JUL. 2020

A Mathematical Model of COVID-19 Transmission

Y. Tang*[1], R. Jayatilaka[1], R. Patel[1], M. Brar[1], H. (F). Li[1], N.M. Jisrawi[1,2], S.R. Valluri[1,2]

[1]*Department of Physics & Astronomy, Western University, 1151 Richmond Street, London, Ontario, Canada*

[2]*King's University College, Western University, 266 Epworth Avenue, London, Ontario, Canada*

WOSU, 2020

1918 “Spanish” Influenza

NCR, 2020

2020 COVID-19

How do we model disease to better understand disease transmission, and better prepare for disease burden?

Foundation of Epidemics

The goal of epidemics is to figure out what causes different health outcomes in different groups of people

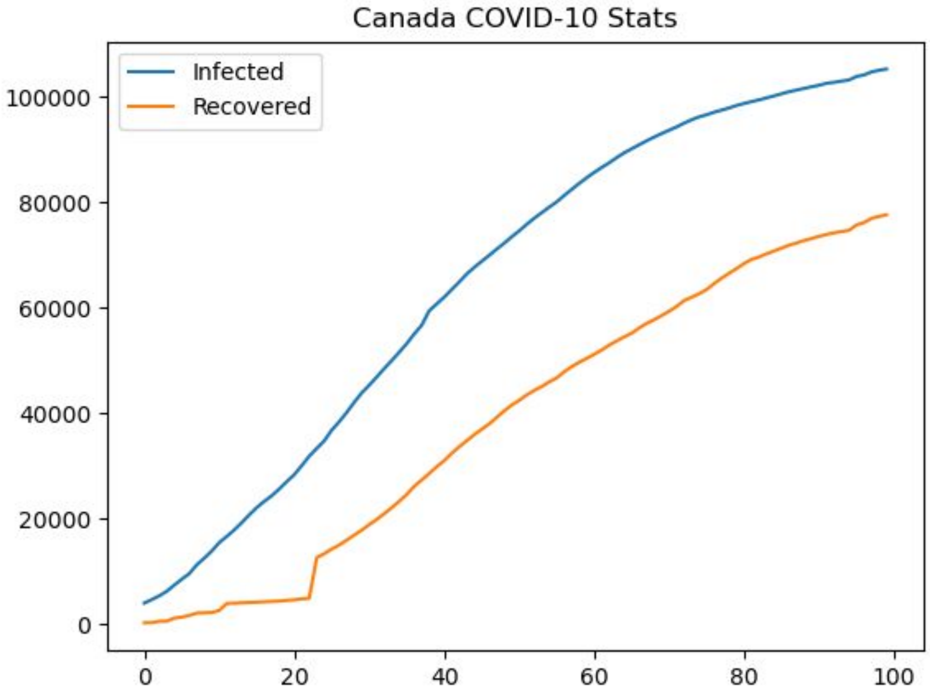
Importance of Epidemics

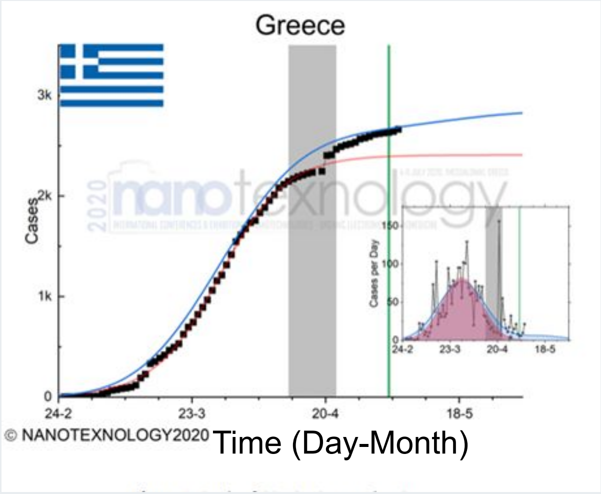
Disease distribution
Source of disease
Cause of disease
Methods of disease control

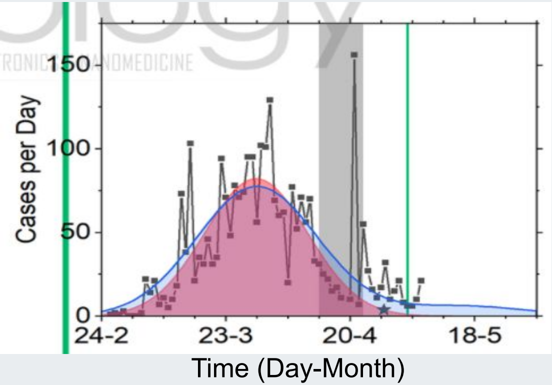
Epidemiology plays a very crucial aspect in studying the transmission of diseases since it provides a strong foundation

It is an inexact science, but the various branches of study encompass essential aspects of accurately understanding and predicting disease transmission

Discussion - Recent Covid-19 Development







What is the SIR Model?

In an SIR model for contagious diseases, an individual can be categorized as:

Susceptible (S(t)):
$s(t)=S(t)/N$

Infected (I(t)):
$i(t)=I(t)/N$

Removed (R(t)) (dead or cured):
$r(t)=R(t)/N$

along an independent variable, time, in a closed population, where (S(t)), (I(t)) and (R(t)) represent a numerical value over a selected population (N).

Overall, these equations must add to 1: $\mathbf{s(t)/N+i(t)/N+r(t)/N= 1}$

The Susceptible Equation:
$dS/dt= -\boldsymbol{\beta}\, s(t)\, i(t)$
Where $\boldsymbol{\beta}$ represents the infection rate

The Recovered Equation:
$dR/dt=\boldsymbol{\gamma}\, i(t)$
Where $\boldsymbol{\gamma}$ represents the recovery rate

The Infected Equation:
$dI/dt= -\beta\, s(t)\, i(t) - \boldsymbol{\gamma}\, i(t)$

What is the Lambert W function?

$$W(z)e^{W(z)} = z$$

Quotetab, 2020

JOHANN lAMBERT

Multivalued inverse of function:

$$w \to we^{w}$$

Implicitly elementary

Wide range of applications

How to apply SIR Model to COVID-19?

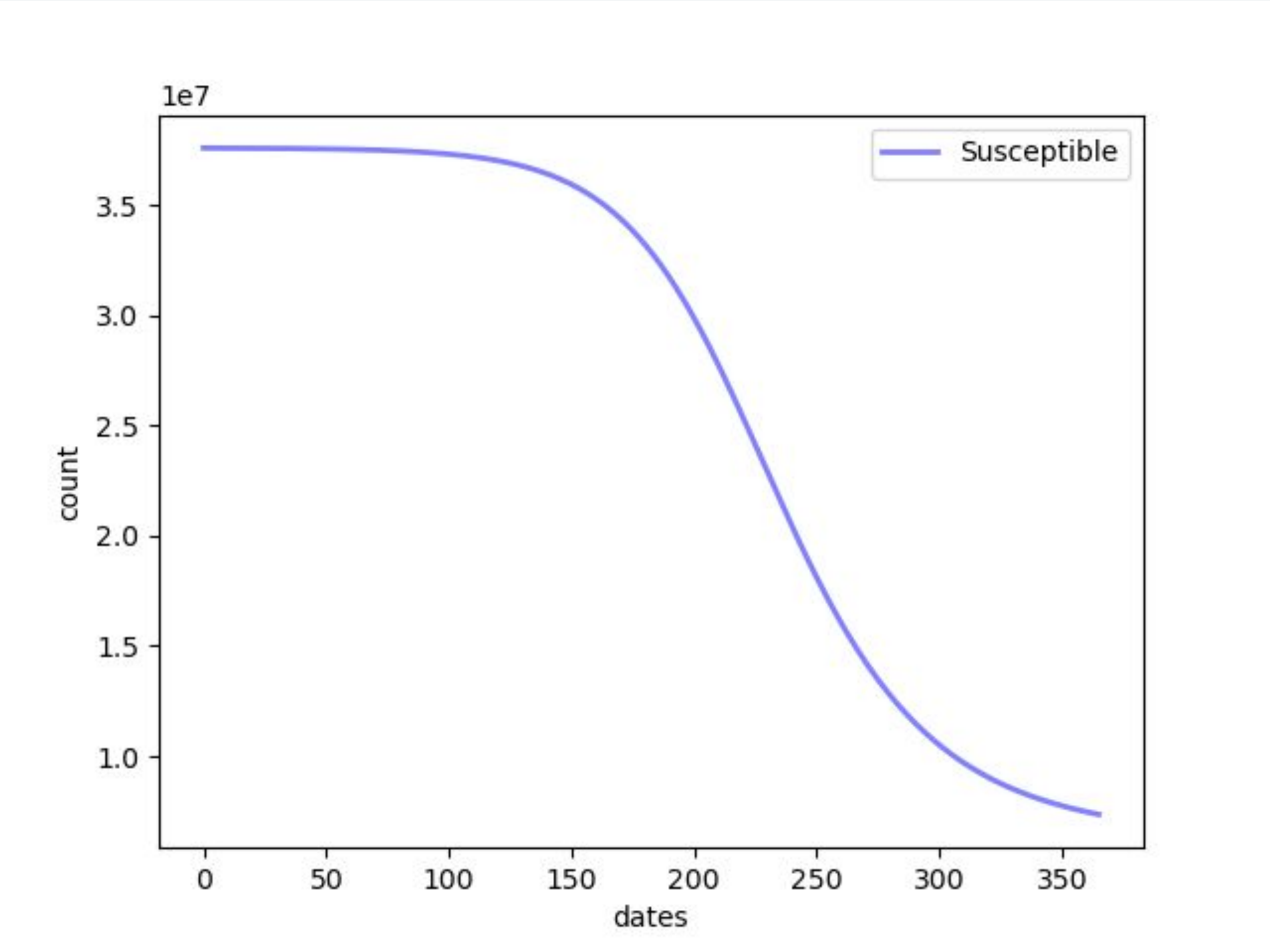
Gather data from Government of Canada's Health Infobase:

- Susceptible cases = total population - infected population
- Infected cases = confirmed cases
- Removed cases = recovered cases + deceased cases

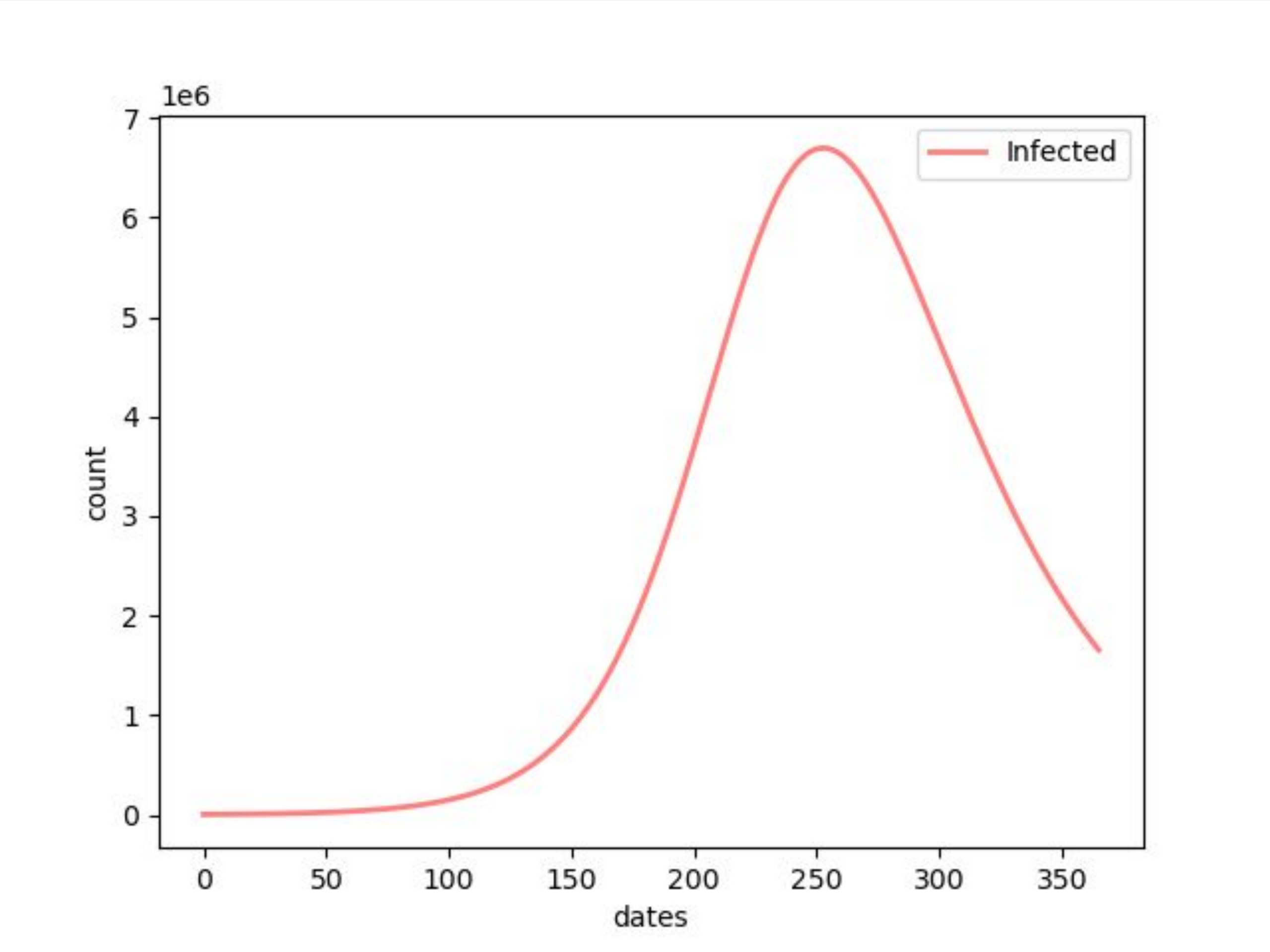
- Estimate initial values (beta, gamma)
- Numerically solve SIR equation

Utilize LambertW function to identify phase transition point

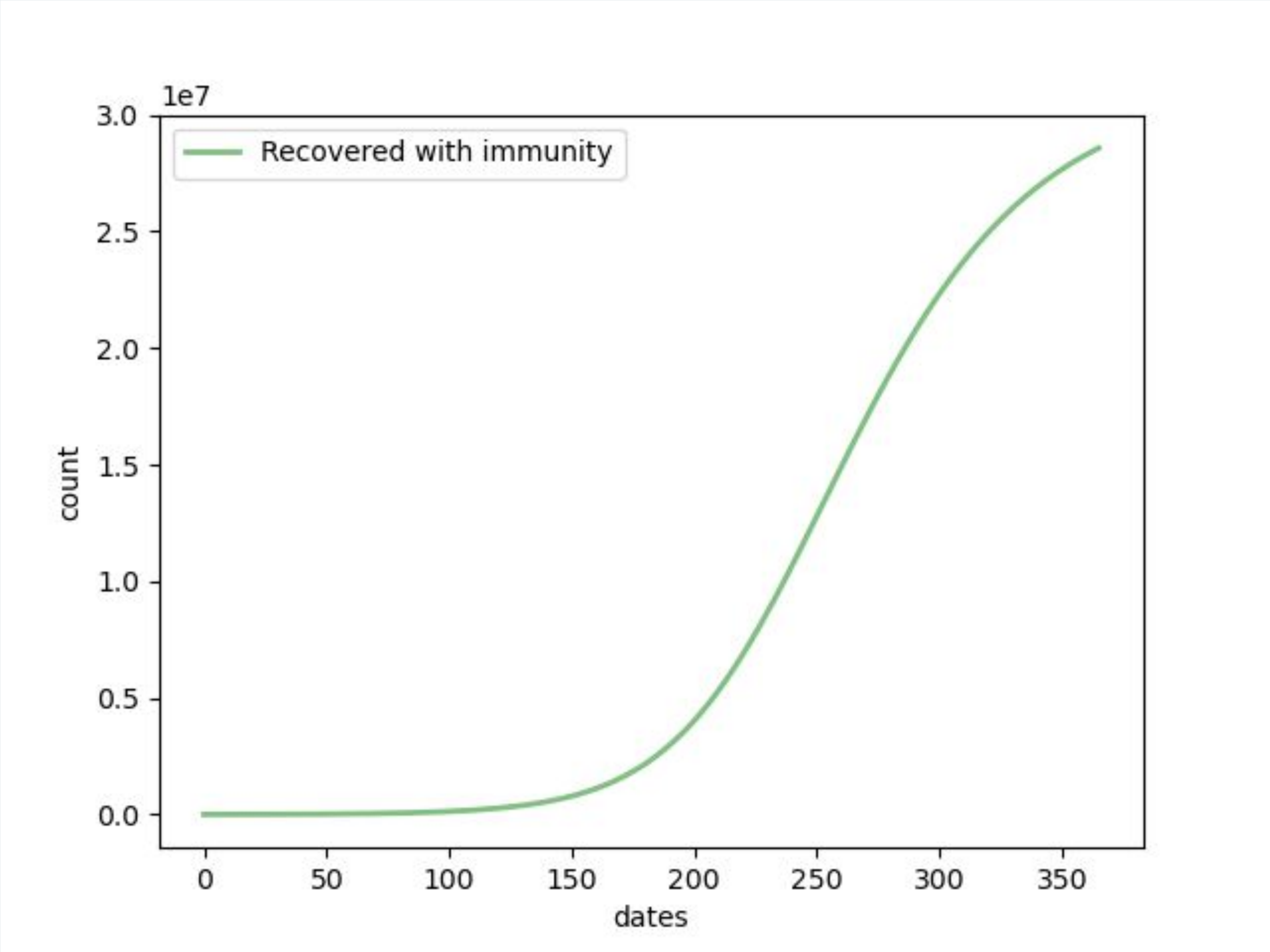
Results/Interpretation - S Curve



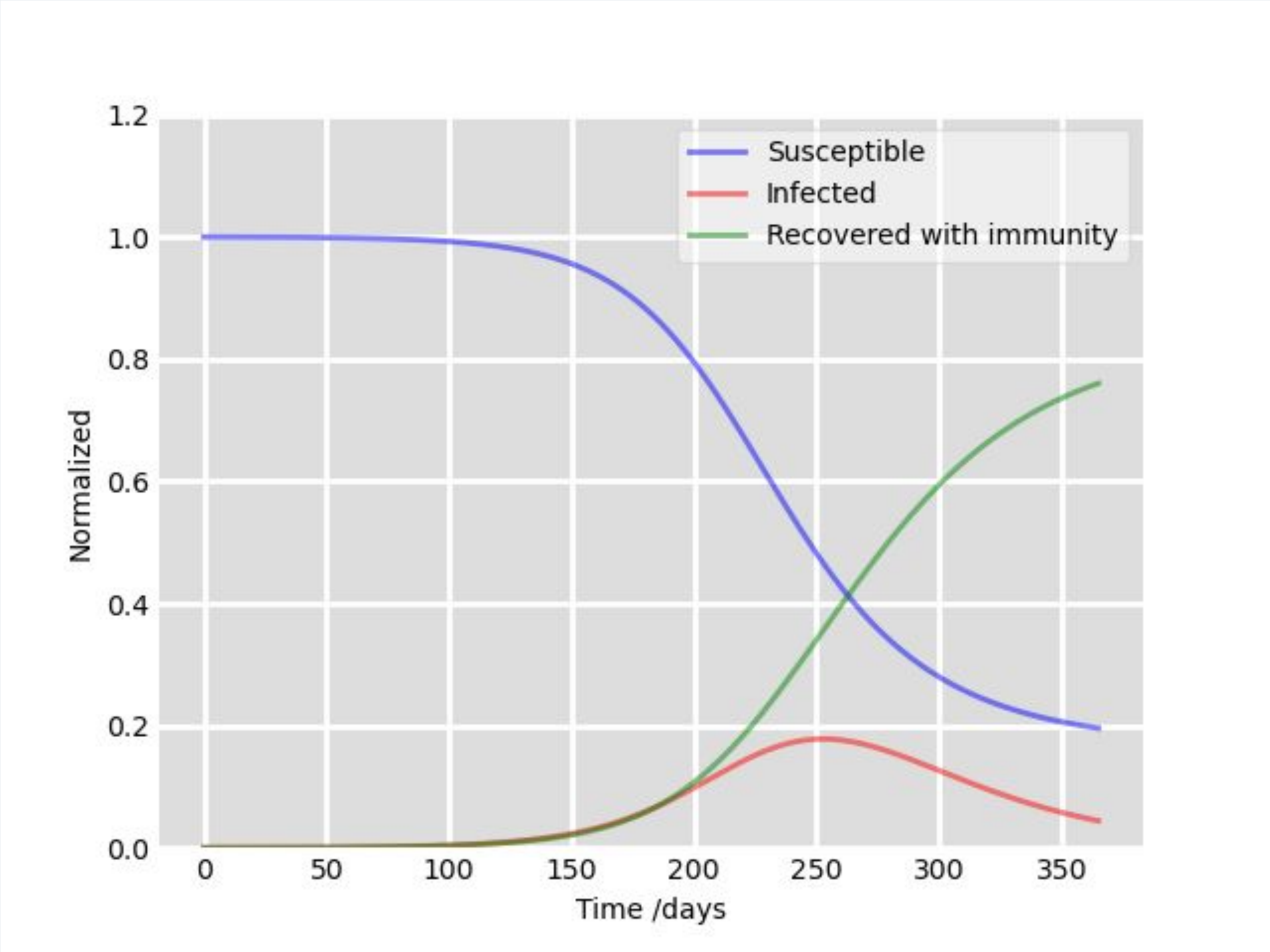
Results/Interpretation - I Curve



Results/Interpretation - R Curve



Results/Interpretation - SIR



Results/Interpretation - Solving **Lambert W Function**

From the two equations,

$$ds(t)/dt = -k * s(t) * i(t)$$

and

$$dr(t)/dt = l * i(t)$$

We can obtain

$$s(t) = s_0 * e^{-k*r(t)/l}$$

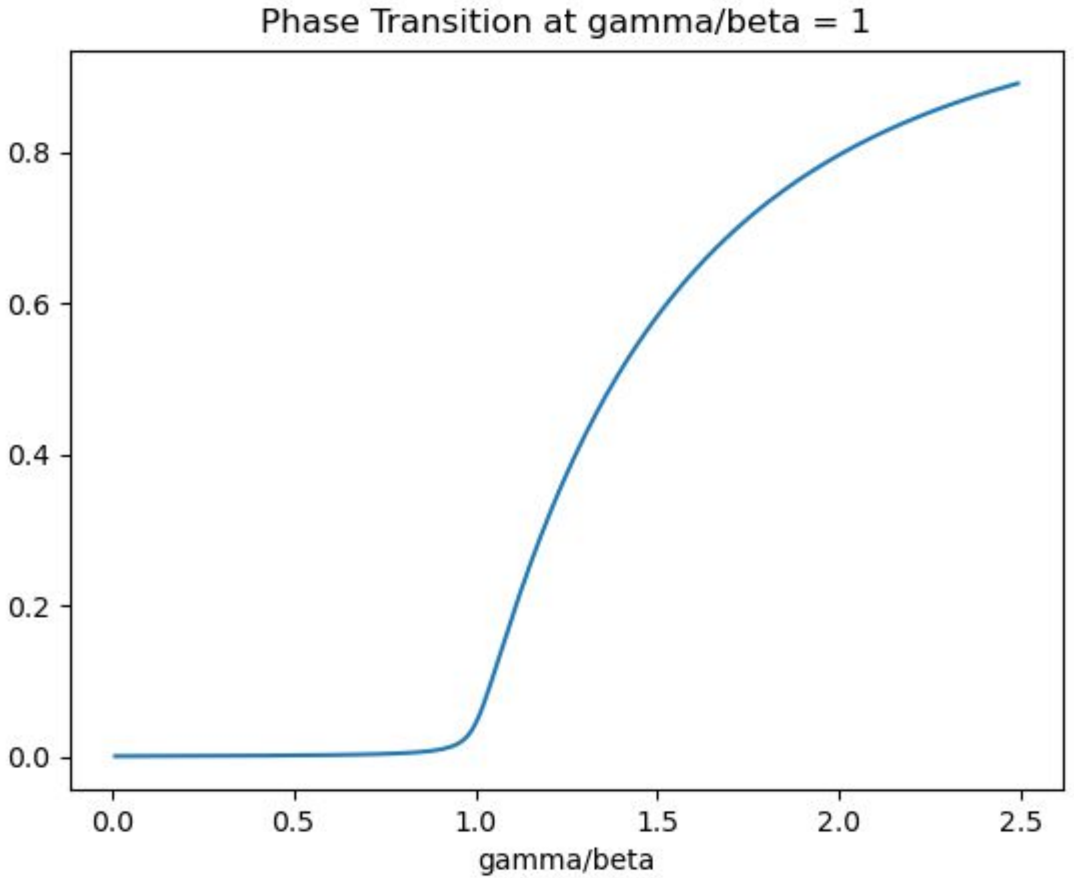
Therefore,

$$dr(t)/dt = l * (N - r(t) - s_0 * e^{-k*r(t)/l})$$

We obtain the Lambert function

$$l * LambertW(-999/1000 * k * e^{-k/l}/l)/k + 1$$

Results/Interpretation - Solving **Lambert W Function**



Work in Progress - Effect of Social/Physical Distancing

- By limiting social contact in a physical aspect, one can reduce opportunity for disease to spread
- Matrajt and Leung :
  - Social distancing interventions earlier in epidemic delay curve while later interventions flatten curve
  - Epidemic rebounded when interventions ended
- Chen et. al:
  - Interventions could reduce reproduction number of the disease.

Conclusion

- In the SIR model, the asymptotic value of the recovered class can be expressed as a Lambert W function. From the plot of the W function, we can see that the phase transition happens when gamma equals to beta.
- Future steps: include the effect of social distancing into our models

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