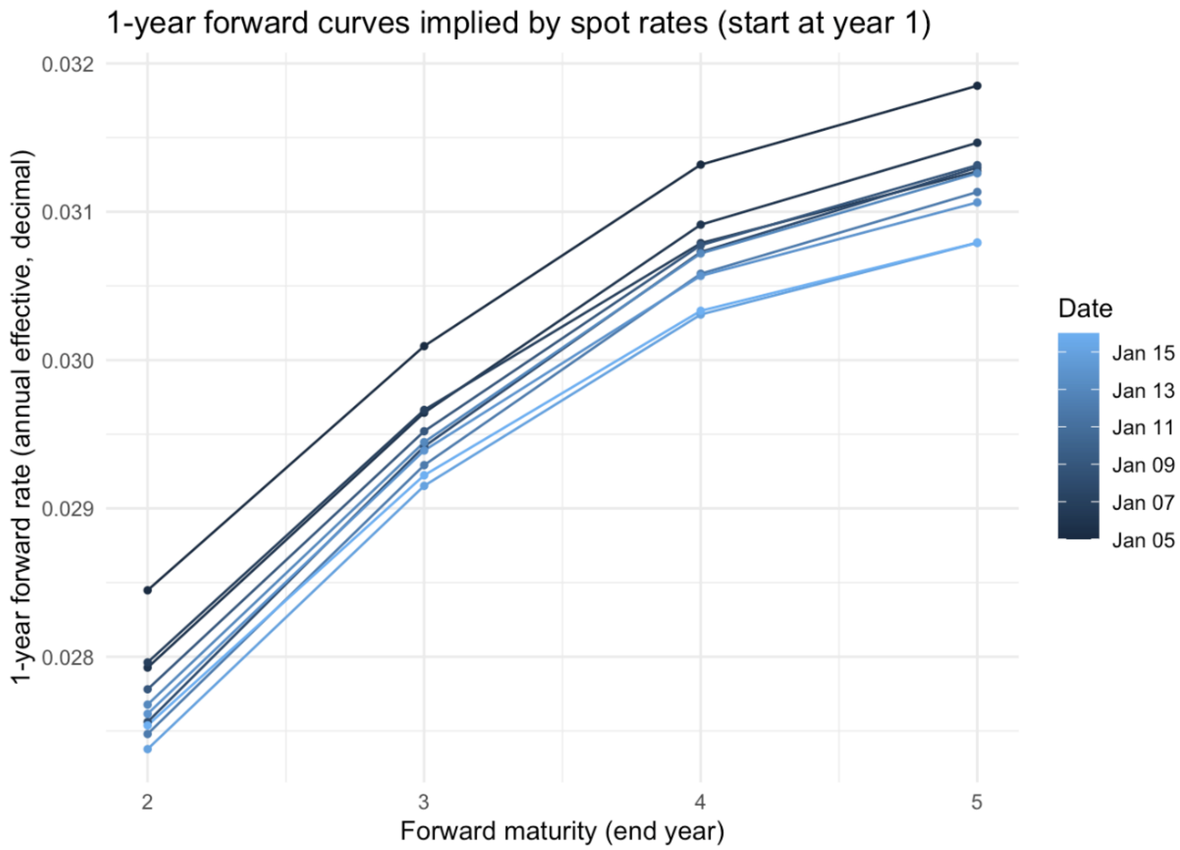

Figure 3: 1-year forward curves (2–5Y terms), daily (Jan 5–19, 2026).

```
##     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.02738 0.02898 0.03020 0.02979 0.03082 0.03185
```

Figure 4: Summary of daily log-returns used in covariance estimation.

```
##                  y1            y2            y3           y4           y5
## y1  3.406046e-05 -3.261099e-07 -1.765341e-06 5.387681e-06 1.114311e-05
## y2 -3.261099e-07  2.270148e-05  1.969717e-05 1.832577e-05 1.723540e-05
## y3 -1.765341e-06  1.969717e-05  2.074494e-05 1.713223e-05 1.416184e-05
## y4  5.387681e-06  1.832577e-05  1.713223e-05 1.803037e-05 1.878707e-05
## y5  1.114311e-05  1.723540e-05  1.416184e-05 1.878707e-05 2.262965e-05
```

Figure 5: Covariance matrix of daily log-returns for yields $(y_1, \ldots, y_5)$.

```
##              f_1y1y       f_1y2y       f_1y3y       f_1y4y
## f_1y1y 9.792245e-05 6.658668e-05 4.380086e-05 3.378003e-05
## f_1y2y 6.658668e-05 4.705242e-05 3.295341e-05 2.674269e-05
## f_1y3y 4.380086e-05 3.295341e-05 2.944258e-05 2.786381e-05
## f_1y4y 3.378003e-05 2.674269e-05 2.786381e-05 2.831382e-05
```

Figure 6: Covariance matrix of daily log-returns for forward rates (1Y-1Y to 1Y-4Y).

5.

```
## [1] 7.506226e-05 3.648097e-05 4.814021e-06 1.808636e-06 1.024323e-09
```

Figure 7: Eigenvalues of the yield covariance matrix.

```
##           [,1]        [,2]       [,3]       [,4]         [,5]
## [1,] 0.1772295  0.92430371 -0.3308903 -0.0689504  0.003129533
## [2,] 0.5103602 -0.22248344 -0.1791173 -0.8111399 -0.001660979
## [3,] 0.4645893 -0.26121340 -0.5796629  0.4912010  0.372339691
## [4,] 0.4867547 -0.02543223  0.1219521  0.2879760 -0.815242656
## [5,] 0.5053206  0.16518032  0.7124234  0.1144088  0.443542572
```

Figure 8: Eigenvectors of the yield covariance matrix.

```
## [1] 1.822639e-04 1.981072e-05 6.558689e-07 7.389255e-10
```

Figure 9: Eigenvalues of the forward-rate covariance matrix.

```
##            [,1]       [,2]        [,3]          [,4]
## [1,] -0.7152009  0.4777404 -0.51014834 -0.0007246672
## [2,] -0.5041294  0.1209791  0.81971047  0.2435001976
## [3,] -0.3705010 -0.4722686  0.07828639 -0.7959664672
## [4,] -0.3115610 -0.7308150 -0.24838410  0.5542061997
```

Figure 10: Eigenvectors of the forward-rate covariance matrix.

```
## [1] 6.352224e-01 3.087240e-01 4.073916e-02 1.530577e-02 8.668442e-06
```

Figure 11: Variance explained by yield PCs.

```
## [1] 8.990421e-01 9.771911e-02 3.235164e-03 3.644852e-06
```

Figure 12: Variance explained by forward PCs.

6. In both cases, the first (largest) eigenvalue and its eigenvector represent the dominant daily co-movement factor; empirically this is typically a level factor (roughly same-signed loadings across maturities), meaning most variation comes from near-parallel shifts of the curve.

References and GitHub Link to Code

Business Insider bond data (Frankfurt exchange), accessed January 2026.
The code and data for this assignment are available at: `https://github.com/YiyunGao/APM466_Assignment1`