

vectorization

getting rid of for loops in calculations

example

↳ effectuer des opérations mathématiques sur l'ensemble des données

$$z = w^T x + b \quad w, x \in \mathbb{R}^n$$

non vectorized implementation

$$z = 0$$

for i in range(n)

$$z += w[i] * x[i]$$

$$z += b$$

vectorized implementation

$$z = np.dot(w, x) + b$$

example 2

$$u = Av$$

$$u_i = \sum_j A_{ij} \cdot v_j$$

non vectorized implementation

$$u = np.zeros((n, 1))$$

for i in range(n):

for j in range(n):

for j in range(n)

$$u[i] += A[i, j] * v[j]$$

vectorized implementation

$$u = np.dot(A, v)$$

example 3: vectors and matrix valued functions

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \quad u = \begin{bmatrix} e^{v_1} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

non vectorized

$$u = np.zeros((n, 1))$$

for i in range(n)

$$u[i] = math.exp(v[i])$$

vectorized

$$u = np.exp(v)$$

$$ab - (a+b) + bc$$

$$b(a+c) - (a+bc)$$

python / numpy vectors

$a = \text{np.random.randn}(5)$

$a.\text{shape} = (5,)$: rank 1 arrays

~~$a.\text{shape} = (5,)$~~

$a = [0,5; -0,2; 0,95; 0,8; 0,7]$

$a.T$



$[0,5; -0,2; 0,95; 0,8; 0,7]$

pas de transpose

or $\text{np.dot}(a, a.T) \in \mathbb{R}$

$a = \text{np.random.randn}(5,1)$

$a.\text{shape} = (5,1)$

$a.T.\text{shape} = (1,5)$

$\sim \mathbb{R}_g(0,0)$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 21 \\ 13 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}$$

$$(3,4) + \dots = (3,4)$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}$$

$$ab + ac \neq b+c = a(b+c)$$

$(8,1)$

$$- \sum_{i=1}^m y^{(i)} \log(a^{[2]}(i))$$

$$Y = [y^{(1)}, \dots, y^{(m)}]$$

$$A^{[2]} = A_2 = [A^{[2]}(1), \dots, A^{[2]}(m)]$$

first method

$$\text{logprobs} = \text{np.multiply}(\text{np.log}(A_2), Y)$$

$$\text{cost} = -\frac{1}{m} \sum (\text{logprobs})$$

second method

$$\text{cost} = -\frac{1}{m} \sum (\text{np.dot}(\text{np.log}(A_2), Y)) \cdot \text{squeeze}()$$

intuition about derivatives

$$f(a) = 3a$$

$$a = 2 \Rightarrow f(2) = 6$$

$$a = 2.001 \quad f(a) = 6.003$$

$$\frac{df(a)}{da} = 3 : \text{slope}$$

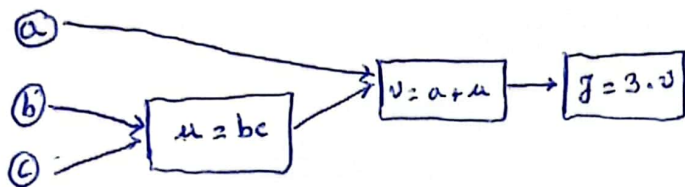
Computation graphs

$$f(a, b, c) = 3(a + bc)$$

$$u = bc$$

$$v = a + u$$

$$f = 3 \cdot v$$



derivatives with a computation graph

$$\frac{df}{dv} ? \quad f = 3 \cdot v$$

$$\frac{df}{dv} = 3$$

$$\frac{\partial f}{\partial a} ? \quad \frac{df}{da} = \frac{df}{dv} \cdot \frac{\partial v}{\partial a} = 3 \cdot 1 = 3$$

$$\frac{\partial f}{\partial u} = \frac{df}{dv} \cdot \frac{\partial v}{\partial u} = 3 \cdot 1 = 3$$

$$\frac{\partial f}{\partial b} = \frac{df}{dv} \cdot \frac{\partial v}{\partial u} \cdot \frac{\partial u}{\partial b} = 3 \cdot 1 \cdot c = c$$

$$\frac{\partial f}{\partial c} = \frac{df}{dv} \cdot \frac{\partial v}{\partial u} \cdot \frac{\partial u}{\partial c} = 3 \cdot 1 \cdot b = b$$

Broadcasting in numpy

	apples	beef	Eggs	potatoes
carb	56.0	0.0	4.4	68.0
protein	1.2	104.0	52.0	8.0
Fat	1.8	135.0	99.0	0.9

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + 100 \Leftrightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

calculate % of calories from carb, prot, fat
can we do it without explicit for loop

sum of the columns

$$\text{sums} = A.\text{sum}(\text{axis}=0)$$

~~percentages~~

$$[59, 239, 155.4, 76.9]$$

$$\text{percentages} = 100 \times A / \text{sums}.\text{reshape}(1,4)$$

$$[1, 2, 3, 4] + 100 \Leftrightarrow [1, 2, 3, 4] + [100, 100, 100, 100]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + [100 \ 200 \ 300] \Leftrightarrow$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 100 & 200 & 300 \\ 100 & 200 & 300 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 100 \\ 200 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 100 & 100 & 100 \\ 200 & 200 & 200 \end{bmatrix}$$

In general

$$\begin{matrix} (m,n) \\ \text{matrix} \end{matrix} \begin{matrix} + \\ \times \\ / \end{matrix} \begin{matrix} (1,n) \\ \\ \end{matrix} \Leftrightarrow \begin{matrix} (m,n) \\ \text{matrix} \end{matrix} \begin{matrix} + \\ \times \\ / \end{matrix} \begin{matrix} (m,n) \\ \\ \end{matrix}$$

$$\begin{matrix} (m,n) \\ \text{matrix} \end{matrix} \begin{matrix} + \\ \times \\ / \end{matrix} \begin{matrix} (m,1) \\ \\ \end{matrix} \Leftrightarrow \begin{matrix} (m,n) \\ \text{matrix} \end{matrix} \begin{matrix} + \\ \times \\ / \end{matrix} \begin{matrix} (m,n) \\ \\ \end{matrix}$$