MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Governments issue bonds to finance spending by borrowing from investors without permanently increasing the money supply, which helps avoid persistent inflation associated with printing money.

   (b) If markets expect future policy-rate cuts while the current policy rate remains high, short maturities stay elevated, but demand for longer bonds increases (anticipating lower future rates), pushing long yields down and flattening the long end of the curve.

   (c) Quantitative easing (QE) is large-scale central-bank purchases of longer-maturity securities to reduce longer-term yields and support market functioning; for example, since early 2020 the U.S. Fed purchased Treasuries and agency MBS to stabilize markets and ease financial conditions.

2. I construct 0–5Y curves using ten nominal Government of Canada (GoC) bonds with semi-annual coupons. These issues are suitable because (i) they share the same issuer and credit risk (GoC), so prices are comparable across maturities; (ii) they are actively traded benchmark-style issues, reducing pricing noise; and (iii) their coupon schedules and maturity dates provide cashflows spanning the short and medium horizon, allowing bootstrapping of discount factors (and hence spot rates) up to 5 years on each business day:

   - CAN 4.5 Feb 26 (ISIN CA135087R226; issued 11/1/2023; matures 2/1/2026)
   - CAN 0.25 Mar 26 (ISIN CA135087L518; issued 10/9/2020; matures 3/1/2026)
   - CAN 4 May 26 (ISIN CA135087R556; issued 2/12/2024; matures 5/1/2026)
   - CAN 1.5 Jun 26 (ISIN CA135087E679; issued 7/21/2015; matures 6/1/2026)
   - CAN 1 Sep 26 (ISIN CA135087L930; issued 4/16/2021; matures 9/1/2026)
   - CAN 4 Aug 26 (ISIN CA135087R978; issued 5/6/2024; matures 8/3/2026)
   - CAN 1 Jun 27 (ISIN CA135087F825; issued 8/3/2016; matures 6/1/2027)
   - CAN 3.25 Aug 27 (ISIN CA135087P733; issued 12/2/2022; matures 8/24/2027)
   - CAN 3.25 Sep 28 (ISIN CA135087Q491; issued 4/21/2023; matures 9/1/2028)
   - CAN 0.5 Dec 30 (ISIN CA135087L443; issued 10/5/2020; matures 12/1/2030)

3. Given a covariance matrix of curve points (e.g., daily log-returns of yields), eigenvectors represent the principal directions of co-movement (principal components), while eigenvalues measure the variance explained by each direction; the largest eigenpair therefore identifies the dominant factor driving day-to-day curve changes.

Empirical Questions - 75 points

4.

(a) I construct the daily 1–5Y YTM curves by computing yields-to-maturity for each bond on each business day and then interpolating across maturities to obtain the five annual points (1,2,3,4,5 years). I use piecewise linear interpolation in maturity (linear between adjacent maturities), which is appropriate here because the target grid is coarse (annual points) and the curve is smooth over the short horizon; linear interpolation is stable and avoids overfitting relative to higher-order splines.

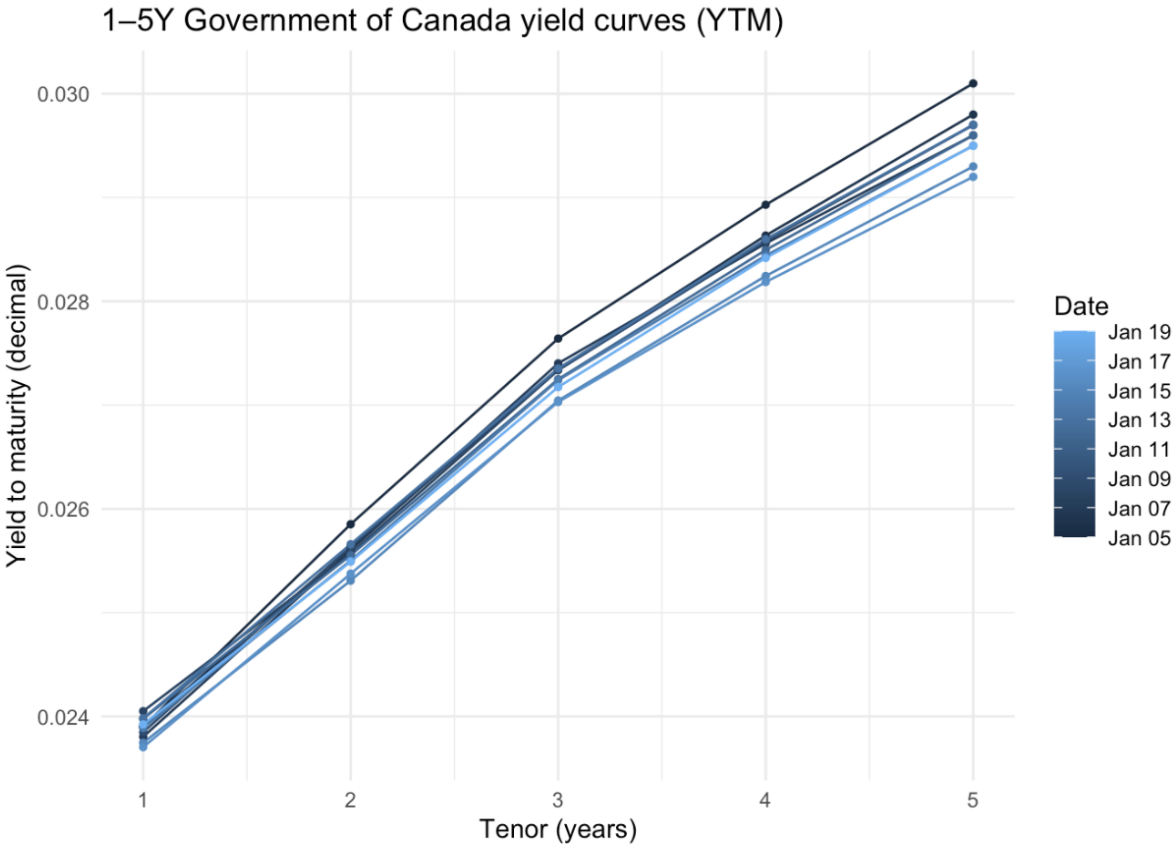

Figure 1: 1–5Y Government of Canada yield curves (YTM), daily (Jan 5–19, 2026).

(b) Pseudo-code to derive the 1–5Y spot curve (bootstrap, semi-annual coupons):

i. For each day, take the set of bond prices and the full cashflow schedules (coupon and principal).

ii. Sort all distinct cashflow dates in increasing order and initialize discount factors $D(t)$ at those dates as unknown.

iii. For each bond, write the pricing equation

$$P = \sum_i CF_i \, D(t_i),$$

where CF_i is the cashflow at date t_i.

iv. Proceed in increasing cashflow-date order: when reaching a new date t_k, subtract the present value of all earlier cashflows using already-solved $D(t_i)$, then solve the remaining linear equation for the new unknown $D(t_k)$.

v. Convert discount factors to spot rates using the assignment convention (same day-count/compounding used in the code).

vi. Interpolate spot rates to obtain annual spot points at exactly 1,2,3,4,5 years for plotting.

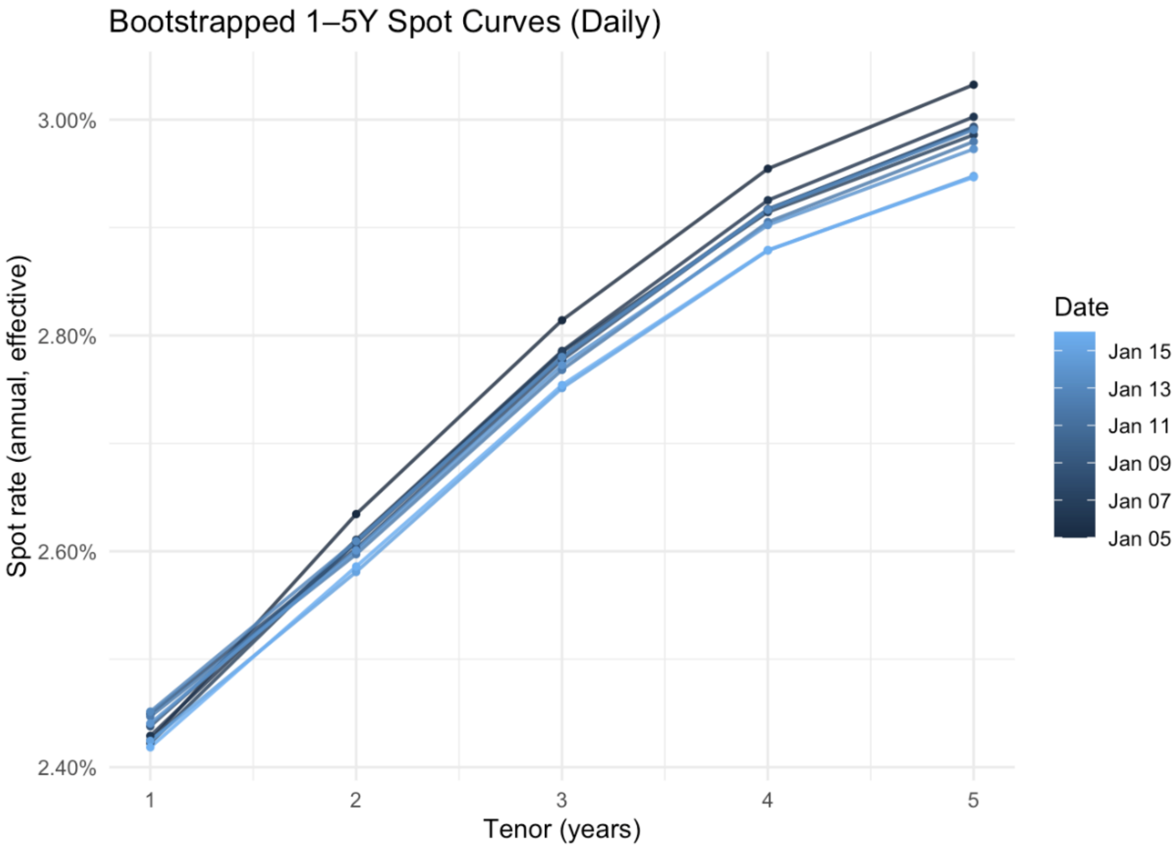

Figure 2: 1–5Y Government of Canada spot curves, daily (Jan 5–19, 2026).

(c) Pseudo-code to derive the 1-year forward curve (terms 2–5Y):

i. For each day, start from annual spot rates $S(1), S(2), S(3), S(4), S(5)$.

ii. For each $n \in \{1, 2, 3, 4\}$, compute the 1-year forward rate starting at 1 year and ending at $1 + n$ years using the spot-to-forward identity under the assignment convention.

iii. Collect forward points: (1Y-1Y), (1Y-2Y), (1Y-3Y), (1Y-4Y) and plot across days.

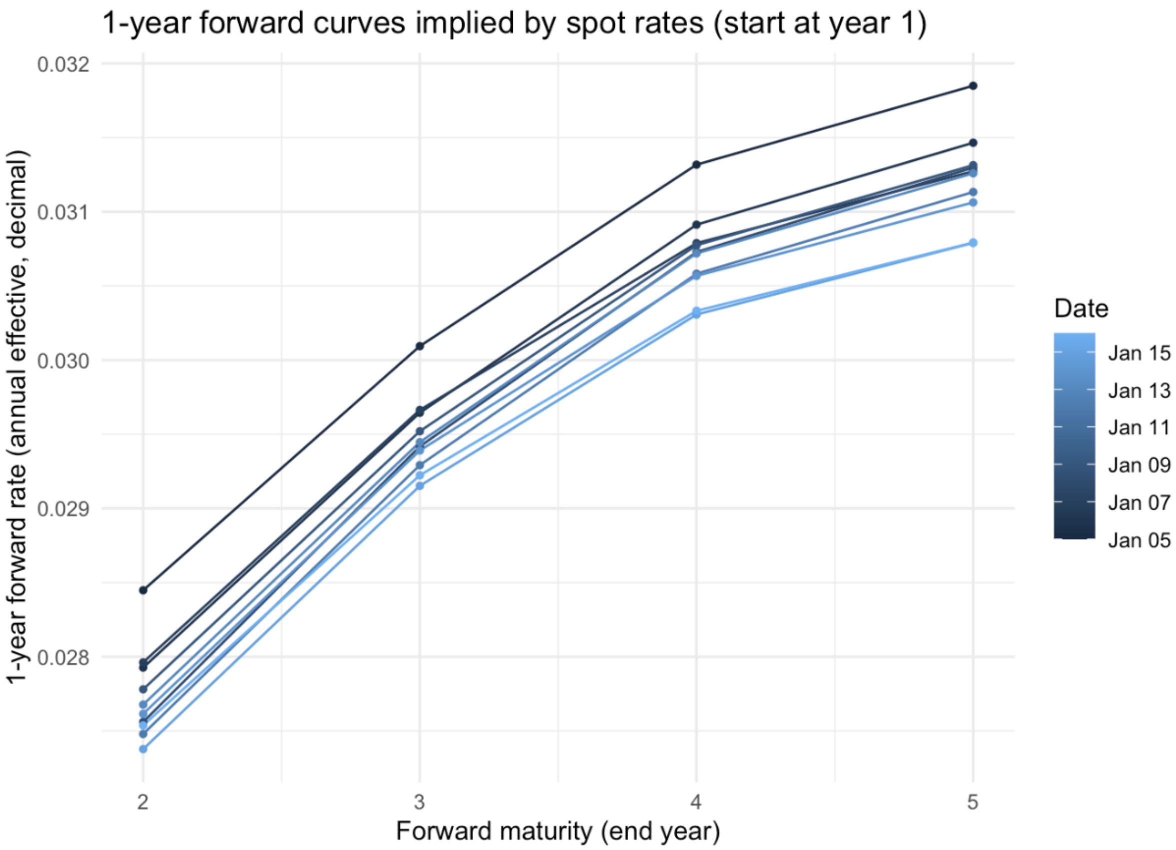

Figure 3: 1-year forward curves (2–5Y terms), daily (Jan 5–19, 2026).

5.

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.02738 0.02898 0.03020 0.02979 0.03082 0.03185
```

Figure 4: Summary of daily log-returns used in covariance estimation.

```
##                 y1            y2            y3           y4           y5
## y1  3.406046e-05 -3.261099e-07 -1.765341e-06 5.387681e-06 1.114311e-05
## y2 -3.261099e-07  2.270148e-05  1.969717e-05 1.832577e-05 1.723540e-05
## y3 -1.765341e-06  1.969717e-05  2.074494e-05 1.713223e-05 1.416184e-05
## y4  5.387681e-06  1.832577e-05  1.713223e-05 1.803037e-05 1.878707e-05
## y5  1.114311e-05  1.723540e-05  1.416184e-05 1.878707e-05 2.262965e-05
```

Figure 5: Covariance matrix of daily log-returns for yields $(y_1, \ldots, y_5)$.

```
##               f_1y1y       f_1y2y       f_1y3y       f_1y4y
## f_1y1y 9.792245e-05 6.658668e-05 4.380086e-05 3.378003e-05
## f_1y2y 6.658668e-05 4.705242e-05 3.295341e-05 2.674269e-05
## f_1y3y 4.380086e-05 3.295341e-05 2.944258e-05 2.786381e-05
## f_1y4y 3.378003e-05 2.674269e-05 2.786381e-05 2.831382e-05
```

Figure 6: Covariance matrix of daily log-returns for forward rates (1Y-1Y to 1Y-4Y).

6.

```
## [1] 7.506226e-05 3.648097e-05 4.814021e-06 1.808636e-06 1.024323e-09
```

Figure 7: Eigenvalues of the yield covariance matrix.

```
##           [,1]        [,2]       [,3]       [,4]         [,5]
## [1,] 0.1772295  0.92430371 -0.3308903 -0.0689504  0.003129533
## [2,] 0.5103602 -0.22248344 -0.1791173 -0.8111399 -0.001660979
## [3,] 0.4645893 -0.26121340 -0.5796629  0.4912010  0.372339691
## [4,] 0.4867547 -0.02543223  0.1219521  0.2879760 -0.815242656
## [5,] 0.5053206  0.16518032  0.7124234  0.1144088  0.443542572
```

Figure 8: Eigenvectors of the yield covariance matrix.

```
## [1] 1.822639e-04 1.981072e-05 6.558689e-07 7.389255e-10
```

Figure 9: Eigenvalues of the forward-rate covariance matrix.

```
##            [,1]       [,2]        [,3]          [,4]
## [1,] -0.7152009  0.4777404 -0.51014834 -0.0007246672
## [2,] -0.5041294  0.1209791  0.81971047  0.2435001976
## [3,] -0.3705010 -0.4722686  0.07828639 -0.7959664672
## [4,] -0.3115610 -0.7308150 -0.24838410  0.5542061997
```

Figure 10: Eigenvectors of the forward-rate covariance matrix.

```
## [1] 6.352224e-01 3.087240e-01 4.073916e-02 1.530577e-02 8.668442e-06
```

Figure 11: Variance explained by yield PCs.

```
## [1] 8.990421e-01 9.771911e-02 3.235164e-03 3.644852e-06
```

Figure 12: Variance explained by forward PCs.

The first (largest) eigenvalue and its associated eigenvector represent the dominant daily co-movement factor. In these short-maturity curves, the leading eigenvector typically has same-signed loadings across maturities, corresponding to a level factor (near-parallel shifts), which explains the largest share of the day-to-day variation.

References and GitHub Link to Code

Business Insider bond data (Frankfurt exchange), accessed January 2026.
The code and data for this assignment are available at: `https://github.com/YiyunGao/APM466_Assignment1`