Homework 7

Vincent La
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1. Show that
$$Res_{z=\pi i}\frac{\exp zt}{\sinh z} + Res_{z=-\pi i}\frac{\exp zt}{\sinh z} = -2\cos \pi t$$

   Proof. Let us tackle this problem by separating evaluating the numerators and denominators.

   First, let us evaluate the denominators. In general, because $\sinh z = -i \sin iz$ it follows that $\frac{d}{dz}\sinh z = -i^2 \cdot \cos iz = \cos iz$ Therefore, the denominators are
$$\begin{aligned}\cos i(\pi i) &= \cos \pi i^2 = \cos -\pi \\ \cos i(-\pi i) &= \cos -\pi i^2 = \cos \pi\end{aligned}$$

   Both of these expressions are -1, so they share a common denominator.

   Then, recall the identity $\exp z = \exp x \cos y + i \exp x \sin y$ where x and y are the real and imaginary parts of z respectively. Therefore,
$$\begin{aligned}\exp \pi it &= \exp 0 \cos \pi t + i \exp 0 \sin \pi t \\ &= \cos \pi t + i \sin \pi t \\ \exp -\pi it &= \exp 0 \cos -\pi t + i \exp 0 \sin -\pi t \\ &= \cos -\pi t + i \sin -\pi t\end{aligned}$$

   Simplifying the original expression, we thus get
$$Res_{z=\pi i}\frac{\exp zt}{\sinh z} + Res_{z=-\pi i}\frac{\exp zt}{\sinh z} = \frac{(\cos \pi t + \cos -\pi t) + i(\sin \pi t + \sin -\pi t)}{-1}$$

   Because cos is an even function, and sin is an odd function, this simplifies to $-2\cos \pi t$ as we set out to prove. $\square$

2. Evaluate the integral
$$\int_C \frac{dz}{\sinh 2z}$$
   about $C : |z| = 2$.