

# Gender Ratio Problem

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# 1 Problem Statement

Theoretically, a mother gives birth to a boy or girl with the same probability. The gender ratio of boys and girls is thus expected to be 0.5 : 0.5 in a large population. What if every mother will not stop having a new child until she has a boy? Will the ratio be changed?

# 2 Solution

Let  $N$  denote the number of mothers and assume they have new children simultaneously. There will be  $N/2$  boys and girls in the first birth wave. For the  $N/2$  mothers who do not have boys will have new  $N/2/2$  boys and girls, and so on.

The total number of girls  $N_G$  can be obtained by the geometric series sum formula:

$$N_G = \sum_i \frac{1}{2^i} N = \left(1 - \left(\frac{1}{2}\right)^i\right) N.$$

Since each mother eventually has one and only one boy, the total number of boys is the same as mothers, i.e.,  $N_B = N$ .

The gender ratio  $R$  is thus

$$R = \frac{N_G}{N_B} = \left(1 - \left(\frac{1}{2}\right)^i\right).$$

We list the ratio values below for small  $N$ .

$N$	1	2	5	10	100
$R$	0.33 : 0.67	0.43 : 0.57	0.49 : 0.51	0.50 : 0.50	0.50 : 0.50

Counter-intuitively, as  $N$  grows, the ratio quickly approaches 0.5 : 0.5. Therefore, in a large population, the evil reproductive policy above will not change gender ratio.

However, since boys are always the last child in a family, the average age of boys will be less than that of girls, which will affect gender ratio in a long term (takes generations).