2
CHAPTER

Mechanical Springs

Springs are mechanical elements that deflect under load, absorb and store energy and release the energy when load is removed. They work as shock absorbers in automobiles, used for mechanical clamping in machine tools, used in inlet and outlet valves in IC engines. Torsion springs are used in watches and toys. Springs have been used in the field of machine design, such as to cushion impact and shock loading, to store energy, to maintain contact between machine members, used in for measuring devices, spring balances and control of vibrations.

2.1 CLASSIFICATION OF SPRINGS

Springs are broadly classified as coil springs, leaf springs and disc springs. Coil springs are further classified as tension springs, compression springs, torsion springs, volute springs etc. Leaf springs are either cantilever type or simply supported type. If there are more than one leaf, then they are called laminated springs. The semi-elliptical laminated springs are widely used in cars, trucks and buses. Disc springs are used where large loads are to be supported with small deflection. They are mainly used in machine tools for slide clamping. Figure 2.1 shows some typical springs.

2.2 HELICAL SPRINGS

Helical springs are made of circular cross-section, rectangular cross-section or square cross-section wires. Most commonly used cross-section is circular. When subjected to axial load, the springs will experience a torsional shear stress and a transverse shear stress. In addition, there is an additional stress effect due to curvature.

Figure 2.2 shows a helical spring under compression due to axial load of F.

Symbols:

d = wire diameter, mm

r = wire radius, mm

D = mean coil diameter, mm

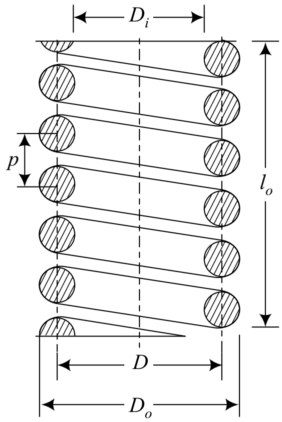


Fig. 2.2

R = Mean radius, mm

D_i = inside diameter of coil, mm

$= D - d$

D_o = outside diameter of coil, mm

$= D + d$

C = spring index = D/d. $\left(\dfrac{20-24}{P20.8}\right)$

It is the ratio of mean coil dia to wire diameter. Commonly used ratios are C = 4 to 10.

F = axial load, N

y = deflection under load, mm

i = no. of active turns.

i' = total no. of coils = $i + n$

n = additional no. of coils depending on the end conditions.

There are four types of end condition as shown in Fig. 2.3.

G = modulus rigidity, MPa

p = pitch, mm

It is the distance between two consecutive turns.

K = Wahl's curvature factor or stress factor

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad \left(\frac{20-23}{P20.8}\right)$$

F_o = Stiffness or spring rate or spring scale, $\dfrac{\text{N}}{\text{mm}}$

Step 3. Deflection $y = \dfrac{8FD^3 i}{Gd^4}$ (20-29)

find deflection 'y' if no. of coils is given.

or find no. of active coils 'i' if deflection is given.

Step 4. Free length of spring

$$l_o \geq (i+n)d + y + a \qquad \left(\frac{20-53}{P20.20}\right)$$

where i = no. of active coils

n = extra no. of coils depending on end conditions

n = 0 for plain or free ends

= 1 for either squared or ground ends

= 2 for both squared and ground ends

(It is a good design practice to use squared and ground ends for which $n = 2$).

$$a = \text{clearance between coils} \simeq 0.25\,y \qquad \left(\frac{20-50b}{P20.19}\right)$$

(25% of working deflections).

Step 5. Pitch (Ref. Fig. 2.3)

$$p = \frac{l_o - 2d}{i} \text{ for squared and ground ends}$$

$$p = \frac{l_o - d}{i} \text{ for plain ends}$$

$$p = \frac{l_o}{i} \text{ for ground ends, and}$$

$$p = \frac{l_o - 3d}{i} \text{ for squared ends}$$

Step 6. Shiftness $F_o = F/y$ $\left(\dfrac{20-30}{P20.9}\right)$

Step 7. Length of wire required to manufacture the spring

$$l_w = \pi D\,(i') \qquad \left(\frac{20-289}{P20.8}\right)$$

where i' = Total no. of coils = $(i + n)$

$\therefore$ $l_w = \pi D\,(i + n)$

2.8 NATURAL FREQUENCY OF VIBRATION OF HELICAL SPRINGS

Helical compression springs are widely used in inlet and outlet valves of IC engines for opening and closing that operate at very high frequencies. If the operating frequency is close to natural frequency of vibration, then resonance occurs leading to vibrations and subsequent failure of springs. Hence, it is sometimes necessary to calculate the natural frequency of vibration of helical springs.

(iii) Free length of spring

$$l_o \geq (i+n)\,d + y + a$$

$$i = 10,\ n = 2,\ d = 6.3,\ y = 112.51$$

assume $a = 0.25y = 0.25 \times 112.51 = 28.13$ mm.

$\therefore$ $l_o \geq (10 + 2)\,6.3 + 112.51 + 28.13$

$\geq 216.24 \simeq 217$ mm.

(iv) Critical frequency of vibration, when one end is at rest

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2F_o g}{W}} \qquad \left(\frac{20-75}{P20.23}\right)$$

Stiffness $F_o = 11.875$ N/mm

$g = 9810$ mm/sec^2

W = weight of spring = vol × SP. weight

= (Area × length of wire) × w

$$W = \left(\frac{\pi}{4}d^2 \times l_w\right)w$$

Length of wire $l_W = \pi D i' = \pi D\,(i+n)$

$l_w = \pi \times 51 \times 12 = 1922.65$ mm

$$W = \left(\frac{\pi}{4} \times 6.3^2 \times 1922.65\right) \times 76.518 \times 10^{-6}$$

$$W = 4.586 \text{ N}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2 \times 11.875 \times 9810}{4.586}} = 35.873 \text{ H}_z$$

Problem 3 Design a helical compression spring for a spring loaded safety valve for the following data:

Operating pressure	= 1 MPa
Maximum pressure when the valve blows off freely	= 1.1 MPa
Maximum list of valve when the pressure is 1.1 MPa	= 6 mm
Dia of valve seat	= 100 mm
Maximum shear stress	= 360 MPa
Modulus of rigidity	= 84 GPa
Spring index	= 5.5

Step 2. Mean coil diameter $D = Cd = 5.5 \times 21 = 115.5$ mm

Inside dia of coil $D_i = D - d = 115.5 - 21 = 94.5$ mm

Outside dia of coil $D_o = D + d = 115.5 + 21 = 136.5$ mm

Step 3. No. of coils $i = \dfrac{yGd^4}{8FD^3}$

$$i = \frac{66 \times 84 \times 10^3 \times 21^4}{8 \times 8639.38 \times 115.5^3} = 10.12 \simeq 11 \text{ coils}$$

Step 4. Free length $l_o \geq (i + n)d + y + a$

$n = 2$ assuming squared and ground ends

assume clearance $a = 0.25 \quad y = 0.25 \times 66 = 16.5$ mm

$l_o \geq (11 + 2)\ 21 + 66 + 16.5$

$l_o \geq 355.5 \approx 356$ mm

(Rounded off to nearest higher integer)

Step 5. Pitch $p = \dfrac{l_o - 2d}{i} = \dfrac{356 - 2 \times 21}{11} = 28.36$ mm

Step 6. Stiffness $F_o = \dfrac{F}{y} = \dfrac{8639.38}{66} = 130.9$ N/mm

Step 7. Length of wire $l_w = \pi D\,(i + n)$

$l_w = \pi \times 115.5\ (11 + 2)$

$l_w = 4717.1$ mm.

Problem 4 A spring loaded safety valve for a boiler is required to blow off at a pressure of 0.8 MPa. The dia of valve seat is 90 mm and maximum lift of valve is of 10 mm. Design a suitable spring for the valve assuming the spring index as 7. Provide an initial compression of 30 mm. Take alowable shear stress as 420 MPa.

Solution Given data:

Blow off pressure $p_2 = 0.8$ MPa

Dia of valve seat $D_V = 90$ mm

Lift $y' = 10$ mm

Initial compression $y_1 = 30$ mm

Spring index $C = 7$

Final compression $y = y_2 = y_1 + y' = 30 + 10 = 40$ mm

Allowable shear stress $\tau = 420$ MPa

Select modulus of rigidity for steels $G = 80$ GPa $= 80 \times 10^3$ MPa

Force on the spring = Force on valve seat = Area of valve × pressure

Free length, $l_o \geq (i+n)d + y + a$

Given clearance between coils = $\frac{1}{4}$ of wire dia = $\frac{d}{4}$

∴ Total clearance $a = i \times \frac{d}{4} = 6 \times \frac{12.5}{4} = 18.75$ mm

$$l_o \geq (6+2)12.5 + 30 + 18.75$$

$$l_o \geq 148.75 \approx 149 \text{ mm}$$

$$p = \frac{149 - 2 \times 12.5}{6} = 20.67 \text{ mm}$$

(d) Natural frequency of vibration when both ends fixed

$$f_n = \frac{1}{\pi}\sqrt{\frac{2 f_o g}{W}} \qquad \left(\frac{20.76}{P20.24}\right)$$

$$F_o = \text{stiffness} = \frac{F}{y} = \frac{5000}{30} = 166.67 \text{ N/mm}$$

$$g = 9810 \text{ mm/sec}^2$$

W = weight of spring = volume × specific weight

W = (Area of C/S × length of wire) × w

For steel wire, specific weight

$$w = 76.6 \times 10^{-6} \text{ N/mm}^3 \qquad \text{(Table 2-10, } P\,2.38)$$

length of wire = $\pi D i'$

$$W = \left(\frac{\pi}{4} d^2 l_w\right) w = \frac{\pi}{4} \times 12.5^2 \times (\pi \times 62.5 \times 8) \times 76.6 \times 10^{-6}$$

$$= 14.766 \text{ N}$$

∴ $$f_n = \frac{1}{\pi}\sqrt{\frac{2 \times 166.67 \times 9810}{14.766}} = 149.79 \text{ Hz..}$$

Problem 6 A spring controlled governor of the Hartnell type has two balls, each weighing 2.2 kg and each attached to the arms of a bell crank lever which pivots about a fixed fulcrum. The outer arms of bell crank levers carry rollers which lift the sleeve against the pressure exerted by a spring surrounding the governor spindle. The two arms of the bell crank lever are of equal length and maximum and minimum radii rotation of governor balls are 90 mm and 75 mm. If the sleeve begins to raise at 240 rpm and increase in speed allowed is 7%, find the initial load on the sleeve and the stiffness of the spring.

Design a helical spring for the following data:

- Circular cross-section,
- Maximum working stress = 210 MPa

Minimum force

$$(2F_{C_1})x = F_1 \cdot x$$

$\therefore$ $$F_1 = 2 \times 104.22 = 208.44 \text{ N}$$

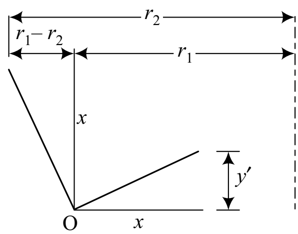
Maximum force

$$(2F_{C_2})x = F_2 \cdot \text{x}$$

$\therefore$ $$F_2 = 2 \times 143.2 = 286.4 \text{ N}.$$

These are the loads acting on sleeve and the same loads are acting on spring at minimum and maximum position of sleeve.

To find lift:



From similar triangles

$$\frac{r_2 - r_1}{x} = \frac{y'}{x}$$

$\therefore$ lift $$y' = r_2 - r_1 = 90 - 75 = 15 \text{ mm}.$$

We know that

stiffness $$\frac{F_1}{y_1} = \frac{F_2}{y_2} = \frac{F_2 - F_1}{y'} \qquad \left(\frac{20-31}{P20.9}\right)$$

$\therefore$ maximum deflection $$y_2 = \frac{F_2 y'}{F_2 - F_1}$$

$$y_2 = \frac{286.4 \times 15}{286.4 - 208.44} = 55.1 \text{ mm}$$

Initial load on sleeve $= F_1 = 208.44$ N.

Stiffness $$= \frac{F_2}{y_2} = \frac{286.4}{55.1} = 5.198 \text{ N/mm}$$

Now consider spring:

Maximum load on spring $F_2 = F = 286.4$ N

Maximum deflection $y = y_2 = 55.1$ mm

Allowable shear stress $\tau = 210$ MPa

Spring index $C = 6$

Rigidity modulus $G = 81 \text{ GPa} = 81 \times 10^3$ MPa.

Solution Given data:

Torque $= 120$ N-m $= 120 \times 10^3$ N mm

Single plate clutch, $\therefore$ no. of friction surfaces

$i = 2$ (Both sides of plate effective)

Diameters ratio $= \dfrac{D_2}{D_1} = 2$

axial pressure $p = 0.18$ MPa.

Coefficient of friction $\mu = 0.3$

No. of springs $= 5$

First consider clutch problem.

Torque $M_t = 120 \times 10^3$ N-mm

Assuming uniform wear, mean diameter of friction lining

$$D_m = \frac{D_1 + D_2}{2} = \frac{D_1 + 2D_1}{2} = 1.5\, D_1 \qquad \left(\frac{19-85b}{P19.9}\right)$$

Axial force acting on friction lining $F_a = \dfrac{1}{2}\pi p D_1 (D_2 - D_1)$ $\left(\dfrac{19-83}{P19.9}\right)$

$$F_a = \frac{1}{2}\pi \times 0.18 \times D_1\,(2D_1 - D_1) = 0.2827\, D_1^2$$

Torque capacity of single plate clutch

$$M_t = \frac{1}{2}\mu\, Fa D_m i \qquad \left(\frac{19-84}{P19.9}\right)$$

$$120 \times 10^3 = \frac{1}{2} \times 0.3 \times 0.2827\, D_1^2 \times 1.5\, D_1 \times 2$$

Inside dia of clutch

$$D_1 = 98.07 \simeq 98 \text{ mm}$$

Outside dia of clutch

$$D_2 = 2D_1 = 2 \times 98 = 196 \text{ mm}$$

$\therefore$ Axial force $F_a = 0.2827\, D_1^2 = 2715$ N

This force is exerted by 5 springs.

$\therefore$ Load on each spring $F_1 = \dfrac{F_a}{5} = \dfrac{2715}{5} = 543$ N

Spring Design

Force $F_1 = 543.8$ N

Spring index $C = 6$

Initial compression $y_1 = 10$ mm

Step 1. Shear stress $\tau = \dfrac{8FCK}{\pi d^2}$

where $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = \dfrac{4\times 6-1}{4\times 6-4} + \dfrac{0.615}{6} = 1.2525$

$$270 = \frac{8\times 6480\times 6\times 1.2525}{\pi\times d^2} \Rightarrow \text{wire dia } d = 21.43 \simeq 22 \text{ mm}$$

Standard dia $d = 22$ mm.

Step 2. Mean coil dia $D = Cd = 6\times 22 = 132$ mm

Inside dia of coil $D_i = D - d = 132 - 22 = 110$ mm

Outside dia of coil $D_o = D + d = 132 + 22 = 154$ mm

Step 3. Deflection $y = \dfrac{8FD^3 i}{Gd^4}$

$\therefore$ No. of active coils $i = \dfrac{yGd^4}{8FD^3}$

$$i = \frac{200\times 84\times 10^3\times 22^4}{8\times 6480\times 132^3} = 33 \text{ coils}$$

Step 4. Free length $l_o \geq (i+n)d + y + a$

Assume squared and ground ends,

$n = 2$

Assume clearance $a = 0.25\, y$

$\therefore$ $l_o \geq (33+2)22 + 200 + 0.25\times 200$

$l_o = 1020$ mm.

Step 5. Pitch $p = \dfrac{l_o - 2d}{i}$ for squared and ground

$$= \frac{1020 - 2\times 22}{33} = 29.58 \text{ mm}$$

Step 6. Stiffness $F_o = \dfrac{F}{y} = \dfrac{6480}{200} = 32.4$ N/mm

Step 7. Length of wire $l_w = \pi Di' = \pi D(i+n)$

$l_w = \pi\times 132\,(33+2) = 3518.58$ mm

Problem 9 A railway car weighing 20 kN and moving with a velocity of 15 kmph is to be stopped by a buffer consisting of 4 helical compression springs in which the maximum compression allowed is 0.3 m. Find the no. of active turns required if the spring is made of 20 mm dia wire and 160 mm mean coil diameter. Also find the maximum shear stress induced in the coils.

$$\frac{F \times 200}{2} = 0.3 \times 10^6 \Rightarrow F = 3000 \text{ N}$$

$\therefore$ $F = 3000$ N, $y = 200$ mm, $C = 5$, $\tau = 420$ MPa, $G = 80 \times 10^3$ MPa

Step 1. Shear stress $\tau = \dfrac{8FCK}{\pi d^2}$

where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C} = \dfrac{4 \times 5 - 1}{4 \times 5 - 4} + \dfrac{0.615}{5} = 1.3105$

$\therefore$ $420 = \dfrac{8 \times 3000 \times 5 \times 1.3105}{\pi d^2} \Rightarrow$ dia of wire $d = 10.917$

Standard dia from Table 2.2,

$$d = 11 \text{ mm}$$

Step 2. Mean coil diameter $D = Cd = 5 \times 11 = 55$ mm

Inside dia of coil $D_i = D - d = 55 - 11 = 44$ mm

Outside dia of coil $D_o = D + d = 55 + 11 = 66$ mm

Step 3. Deflection $y = \dfrac{8FD^3 i}{Gd^4}$

$\therefore$ no. of active coils $i = \dfrac{yGd^4}{8FD^3}$

$$i = \frac{200 \times 80 \times 10^3 \times 11^4}{8 \times 3000 \times 55^3} = 58.67 \approx 59 \text{ coils}$$

Step 4. Free length $l_o \geq (i + n)\, d + y + a$

Assuming squared and ground ends,

$$n = 2$$

Assume clearance $a = 0.25\, y = 0.25 \times 200 = 50$ mm

$\therefore$ $l_o \geq (59 + 2)11 + 200 + 55 \geq 921$

$l_o = 921$ mm.

Step 5. Pitch $p = \dfrac{l_o - 2d}{i} = \dfrac{921 - 2 \times 11}{59} = 15.237$ mm

Step 6. Stiffness $F_o = \dfrac{F}{y} = \dfrac{3000}{200} = 15$ N/mm

Step 7. Length of wire $l_w = \pi D i' = \pi D\,(i + n)$

$l_w = \pi \times 55 \times (59 + 2) = 10540$ mm.

Problem 11 Design the spring for an elevator at the bottom of which 8 identical springs are set in parallel to absorb the shock of the elevator in case of failure. The weight of the elevator is 20 kN and has a free fall of 1.2 m from rest. The spring is made of 20 mm dia wire and has a spring index of 5.5. Each spring has 12 active turns. Neglect the effect of counter weight.

Solution Given data:

No. of springs $= 8$

Weight of elevator $= 20$ kN $= 20000$ N

Counter weight $= 0$ (neglect)

$\therefore$ Net weight $W =$ Weight – counter weight $= 20000$ N

Height of free fall $H = 1.2$ m $= 1200$ mm.

Wire diameter $d = 20$ mm

Spring index $C = 5.5$

$\therefore$ Mean coil diameter $D = Cd = 5.5 \times 20 = 110$ mm

No. of active coils $i = 12$

Assume rigidity modulus $G = 80 \times 10^3$ MPa

Let deflection $= y$

$\therefore$ Total distance moved by load

$$h = H + y = 1200 + y$$

Falling body develops potential energy

$$\text{Total } PE = mgh = Wh = W \cdot (H + y)$$

$$\text{Total } PE = 20000\,(1200 + y)$$

$\therefore$ Energy/spring

$$= \frac{\text{Total } PE}{\text{No. of springs}} = \frac{20000\,(12 + y)}{8} = 2500\,(1200 + y)$$

Equating this to resilience, $U = \dfrac{Fy}{2} = 2500\,(1200 + y)$ ...(1)

But deflection

$$y = \frac{8FD^3 i}{Gd^4} = \frac{8 \times F \times 110^3 \times 12}{80 \times 10^3 \times 20^4}$$

$$y = 9.9825 \times 10^{-3}\, F \qquad \text{...(2)}$$

Substituting in (1)

$$\frac{F \times 9.9825 \times 10^{-3} F}{2} = 2500\,(1200 + 9.9825 \times 10^{-3}\, F)$$

$$4.99 \times 10^{-3} F - 24.9563 F - 3 \times 10^6 = 0$$

Solving

$$F = \frac{24.9563 \pm \sqrt{24.9563^2 - 4 \times 4.99 \times 10^{-3}(-3 \times 10^6)}}{2 \times 4.99 \times 10^{-3}}$$

$$F = 27147.24 \;\text{ or } -22145.98 \text{ N}$$

Minimum force on spring when the valve is closed $F_1 = 360$ N

Maximum force on spring when the valve is open $F_2 = 540$ N

$\therefore$ Maximum deflection $y_2 = \dfrac{F_2 y'}{F_2 - F_1}$ $\left(\dfrac{20-31}{P20.9}\right)$

$$y_2 = \frac{540 \times 10}{540 - 360} = 30 \text{ mm}$$

Select *Cr Va* steel for spring and factor of safety = 3

From Table 2.1, for chrome-vanadium steel, (Table 20-10, *P* 20.15)

torsional ultimate stress $\tau_u = 965$ MPa

and $G = 78.45$ GPa = 78450 MPa

Allowable shear stress $\tau = \dfrac{\tau_u}{Fos} = \dfrac{965}{3} = 321.67$ MPa

Design the spring for maximum load and maximum deflection.

$$F = F_2 = 540 \text{ N}, \quad y = y_2 = 30 \text{ mm}, \quad \tau = 321.67 \text{ MPa}$$

$$G = 78.45 \times 10^3 \text{ MPa}, D_i = 30 \text{ mm}$$

$\therefore$ $D_o = D_i + d = 30 + d$

Since C is not known, K will be unknown and hence use trial and error procedure to solve for wire dia 'd'.

Temporarily assume $K = 1.2$ to 1.4 say $K = 1.3$ (based on earlier problems) to arrive at first trial

$$\tau = \frac{8FDK}{\pi d^3}$$

$$321.67 = \frac{8 \times 540 \times (30 + d) \times 1.3}{\pi \times d^3}$$

$$d^3 = \frac{8 \times 540 \times (30 + d) \times 1.3}{\pi \times 321.67} = 5.56\,(30 + d)$$

$\therefore$ $d^3 - 5.56\,d - 166.8 = 0$

Solving for d, $d \simeq 5.84$

Select standard dia $d = 6$ mm as trial (1)

Trial (1) $d = 6$ mm

$$D = D_i + d = 30 + 6 = 36 \text{ mm}$$

$\therefore$ $C = \dfrac{D}{d} = \dfrac{36}{6} = 6$

$\therefore$ $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = \dfrac{4 \times 6 - 1}{4 \times 6 - 4} + \dfrac{0.615}{6} = 1.2525$

l_{o2} = Free length of inner spring

F_1 = Load on outer spring

F_2 = Load on inner spring

F = Total load = $F_1 + F_2$

τ_1 = Shear stress in outer spring

τ_2 = Shear stress in inner spring

C_1 = Spring index of outer spring = $\dfrac{D_1}{d_1}$

C_2 = Spring index of inner spring = $\dfrac{D_2}{d_2}$

The concentric springs may be of either equal length (height) or unequal length (one spring longer than the other).

Case (i). Springs of the same length

If the free length is same for both the springs, the deflections will be same.

Hence, $y_1 = y_2$

$$\frac{8F_1D_1^3i_1}{GD_1^4} = \frac{8F_2D_2^3i_2}{GD_2^4} \qquad \text{...(1)}$$

and total load $F = F_1 + F_2$ ...(2)

Using these two relations, loads F_1 and F_2 can be calculated.

Case (ii). Springs of different free lengths.

Since one spring is longer than the other, initially the longer spring alone takes the load. After the lengths become equal, the remaining load is shared by both springs for further equal deflection.

Clearance between the Two Springs in a Concentric Spring

When a spring is compressed, the outer diameter of coil will slightly increase. Hence, in a concentric spring, to avoid inner spring coming in contact with outer spring, i.e. to maintain desirable clearance between the springs, the following condition is to be satisfied.

$$\frac{d_1}{d_2} = \frac{C}{C-2}$$

where d_1 = wire dia of outer spring

d_2 = wire dia of inner spring

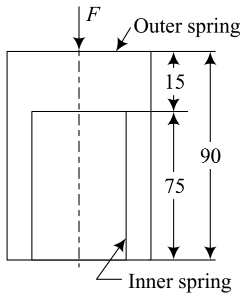
C = spring index = $\dfrac{D_1}{d_1} = \dfrac{D_2}{d_2}$

Problem 13 One helical spring is rested inside another. The dimensions are tabulated. Both springs have same free length and carry a maximum load of 3 kN. Take G = 80 GPa.

No. of active coils	for inner spring	$i_2 = 8$
	outer spring	$i_1 = 10$
Free length for inner spring		$l_{o_2} = 75$ mm
outer spring		$l_{o_1} = 90$ mm

Here the outer spring is longer than inner spring as shown in sketch, by 90 – 75 = 15 mm = y_i.

Since the outer spring is longer by 15 mm, initially, the outer spring alone supports the load. After the lengths become equal (i.e., after the outer spring is compressed by 15 mm), the remaining load will be shared by both springs for further equal deflection.



Let F_i be the load required to compress the longer outer spring by 15 mm.

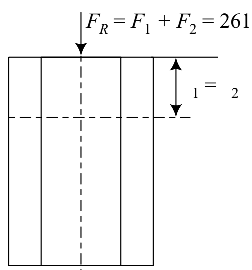
$$y_i = \left(\frac{8FD^3 i}{Gd^4}\right)_{\text{outer spring}}$$

$$15 = \frac{8F_i\, 40^3 \times 10}{84000 \times 4.875^4} \quad \Rightarrow \quad F_i = 138.995 \simeq 139 \text{ N}$$

Remaining load $\quad F_R = F - F_i = 400 - 139 = 261$ N

$F_R = F_1 + F_2 = 261$ N

This load is shared by both springs of equal length (The analysis is same as problem 13).



Here $\quad y_1 = y_2$

$$\frac{8F_1 D_1^3 i_1}{Gd_1^4} = \frac{8F_2 D_2^3 i_2}{Gd_2^4}$$

(ii) It is asked to draw load vs deflection curve upto a deflection of 60 mm of inner spring. At deflection of $y = 40$ mm, the inner spring will compress by $y_2 = 40$ mm, outer spring will compress by $y_1 = 40 - 30 = 10$ mm only.

Find F_1 and F_2 for these deflections of $y_1 = 10$ and $y_2 = 40$.

Total load $\quad F = F_1 + F_2$ at $y = 40$

(iii) Repeat this for $\quad y = 50, \quad$ i.e. $y_2 = 50$ and $\quad y_1 = 50 - 30 = 20$

and $\quad y = 60, \quad$ i.e. $y_2 = 60$ and $\quad y_1 = 60 - 30 = 30$

(iv) Draw a graph for F vs y at these 4 conditions.

Step 1. At $\quad y = y_2 = 30,$

$$y_2 = \frac{8F_2\, D_2^3 i_2}{G d_2^4}$$

SAE 6150 is Cr Va steel for which $G = 78.45$ GPa.

(from Table 2.1) (Table 20-10 P 20.15)

$$30 = \frac{8F_2 \times 60^3 \times 15}{78.45 \times 10^3 \times 6^4} \quad \Rightarrow \quad F_2 = 117.7 \text{ N}$$

∴ Total load at $\quad y = 30$, i.e. $\quad (y_1 = 0, y_2 = 30)$

$$F_{30} = F_1 + F_2 = 0 + 117.7 = 117.7 \text{ N}$$

Step 2. At $\quad y = 40, \quad y_1 = 10$ and $y_2 = 40$

∴ For outer spring $\quad y_1 = \dfrac{8F_1 D_1^3 i_1}{G d_1^4}$

$$10 = \frac{8 \times F_1 \times 100^3 \times 20}{78.45 \times 10^3 \times 10^4} \quad \Rightarrow \quad F_1 = 49.03 \text{ N}$$

and for inner spring $\quad y_2 = \dfrac{8F_2 D_2^3 i_2}{G d_2^4}$

$$40 = \frac{8 \times F_2 \times 60^3 \times 15}{78.45 \times 10^3 \times 6^4} \quad \Rightarrow \quad F_2 = 156.9 \text{ N}$$

∴ $\quad F_{40} = F_1 + F_2 = 49.03 + 156.9 = 205.93$ N

Step 3. At $\quad y = 50, \quad y_1 = 20$ and $y_2 = 50$

For outer spring

$$20 = \frac{8F_1 \times 100^3 \times 20}{78.45 \times 10^3 \times 10^4} \quad \Rightarrow \quad F_1 = 98.06 \text{ N}$$

For inner spring

$$50 = \frac{8F_2 \times 60^3 \times 15}{78.45 \times 10^3 \times 6^4} \qquad \Rightarrow \quad F_2 = 196.13 \text{ N}$$

$\therefore \qquad F_{50} = F_1 + F_2 = 98.06 + 196.13 = 294.19$

Step 4. at $\qquad y = 60, \quad y_1 = 30$ and $y_2 = 60$

For outer spring, $\quad 30 = \dfrac{8\,\mathrm{F}_1 \times 100^3 \times 20}{78.45 \times 10^3 \times 10^4} \qquad \Rightarrow \quad F_1 = 147.09 \text{ N}$

For inner spring $\quad 60 = \dfrac{8F_2 \times 60^3 \times 15}{78.45 \times 10^3 \times 6^4} \qquad \Rightarrow \quad F_2 = 235.35 \text{ N}$

$\therefore \qquad F_{60} = F_1 + F_2 = 147.09 + 235.35 = 382.44 \text{ N}$

Draw a graph for F vs y for combined spring as shown in Fig. 2.10.

$F_{30} = 1117.7$ N at $y = 30$ mm
$F_{40} = 205.93$ N at $y = 40$ mm
$F_{50} = 294.19$ N at $y = 50$ mm
$F_{60} = 382.44$ N at $y = 60$ mm

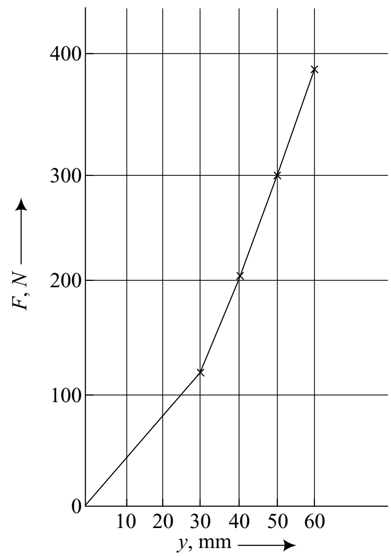


Fig. 2.10 Load vs deflection

Problem 16 Design a set of helical springs to support a load of 3 kN. The spring index is 6 and deflection under load is to be 60 mm. Both springs have same free length and are made of oil tempered steel wire.

Solution Given data: $F = 3000 \text{ N} = F_1 + F_2$

Spring index $C = 6 = C_1 = C_2$

Deflection $y = y_1 = y_2 = 60 \text{ mm}$

Since the free lengths are same, the deflections are same material, oil tempered steel wire. From Table 2.1, (Table 20-10 *P* 20.15)

Ultimate shear stress $\tau_u = 795$ MPa and

$G = 78.45 \text{ GPa} = 78.45 \times 10^3 \text{ MPa}$

Assume a factor of safety $Fos = 3$ based on ultimate stress

$$\therefore \quad \tau = \frac{\tau_u}{Fos} = \frac{795}{3} = 265 \text{ MPa}$$

To maintain optimum clearance between the inner and outer springs,

$$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5$$

$$d_1 = 1.5\, d_2$$

Since the material is same, the allowable shear stress is same, $\tau_1 = \tau_2$

$$\frac{8F_1C_1K_1}{\pi d_1^2} = \frac{8F_2C_2K_2}{\pi d_2^2}$$

$$\therefore \quad \frac{F_1}{d_1^2} = \frac{F_2}{d_2^2} \quad \text{(since } C_1 = C_2 \text{ and } K_1 = K_2\text{)}$$

$$\frac{F_1}{F_2} = \left(\frac{d_1}{d_2}\right)^2 = 1.5^2 = 2.25$$

$$\therefore \quad F_1 = 2.25\, F_2$$

But $F_1 + F_2 = F = 3000 \text{ N}$

$$2.25\, F_2 + F_2 = 3000$$

$$F_2 = 923.08 \text{ N}$$

$$F_1 = 3000 - F_2 = 2076.92 \text{ N}$$

(now design the springs independently)

(a) Design of outer spring: $F = F_1 = 2076.92 \text{ N}$

$y = 60$ mm, $C = 6$, $\tau = 265$ MPa, $G = 78.45 \times 10^3$ MPa

Step 1. Shear stress $$\tau = \frac{8FCK}{\pi d^2}$$

For $$C = 6,\; K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

Step 5. Pitch $p = \dfrac{l_o - 2d}{i} = \dfrac{313 - 2 \times 8.5}{25} = 11.84$ mm

Step 6. Stiffness $F_o = \dfrac{F}{y} = \dfrac{923.08}{60} = 15.38$ N/mm

Step 7. Length of wire $l_w = \pi Di' = \pi D(i + n)$

$= \pi \times 51(25 + 2) = 4325.97$ mm.

2.11 DESIGN OF SPRING SUBJECTED TO FLUCTUATING LOADS

Springs used in IC engine valves operate several times every minute. The load on spring varies cyclically from a minimum value when the valve is closed to a maximum value when the valve is open.

Such springs may fail in fatigue. Either Soderberg's or Goodman's criteria may be used while designing these springs.

Soderberg's relation:

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1 \text{ and} \qquad \left(\frac{20-62}{P20.22}\right)$$

Goodman's relation:

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{ud}} = 1 \qquad (20\text{-}61)$$

where τ_{-1d} = design endurance stress $= \dfrac{\tau_{-1}}{FoS}$

τ_{-1} = torsional endurance stress $\approx 0.6\, \tau_u$

τ_{yd} = design yield stress in shear $= \dfrac{\tau_y}{FoS}$

τ_y = torsional yield stress

τ_{ud} = design ultimate stress $= \dfrac{\tau_u}{FoS}$

τ_u = ultimate shear stress

FoS = factor of safety (usually 2 to 4)

τ_a = amplitude stress

$$= K_w \frac{8D}{\pi d^3}\left(\frac{F_{\max} - F_{\min}}{2}\right) \qquad \left(\frac{20-58}{P20.21}\right)$$

τ_m = mean stress

$$K_\tau = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{6} = 1.083$$

$$K_C = 1.15 \text{ for } C = 6 \qquad \text{(Table 20-15 } P\text{ 20.19)}$$

$$\therefore \quad K_w = K_\tau \cdot K_c = 1.083 \times 1.15 = 1.245$$

$$\tau_a = k_w \frac{8D}{\pi d^3}\left(\frac{F_{\max} - F_{\min}}{2}\right) \qquad \left(\frac{20-58}{P20.21}\right)$$

put $$\frac{D}{d} = C = 6$$

$$\tau_a = 1.245 \times \frac{8 \times 6}{\pi d^2}\left(\frac{450-250}{2}\right) = \frac{1902.22}{d^2}$$

$$\tau_m = k_\tau \frac{8D}{\pi d^3}\left(\frac{F_{\max} + F_{\min}}{2}\right) \qquad \left(\frac{20-59}{P20.21}\right)$$

$$\tau_m = 1.083 \ \frac{8 \times 6}{\pi d^2}\left(\frac{450+250}{2}\right) = \frac{5791.46}{d^2}$$

Substituting in Soderberg's relation

$$\frac{\tau_a}{\tau_{-ld}} + \frac{\tau_m}{\tau_{yd}} = 1 \qquad \left(\frac{20-62}{P20.22}\right)$$

$$\frac{1902.22}{d^2 \times 300} + \frac{5791.46}{d^2 \times 400} = 1$$

$$d = 4.56 \text{ mm}$$

From Table 2.2, select standard dia $d \simeq 4.5$ mm.

Step 2. Mean coil dia $D = Cd = 6 \times 4.5 = 27$ mm

Inside dia of coil $D_i = D - d = 27 - 4.5 = 22.5$ mm

Outside dia of $D_o = D + d = 27 + 4.5 = 31.5$ mm

Step 3. No. of active coils $i = \dfrac{yGd^4}{8FD^3} = \dfrac{18 \times 80 \times 10^3 \times 4.5^4}{8 \times 450 \times 27^3}$

$$i = 8.33 \simeq 9 \text{ coils}$$

Step 4. Free length $l_o \geq (i + n)\, d + y + a$

Selecting squared and ground ends, $n = 2$

Assume clearance $a = 0.25 \quad y = 0.25 \times 18$

$$\therefore \quad l_o \geq (9 + 2)\, 4.5 + 18 + 18 \times 0.25 = 72 \text{ mm}$$

$$K = \frac{4C-1}{4C-4} + \frac{0.165}{C} \qquad \left(\frac{20-36}{P20.9}\right)$$

Procedure:

Step 1. Shear stress $\tau = \dfrac{KFD\,(1.5h + 0.9b)}{b^2 h^2}$ $\left(\dfrac{20-34a}{P20.9}\right)$

final 'b' and 'h', knowing their ratio.

Step 2. Mean coil dia $D = Cb$ or Ch

(depending on $C = \dfrac{D}{b}$ if $b < h$ $\left(\dfrac{20-38}{P20.11}\right)$

$C = \dfrac{D}{b}$ if $h < b$)

Inside dia of coil $D_i = D - b$

Outside dia of coil $D_o = D + b$

Step 3. Deflection $y = \dfrac{2.83\,iFD^3(b^2 + h^2)}{b^3 h^3 G}$ $\left(\dfrac{20-39}{P20.11}\right)$

If y is known, then find no. of active coils 'i'.

Step 4. Free length $l_o \geq (i + n)\,h + y + a$ $\left(\dfrac{20-53}{P20.21}\right)$

n = extra no. of coils depending on end conditions
= 0 for plain ends
= 1 for squared or ground ends
= 2 for squared and ground ends
y = deflection
a = clearance $\simeq$ 0.25 y

Step 5. pitch $p = \dfrac{l_o - 2h}{i}$ for squared and ground ends

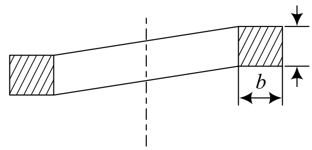
(For others refer Fig. 2.3)

Step 6. Stiffness $F_o = \dfrac{F}{y}$ $\left(\dfrac{20-30}{P20.9}\right)$

Step 7. Length of wire $l_w = \pi D i'$

when i' = total no. of coils = $i + n$

Problem 18 Design a rectangular section helical spring to mount to a buffer to sustain a load of 30 kN. The deflection under load is 90 mm. The spring is made of Z-Nickle. The longer side of rectangle is twice the shorter side and the spring is wound with longer side of rectangle parallel to the axis. The spring index is 10. Take factor of safety = 2.5.



Sides ratio $\frac{b}{h} = 1.5 \Rightarrow b = 1.5h$

Minimum pressure $p_1 = 0.5$ MPa

Maximum pressure $p_2 = 0.6$ MPa

Lift $y' = 10$ mm

Spring index $C = 8 = \frac{D}{h}$ since $h < b$

$\therefore$ $D = 8h$

$\tau = 420$ MPa

$G = 80 \times 10^3$ MPa

Dia of valve seat $D_V = 65$ mm

(Ref Fig. 2.5, force calculations are similar to problem 3.)

$$F_1 = \frac{\pi}{4} D_V^2 \cdot p_1 = \frac{\pi}{4} 65^2 \times 0.5 = 1659.15 \text{ N}$$

$$F_2 = \frac{\pi}{4} D_V^2 \cdot p_2 = \frac{\pi}{4} 65^2 \times 0.6 = 1990.98 \text{ N}$$

Maximum deflection $y = y_2 = \frac{F_2 y'}{F_2 - F_1}$

$$y = y_2 = \frac{1990.98 \times 10}{1990.98 - 1659.15} = 60 \text{ mm}$$

Design the spring for maximum load and maximum deflection.

$F = F_2 = 1990.98$ N

$y = y_2 = 60$ mm

Step 1. For rectangular cross-section, shear stress $\tau = \frac{KFD(1.5h + 0.9b)}{b^2 h^2}$

For $C = 8,\ K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = 1.184$

$$420 = \frac{1.184 \times 1990.98 \times 8h\,(1.5h + 0.9 \times 1.5h)}{(1.5h)^2 \cdot h^2}$$

$$h = 7.54 \simeq 8 \text{ mm}$$

$\therefore$ $$b = 1.5, \quad h = 12 \text{ mm}$$

Step 2. Mean coil diameter $D = 8h = 8 \times 8 = 64$ mm

Inside dia of coil $D_i = D - b = 64 - 12 = 52$ mm

Outside dia of coil $D_o = D + b = 64 + 12 = 76$ mm

Step 3. Deflection $$y = \frac{2.83\, iFD^3\,(b^2 + h^2)}{b^3h^3G}$$

$\therefore$ $$i = \frac{60 \times 12^3 \times 8^3 \times 80000}{2.83 \times 1990.98 \times 64^3\,(12^2 + 8^2)} = 13.82 \simeq 14 \text{ coils}$$

Step 4. Free length $l_o \geq (i + n)h + y + a$

assume $n = 2$ and $a = 0.25y$

$$l_o \geq (14 + 2)8 + 60 + 0.25 \times 60 = 203 \text{ mm}$$

Step 5. Pitch $$p = \frac{l_o - 2h}{i} = \frac{203 - 2 \times 8}{14} = 13.36 \text{ mm}$$

Step 6. Stiffness $$F_o = \frac{F}{y} = \frac{1990.98}{60} = 33.18 \text{ N/mm}$$

Step 7. Length of wire $l_w = \pi D\,(i + n)$

$$= \pi \times 64 \times (14 + 2)$$

$$l_w = 3216.99 \simeq 3217 \text{ mm}$$

2.13 SQUARE CROSS-SECTION HELICAL SPRINGS

For square cross-section, $h = b$, Fig. 2.12 shows a schematic diagram of a square cross-section helical spring.

Here $$h = b$$

$$C = \frac{D}{h}$$

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \quad \text{(same)}$$

Material Si-Mn steel from Table 2.1, torsional ultimate stress (Table 20-10 *P* 20-15)

$$\tau_u = 965 \text{ MPa}$$

$$G = 78.45 \text{ GPa}$$

Assume $FoS = 3$ based on ultimate stress

$$\therefore \qquad \tau = \frac{\tau_u}{FoS} = \frac{965}{3} = 321.67 \text{ MPa}$$

Assume spring index $C = 6$ (C is between 4 to 10)

$$C = \frac{D}{h} = 6 \Rightarrow D = 6h$$

Step 1. For square cross-section $\tau = \dfrac{2.4\,KFC}{h^2}$ $\left(\dfrac{20-43}{P20.12}\right)$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

$$321.67 = \frac{2.4 \times 1.2525 \times 1800 \times 6}{h^2} \Rightarrow h = 10.05 \simeq 10 \text{ mm}$$

Step 2. Mean coil dia $D = Ch = 6 \times 10 = 60$ mm

Inside dia of coil $D_i = D - h = 60 - 10 = 50$ mm

Outside dia of coil $D_o = D + h = 60 + 10 = 70$ mm

Step 3. Deflection for square cross-section

$$y = \frac{5.66\,iFD^3}{Gh^4} \qquad \left(\frac{20-44}{P20.12}\right)$$

$$45 = \frac{5.66 \times i \times 1800 \times 60^3}{78.45 \times 10^3 \times 10^4}$$

No. of active coils $i = 16.04 \simeq 16$ coils

Step 4. Free length $l_o \geq (i+n)h + y + a$

Assume squared and ground ends, $n = 2$

Assume clearance $a = 0.25\,y = 0.25 \times 45$

$$l_o \geq (16+2)10 + 45 + 0.25 \times 45 = 236.25$$

$$l_o = 237 \text{ mm}$$

$$F = 30\,y = 30 \times 1128.8 = 33864 \text{ N}$$
$$F = 33864 \text{ N}, \quad y = 1128.8 \text{ mm}, \quad C = 5, \quad \tau = 600, \quad G = 81 \times 10^3$$

Step 1. Shear stress for square cross-section $\tau = \dfrac{2.4\,KFD}{h^3}$ $\left(\dfrac{20-43}{P20.12}\right)$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.3105$$
$$D = Ch = 5h$$
$$600 = \frac{2.4 \times 1.3105 \times 33864 \times 5}{h^2} \Rightarrow h = 29.79 \simeq 30 \text{ mm}$$

Step 2. Mean coil diameter $D = Ch = 5 \times 30 = 150$ mm

Inside dia of coil $D_i = D - h = 150 - 30 = 120$ mm

Outside dia of coil $D_o = D + h = 150 + 30 = 180$ mm

Step 3. Deflection for square cross-section

$$y = \frac{5.66\,iFD^3}{b^4 G} \qquad \left(\frac{20-44}{P20.12}\right)$$
$$1128.8 = \frac{5.66 \times i \times 33864 \times 150^3}{30^4 \times 81000}$$
$$i = 114.49 \simeq 115 \text{ coils}$$

Step 4. Free length $l_o \geq (i+n)h + y + a$

Assume $n = 2$ and $a = 0.25\,y$

$$l_o \geq (115+2)30 + 1128.8 + 0.25 \times 1128.8$$
$$l_o = 4921 \text{ mm}$$

Step 5. Pitch $p = \dfrac{l_o - 2h}{i}$

$$= \frac{4921 - 2\times30}{115} = 42.27 \text{ mm}$$

Step 6. Stiffness $F_o = \dfrac{F}{y} = 30$ N/mm (given)

Step 7. Length of wire $l_w = \pi D\,(i+n)$

$$l_w = \pi \times 150\,(115+2) = 55134.95 \text{ mm.}$$

2.14 FLAT SPRINGS OR LEAF SPRINGS

Flat springs are of two types, cantilevers and simply supported, both having one leaf or multi-leaves. If the spring has multileaves, then it is called laminated spring.

Problem 22 Determine the width and thickness of a flat spring carrying a central load of 3 kN, the deflection in which is limited to 60 mm. The spring is supported at both ends at a distance of 1.2 m and is of constant thickness and varying width type. Allow a stress of 450 MPa. Take $E = 206$ GPa.

Solution Given data:

Constant thickness, varying breadth type simply supported beam is as shown here.

Refer Table 2.3 Fig. (3) (Table 20-1 *P* 20.2)

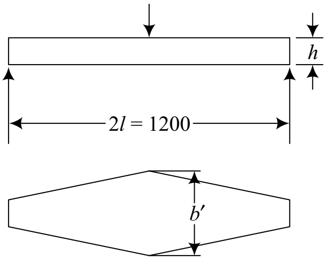


Fig. 2.14

$$2l = 1200 \text{ mm} \quad \therefore \quad l = 600 \text{ mm}$$

$$F = 3000 \text{ N} \qquad \sigma = 450 \text{ mm} \quad E = 206 \times 10^3 \text{ MPa}$$

$$y = 60 \text{ mm} \qquad i = 1 \text{ (single leaf)}$$

From Fig. (3) Table 2.3

$$C_1 = 3$$

$$C_2 = 3$$

Step 1. Stress $\sigma = \dfrac{C_1 F l}{i b' h^2} \Rightarrow 450 = \dfrac{3 \times 3000 \times 600}{1 \times b' h^2}$ $\left(\dfrac{20-4}{P20.3}\right)$

$\therefore \quad b'h^2 = 12000$ ...(1)

Step 2. Deflection $y = \dfrac{C_2 F l^3}{E i\, b' h^3} \Rightarrow 60 = \dfrac{3 \times 300 \times 600^3}{206 \times 10^3 \times 1 \times b' h^3}$ $\left(\dfrac{20-5}{P20.3}\right)$

$\therefore \quad b'h^3 = 157281.55$ ...(2)

Step 3. $\therefore$ Thickness $h = \dfrac{b'h^3}{b'h^2} = \dfrac{157281.55}{12000} = 13.11$ mm

$\therefore$ Width $b' = \dfrac{b'h^3}{h^3} = \dfrac{157281.55}{13.11^3} = 69.8$ mm

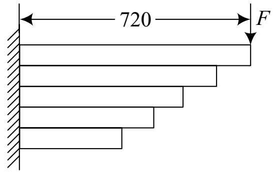


Fig. 2.17

Assume $E = 206 \times 10^3$ MPa

Ref Fig.(5) Table 2.3 (constant width, varying depth) (Table 20-1 *P* 20.2)

$$C_1 = 6 \text{ and } C_2 = 8$$

Step 1. Deflection $$y = \frac{C_2 F l^3}{E i b' h^3} \Rightarrow 90 = \frac{8 \times F \times 720^3}{206 \times 10^3 \times 5 \times 120 \times 6^3} \qquad \left(\frac{20-5}{P20.3}\right)$$

Load $F = 804.69$ N

Step 2. Bending stress $$\sigma = \frac{C_1 F l}{i b' h^2} \qquad (20\text{-}4)$$

$$\sigma = \frac{6 \times 804.69 \times 720}{5 \times 120 \times 6^2} = 160.94 \text{ MPa}$$

2.15 SEMI-ELLIPTICAL LAMINATED SPRING (TRUCK SPRING OR AUTOMOBILE SPRING)

Figure 2.18 shows a semi-elliptical laminated spring that is commonly used in trucks and automobiles.

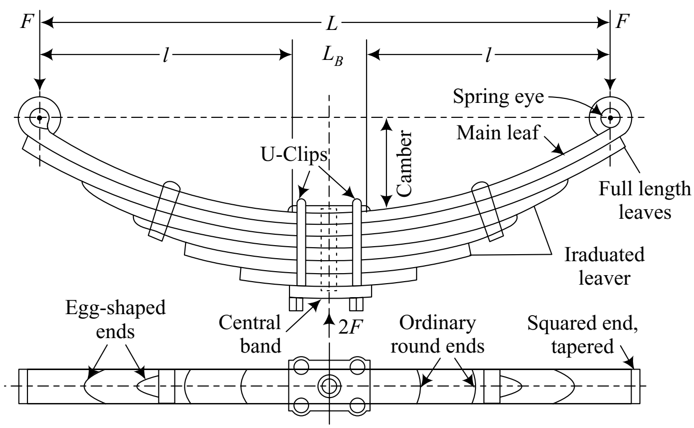


Fig. 2.18 Laminated springs for automobiles

Symbols:

L = Length between supports (Total length)

L_B = Width of central band

l = Effective length = $\dfrac{L - L_B}{2}$

$2F$ = Central load

C = Camber or height of curvature

y = Deflection

E = Modulus of elasticity

σ_f = Stress in full length leaves

σ_g = Stress in graduated leaves

σ = Equalising stress

i_f = No. of full length leaves (usually 2 nos)

i_g = No. of graduated leaves (usually 6 to 12)

i = Total no. of leaves = $i_g + i_f$

b^1 = Width of each leaf

h = Thickness of each leaf

F_b = Load exerted on central bandor U-bolts

It can be seen that the stress in full length leaves is more than that in graduated leaves due to longer span. The full length leaves experience 50% greater stress than that of graduated leaves.

2.16 DERIVATION OF $\sigma_f = 1.5\ \sigma_g$

Consider a single cantilever beam loaded as shown in Fig. 2.19(a).

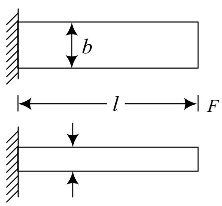


Fig. 2.19(a)

b = width of beam

h = thickness of beam

l = length of beam

F = load

l = moment of inertia = $\dfrac{bh^3}{12}$ and $C = \dfrac{h}{2}$

Maximum bending moment $M_b = F \cdot l$

Bending stress
$$\sigma = \frac{M_b}{\left(\frac{I}{C}\right)} = \frac{Fl}{\left(\frac{bh^2}{6}\right)} = \frac{6Fl}{bh^2} \quad ...(1)$$

Maximum deflection
$$y = \frac{Fl^3}{3EI}$$

$$y = \frac{Fl^3}{3E\left(\frac{bh^3}{12}\right)} = \frac{4Fl^3}{Ebh^3} \quad ...(2)$$

$$y = \frac{2}{3}\left(\frac{6Fl}{bh^2}\right)\frac{l^2}{Eh} = \frac{2\sigma l^2}{3Eh}$$

If the plate is cut into a series of 'i' no of strips of width b' and are assembled as shown in Fig. 2.19(b). Then equations (1) and (2) can be written as follows.

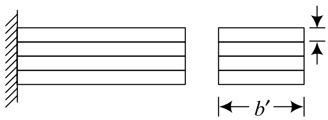


Fig. 2.19(b)

$$\sigma_f = \frac{6Fl}{(ib')h^2} \quad ...(3)$$

and
$$y = \frac{4Fl^3}{Eib'h^3} = \frac{2\,\sigma\,l^2}{3Eh} \quad ...(4)$$

If a triangular plate is used as shown in Fig. 2.19(c), then the stress will be uniform throughout (constant strength beam). If this triangular plate is cut into strips of uniform width and placed one below the other as shown in Fig. 2.19(d) to form a graduated leaf spring,

then
$$\sigma_g = \frac{6\,Fl}{ib'h^2} \quad ...(5)$$

and
$$y = \frac{6\,Fl^3}{Eib'h^3} = \frac{\sigma\,l^2}{Eh} \quad ...(6)$$

From equations (4) and (6) we can see that for the same deflection, the stress in the full length leaves is 50% greater than that in the graduated leaves.

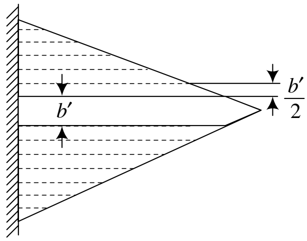


Fig. 2.19(c)

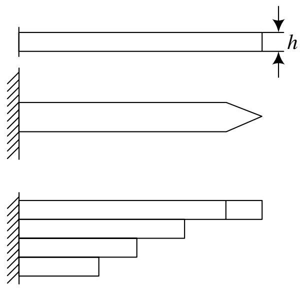


Fig. 2.19(d)

$$\sigma_f = \frac{3}{2}\,\sigma_g$$

$$\frac{6F_f l}{i_f b' h^2} = \frac{3}{2}\,\frac{f_g l}{i_g b' h^2}$$

$\therefore$ $$\frac{F_f}{F_g} = \frac{3i_f}{2i_g} \qquad ...(7)$$

Adding '1' on both sides

$$\frac{F_f}{F_g} + 1 = \frac{3i_f}{2i_g} + 1$$

$$\frac{F_f + F_g}{F_g} = \frac{3i_f + 2i_g}{2i_g} \Rightarrow F_g = (F_f + F_g)\left(\frac{2i_g}{3i_f + 2i_g}\right)$$

But $F_f + F_g = F =$ Total load

$$F_g = \frac{F\,2i_g}{3i_f + 2i_g}$$

From (7) rewriting $$\frac{F_g}{F_f} = \frac{2i_g}{3i_f}$$

Adding '1' on both sides and simplifying like above

$$F_f = \frac{F \cdot 3i_f}{(2i_g + 3i_f)}$$

Stress in full length leaves, $\sigma_f = \dfrac{6F_f l}{i_f b' h^2} = \dfrac{6l}{i_f b' h^2}\left(\dfrac{F \cdot 3i_f}{2i_g + 3i_f}\right)$

$$\sigma_f = \frac{18Fl}{b'h^2(2i_g + 3i_f)} \qquad ...(8)$$

Stress in graduated leaves, $\sigma_g = \dfrac{2}{3}\,\sigma_f = \dfrac{12Fl}{b'h^2(2i_g + 3i_f)}$ ...(9)

Deflection: Consider anyone, say full length leaves.

$$y = \frac{2\sigma_f l^2}{3Eh} = \frac{2l^2}{3Eh}\left(\frac{18Fl}{b^1h^2(2i_g + 3i_f)}\right)$$

$$y = \frac{12Fl^3}{b^1h^3(2i_g + 3i_f)} \qquad ...(10)$$

2.17 DESIGN PROCEDURE FOR SEMI-ELLIPTICAL LAMINATED SPRINGS

Step 1. For the purpose of analysis, consider the stress in full length leaves which are heavily stressed

$$\sigma_f = \frac{18Fl}{b'h^2(2i_g + 3i_f)}$$

or equalising stress $\sigma = \dfrac{6Fl}{ib'h^2}$

- If the leaves are not prestressed, then since full length leaves experience more stress, use σ_f equation.
- If the leaves are prestressed so that all the leaves have the same stress after full load has been applied, then use σ equation.

Find $b'h^2 = ?$ ...(1)

Step 2. Deflection $y = \dfrac{12Fl^3}{b'h^3E(2i_g + 3i_f)}$

Find $b'h^3 = ?$ ...(2)

Step 3. Thickness of leaves $h = \dfrac{b'h^3}{b'h^2} = \dfrac{(2)}{(1)} = ?$

Find width from equation (2)

$$b' = \frac{b'h^3}{h^3} = ?$$

Step 4. Load on band

$$F_b = \frac{2Fi_g i_f}{i(2i_g + 3i_f)}$$

Step 5. Nip or camber $C = \dfrac{2Fl^3}{Eib'h^3}$

(Gap between full length and graduated leaves)

Problem 26 Design a leaf spring for the following specifications for a truck

Maximum load on springs = 120 kN
No. of springs = 4
Material of springs = Cr Va steel
Span of spring = 1200 mm
Width of central band = 200 mm
Permissible deflection = 120 mm

Solution Given data:

Total load on 4 springs = 120 kN
No. of springs = 4

Load on each spring $2F = \dfrac{120 \times 10^3}{4} = 30 \times 10^3$ N

$$F = \frac{30000}{2} = 15000 \text{ N}$$

Material = Cr Va steel

From Table 2.1, ultimate tensile stress, $\sigma_u = 1380$ MPa (Table 20-10 Pg 20.15)

$$E = 206 \text{ GPa} = 206 \times 10^3 \text{ MPa}$$

Assume FoS = 3

$\therefore$ allowable stress $= \dfrac{\sigma_u}{Fos} = \dfrac{1380}{3} = 460$ MPa

Since prestress condition is not given, the allowable stress is taken as σ_f

Span = length of spring = L = 1200 mm

Width of central band $= L_B = 200$ mm

$\therefore$ effective length $\quad l = \dfrac{L - L_B}{2} = \dfrac{1200 - 200}{3} = 500 \text{ mm}$

Deflection $\quad y = 120 \text{ mm}$

Assume 2 full length and 6 graduated leaves, $\quad i_f = 2, \quad i_g = 6,$

$\therefore \quad i = i_g + i_f = 2 + 6 = 8$

Step 1. Stress
$$\sigma_f = \frac{18Fl}{b'h^2(2i_g + 3i_f)}$$

$$460 = \frac{18 \times 15000 \times 500}{b'h^2(2 \times 6 + 3 \times 2)}$$

$$\therefore \quad b'h^2 = 16304.34 \qquad \text{...(1)}$$

Step 2. Deflection
$$y = \frac{12\,Fl^3}{b'\,h^3 E(2i_g + 3i_f)}$$

$$120 = \frac{12 \times 15000 \times 500^3}{b'\,h^3 \times 206 \times 10^3\ (2 \times 6 + 3 \times 2)}$$

$$\therefore \quad b'h^3 = 50566.34 \qquad \text{...(2)}$$

Step 3. $\therefore$
$$h = \frac{b'h^3}{b'h^2} = \frac{(2)}{(1)} = \frac{50566.34}{16304.35} = 3.10 \text{ mm}$$

$$\therefore \quad b' = \frac{b'h^3}{h^3} = \frac{50566.34}{3.1^3} = 1697.4 \text{ mm}$$

(Since the spring is not prestressed, no load will be exerted on band)

Step 4. Camber
$$C = \frac{2Fl^3}{Eib'h^3} = \frac{2 \times 15000 \times 500^3}{206 \times 10^3 \times 8 \times 1697.4 \times 3.1^3}$$

$$C = 45 \text{ mm}$$

Problem 27 A multi-leaf spring is fitted to the chasis of an automobile over a span of 1.25 m to absorb the shock due to a max load of 24 kN. The spring material can sustain a stress of 540 MPa if all the leaves in spring were to receive the same stress. The spring is required at least with 2 full length leaves and remaining graduated. The leaves are assembled with bolts over a span of 100 mm at the middle. The maximum deflection is limited to 90 mm. Determine

(i) The width and thickness of leaves

(ii) The load exerted on the bolts

(iii) The nip that should be provided between full length and graduated leaves before the band is fixed.

Solution Given data:

Central load $2F = 24\text{ kN} = 24 \times 10^3\text{ N}$

$\therefore$ $F = 12000\text{ N}$

Length $L = 1250\text{ mm}$

Width of central band $L_B = 100\text{ mm}$

$\therefore$ Effective length $l = \dfrac{L - L_B}{2} = \dfrac{1250 - 100}{2} = 575\text{ mm}$

No. of full length leaves $i_f = 2$

Assume no. of graduated leaves $i_g = 6$

$\therefore$ Total no. of leaves $i = i_g + i_f = 8$

Deflection $y = 90\text{ mm}$

Equalising stress $\sigma = 540\text{ MPa}$

(given all leaves are stressed to same value)

Assume $E = 206\text{ GPa} = 206 \times 10^3\text{ MPa}$

Step 1. Equalising stresses $\sigma = \dfrac{6Fl}{ib'h^2}$

$$540 = \frac{6 \times 12000 \times 575}{8b'h^2} \qquad \Rightarrow \quad b'h^2 = 9583.33 \quad \text{...(1)}$$

Step 2. Deflection $y = \dfrac{12Fl^3}{b'h^3E(2i_g + 3i_f)}$

$$90 = \frac{12 \times 1200 \times 575^3}{b'h^3 \times 206 \times 10^3\,(2 \times 6 + 3 \times 2)} \qquad \Rightarrow \quad b'h^3 = 82032.1 \text{ ...(2)}$$

Step 3. Thickness of leaf $h = \dfrac{b'h^3}{b'h^2} = \dfrac{(2)}{(1)} = \dfrac{82032.1}{9583.33} = 8.56\text{ m}$

$\therefore$ width of leaf $b' = \dfrac{b'h^3}{h^3} = \dfrac{82032.1}{8.56^3} = 130.79\text{ mm}$

Step 4. Load exerted on bolts $F_b = \dfrac{2F\,i_f\,i_g}{i\,(2i_g + 3i_f)}$

$$= \frac{2 \times 12000 \times 2 \times 6}{8(2 \times 6 + 3 \times 2)} = 2000\text{ N}$$

Step 5. Nip or camber $C = \dfrac{2Fl^3}{Eib'h^2}$

$$C = \frac{2 \times 12000 \times 575^3}{206 \times 10^3 \times 8 \times 130.79 \times 8.56^3} = 33.75 \text{ mm}$$

Problem 28 A truck spring has 10 leaves of graduated length. The spring supports are 1060 mm apart and the central load is 5.4 kN. The width of central band is 80 mm. The allowable stress is 270 MPa. Determine the width and thickness of leaves. The spring should have a ratio of total depth to width as about 2.5. To what radius should the leaves be initially bent for the spring to be flat under the given load. Take $E = 210$ GPa.

Solution Given data:

Central load $2F = 5400$ N

$$\therefore \quad F = \frac{5400}{2} = 2700 \text{ N}$$

Total no. of leaves $i = 10$

Since all 10 are graduated leaves, only one will be full length extending between supports and remaining 9 are graduated.

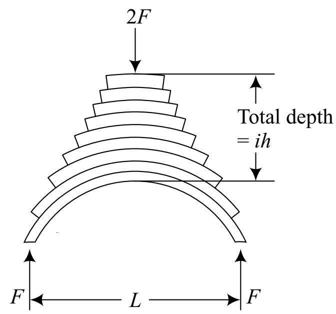


Fig. 2.20

$$\therefore \quad i_f = 1$$

$$i_g = 9$$

$$i = i_f + i_g = 10$$

Total depth $= ih = 10h$

Width $= b'$

$$\frac{\text{Depth}}{\text{Width}} = 2.5 = \frac{10h}{b'} \quad \therefore b' = \frac{10h}{2.5} = 4h$$

$$E = 210 \text{ GPa} = 210 \times 10^3 \text{ MPa}$$

$\sigma_f = 270$ (since no prestress condition is given, the allowable stress is taken as σ_f)

$L = 1060$ mm $\quad L_B = 80$ mm

$$\therefore \quad \text{effective length} \quad l = \frac{L - L_B}{2} = \frac{1060 - 80}{2} = 490 \text{ mm}$$

Step 1. Stress $$\sigma_f = \frac{18\,Fl}{b'h^2(2i_g + 3i_f)}$$

$$270 = \frac{18 \times 2700 \times 490}{(4h)\,h^2\ (2 \times 9 + 1 \times 1)} \Rightarrow h = 10.5 \text{ mm}$$

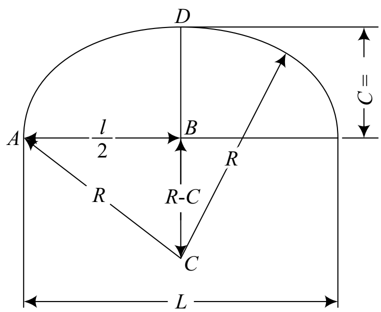
∴ $$b' = 4h = 4 \times 10.5 = 42 \text{ mm}$$

Step 2. Deflection $$y = \frac{12\,Fl^3}{b'\,h^3 E(2i_g + 3i_f)}$$

$$= \frac{12 \times 2700 \times 490^3}{210 \times 10^3 \times 42 \times 10.5^3\ (2 \times 9 + 1 \times 1)} = 19.65 \text{ mm}$$

Step 3. To find the radius of leaves

For the spring to be flat under load, the camber should be equal to the deflection.



Consider triangle ABC, C is centre of curvature

$$AB = \frac{L}{2},\ AC = R = CD$$

∴ $$BC = CD - BD = R - C$$

$$AC^2 = AB^2 + BC^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + (R - C)^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + R^2 + C^2 - 2\,RC$$

∴ $$2\,RC = \left(\frac{L}{2}\right)^2 + C^2$$

Radius $$R = \frac{\left(\frac{L}{2}\right)^2 + C^2}{2C}$$

$$R = \frac{\left(\frac{1060}{2}\right)^2 + 19.65^2}{2 \times 19.65} = 7157.4 \text{ mm}$$

The leaves are to be initially bent to a radius of 7157.4 mm.

Problem 29 A truck spring has a overall length of 1.2 m and sustain a load of 60 kN. The spring has 3 full length and 15 graduated leaves. All the leaves are stressed to 360 MPa when fully loaded. The ratio of total depth to width is 2. Take E = 206 GPa. Determine the width and thickness of leaves, the camber and load exerted on band.

Solution Given data:

$$L = 1200 \text{ mm}, \quad 2F = 60 \text{ kN} = 60000 \quad i_f = 3 \quad i_g = 15$$

$\therefore$ $$i = i_f + i_g = 3 + 15 = 18$$

Equalising stress $\sigma = 360$ MPa

$$\frac{\text{Depth}}{\text{Width}} = \frac{ih}{b'} = 2 \quad \Rightarrow \frac{18h}{b'} = 2$$

$\Rightarrow$ $b' = 9\text{h}$ $\quad E = 206 \times 10^3$ MPa

Assume width of central band $L_B = 200$ mm

$\therefore$ effective length $$l = \frac{L - L_B}{2} = \frac{1200 - 200}{2} = 500 \text{ mm}$$

$$F = \frac{60000}{2} = 30{,}000 \text{ N}$$

Step 1. Equalising stress $$\sigma = \frac{6Fl}{ib'h^2} \Rightarrow 360 = \frac{6 \times 30000 \times 500}{18 \times 9h \times h}$$

Thickness $h = 11.56$ mm

Width $b' = 9h = 9 \times 11.56 = 104.04$ mm.

Step 2. Deflection $$y = \frac{12Fl^3}{Eb'h^3(2i_g + 3i_f)}$$

$$= \frac{12 \times 30000 \times 500^3}{206 \times 10^3 \times 104.04 \times 11.56^3 \ (2 \times 15 + 3 \times 3)}$$

$$y = 34.85 \text{ mm}$$

Step 3. Camber $$C = \frac{2Fl^3}{Eib'h^3} = \frac{2 \times 30000 \times 500^3}{206 \times 10^3 \times 18 \times 104.04 \times 11.56^3}$$

$$C = 12.58 \text{ mm}$$

Step 4. Load on band $$F_B = \frac{2Fi_g i_f}{i(2i_g + 3i_f)} = \frac{2 \times 30000 \times 15 \times 3}{18(2 \times 15 + 3 \times 3)}$$

$$F_B = 3846.15 \text{ N.}$$

Problem 30 A car weighing 10000 N without passengers has a seating capacity of 4 passengers each weighing 1000 N. The wheel base is 2.2 m and the *cg* of loaded car is 1.2 m behind front axle. The car is to be supported on four similar longitudinal semi-elliptical carriage springs of each 900 mm between supports. Design a suitable spring using a *FoS* of 1.8 based on proof stress of 810 MPa. The static loads are multiplied with a factor of 2.2 to allow for impacts. The spring should have a deflection of 60 mm

Solution Given data:

Wt of car $W_C = 10000$ N

Wt of each passenger, $W_P = 1000$ N

No. of passengers $= 4$

∴ Total weight $= W_C + 4\,W_P = 10000 + 4 \times 1000$

$= 14000$ N

Impact factor $= 2.2$

∴ $W = 2.2 \times 14000 = 30{,}800$ N

Wheel base $= 2.2$ m $= 2200$ mm

Distance to c.g. $= 1200$ m

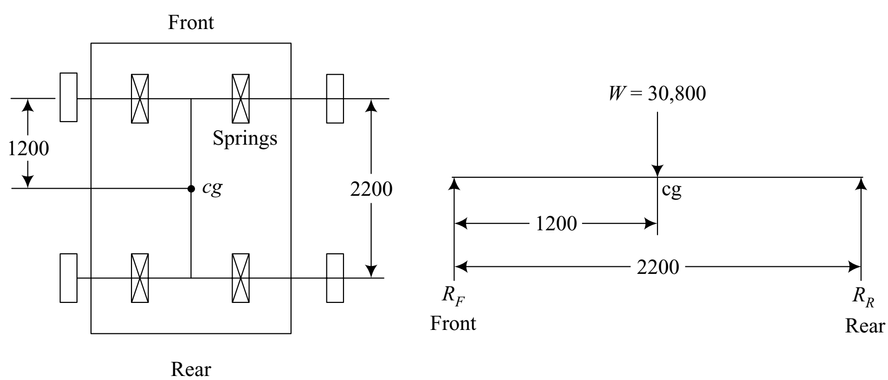


Fig. 2.21

The load W acts at cg

$$R_F + R_R = 30{,}800 \text{ N}$$

Taking moments about front axle, $W \times 1200 = R_R \times 2200$

$$\therefore \quad R_R = \frac{30{,}800 \times 1200}{2200} = 16{,}800 \text{ N}$$

$$\therefore \quad R_F = 30{,}800 - R_R = 14{,}000 \text{ N}$$

The load on front axle $= \text{Reaction } R_F = 14{,}000 \text{ N}$

Load on each front spring $= \dfrac{R_F}{2} = 7000 \text{ N}$

The load on rear axle $= \text{Reaction } R_R = 16{,}800 \text{ N}$

Load on each rear spring $= \dfrac{R_R}{2} = 8{,}400 \text{ N}$

Maximum of these two, load on spring $2F = 8{,}400 \text{ N}$

$$F = \frac{8400}{2} = 4200 \text{ N}$$

Length of spring $L = 900$ mm. Neglect central band, $L_B = 0$ (Not given)

$\therefore$ effective length $l = \dfrac{L - L_B}{2} = \dfrac{900 - 0}{2} = 450 \text{ mm}$

Proof stress $\sigma_y = 810 \text{ MPa}$ $FoS = 1.8$

$\therefore$ Allowable stress $= \dfrac{\sigma_y}{FoS} = \dfrac{810}{1.8} = 450 \text{ MPa}$

Since the leaves are not pre-stressed, take this allowable stress as $\sigma_f = 450$ MPa.

Deflection $y = 60$ mm Assume $E = 206 \times 10^3$ MPa

Select $i_f = 2$ and $i_g = 6$ $\therefore$ $i = i_f + i_g = 2 + 6 = 8$

Step 1.

$$\sigma_f = \frac{18\,Fl}{b'h^2(2i_g + 3i_f)} = \frac{18 \times 4200 \times 450}{b'h^2\,(2 \times 6 + 3 \times 2)} = 450$$

$$b'h^2 = 4200 \qquad \text{...(1)}$$

Step 2. Deflection

$$y = \frac{12\,Fl^3}{E\,b'h^3(2i_g + 3i_f)}$$

$$60 = \frac{12 \times 4200 \times 450^3}{206 \times 10^3 \times b'h^3\,(2 \times 6 + 3 \times 2)} \Rightarrow b'h^3 = 20643.2 \qquad \text{...(2)}$$

Step 3. $$h = \frac{b'h^3}{b'h^2} = \frac{(2)}{(1)} = \frac{20643.2}{4200} = 4.915 \simeq 5 \text{ mm}$$

$$b' = \frac{b'h^3}{h^3} = \frac{20643.2}{53} = 165.14 \simeq 160 \text{ mm}$$

Problem 31 A truck spring has 3 full length and 9 graduated leaves. The span is 1.1 m and width of central band is 150 mm. The load on spring is 9000 N. The allowable stress is 500 MPa in all the leaves. The deflection under load is 72 mm. Design the spring.

Solution Given data:

$$i_f = 3, i_g = 9 \quad \therefore \quad i = i_f + i_g = 3 + 9 = 12 \text{ leaves}$$

Span $L = 1100$ mm Central band $L_B = 150$ mm

$\therefore$ effective length $$l = \frac{L - L_B}{2} = \frac{1100 - 150}{2} = 475 \text{ mm.}$$

Central load $2F = 9000$ N $\Rightarrow F = 4500$ N

Equalising stress $\sigma = 500$ MPa

Deflection $y = 72$ mm.

Assume $E = 206 \times 10^3$ MPa

Step 1. Equalising stress $$\sigma = \frac{6Fl}{ib'h^2}$$

$$500 = \frac{6 \times 4500 \times 475}{12\, b'h^2} \quad \Rightarrow \quad b'h^2 = 2137.5 \qquad \text{...(1)}$$

Step 2. Deflection $$y = \frac{12\, Fl^3}{E\, b'h^3\, (2i_g + 3i_f)}$$

$$72 = \frac{12 \times 4500 \times 475^3}{206 \times 10^3 \times b'h^3\, (2 \times 9 + 3 \times 3)} \quad \Rightarrow \quad b'h^3 = 14451.44 \qquad \text{...(2)}$$

Step 3. $$h = \frac{b'h^3}{b'h^2} = \frac{(2)}{(1)} = \frac{14451.44}{2137.5} = 6.7\text{mm} \simeq 7 \text{ mm}$$

$$b' = \frac{b'h^3}{h^3} = \frac{14451.44}{7^3} = 42.13 \simeq 43 \text{ mm}$$

Step 4. Camber $$C = \frac{2Fl^3}{E\, ib'\, h^3} = \frac{2 \times 4500 \times 475^3}{206 \times 10^3 \times 12 \times 43 \times 7^3} = 26.45 \text{ mm}$$

Step 5. Load on band $$F_B = \frac{2F\, i_g i_f}{i\, (2i_g + 3i_f)} = \frac{2 \times 4500 \times 9 \times 3}{12\, (2 \times 9 + 3 \times 3)} = 750 \text{ N}$$

Problem 32 Design a semi-elliptical laminated spring having effective length of 500 mm with 6 graduated and 2 full length leaves each 5 mm thick and 25 mm wide. The permissible stress of 200 MPa. Find (i) the central load (ii) the deflection.

Solution effective length $l = 500$ mm

$i_g = 6 \quad i_f = 2 \quad \Rightarrow \quad i = 2 + 6 = 8$

$h = 5$ mm $\quad b' = 25$ mm $\quad \sigma_f = 200$ MPa (no pre-stress)

Assume $E = 206 \times 10^3$ MPa

$$\sigma_f = \frac{12\,Fl}{b'h^2\,(2i_g + 3i_f)} \Rightarrow 200 = \frac{12 \times F \times 500}{25 \times 5^2\,(2 \times 6 + 3 \times 2)}$$

$F = 375$ N

∴ Central load $2F = 2 \times 375 = 750$ N

Deflection
$$y = \frac{12\,Fl^3}{E\,b'h^3\,(2i_g + 3i_f)} = \frac{12 \times 375 \times 500^3}{206 \times 10^3 \times 25 \times 5^3\,(2 \times 6 + 3 \times 2)}$$

$y = 48.54$ mm.

Problem 33 A semi-elliptical spring has effective length of 1 m. The spring has to sustain a load of 75 kN. The spring has 3 full length and 16 graduated leaves. The leaves are prestressed such that all the leaves are stressed to same value of 400 MPa when fully loaded. The width is 9 times the thickness. $E = 210$ GPa. Determine:

(i) The width and thickness of leaves

(ii) The initial gap that should be provided between full length and graduated leaves, and

(iii) Load on clips to close the initial gap

Solution Given data:

Effective length $l = 1000$ N

Central load $2F = 75000$ N $\Rightarrow$ $F = 37500$ N

$i_f = 3 \quad i_g = 16 \quad \therefore \quad i = i_f + i_g = 3 + 16 = 19$

Equalising stress $\sigma = 400$ MPa

$E = 210 \times 10^3$ MPa $\quad b' = 9h$

Step 1. Equalising stress $\sigma = \dfrac{6\,Fl}{i\,b'h^2}$

$$400 = \frac{6 \times 37500 \times 1000}{19 \times 9h \times h^2} \Rightarrow \text{thickness } h = 14.87 \approx 15 \text{ mm}$$

∴ width $b' = 9h = 135$ mm

Step 2. Deflection $$y = \frac{12Fl^3}{Eb'h^3(2i_g + 3i_f)} = \frac{12 \times 37500 \times 1000^3}{210 \times 10^3 \times 135 \times 15^3 \times (2 \times 16 + 3 \times 3)}$$

$$y = 114.71 \text{ mm.}$$

Step 3 Nip or camber or gap $$C = \frac{2Fl^3}{Eib'h^3}$$

$$= \frac{2 \times 37500 \times 1000^3}{210 \times 10^3 \times 19 \times 135 \times 15^3} = 41.26 \text{ mm.}$$

Step 4. Load on clips $$= F_B = \frac{2Fi_g i_f}{i(2i_g + 3i_f)}$$

$$F_B = \frac{2 \times 37500 \times 16 \times 3}{19(2 \times 16 + 3 \times 3)} = 4621.3 \text{ N}$$

Problem 34 Design a semi-elliptical laminated spring 1.2 m between supports has a central band of 200 mm wide. The central load is 6 kN. Allowable stress is 480 MPa. Find the no. of leaves, width and thickness of leaves if the deflection is 90 mm, take the width to thickness ratio of 12. The spring should have at least 2 full length leaves and remaining graduated.

Solution Given data:

$$L = 1200 \text{ mm} \quad L_B = 200 \text{ mm}$$

∴ effective length $$l = \frac{L - L_B}{2} = \frac{1200 - 200}{2} = 500 \text{ mm.}$$

Central load $2F = 6000$ N $\Rightarrow$ $F = 3000$ N

Allowable stress $\sigma_f = 480$ MPa (no pre-stress condition)

Deflection $y = 90$ mm $\quad \frac{b'}{h} = 12 \Rightarrow b' = 12h$

$i_f = 2$ find $i_g = ?$ Assume $E = 206$ GPa $= 206 \times 10^3$ MPa

Step 1. $$\sigma_f = \frac{12Fl}{b'h^2(2i_g + 3i_f)}$$

$$480 = \frac{12 \times 3000 \times 500}{12h \cdot h^2(2 \times i_g + 3 \times 2)}$$

$$h^3(2i_g + 6) = 3125 \quad \text{...(1)}$$

Step 2. $$y = \frac{12Fl^3}{Eb'h^3(2i_g + 3i_f)}$$

$$90 = \frac{12 \times 3000 \times 500^3}{206 \times 10^3 \times 12h \times h^3 \, (2 \times i_g + 3 \times 2)}$$

$$h^4 \, (2i_g + 6) = 20226.54 \qquad \text{...(2)}$$

$\therefore$ Thickness $$h = \frac{h^4 \, (2i_g + 6)}{h^3 \, (2i_g + 6)} = \frac{(2)}{(1)} = \frac{20226.54}{3125} = 6.47 \simeq 6.5 \text{ mm}$$

$\therefore$ width $\quad b' = 12h = 12 \times 6.5 = 78$ mm

Step 3. To find i_g, substitute h value in (2)

$$6.5^4 \, (2\, i_g + 6) = 20226.54$$

No. of graduated leaves $i_g = 2.67 \simeq 3$ leaves $\quad \therefore \quad$ Total no. of leaves $i = i_g + i_f = 3 + 2 = 5$

Problem 35 A truck spring has 12 leaves 3 of which are of full length. The effective length of the spring is 600 mm. The width and thickness of each leaf are 90 mm and 6 mm respectively. The allowable stress is 300 MPa if all the leaves are stressed to same value. Take $E = 210$ GPa. Find the central load and deflection under this load.

Solution Given data:

	$i = 12,$	$i_f = 3 \quad \therefore$	$i_g = 12 - 3 = 9$
Effective length	$l = 600$ mm,	$b' = 90$ mm	$h = 6$ mm
Equalising stress	$\sigma = 300$ MPa	$E = 210 \times 10^3$ MPa	

(i) Equalising stress $\quad \sigma = \dfrac{6Fl}{ib'h^2} \Rightarrow 300 = \dfrac{6 \times F \times 600}{12 \times 90 \times 6^2}$

$\therefore$ Load $\quad F = 3240$ N

$\therefore$ Central load $\quad 2F = 2 \times 3240 = 6480$ N

(ii) Deflection $$y = \frac{12Fl^3}{Eb'h^3 \, (2i_g + 3i_f)} = \frac{12 \times 3240 \times 600^3}{210 \times 10^3 \times 90 \times 6^3 \, (2 \times 9 + 3 \times 3)}$$

$$y = 76.19 \text{ mm}$$

2.18 COMBINATION OF SPRINGS

Problem 36 The free end of a horizontal constant strength steel cantilever spring is directly over and in contact with a vertical coil spring as shown in Fig. 2.22. The width at the fixed end is 600 mm and its thickness is 12.5 mm. Its length is 750 mm. The coil spring has 10 active coils of 15 mm wire diameter and has an outside dia of 105 mm. Shear modulus is 80 GPa.

(i) What force Q, if gradually applied at the end of cantilever spring is required to cause a deflection of 40 mm?

(ii) What is the bending stress in the cantilever?

(iii) What is the shear stress in coil?

(iv) What are the energies stored by each spring?

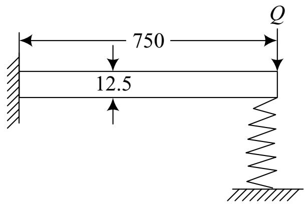


Fig. 2.22

Solution Given data:

Cantilever			Coil	
No. of leaves	$i = 1$		No. of coils	$i = 10$
Length	$l = 750$ mm		wire dia	$d = 15$ mm
Width	$b' = 600$ mm		outside dia of coil	$D_o = 105$ mm
Thickness	$h = 12.5$ mm	$\therefore$	Mean coil dia	$D = D_o - d = 90$ mm
Deflection	$y_1 = 40 \text{ mm} = y$		Spring index	$C = \dfrac{D}{d} = \dfrac{90}{15} = 6$
Assume	$E = 20 \times 10^3$ MPa			$y_2 = 40 \text{ mm} = y$
Refer Fig. 6, Table 2.3 (Table 20-1 *P* 20.2)				$G = 80 \times 10^3$ MPa
h constant, b' varying				$K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = 1.2525$
	$C_1 = 6, C_2 = 6$			Load $= F_2$
				Load $= F_1$

Since the cantilever is directly over and in contact with coil,

Deflection in cantilever = Deflection in coil

$$y_1 = y_2 = 40 \text{ mm.}$$

Consider cantilever:

Deflection $$y_1 = \frac{C_2 F_1 l^3}{E\, i b' h^3} \Rightarrow 40 = \frac{6 \times F_1 \times 750^3}{210 \times 10^3 \times 1 \times 600 \times 12.5^3} \qquad \left(\frac{20-5}{P20.3}\right)$$

Load shared by cantilever $F_1 = 3888.9$ N

Consider coil spring:

Deflection $$y_2 = \frac{8 F_2 D^3 i}{G d^4} \Rightarrow 40 = \frac{8 \times F_2 \times 90^3 \times 10}{80 \times 10^3 \times 15^4} \qquad \left(\frac{20-29}{P20.8}\right)$$

Load shared by coil $F_2 = 2777.8$ N

(i) Total load $Q = F_1 + F_2 = 3888.9 + 2777.8 = 6666.7$ N

(ii) Bending stress in cantilever $\sigma = \dfrac{C_1 F_1 l}{i b' h^2} = \dfrac{6 \times 3888.9 \times 750}{1 \times 600 \times 12.5^2}$ $\left(\dfrac{20-4}{P20.3}\right)$

$\sigma = 186.67$ MPa

(iii) Shear stress in coils $\tau = \dfrac{8 F_2 C K}{\pi d^2} = \dfrac{8 \times 2777.8 \times 6 \times 1.2525}{\pi \times 15^2}$ $\left(\dfrac{20-22}{P20.8}\right)$

$\tau = 236.26$ MPa

(iv) Energies stored by each spring $U = \dfrac{Fy}{2}$

(a) Cantilever $U_1 = \dfrac{F_1 y_1}{2} = \dfrac{3888.9 \times 40}{2} = 77778$ N mm

(b) Coil spring $U_2 = \dfrac{F_2 y_2}{2} = \dfrac{2777.8 \times 40}{2} = 55556$ N mm

Problem 37 A 90 mm outside dia steel coil spring having 8 active coils of 15 mm wire dia is placed on the top of a steel cantilever spring having 6 graduated leaves each 120 mm wide and 9 mm thick as shown in Fig. 2.23.

(i) What force F if gradually applied to the top of coil will cause cantilever to deflect by 60 mm. Take $E = 206$ GPa and $G = 80$ GPa.

(ii) What is the bending stress in the leaves?

(iii) What is the shear stress in the coils?

(iv) What are the energies stored by each spring?

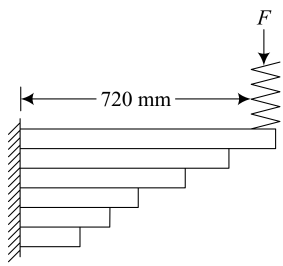


Fig. 2.23

Solution Given data:

Cantilever width const, depth varying	Coil
$i = 6$	$i = 8$
$l = 720$ mm	$d = 15$ mm
$b' = 120$ mm	$D_o = 90$ mm
$h = 9$ mm	$D = D_o - d = 90 - 15 = 75$ mm

$$y_1 = 60 \text{ mm} \qquad C = \frac{D}{d} = \frac{75}{15} = 5$$

$$E = 206 \times 10^3 \text{ MPa} \qquad G = 80 \times 10^3 \text{ MPa}$$

Load $F_1 = F$ Force $F_2 = F$

Ref Fig. 5, Table 2.3 (T 20-1 *P* 20.1)   Deflection $= y_2$

$$C_1 = 6, \quad C_2 = 8 \qquad K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.3105$$

Since the coil is on the top of cantilever, the load applied on coil goes to cantilever directly. Hence, Load on coil = load on cantilever

$$F_1 = F_2 = F$$

(i) Consider cantilever, $y_1 = \dfrac{C_2 F l^3}{E i b' h^3} \Rightarrow 60 = \dfrac{8 \times F \times 720^3}{206 \times 10^3 \times 6 \times 120 \times 9^3}$

Load $F = F_1 = F_2 = 2172.6$ N

(ii) Bending stress in cantilever $\sigma = \dfrac{C_1 F l}{i b' h^2} = \dfrac{6 \times 2177.6 \times 720}{6 \times 120 \times 9^2}$

$$\sigma = 160.93 \text{ MPa}$$

(iii) Shear stress in coils $\tau = \dfrac{8FCK}{\pi d^2} = \dfrac{8 \times 2172.6 \times 5 \times 1.3105}{\pi \times 15^2}$

$$\tau = 161.12 \text{ MPa}$$

(iv) Energy stored $U = \dfrac{Fy}{2}$

(a) For leaf spring $U_1 = \dfrac{Fy_1}{2} = \dfrac{2172.6 \times 60}{2} = 65178$ N mm

(b) For coil $U_2 = \dfrac{Fy_2}{2}$

where deflection $y_2 = \dfrac{8FD^3 i}{Gd^4} = \dfrac{8 \times 2172.6 \times 75^3 \times 8}{80000 \times 15^4}$

$$y_2 = 14.48 \text{ mm}$$

$\therefore$ Energy $U_2 = \dfrac{2172.6 \times 14.48}{2} = 15729.62$ N mm

Problem 38 A semi-elliptical leaf spring has a span of 1.8 m. The spring carries a helical spring upon which is imposed an impact of 3 kN m. The laminated spring has 8 graduated and 3 full length leaves each 60 mm wide and 6 mm thick. The coil spring has 9 coils of 12.5 mm wire dia and a spring index of 7. Find the stresses induced in each spring.

Solution Given data: Ref Fig. 2.24

Cantilever

$L = 1800$ mm

Assume $L_B = 0$

$\therefore$ $L = \dfrac{L - L_B}{2} = 900$ mm

$b' = 60$ mm

$h = 6$ mm

$i_f = 3$

$i_g = 8$

$i = i_f + i_g = 11$

Assume $E = 206 \times 10^3$ MPa

$U = 3$ kN m $= 3 \times 10^6$ N mm

coil

$i = 9$ coils

$d = 12.5$ mm

$C = 7 = \dfrac{D}{d}$

$\therefore$ $D = Cd = 7 \times 12.5 = 87.5$ mm

Assume $G = 80 \times 10^3$ MPa

$U = 3$ kN m $= 3 \times 10^6$ N mm

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = 1.2129$$

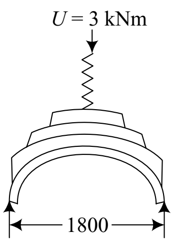


Fig. 2.24

The energy is same for both springs.

(i) Consider cantilever, $U_1 = \dfrac{F_1 y_1}{2} \Rightarrow 3 \times 10^6 = \dfrac{F_1 y_1}{2}$ ...(1)

Deflection
$$y_1 = \frac{12 F_1 l^3}{Eb'h^3\ (2i_g + 3i_f)}$$

$$y_1 = \frac{12 F_1 \times 900^3}{206 \times 10^3 \times 60 \times 6^3\ (2 \times 8 + 3 \times 3)} = 0.13017\, F_1$$

Substituting in (1),

$$\frac{F_1 \times 0.13017\, F_1}{2} = 3 \times 10^6$$

$$F_1 = 6765.87 \text{ N}$$

Maximum stress = stress in full length leaves $\sigma_f = \dfrac{12 F_1 l}{b'h^2\ (2i_g + 3i_f)}$

$$\sigma_f = \frac{12 \times 6765.87 \times 900}{60 \times 6^2\ (2 \times 8 + 3 \times 3)} = 1353.17 \text{ MPa}$$

(ii) Consider coil $\quad U_2 = \frac{F_2 y_2}{2} \Rightarrow 3 \times 10^6 = \frac{F_2 y_2}{2}$ ...(2)

For coil $\quad y_2 = \frac{8F_2 D^3 i}{Gd^4}$

$$y_2 = \frac{8F_2 \times 87.5^3 \times 9}{80 \times 10^3 \times 12.5^4} = 0.0247\ F_2$$

Substituting in (2),

$$\frac{F_2 \cdot 0.0247\ F_2}{2} = 3 \times 10^6 \Rightarrow F_2 = 15587 \text{ N}$$

$\therefore$ Shear stress $\quad \tau = \frac{8F_2 CK}{\pi d^2} = \frac{8 \times 15587 \times 7 \times 1.2129}{\pi \times 12.5^2}$

$$\tau = 2156.8 \text{ MPa}$$

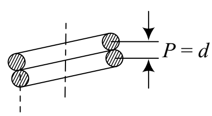
2.19 TENSION SPRING (Refer Fig. (i) of 2.1)

The design procedure is same as compression spring. It is generally wound close coiled without any gap between the coils, i.e. clearance almost zero. The tension spring will have one loop on each side, hence two extra coils are needed.

Hence, the free length $\quad l_o = id + 2D$

pitch $\quad p =$ the wide dia itself $= d$

For some reason, it there is a clearance of a' between each coil, then



$$l_o = id + 2D + a$$

Total clearance $\quad a = i \times a'$

and pitch $\quad p = d + a'$

Problem 39 Design a helical spring to support a tensile load of 6 kN and to have a stiffness of 100 N/mm. The spring index is 6. The spring is made of steel having allowable stress of 300 MPa. Take $G = 80$ GPa.

Solution Given data:

Tensile load hence design a tension spring

$$F = 6000 \text{ N}, \quad F_o = 100 \text{ N/mm} = \frac{F}{y}$$

$\therefore$ Deflection $\quad y = \dfrac{F}{F_o} = \dfrac{6000}{100} = 60 \text{ mm}$

$$C = 6, \qquad \tau = 300 \text{ MPa}, \quad G = 80 \times 10^3 \text{ MPa}$$

Step 1. Shear stress $\quad \tau = \dfrac{8FCK}{\pi d^2}$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

$$300 = \frac{8\times 6000\times 6\times 1.2525}{\pi d^2} \quad \Rightarrow \quad d = 19.56 \simeq 20 \text{ mm}$$

Step 2. Mean coil dia $\quad D = Cd = 6 \times 20 = 120$ mm

Inside dia of coil $\quad D_i = D - d = 120 - 20 = 100$ mm

Outside dia of coil $\quad D_o = D + d = 120 + 20 = 140$ mm

Step 3. Deflection $\quad y = \dfrac{8FD^3 i}{Gd^4} = \dfrac{8\times 6000\times 120^3\times i}{80\times 10^3\times 20^4} = 60$

$\therefore$ No. of active coils $\quad i = 9.26 \simeq 10$ coils

Step 4. Free length $\quad l_o = (id) + 2D$

(assuming no clearance between coils)

$$l_o = 10 \times 20 + 2 \times 120 = 440 \text{ mm}$$

Step 5. Pitch $\quad p = d = 20$ mm

Step 6. Stiffness $\quad F_o = \dfrac{F}{y} = 100$ N/mm

Step 7. Length of wire $\quad = \pi Di' = \pi D(i + 2)$

$$l_w = \pi \times 120\ (10 + 2) = 4523.9 \text{ mm}$$

2.20 TORSION SPRING

Figure 2.25 shows a helical torsion spring. It is wound similar to tension spring (close coiled) but ends are kept straight to transmit torque.

While transmitting torque, the spring wire is subjected to bending stress due to torque and a direct tensile stress. Each coil of torsion spring is considered a curved beam.

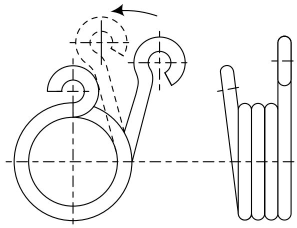


Fig. 2.25 Torsion spring

The stress equation is $$\sigma = \frac{F}{A} + \frac{K'M_b \cdot c}{I}$$

$$F = \frac{M_t}{R} = \frac{2M_t}{D}, \quad M_b = M_t, \quad c = \frac{d}{2}, \quad I = \frac{\pi d^4}{64}$$

$$A = \frac{\pi d^2}{4}$$

$$\sigma = \frac{2M_t}{DA} + \frac{K' M_t}{Z} \qquad \left(\frac{20-88}{P20.27}\right)$$

$$\sigma = \frac{8M_t(4K'D + d)}{\pi d^3 D} \qquad \left(\frac{20-89a}{P20.27}\right)$$

Deflection $$y = \frac{M_t LD}{2EI} \qquad \left(\frac{20-89}{P20.27}\right)$$

L = length of wire = πDi

$$Z = \frac{I}{c} = \frac{\pi d^3}{32}$$

E = Modulus of elasticity

$C = \frac{D}{d}$ spring index

K' = constant depends on C and given below $\left(\frac{\text{Fig. } 20-5}{P20.8}\right)$

C	4	5	6	7	8	9	10
K'	1.25	1.175	1.15	1.12	1.1	1.09	1.08

Problem 40 A helical torsion spring is to support a torque of 12 Nm. The bending stress is limited to 300 MPa. Deflection is to be 30 mm. The spring index is 6. Take E = 206 GPa. Design the spring.

Solution Given data:

Torque $M_t = 12 \times 10^3$ N mm

Bending stress $\sigma = 300$ MPa

Deflection $y = 30$ mm

$$C = 6 = D/d \quad \Rightarrow \quad D = 6d$$
$$E = 206 \times 10^3 \text{ MPa}$$

Step 1. $$\sigma = \frac{8M_t\,(4K'D + d)}{\pi d^3 D} \qquad \left(\frac{20-89a}{P20.27}\right)$$

For $C = 6$, $K' = 1.15$ from above table 2.4

$$D = 6d$$

$$300 = \frac{8 \times 12000 \times (4 \times 1.15 \times 6d + d)}{\pi d^3 \times 6d}$$

$\therefore$ $d = 7.86 \simeq 8$ mm

Step 2. Mean coil dia $D = cd = 6 \times 8 = 48$ mm

Step 3. Deflection $$y = \frac{M_t\,LD}{2EI} \qquad \left(\frac{20-89}{P20.27}\right)$$

$$L = \pi\,Di, \quad I = \frac{\pi d^4}{64}$$

$$30 = \frac{12000 \times (\pi \times 48 \times i) \times 48}{2 \times 206 \times 10^3 \times \left(\frac{\pi \times 84}{64}\right)}$$

$\therefore$ No. of active coils $i = 28.61 \simeq 29$ coils

Deflection in terms of angle

But $$y = \theta \cdot R = \theta \cdot \frac{D}{2}$$

Deflection $$\theta = \frac{2 \times 30}{48} = 1.25 \text{ radians} = 1.25 \times \frac{180}{\pi} = 71.62 \text{ degrees}$$

Problem 41 A helical torsion spring has a mean dia of 60 mm and wire dia of 6 mm. The allowable stress is 420 MPa and deflection is 36 mm, take $E = 210$ GPa. Find the torque that can be resisted by this spring and no. of active coils.

Solution Given data:

$$D = 60 \text{ mm}, \qquad d = 6 \text{ mm} \quad \sigma = 420 \text{ mm} \quad y = 36 \text{ mm}$$

$$E = 210 \times 10^3 \text{ MPa} \;\; C = \frac{D}{d} = \frac{60}{6} = 10$$

For C = 10, $K' = 1.08$ from table 2.4 of torsion spring

(i) Stress $$\sigma = \frac{8M_t(4K'D + d)}{\pi d^3 D}$$

(a) Safe load that can be carried

(b) The deflection at this load and

(c) The stress produced at the outer edge.

Solution Given data:

$$t = 3 \text{ mm} \quad d_i = 50 \text{ mm} \quad d_o = 130 \text{ mm} \quad h = 5 \text{ mm}$$

Maximum stress $\sigma_i = 600$ MPa (allowable stress $= \sigma_i$)

$$\frac{d_o}{d_i} = \frac{130}{50} = 2.6, \text{ from Fig. 2.26, for } \frac{d_o}{d_i} = 2.6$$

$$C_1 = 1.34 \quad C_2 = 1.55 \text{ and } M = 0.75$$

Assume $E = 206$ GPa $= 206 \times 10^3$ MPa and

Poisson ratio $\nu = 0.3$ for steel

(i)
$$\sigma_i = \frac{4\,Ey}{(1-v^2)d_o^2}\left[C_1\left(h - \frac{y}{2}\right) + C_2 t\right]$$

$$600 = \frac{4 \times 206 \times 10^3 \times y}{(1-0.3^2)130^2}\left[1.34\left(5 - \frac{y}{2}\right) + 1.55 \times 3\right]$$

$$600 = 53.58y\,[6.7 - 0.67y + 4.65]$$

$$600 = 608.13y - 35.9y^2$$

$$35.9y^2 - 608.13y + 600 = 0$$

$$y = 15.888 \text{ or } y = 1.05 \text{ mm}$$

Since y cannot be more than h, take $y = 1.05$ mm,

(ii) $\therefore$ Force
$$F = \frac{4\,Ey}{(1-v^2)\,Md_o^2}\left[(h-y)\,(h - \frac{y}{2})t + t^3\right]$$

$$F = \frac{4 \times 206 \times 10^3 \times 1.05}{(1-0.3^2)\,0.75 \times 130^2}\left[(5-1.05)\left(5 - \frac{1.05}{2}\right)3 + 3^3\right]$$

$$F = 6003.07 \text{ N}$$

(iii) Stress at outer radius

$$\sigma_o = \frac{4\ Ey}{(1-v^2)d_0^2}\left[a\left(h - \frac{y}{2}\right) - C_2 t\right]$$

$$= \frac{4 \times 206 \times 10^3 \times 1.05}{(1-0.3^2)130h^2}\left[1.34\left(5 - \frac{1.05}{2}\right) - 1.55 \times 3\right]$$

$$\sigma_o = 75.75 \text{ MPa}$$

Problem 43 A disc spring is made of 2 mm steel with outside and inside diameters of 100 mm and 40 mm. The spring has a inner height of 4 mm. Determine the force required to deflect the spring by 1.5 mm. Also find the resulting stresses at the inner and outer edges. Take Poisson ratio = 0.28 and modulus of elasticity = 2 × 10^5 MPa.

Solution Given data:

$$t = 2 \text{ mm} \quad d_o = 100 \text{ mm} \quad d_i = 40 \text{ mm}$$
$$h = 4 \text{ mm} \quad y = 1.5 \text{ mm} \quad \nu = 0.28 \quad E = 2 \times 10^5 \text{ MPa}$$

$$\frac{d_o}{d_i} = \frac{100}{40} = 2.5, \text{ from Fig. 2.26, for } \frac{d_o}{d_i} = 2.5$$

$$C_1 = 1.32 \quad C_2 = 1.55 \text{ and } M = 0.75$$

(i)
$$F = \frac{4\,Ey}{(1-\nu^2)\,M\,d_o^2}\left[(h-y)\left(h-\frac{y}{2}\right)t + t^3\right]$$

$$F = \frac{4 \times 2 \times 10^5 \times 1.5}{(1-0.28^2)0.75 \times 100^2}\left[(4-1.5)\left(4-\frac{1.5}{2}\right) \times 2 + 2^3\right]$$

$$F = 4210.07 \text{ N}$$

(ii) Stress at inner circumference

$$\sigma_i = \frac{4\,Ey}{(1-\nu^2)d_o^2}\left[C_1\left(h-\frac{y}{2}\right) + C_2 t\right]$$

$$= \frac{4 \times 2 \times 10^5 \times 1.5}{(1-0.28^2)100^2}\left[1.32\left(4-\frac{1.5}{2}\right) + 1.55 \times 2\right]$$

$$= 962.24 \text{ MPa}$$

(iii) Stress at outer circumference

$$\sigma_o = \frac{4\,Ey}{(1-\nu^2)d_o^2}\left[C_1\left(h-\frac{y}{2}\right) - C_2 t\right]$$

$$= \frac{4 \times 2 \times 10^5 \times 1.5}{(1-0.28^2)100^2}\left[1.32\left(4-\frac{1.5}{2} - 1.55 \times 2\right)\right]$$

$$= 154.95 \text{ MPa.}$$

EXERCISES

1. Design a helical compression spring for a maximum load of 1000 N and for a deflection of 25 mm. The maximum permissible shear stress for spring material is 420 N/mm^2. Modulus of rigidity is 0.84 × 10^5 N/mm^2. Spring index is 6.