Complex Exponential Function

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1 Complex Sine and Cosine

Let $z \in \mathbb{C}$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

1.1 Derivatives

$$\frac{d}{dz}\cos(z) = -\sin(z)$$

Proof.

$$\begin{aligned}
\frac{d}{dz}\cos(z) &= \frac{d}{dz}\frac{e^{iz} + e^{-iz}}{2} \\
&= \frac{ie^{iz} - ie^{-iz}}{2} \\
&= \frac{i(e^{iz} - e^{-iz})}{2} \cdot \frac{i}{i} \\
&= \frac{i^2(e^{iz} - e^{-iz})}{2i} \\
&= -(\frac{e^{iz} - e^{-iz}}{2i}) = -\sin(z)
\end{aligned}$$

□